

THERMAL ANALYSIS OF PRESSURIZED WATER REACTORS

THIRD EDITION

L. S. TONG & JOEL WEISMAN



THERMAL ANALYSIS OF PRESSURIZED WATER REACTORS

Third Edition

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Library of Congress Cataloging-in-Publication Data

Tong, L. S. (Long-sun), 1915–

Thermal analysis of pressurized water reactors / L.S. Tong, Joel
Weisman. — 3rd ed.

p. cm.

Includes bibliographical references and index.

ISBN 0-89448-038-3

1. Pressurized water reactors—Thermodynamics. 2. Heat—
Transmission. I. Weisman, Joel, 1928– II. Title.

TK9203.P7T6 1996

621.48'34—dc20

95-45359

CIP

ISBN: 0-89448-038-3

Library of Congress Catalog Card Number: 95-45359

ANS Order Number: 300028

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555 North Kensington Avenue

La Grange Park, Illinois 60525 USA

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Production/Design: Lorretta Palagi
Composition: Parkwood Composition

Printed in the United States of America—First Printing

To the memory of
Jay Weisman,
a young man full of promise.

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PREFACE

to the Third Edition

The basic objective of this book is to present the principles underlying the thermal and hydraulic design of pressurized water reactors. In addition, the empirical data, engineering properties, and computer techniques required for design, but not available in conventional handbooks, are presented or referenced. This book is intended to provide an overview for nuclear engineering graduate students and to serve as a reference for engineers working in the nuclear power industry. This work is not intended to be a design manual.

While the foregoing objectives are essentially the same as those motivating preparation of the first (1970) and second (1979) editions, it was believed that a new edition was needed because of the many advances and changes that have occurred since 1979. The extensive improvements that have occurred in both understanding the phenomena involved and in calculational techniques are reflected in the substantial additions to the text.

Since the previous editions of this monograph have apparently been found useful by a number of readers, the basic format of previous editions has been essentially retained. However, an additional chapter has been added to accommodate the many developments in the area of safety analysis. We hope the revisions have enhanced the usefulness of this monograph.

August 1995

L. S. Tong
Joel Weisman

NOMENCLATURE

This book follows the procedure of defining pertinent symbols after each equation. However, there are a number of symbols so frequently used that they have not been defined at each location of use. It is these symbols that are defined in this list.

a_l, a_v, a_{TP} = liquid, vapor, and two-phase sonic velocities,
respectively

a_1 = sonic velocity, length/time

A = flow area, length²

= specific heats at constant pressure and volume,
respectively, energy/mass deg

D = diameter, length

D' = extrapolated diameter of reactor, length

D_1 = diffusion coefficient, length²

De = equivalent diameter = $4A/\text{perimeter}$, length

e = eddy diffusivity, area/time

E = internal energy, energy/mass

f = flux depression factor, dimensionless

f = friction factor, dimensionless

F_Q^N, F_R^N, F_Z^N = nuclear hot-channel factors for overall heat flux, radial
heat flux variation, and axial variation, respectively

$F_q, F_{\Delta H}$ = overall hot-channel factors for heat flux and enthalpy
rise, respectively

$F_q^E, F_{\Delta H}^E$ = engineering hot-channel factors for heat flux and
enthalpy rise, respectively

g_c = gravitational conversion factor, (mass/weight)(length/
time²)

g = gravitational acceleration, length/time²

G = mass flux, mass/time area

h = heat transfer coefficient, energy/time area deg

h_g = gap conductance, energy/time area deg

H = enthalpy, energy/mass

H_{fg} = enthalpy change on evaporation, energy/mass

H_v = enthalpy of saturated vapor, energy/mass

- J = mechanical equivalent of heat, mechanical energy / thermal energy
 k = thermal conductivity, energy / time length deg
 k_{eff} = effective multiplication factor
 $k_{\text{ex}} = k_{\text{eff}} - 1$
 k_{∞} = infinite lattice multiplication factor
 K = constant, form friction factor
 L = length
 L, L' = height and extrapolated height of reactor, respectively, length
 p = probability, dimensionless
 p = pitch, length
 p' = perimeter, length
 P = pressure, force / area
 P' = power, energy / time
 Pr = Prandtl number = $c_p \mu / k$
 q = total heat production rate, energy / time
 q' = linear heat output, energy / time length
 q'' = surface heat flux, energy / time length²
 q''_{crit} = critical heat flux, energy / time length²
 q''' = volumetric heat flux, energy / time length³
 Q = volumetric flow, volume / time
 r, R = radius, length
 R = ideal gas constant
 Re = Reynolds number = DeG / μ
 S = slip ratio, vapor velocity / liquid velocity
 t = time
 t_1 = standard normal variate, dimensionless
 T = temperature
 u, v = velocity components, length / time
 u_{gl} = drift velocity, length / time
 v = specific volume, volume / mass
 V = velocity, length / time
 w' = transverse-turbulent mixing flow, mass / length time
 x, y, z = distance in coordinate directions, variables
 χ = quality = mass flow rate of vapor / total mass flow rate
 Z = axial distance, length

NOMENCLATURE

α_a, α_b = thermal expansion coefficients of cladding and fuel,
length/length deg

α = void fraction, porosity

α = parameter in escape probability calculation

γ = Poisson's ratio

$\gamma = c_p/c_v$

$\kappa_0 = (\Sigma_a/D_1)^{1/2}$

μ attenuation coefficient, length⁻¹

μ = viscosity, mass/length time

$\bar{\mu}$ = mean value

ρ = density, mass/volume

σ = stress, force/area

σ standard deviation

σ' surface tension

Σ = macroscopic cross section, length⁻¹

ϕ = neutron flux, neutrons/area time

GLOSSARY OF ACRONYMS

AO	axial offset
AOO	anticipated operating occurrence
ATWS	anticipated transient without scram
BOL	beginning of life
BWR	boiling water reactor
CANDU	Canadian deuterium (reactor)
CCFL	countercurrent flooding limit
CCI	core-concrete interaction
CHF	critical heat flux
CPR	critical power ratio
DNB	departure from nucleate boiling
DNBR	DNB ratio
ECCS	emergency core cooling system
EOL	end of life
FCI	fuel-concrete interaction
HPSI	high-pressure safety injection
IRWST	in-containment refueling water storage tank
LBLOCA	large-break LOCA
LCD	late core disassembly
LCO	limiting condition of operation
LHGR	linear heat generation rate
LOCA	loss-of-coolant accident
LOFA	loss-of-flow accident
LPSI	low-pressure safety injection
LSS	limiting safety setting
NDT	nil-ductility temperature
NRC	Nuclear Regulatory Commission
PCI	pellet-cladding interaction
PORV	power-operated relief valve
PRA	probabilistic risk assessment
PWR	pressurized water reactor
RCC	rod cluster control
RPS	reactor protection system
RSS	root-sum-square
SAFDL	safe acceptable design limit
SBLOCA	small-break LOCA
SDL	statistical design limit
VVER	PWR of Russian design

CHAPTER ONE

Power Generation

A pressurized water reactor (PWR) is a water-cooled nuclear reactor under sufficient pressure to limit steam generation at the core exit and where the large quantity of heat produced is transferred to a secondary system via a heat exchanger. In this chapter, we briefly delineate the major reactor systems falling under this definition and then describe, from a thermal analyst's viewpoint, the basic power distributions found within these reactors. The purpose of this discussion is to indicate the nature of the phenomena involved and some of the approximations that can be made.

1.1 REACTOR CONFIGURATIONS

1.1.1 Basic Concept (Conventional PWR)

1.1.1.1 *Overall System*

The basic PWR concept refers to the original (conventional) design now used in the United States, Europe, and East Asia. Alternative concepts, which are less widely used, are described subsequently (Sec. 1.1.2).

In basic PWR design, coolant flowing through the core is cooled by heat exchange with a secondary fluid. The overpressure adequate to limit bulk boiling is maintained by an electrically heated pressurizer. In a typical reactor system (Fig. 1.1),^{1,2} the primary coolant is circulated through the reactor core by one or more high-pressure pumps. It proceeds to the heat exchanger where steam is generated and is then pumped back to the reactor core inlet. In the secondary system, feedwater is evaporated in the heat exchanger. Saturated steam goes through the turbine where it gives up its

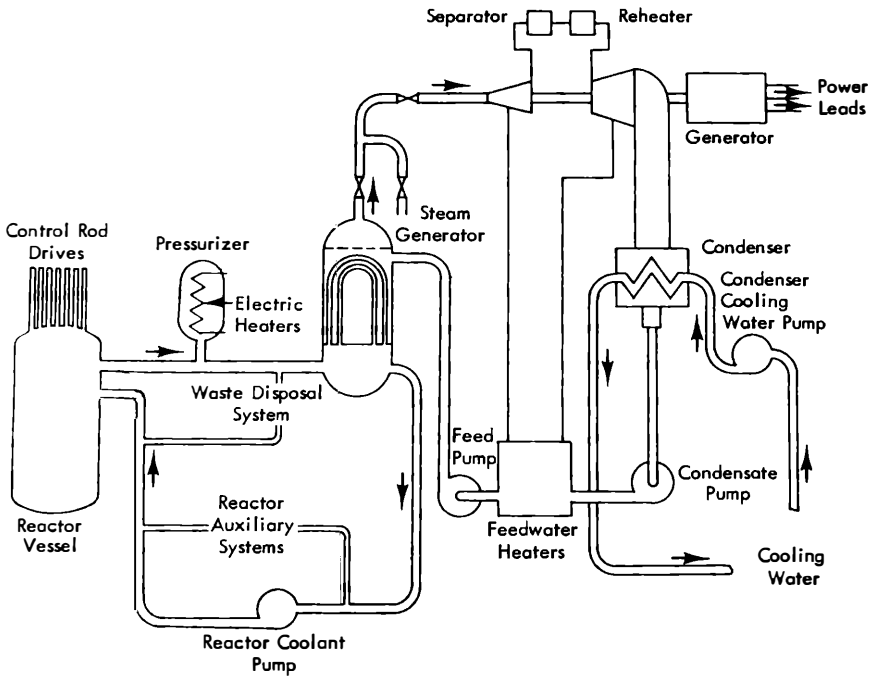


Fig. 1.1 Typical PWR system (from Ref. 1).

energy. It is then condensed in the condenser and returned to the heat exchanger by boiler feed pumps.

In commercial reactors built for central station power, the reactor core, contained in a large pressure vessel with a removable head, is moderated and cooled by light water. Shielding around the vessel maintains low neutron flux levels at the primary loop components; the reactor vessel, primary loop components, and auxiliaries are located within a structure (Fig. 1.2) that can contain the steam and fission products released in the highly unlikely event of a large break in the primary piping.³ The primary system coolant in a PWR is completely isolated from the turbine; this is referred to as an *indirect cycle*. In the boiling water reactor (BWR), the steam produced in the core goes directly to the turbine; this is called a *direct cycle*. Thus, the indirect cycle has the advantage of providing a radioactively clean secondary system, but requires expensive heat exchangers. Such factors as lower contained volume and simpler control requirements of the PWR compensate for this and make the system economically competitive.

The earliest fully commercial Western PWRs produced only about 120 MW of electrical power and the primary system operated at approximately

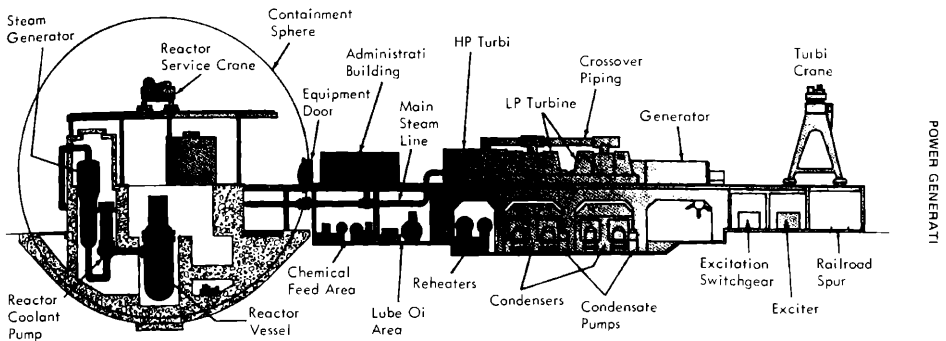


Fig. 1.2 System arrangement for PWR power plant [from *Westinghouse Engineer*, 25, 5, 145 (1965)].

2000 psi. By the 1970s, plant designs had evolved so that most conventional plants were designed to produce 1000 to 1300 MW of electrical power with the primary system operating at pressures of about 2250 psi.* Although several newer (1994) designs are still in the 1000-MW range, some advanced designs will only produce about 600 MW of electrical power.

Although the major application for pressurized water reactors is the production of central station power, they originated as a result of a desire to provide power for naval propulsion units. PWRs have been very successful in this role and continue to be extensively used by the military.

1.1.1.2 Core Configuration

To date, fuel elements for central station power plants have typically consisted of partly enriched uranium dioxide encapsulated in metal tubes. The tubes are assembled into bundles and cooled by water flowing parallel to their axes. Figure 1.3 shows a horizontal cross section of a typical reactor.⁴ The fuel assemblies are placed in a configuration approximating a right circular cylinder; the larger the reactor, the more closely a circular cross section is approached. The assemblies are retained in position by a core baffle attached to the core barrel, which is both the structural support for the baffle and the core support plate on which the fuel assemblies rest. The core barrel is surrounded by a thicker ring of metal designated as the thermal shield. The shield and water gaps between the core and vessel thermalize and attenuate the fast neutron flux emanating from the core. In addition, the shield attenuates the gamma flux; this serves to keep thermal

*Although the Russian VVER plants generating 1000 MWe operate at 2275 psia, the earlier 440-MWe plants operate at only 1800 psia.

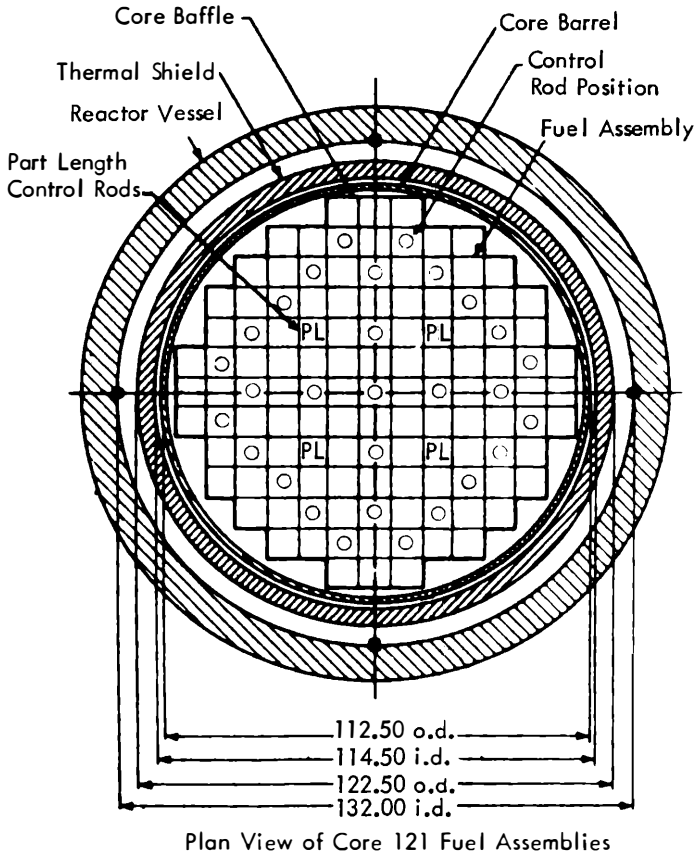


Fig. 1.3 Cross section of PWR reactor vessel [from *Proc. Am. Power Conf.*, 30, 298 (1968)].

stresses, due to gamma and neutron heating, and total neutron exposure of the reactor vessel within acceptable limits.

In American and West European designs, the fuel rods are assembled into large square arrays,* which are held in position by spring clips on egg-crate grids spaced about 2 ft apart along the length of the assembly (Fig. 1.4). Fuel tubes are omitted in a number of locations and are replaced by

*Note that the cores of the Russian VVER reactors utilize fuel assemblies in which the fuel rods are arranged in an equilateral triangular array. In addition, the VVER cores generally did not use rod cluster control but have utilized control elements designed as fuel assembly followers. When the control elements are fully inserted, a number of fuel positions are actually occupied by control assemblies.

POWER GENERATION

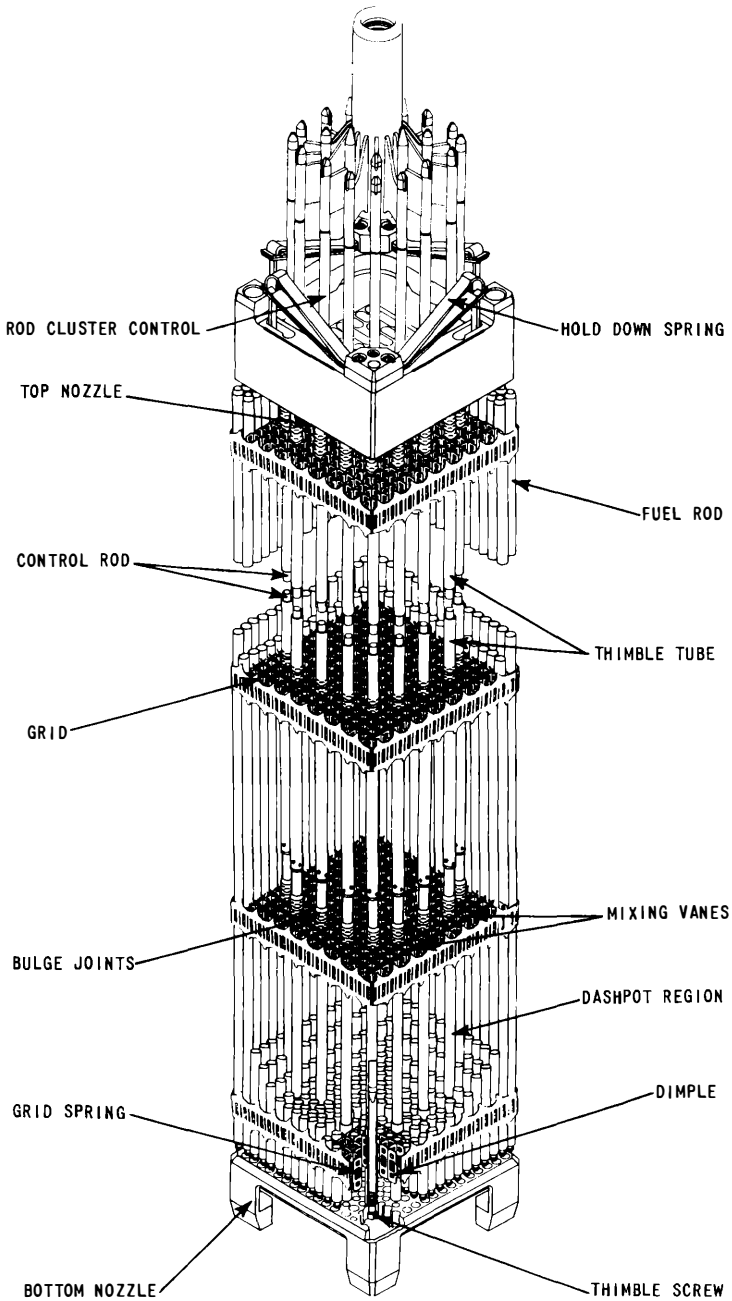


Fig. 1.4 Fuel assembly structure (courtesy Westinghouse Electric Corp.).

hollow guide tubes that provide the structural support needed to tie the assembly together. The top and bottom nozzles are attached to these guide tubes.

The control rods are clusters of absorber rods that fit into the hollow guide tube. Each control rod member of a cluster consists of a sealed stainless-steel tube filled with a neutron-absorbing material such as boron carbide or a silver-indium-cadmium alloy. All fuel assemblies are identical and, hence, each has hollow guide tubes, although only a fraction of the assemblies are under control rod positions.

In some designs, four adjacent rods are omitted from the lattice and the resulting space filled by a single guide tube. This results in a smaller number of larger control rods.

The guide tubes that do not contain control rods are most commonly filled with plugging clusters to prevent unnecessary coolant bypass through the tubes. In some cases, the guide tubes of a number of assemblies are filled with burnable poison rods. These rods can provide the additional reactivity hold-down needed at the beginning of the cycle when the fuel loading scheme used leads to high discharge burnups.

In earlier designs, cruciform-shaped control rods, fitted into cutouts at the edges of the fuel assemblies, were used. To prevent large water gaps when the control rods were withdrawn, the rods were fitted with nonabsorbing extensions. When the control rods were inserted, the extensions projected below the core, necessitating a longer reactor vessel. The individual water slots left by the newer cluster design are small and rod extensions are not needed.

The vertical reactor vessel cross section (Fig. 1.5) illustrates the structural arrangements and coolant flow path.⁵ The core barrel is attached to the lower core support barrel, which is hung from a ledge near the top of the vessel. The upper core support plate and upper control rod shrouds are attached to the upper core support barrel, which fits closely within the lowest support barrel and is supported by it. Both upper and lower core support barrels are designed so they can be removed. Normally, the lower support barrel remains in place throughout plant life.

The primary coolant enters through the inlet nozzle and flows downward through the passages between the core barrel and reactor vessel. After turning in the plenum area at the bottom of the pressure vessel, it flows upward through the core and exits through outlet nozzles via close fitting adapters provided as part of the lower core support barrel.

Core design is treated in detail in Chapter 5 and the thermal design parameters for a number of PWR reactors are tabulated in Table 5.V

1.1.1.3 Steam Generation

Heat transferred to the secondary fluid is used to generate steam. Consequently, the heat exchanger where this occurs is called a steam generator.

POWER GENERATION

POWER GENERATION

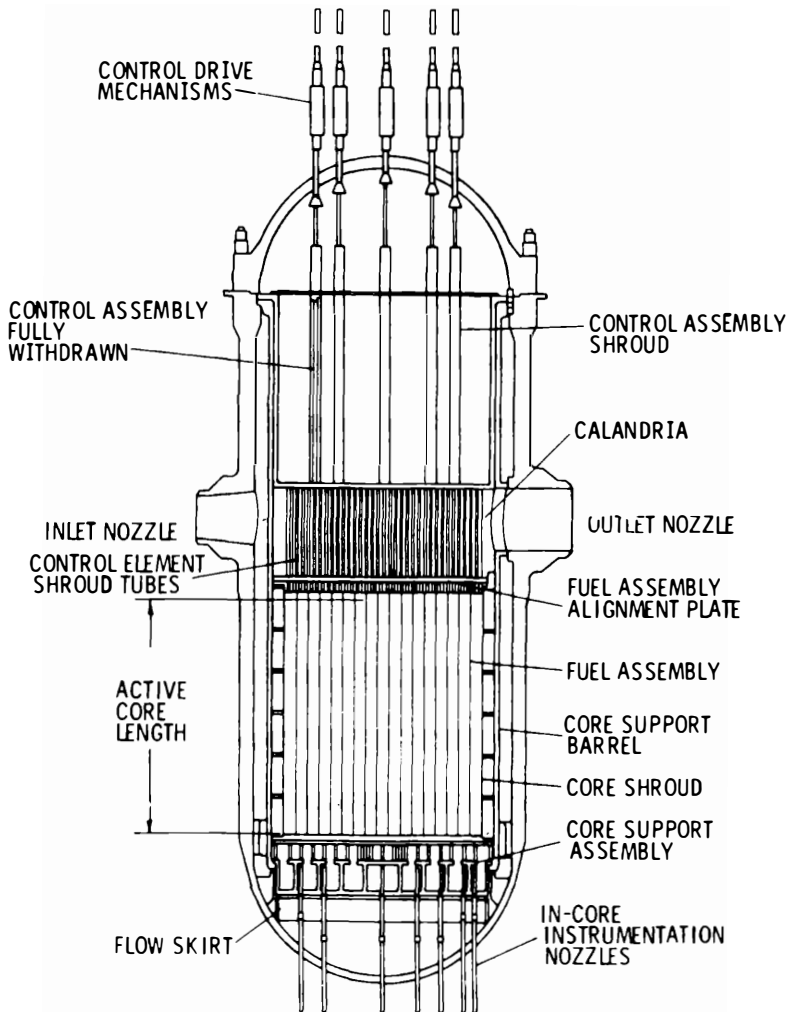


Fig. 1.5 Vertical cross section of large PWR vessel (courtesy Combustion Engineering).

A number of steam generator configurations have been used and, in all, the primary coolant has been circulated through the tubes and the secondary fluid contained within the shell. The low corrosion rates required to limit primary loop contamination led to the use of high-nickel alloys for the tubes. Initially, 300-series stainless steel was used exclusively for this purpose, but its sensitivity to stress corrosion led to the use of Inconel in many more recent units. Excessive corrosion rates have also been encountered with Inconel under some conditions. The difficulty appears to be overcome by replacing the phosphate water treatment, originally used for the secondary system, by all-volatile chemistry (ammonia for pH control, hydrazine for oxygen scavenging). However, in some units that operated with phosphate chemistry for an appreciable time, it was found that some corrosion difficulties persisted even after changing to all-volatile chemistry. Fletcher and Malinowski⁶ and Styrikovich *et al.*⁷ review typical steam generator operating experience. Section 4.6.1 of this text provides further discussion of the problems encountered in steam generator operation and the design changes devised to ameliorate these difficulties.

Horizontal steam generator units are closest to non-nuclear boiler practice. Single-drum or reboiler units (Fig. 1.6), where steam separation takes place in the same drum as the bundle, can be used. Units of this type are still used in the Russian VVER designs. However, capacity is limited by the maximum drum size that can be fabricated or shipped and by the achievable circulation ratio. Higher capacity can be obtained by using multiple-drum units such as those shown in Figs. 1.7 and 1.8. Here, steam is generated in the lower drum containing the tube bundle and a two-phase mixture flows upward through the risers to the upper drum where the

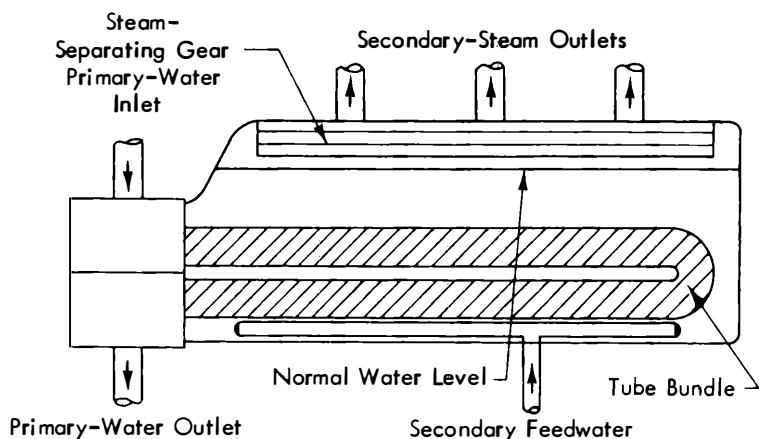


Fig. 1.6 Horizontal U-tube steam generator (From *Nucleonics*, 19, 7, 71 (1961)).

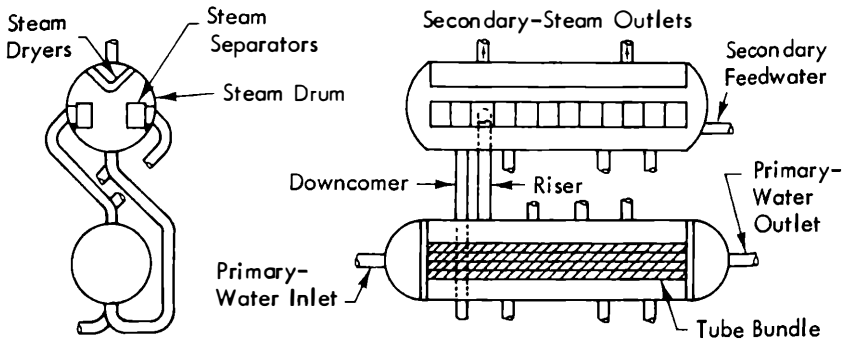


Fig. 1.7 Horizontal straight-tube multiple-drum steam generator [from *Nucleonics*, 19, 7, 71 (1961)].

steam is separated. Primary separation takes place at the vortex created in the separators at the entrance to the drum; water is returned to the lower drum via the downcomers. Units of this type were used at Shippingport; the first PWR in commercial service. Although current (1995) U.S. and West European plants have abandoned horizontal steam generators, a Japanese design of a simplified PWR system has proposed returning to these units.⁸ Such units are stated to have the advantages of freedom from sludge buildup on tube plates and a better capability for withstanding seismic events.

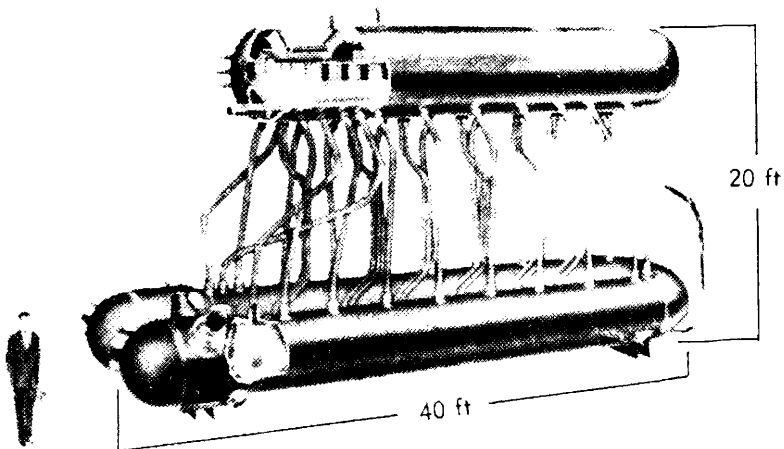


Fig. 1.8 Horizontal U-tube steam generator with contour shell and multiple steam drums [from *Nucleonics*, 19, 7, 71 (1961)].

Unitized vertical steam generators were originally developed to provide compact, high-efficiency units for marine propulsion systems. Nuclear steam generators of this type account for the majority of units installed in large central station plants through 1994. All these units have used a U-tube bundle configuration.

A cross-sectional view of a typical vertical natural recirculation unit is shown in Fig. 1.9. The two-phase mixture produced in the tube bundle rises to the primary cyclone separators. Each separator contains a helical propeller, which establishes a centrifugal field that produces a free vortex. Wet steam exits through the central port and enters the secondary steam separator in the steam drum, while the separated water flows down around the cyclone separators. Feedwater is added through a ring just above the tube bundle; the subcooled water flows downward in the space between the tube bundle and shell, completing the recirculation loop.

Some of the later model steam generators are equipped with an integral preheater located on the cold leg of the tube bundle. Such a preheater increases the secondary surface temperature in the boiling region and produces higher pressure steam. Feedwater is introduced into the preheater through a nozzle in the lower shell. The feedwater flows through a specially baffled region before it joins the main recirculating flow.

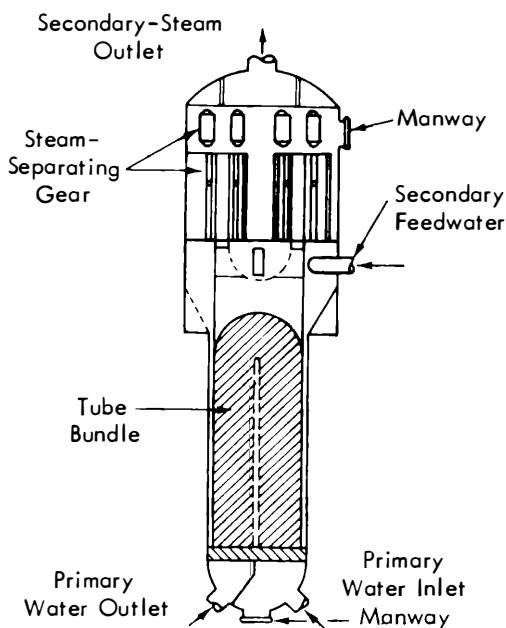


Fig. 1.9 Vertical U-tube single-drum steam generator [from *Nucleonics*, 19, 7, 71 (1961)].

Steam generators for Canadian deuterium-uranium (CANDU) reactors are vertical U-tube units generally similar to the units used for vessel PWRs. In most cases, the steam separators are located in the upper section of the vertical unit. However, in the Bruce A generating stations, the outlets of two vertical U-tube steam generators are connected to a single horizontal separating drum.

The desire to produce superheated steam has led to the development of the *once-through* design. A sectional side view of such a unit⁹ is shown in Fig. 1.10. To provide a comparison of relative size, the outline of a vertical natural-circulation boiler designed for the same conditions is superimposed on it. The reactor coolant enters the once-through generator at the upper plenum, flows through Inconel tubes, and exits from the lower plenum. The entering feedwater is sprayed from an annular ring and mixed with steam bled from the upper end of the boiler section. The mixture flows down the annular downcomer and into the tube bank. No distinct water level is present since the mixture quality gradually increases to 100% and superheating starts. The superheater region is baffled to obtain the cross-flow and high velocities needed for efficient heat transfer to steam. The superheated steam exits through outlet nozzles near the upper end of the shell.

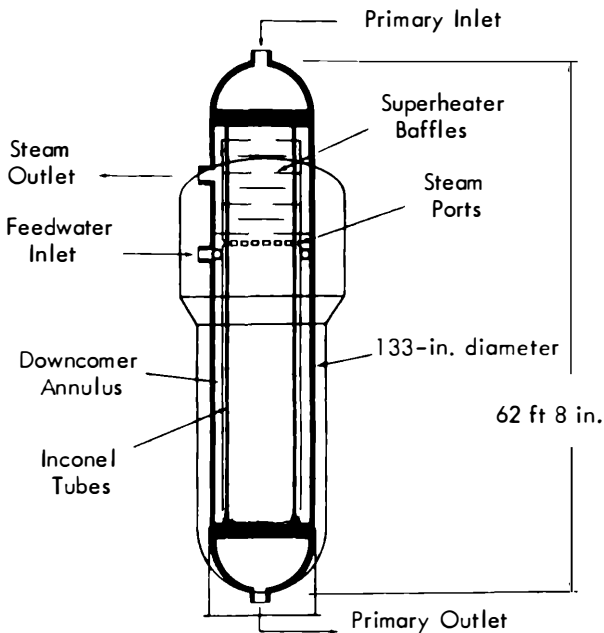


Fig. 1.10 Once-through steam generator; recirculating unit shown in outline [from Ref. 9].

1.1.1.4 *Other Primary System Components*

In addition to the reactor vessel and steam generators, the primary system contains a pressurizer, large reactor-coolant circulating pumps, and connecting piping. In the most common arrangement, each coolant circulating loop contains a steam generator and pump. In modern designs based on the use of a vertical U-tube steam generator, a medium size plant (e.g., 600 MWe) would contain two loops whereas a large plant (e.g., 1000 MWe) would contain four such loops. The pumps are located on the cold legs (primary fluid outlet lines from the steam generators). In plants using once-through steam generators, the outlet plenum of each steam generator is connected to two circulating pumps. A large PWR of this type would have four circulating pumps, but only two steam generators.

The early Russian reactor designs utilized relatively small horizontal steam generators. These plants, with a maximum capacity of 440 MWe, required six loops. Later designs have used larger horizontal steam generators and the 1000-MWe VVER requires only four loops.

Primary Coolant Pumps. In a reactor system for submarine propulsion, leakage from the primary system cannot be tolerated. Because all mechanical pump seals allow at least some small leakage, canned motor pumps, which have no shaft seals, are used. In such a pump, the entire pump and driving motor are completely enclosed within a pressurized casing. To keep the motor windings dry, the stator and rotor are canned within thin sheets of nonmagnetic material (i.e., 300-series stainless steel). An auxiliary impeller circulates a small amount of water in the narrow space between the stator and rotor liners so as to (1) provide lubrication for the journal and thrust bearing and (2) carry away motor heat. This coolant exits through a tube that is wrapped around the motor casing. External cooling water is circulated over this coil in the space between the coil and an outer jacket. The rotor is directly connected to the pump impeller. A labyrinth type seal limits the interchange between the fluid being pumped and the fluid in the space between the stator and rotor.

Since the first central station PWRs were derived from naval reactor designs, the early PWRs used canned motor pumps. However, leakage requirements in a central station plant are much less stringent than for a naval reactor and mechanical seals with low leakage rates are acceptable. Since large pumps with mechanical seals are both less expensive and more efficient than canned motor pumps, all second and subsequent generation plants in the United States and Western Europe used mechanical seals.*

*Both first- and second-generation VVER plants used canned rotor pumps. However, the third-generation VVER 1000 plants employ shaft-sealed pumps. These shaft-sealed pumps are attached to a flywheel to provide extended flow coastdown in case of a power loss.

In the usual main circulating pump design (Fig. 1.11), high-pressure injection water is used to keep the major leakage through the seal inward rather than outward. A typical 90,000-gpm pump requires of the order of 8 gpm of injection water. About 5 gpm flows downward along the shaft removing heat conducted by the shaft and parts of the thermal barrier. The remaining injection flow passes upward through and around the water-lubricated radial bearing of the seal assembly.

A typical shaft seal consists of three face seals operating in a series arrangement. Although the leakage rate for the first (primary) seal may be as high as 3 gpm, the leakage rate from the third (outermost) seal is designed to be of the order of 100 cm³/hr of injection water at operating conditions.

Each of the four pumps for a 900-MWe plant would have a capacity of about 90,000 gpm and be operated by a 9000-hp induction motor. The pumps for a 1300-MWe unit would each have a capacity of about 110,000 gpm and be operated by a 12,500-hp motor.

The absence of high-pressure injection water leads to pump seal failure. Such a failure produces a small loss-of-coolant accident (LOCA). In view of this possibility, some advanced PWR designs (1994) call for the use of canned motor pumps.

Pressurizer. The pressurizer is placed on a pipeline connected to the hot leg of one of the primary circulation loops. A cutaway drawing of a typical unit is shown in Fig. 1.12. During steady-state operations, approximately 55 to 60% of the pressurizer volume is occupied by water and 40 to 45% by steam. Electric immersion heaters, located in the lower section of the vessel, keep the water at saturation temperature and maintain a constant system operating pressure.

When the volume of the water in the system increases, the entry of the primary coolant into the pressurizer raises the water level and compresses the steam. The attendant increase in pressure actuates the valves in the spray line. Cold-leg reactor coolant from the outlet side of the pumps then sprays into the steam space and condenses a portion of the steam.

In the face of a coolant volume contraction, water from the pressurizer flows outward, thus reducing the pressurizer level and pressure. At the same time, some of the water in the pressurizer flashes to steam. In addition, the immersion heaters come on and further limit the pressure reduction.

A power-operated relief valve is connected to a nozzle at the top of the pressurizer to protect against pressure transients that are beyond the capacity of the pressurizer. In such a case, the relief valve discharges steam into a pressurizer relief tank, which is partly filled with water and is where the steam condenses. If the system pressure were to continue to rise, a spring-loaded safety valve would open and thus prevent system failure.

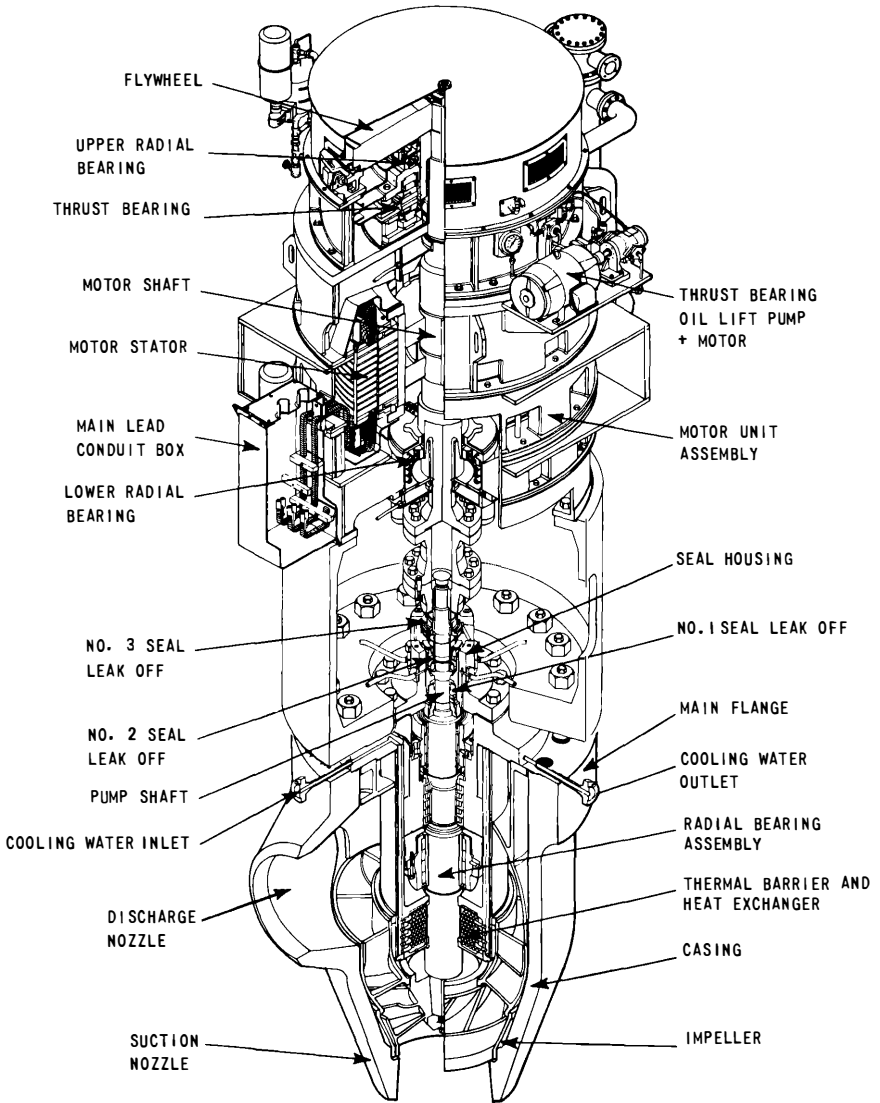


Fig. 1.11 Cutaway of typical reactor coolant pump (courtesy Westinghouse Electric Corp.).

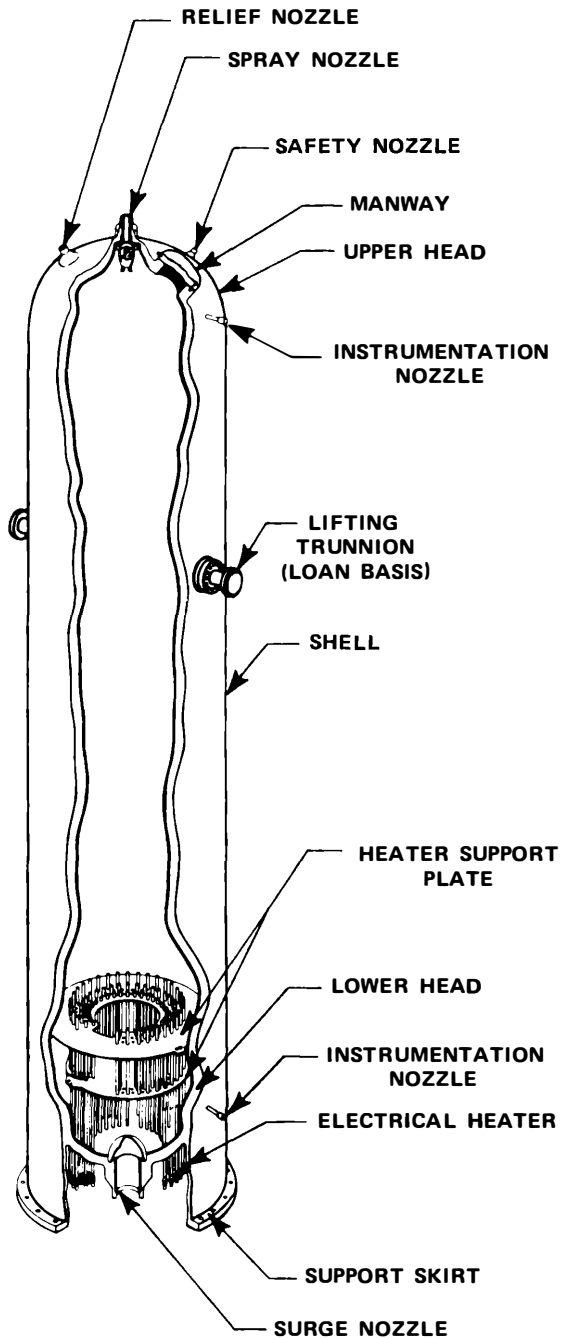


Fig. 1.12 Cutaway of typical pressurizer (courtesy Westinghouse Electric Corp.).

A typical pressurizer for a 1300-MWe plant would be about 41 ft long, 10 ft in diameter, and have a 5- to 6-in.-thick shell. The electric heater capacity would be about 1.75 MW.

1.1.1.5 Auxiliary Systems

In addition to the primary loop, several auxiliary systems are necessary for the proper functioning of the plant during normal operation and to provide adequate core cooling during shutdown and accident situations. The major systems and their functions are as follows:

1. *Chemical and volume control system*: Fills and pressurizes system, maintains water level, reduces concentration of contaminants, adjusts concentration of chemical poisons added to primary coolant.
2. *Residual heat removal system*: Transfers thermal energy from the reactor coolant during shutdown and refueling operations. System used in conjunction with safety systems during LOCA to remove heat from water being recirculated by injection system.
3. *Safety injection system*: Rapidly injects water from gas pressurized accumulators during early phase of large LOCA, adds borated water from high head pumps (needed during small LOCA where depressurization is slow), adds large volume of borated water from low head pumps during LOCA (needed during large LOCA with rapid depressurization).
4. *Fuel handling system*: Provides for fuel insertion and removal from the core, provides for fuel storage.
5. *Containment system*: Provides a vessel capable of containing the pressures and temperatures generated by a large LOCA, provides a spray of cold water to limit the containment pressure and to remove iodine from the containment system, provides heat removal from the containment during normal and accident conditions.

1.1.1.6 The Saturated Steam Turbine Cycle

Large, modern, fossil-fueled steam plants produce steam at pressures above 3200 psig and temperatures around 1000 to 1050°F. Steam conditions for PWR power plants are vastly different; the relatively low temperature of the primary system coolant, coupled with the necessity of providing an appreciable temperature difference across the heat exchangers, leads to steam pressures in the range of 400 to 950 psig (newer plants are at the upper end of the range).

The majority of PWR plants produce dry and saturated steam. The main drawback of a saturated steam cycle is its excessive endpoint moisture (20 to 24%), which leads to blade erosion and blade efficiency losses that reduce thermal efficiency. Therefore, some method of moisture removal must be

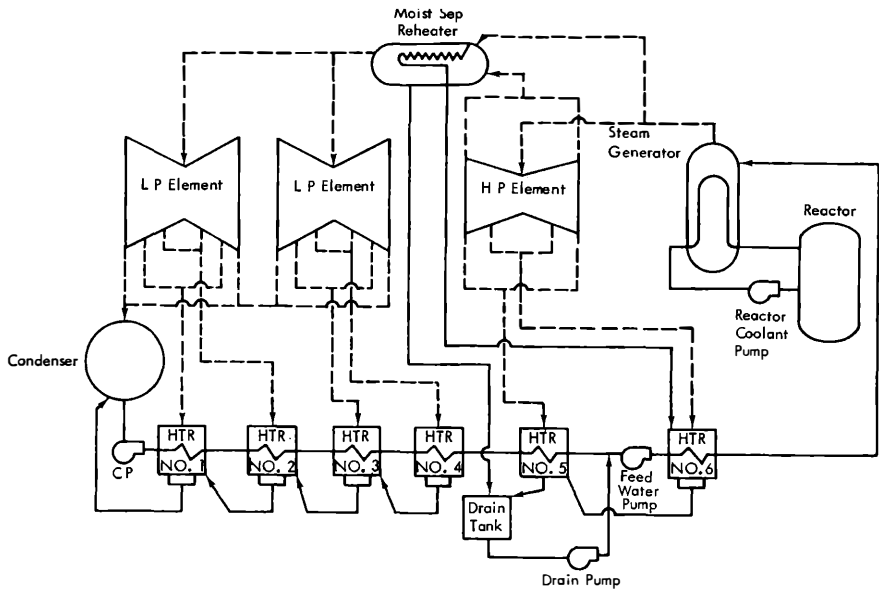


Fig. 1.13 The PWR steam cycle.

used. Evaporation by constant temperature throttling reduces the steam pressure and is detrimental to cycle efficiency. However, the mechanical steam separator has proved to be efficient and desirable since such moisture separators both reduce endpoint moisture and improve turbine efficiency.

Efficiency improvement is enhanced when moisture separation is combined with reheating. Figure 1.13 shows the general arrangement of the elements of a power generation cycle for a medium-size PWR using a combination moisture separator/live-steam reheater, which is placed in the crossover piping between the high- and low-pressure turbines. By means of a finned tube heat exchanger, this device removes moisture and transfers heat from steam bypassed ahead of the throttle valve to the main stream.

Since superheat increases the inlet temperature of a steam turbine and reduces endpoint moisture, it improves cycle efficiency considerably. In a closed cycle, superheating can be achieved by using either fossil fuel or the once-through steam generator previously described. The secondary steam from two early demonstration power reactors, the Carolina-Virginias Test Reactor (CVTR) and Indian Point 1, was superheated by fossil fuel. However, the economics of fossil-fired superheat has not proved attractive and the more modern plants have used saturated steam or superheated steam from a nuclear steam generator. Since the primary heat source is at a relatively low temperature, the amount of superheat that can be supplied by such a steam generator is low; i.e., on the order of 50°F. Thus, the steam

cycle followed is quite similar to the dry and saturated cycle previously described. Moisture separation or moisture separation plus reheat is still required to prevent excessive endpoint moisture.

The steam cycle for a large PWR is very similar to that shown in Fig. 1.11. However, a double-flow intermediate pressure turbine would be located before the two low-pressure units.

An extensive discussion of thermodynamic cycles for nuclear power plants is provided by Kalafati,¹⁰ conventional equipment is described by Artusa,¹¹ El-Wakil,¹² and Weisman and Eckart.¹³

1.1.2 Other Reactor Concepts

1.1.2.1 Compact, Light-Water-Moderated Reactor Systems

A compact, low-weight system is of paramount importance in plants designed for ship propulsion. One method¹⁴ achieves this by enclosing the reactor core and steam generator within the same pressure vessel. Coolant flows up through the core and chimney and down through the heat exchangers; it exits through the outer annulus of the double pipes leading to the pumps. The pumps are closely coupled to the reactor, but outside the primary vessel for ease of maintenance. A pressurizer is eliminated by simply maintaining a steam space above the heat exchangers. Since this steam is in equilibrium with water at the average core exit temperature, the quality of the fluid at the hot channel exit is significant.

A compact reactor design of this general type has operated successfully in the merchant ship *Otto Hahn*. Reactors of this type have also been proposed for combined desalination and power production applications.

In meeting the various military requirements, such as those for small plants in remote areas, weight and size can be of even greater importance. Here, highly enriched fuel can be used. Typical of this reactor group is the PM series of power plants that can be transported by air. The fuel elements are ~0.5-in.-diameter hollow tubes made up of a highly enriched UO_2 -stainless dispersion between stainless-steel claddings. The fuel is cooled by water flowing both inside and outside the fuel tubes. In PM-1 (Ref. 15), the fuel tubes are assembled into six pie-shaped assemblies and one small central assembly, which are held together by brazed ferrules and end plates.

Because of high costs and safety considerations, there is essentially no current interest in compact plants for merchant ships or land-based military units. Of course, the military continues to use compact nuclear reactors for naval vessel power plants. In addition, the Russians operate icebreaker vessels powered by such reactors. Proposals have also been made for commercial submersible vessels powered by compact nuclear plants.

1.1.2.2 Heavy-Water-Moderated Reactors

Although natural uranium alone cannot bring a light-water-moderated reactor to criticality, it is a satisfactory fuel for large reactors moderated and

cooled by heavy water. In areas such as Canada, where enrichment facilities are lacking but heavy water and natural uranium are available, attention has turned to this type of reactor.

The simplest design has the moderator and coolant in the same circuit and is similar in many respects to a light water reactor (LWR). However, the greater distance (i.e., the greater slowing-down length) required to thermalize fission neutrons in heavy water requires a much greater moderator-to-fuel ratio. One proposed method would achieve this by using fuel elements consisting of concentric rings with wide spacing between the rings and fuel elements. Alternatively, clusters of cylindrical fuel rods with wide spacing between clusters can be used. The large-size cores that result lead to reactor vessels far larger than those required for a light water plant of the same rating. The concept is most readily adapted to small- and medium-sized reactors. The 65-MWth Agesta reactor¹⁶ in Sweden was an early example of this concept. The Argentinian vessel type pressurized heavy water reactors (PHWRs) are more recent examples of this approach.

In the vessel-type PHWR design,¹⁷ the large high-pressure reactor vessel contains a moderator tank, which is pierced by a large number of vertical fuel channels (a 320-MWe reactor would contain about 250 such channels, whereas a 750-MWe reactor would contain about 450 channels). A fuel assembly running the full length of the core is inserted into each of the channels. A typical fuel assembly would contain 37 rods arranged on a triangular pitch. Because moderator and coolant are maintained at the same pressure, thin-walled zirconium alloy channels can be used.

Holes at the upper end of the coolant channels interconnect the channels with an outlet plenum. However, the top of each channel is connected to a thick-walled pressure tube, which pierces the head of the vessel. This enables on-line refueling using an automated refueling machine. Such an arrangement is necessary because the natural uranium fuel allows average discharge burnups of only about 6000 MWd/tonne.

By partially separating the moderating and coolant functions, the difficulties imposed by the greater slowing-down length of heavy water can be surmounted. The fuel-containing channels are sufficiently separated to provide enough distance between them for neutron moderation.

A further separation between moderating and coolant functions occurs in pressure-tube designs. In these designs, no large high-pressure reactor vessel is employed but the fuel clusters are contained within heavy-walled pressure tubes. These tubes must have high strength and good corrosion resistance. To maintain good neutron economy, the pressure-tube material must also have a low neutron absorption cross section. All reactors of this type have used zirconium alloy (usually zirconium with a small amount of niobium) pressure tubes to meet these requirements.

Each pressure tube is insulated from the moderator by a narrow gas gap between the pressure tube and an outer tube of Zircaloy. This allows the moderator to be contained in a large low-pressure tank called a *calandria*.

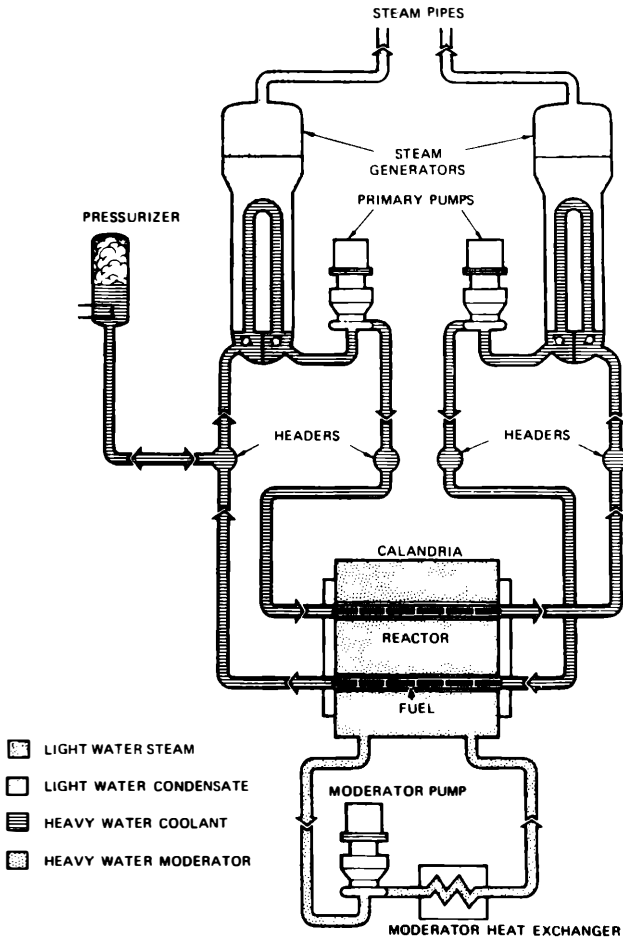


Fig. 1.14(a) CANDU reactor heat transport circuit [from P. Gulshami, *Trans. Am. Nucl. Soc.*, 55, 459 (1987)].

In the early CVTR reactor,¹⁸ the calandria contained vertical U-tubes so that all pressure tube connections were made at the top. In CANDU reactors, the pressure tubes are placed horizontally and are of the once-through type.¹⁹ The CANDU reactors represent the only heavy-water-moderated pressure-tube design of major industrial importance.

In the CANDU reactors, the fuel channels are generally arranged in one or two loops. Each loop, as shown in Fig. 1.14(a), contains two pumps, two steam generators, two inlet headers, and two outlet headers. The fuel channels are connected in parallel to the headers so that they provide two passes through the core. The bidirectional flow that this provides enables the flow to be in opposite directions in adjacent pressure tubes. This flow arrange-

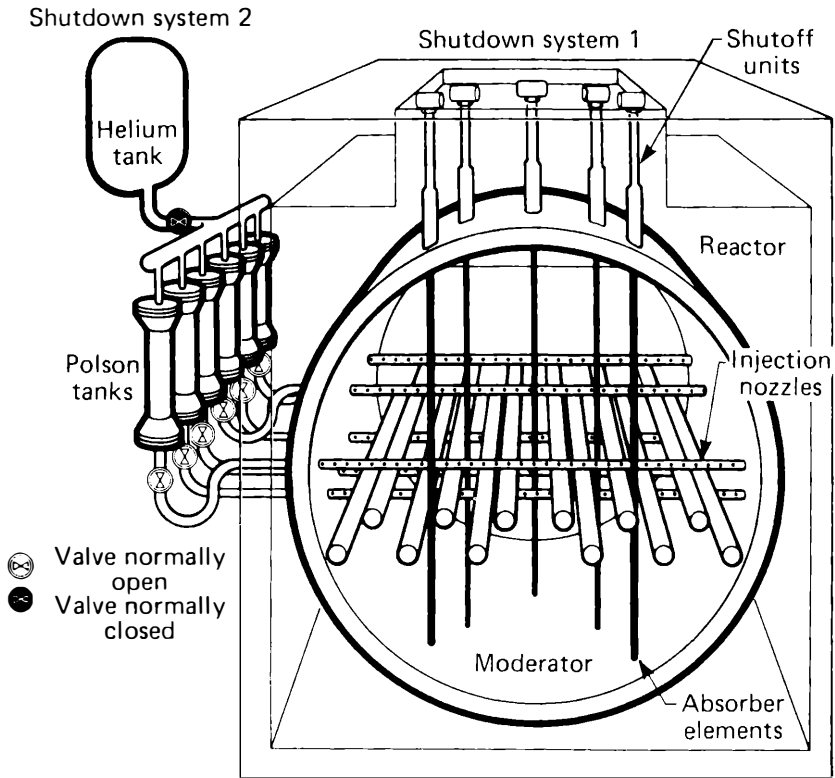


Fig. 1.14(b) CANDU control and shutdown arrangement [from R. Atchinson, F. Bogdand, Z. Domaratzki, *Nucl. Safety*, **24**, 452 (1983)].

ment, together with bidirectional refueling, avoids a reactivity tilt toward one end of the core.

Control and shutdown of CANDU reactors are achieved by insertion of vertical control rods into the low-pressure moderator. A backup shutdown mechanism is provided by a boric acid injection system. The general arrangement of these systems is shown schematically in Fig. 1.14. Note that only a small fraction of the total number of pressure tubes is shown in Fig. 1.14(b).

From a thermal-hydraulic point of view, some of the significant differences between the CANDU reactors and the basic PWR (conventional) design follow:

1. Fuel elements placed in a hexagonal array instead of a square array
2. Coolant channels connected only at core exit and outlet instead of along entire core length
3. Horizontal coolant flow instead of upward flow

4. Primary system operation at pressures in the 1300- to 1400-psi range instead of the 2000- to 2250-psi range
5. Thermodynamic and transport properties of heavy water that differ slightly from those of light water
6. Core lengths of 19 to 20 ft in contrast to the 10 to 12 ft of conventional designs.

The use of pellets of UO_2 produced from natural uranium as the fuel for all current (1994) pressure-tube reactors leads to on-line refueling of CANDU reactors just as it did in the PHWR. Fresh fuel is added at one end of the channel and burned fuel is removed at the other end. Reactors fueled by thorium slightly enriched by ^{223}U have been proposed.²⁰ If fabrication and reprocessing losses can be held low, a self-sufficient cycle might be possible. That is, the bred ^{233}U might be sufficient to replace the ^{233}U consumed in the operation. More recently,²¹ attention has shifted to the possibility of fueling CANDU cores with depleted (enrichment plant tails) UO_2 pellets containing small amounts of PuO_2 (mixed oxide fuel). Another fuel cycle that has been examined uses UO_2 pellets obtained from the slightly enriched uranium chemically separated from discharged LWR fuel.²² This reuse would improve overall utilization of uranium resources. Any of the advanced fuel cycles could also be used with a vessel-type PHWR.

Use of oxide fuel with a fissile material content in excess of that of natural uranium would increase allowable burnup and thus decrease (1) the number of fuel assemblies that must be fabricated and (2) waste disposal costs. However, the presence of plutonium would substantially increase the cost of fabricating an individual assembly. Similarly, the cost of fabrication of reprocessed light water fuel may be increased by the presence of trace quantities of fission products.

The heavy-water-moderated reactors used for production of plutonium and tritium at the U.S. Savannah River Plant may also be considered as pressure-tube PWRs. Although they operate at low pressures and all the heat they produce is dissipated without any power production, these reactors do meet the definition of a PWR since no boiling occurs in the reactor. The reactor coolant is pressurized slightly and contained within vertical pressure tubes arranged in a low-pressure calandria. At present (1992), only a single reactor is operated for tritium production. The pressure tubes of this reactor hold a series of concentric annular elements, which alternately contain fuel (U-Al clad in Al) and target material. In contrast to the upward flow design of conventional PWRs, the fuel elements here are cooled by a downward flow of heavy water.

1.1.2.3 Graphite-Moderated Reactors

Graphite-moderated and light-water-cooled reactors have a long history in the production of plutonium. It is natural that this concept be adapted to

power generation by use of higher coolant temperatures. Reactors of this type that maintain an overpressure sufficient to prevent bulk boiling fit our definition of a PWR. The first atomic power station in the former Soviet Union was of this nature. The reactor is encased in a sealed cylindrical steel jacket filled with graphite bricks pierced by fuel channels; an atmosphere of helium or nitrogen is maintained to prevent graphite oxidation. The fuel element design²³ was rather unusual: The active section of each fuel channel consisted of a graphite cylinder pierced by five tubes. The coolant flowed down the central tube, then upward through passages inside the annular fuel elements. The fuel elements consisted of slightly enriched hollow uranium-metal cylinders clad on both sides by thin stainless steel. The complexity of the fuel element design makes this reactor ill suited for large central station application.

A large, dual-purpose reactor (power and plutonium production) was erected at what was then the Hanford laboratory of the U.S. Atomic Energy Commission (AEC). The reactor, designated the NPR, consisted of a very large, 33- $\times$ 33- $\times$ 39-ft rectangular stack of graphite, penetrated by $\sim$ 1000 horizontal pressure tubes.²⁴ The 2.7-in.-i.d. Zircaloy pressure tubes contained annular fuel elements of uranium metal clad in Zircaloy. If the reactor had been converted to power production alone, UO_2 could have replaced the uranium metal.

Heat was removed from the NPR by six loops, each containing two steam generators. During dual-purpose operation, a portion of the low-pressure steam proceeded to two turbines, while the remainder proceeded to 16 dump condensers. For power production only, the dump condensers could be eliminated and higher pressure steam could be generated.

RMBK reactors, used in the former Soviet Union, have some points of similarity to the NPR design. The RMBK units are also graphite-moderated reactors in which the fuel elements are contained within pressure tubes and are cooled by light water. However, bulk boiling of the coolant occurs in the RMBK design and the reactor has substantially different operating characteristics. Because the nuclear reactor that was the source of the disastrous accident at Chernobyl was of the RMBK design, concern was expressed about other water-cooled, graphite-moderated power reactors. Although analyses of the NPR system indicated that an accident like that at Chernobyl would not occur, the NPR has been shut down.

1.1.2.4 *Supercritical Reactors*

Reactors using supercritical water substance, or water above the critical pressure, as a reactor coolant, have been proposed for future applications. Such systems have the potential advantages of higher efficiency due to higher steam pressure and temperature, higher enthalpy rise through the core, avoidance of limitations imposed by boiling heat transfer, and lower contained energy in the system. Both direct and indirect cycle reactors were considered. Since no phase change can occur at supercritical pressures, both

cycles fall under our definition of a PWR. Supercritical reactors can be of the pressure vessel or pressure tube type.²⁵

Supercritical reactors require the successful development of a high-temperature collapsed-cladding fuel element; cladding is so thin that it relies on the fuel for its support. In addition, water chemistry conditions that will suppress radiolysis, control corrosion, and prevent deposition of solids on the fuel must be determined. There is no current (1995) interest in such systems.

1.1.2.5 District Heating Plants

In addition to the use of nuclear power for the generation of electricity, appreciable consideration has been given to its use for district heating plants. Although the largest district heating demonstration unit has been a natural-circulation BWR, several PWR concepts have been proposed. The SLOWPOKE design²⁶ uses a swimming-pool reactor with just an atmospheric overpressure. However, since the reactor itself is under about 10 m of water, one might marginally consider it a PWR. SLOWPOKE is the only PWR concept for which a demonstration plant has been built.

Other reactor concepts are more like conventional PWRs. The SECURE reactor is a loop-type PWR at 7 bar in a water pool. The GEYSER, HERE, and THERMOS designs²⁷ are all of the so-called "integrated" type; that is, the steam generators and the reactor core are located in the same vessel. THERMOS uses forced circulation while the other integrated designs use natural circulation.

Successful use of district heating reactors requires the reactors to be in proximity to a population center. Public acceptance of such a unit remains to be determined.

1.1.3 Advanced Reactors with Passive Safety Features

1.1.3.1 Approaches Taken

As PWR technology has matured, the understanding of safety issues has advanced substantially. This has led to a gradual increase in the number of and complexity of the safety systems required to cope with all of the accident scenarios that can be hypothesized. Although the resulting level of public safety is believed to be very high, safety in the event of an accident does require the proper startup and operation of complex systems. Although appropriate redundancy is used to minimize failure, this further complicates the reactor system. The desire to simplify the reactor system and to lessen dependence on active systems has led to the development of several new system designs. Tong²⁸ has attempted to codify the requirements for such advanced designs and has developed a set of principles on which such designs should be based. It is hoped that such systems will lead

to lower costs and decreased construction time as well as high safety levels with improved public acceptance.

The advanced designs can be divided into two categories. The first of these has been termed *evolutionary technology reactors*.²⁹ Such reactors use the technology of current (1994) reactors but significant changes in plant design and layout are made. In the event of an accident, safety for most of these designs depends on passive systems and systems started by simple actions, such as the opening of a valve, but systems that are passive in operation (no moving equipment needed for continued safety system operation).

A second category includes future reactors with radical changes in safety systems. Such reactors would have safety systems that require neither active initiation nor active operation of equipment in order to function. Proponents of this approach have used the acronym PRIME²⁹ (Passive safety, Resistant safety, Inherent safety, Malevolence resistance, Extended time for external aid after an accident) to describe such units.

1.1.3.2 Evolutionary Technology Reactors

Reactor designs in this category generally have the following features³⁰:

1. All water needed for heat removal in the primary system can drain by gravity into the reactor core, which is located at the lowest elevation in the plant.
2. Diesel generators, required for injection system pumps in current designs, have been eliminated.
3. Cooling the core after a LOCA is accomplished by depressurizing the reactor and then allowing water to flow in by gravity from overhead tanks.
4. Passive systems are used to cool the reactor containment in the event of an accident.
5. Reactor power densities are reduced, thus increasing the margin of safety.
6. Plant designs have been substantially simplified.

Some of the major PWR designs in this category have power levels in the range of 300 to 600 MWe and continue to use an external loop design with vertical steam generators. Typical of this approach is the two-loop AP-600 reactor³¹ designed to produce 600 MWe* (see Fig. 1.15). Note that in this design the reactor coolant pumps are of the canned motor design and are integral with the heat exchangers. A passive residual heat removal system removes core heat when feedwater fails. This is accomplished by a natural-circulation loop, which exchanges heat between the primary circuit and an

*A 1000-MWe version of this design (AP-1000) has also been proposed.

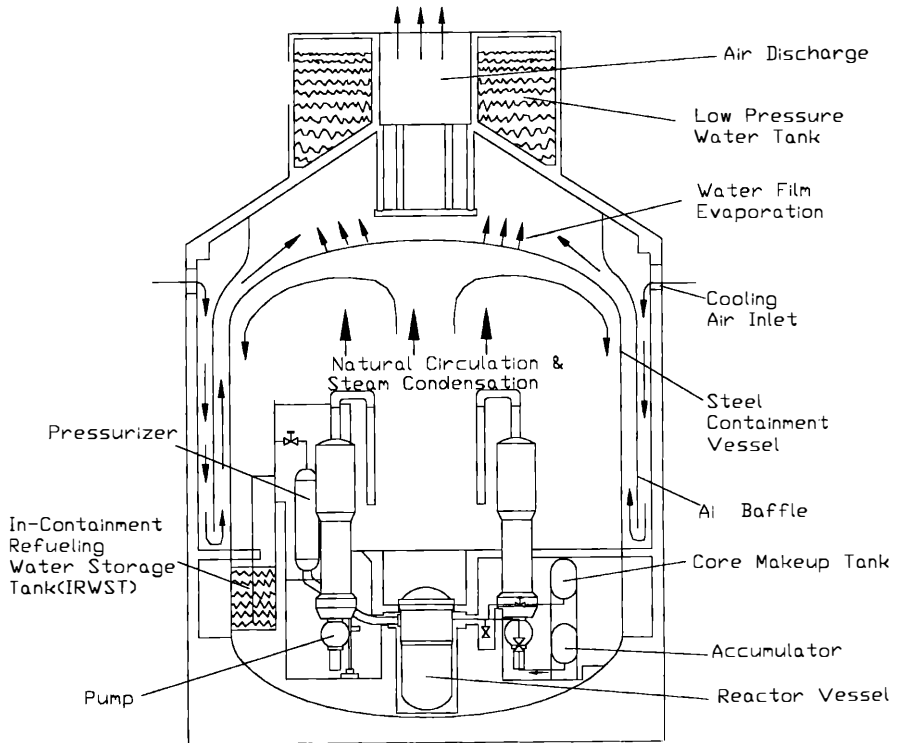


Fig. 1.15 Configuration of advanced reactor (AP-600) with passive safety features (based on information provided by Westinghouse Electric Corp.).

in-containment refueling water storage tank. After a LOCA, water is first supplied by core makeup tanks maintained at system pressure and subsequently by two large accumulators pressurized to 49 bar. Long-term cooling after the system is depressurized is provided by an in-containment refueling water storage tank. This tank has a 10-hr supply of coolant. After this tank has emptied, the containment area will be flooded above the highest elevation of the reactor vessel. Long-term cooling is maintained with condensation of boil-off by the air-cooled vapor container walls.

A more radical departure from present practice is those advanced concepts that use "integrated" designs in which steam generators and pumps are within the reactor vessel. The primary loops are thus eliminated. An arrangement of this nature is proposed³² for the Safe Integral Reactor (SIR). In this system, shown in Fig. 1.16, the core is at the bottom of the pressure vessel and placed under a tall riser to enhance natural circulation. A number of steam generators and sealed primary circulation pumps are located

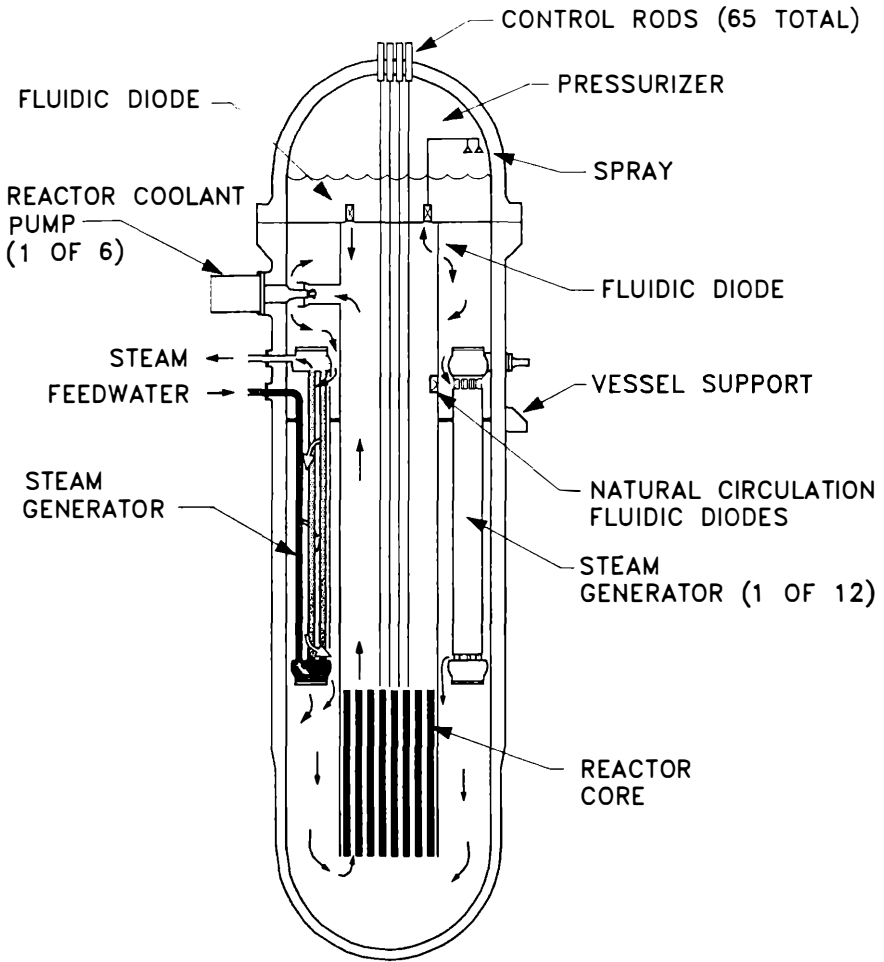


Fig. 1.16 Configuration of safe integral reactor [from Ref. 29].

around the periphery of the vessel. The upper part of the vessel contains a passive pressurizer system that regulates system pressure.

In the SIR design, a large LOCA is considered impossible since there is no primary coolant piping. Potential seismic failure due to differential movement of the vessel and piping in current designs should be eliminated.

A more detailed description of the AP-600 and other advanced designs is provided in Chapter 6.

1.1.3.3 PRIME Reactors

The reactors in this group propose radical departures from current PWR technology. The best known of these designs is the PIUS (Process Inherent

Ultimate Safety reactor) originally proposed by Hannerz.³³ The reactor (shown in Fig. 1.17) is a combination of a PWR and a swimming-pool reactor. The core is placed at the bottom of a very long standpipe or riser located in a large pool of water. Piping at the upper end of the standpipe conducts the circulating flow outside the reactor vessel to one of several steam generators and pumps. The water is then returned to the vessel and flows through an annular downcomer to the core. The lower end of the standpipe containing the core has an opening that connects it to the pool of boric acid water in the pool. Under operating conditions, the pump head keeps boric acid water from entering at the bottom of the standpipe. Although

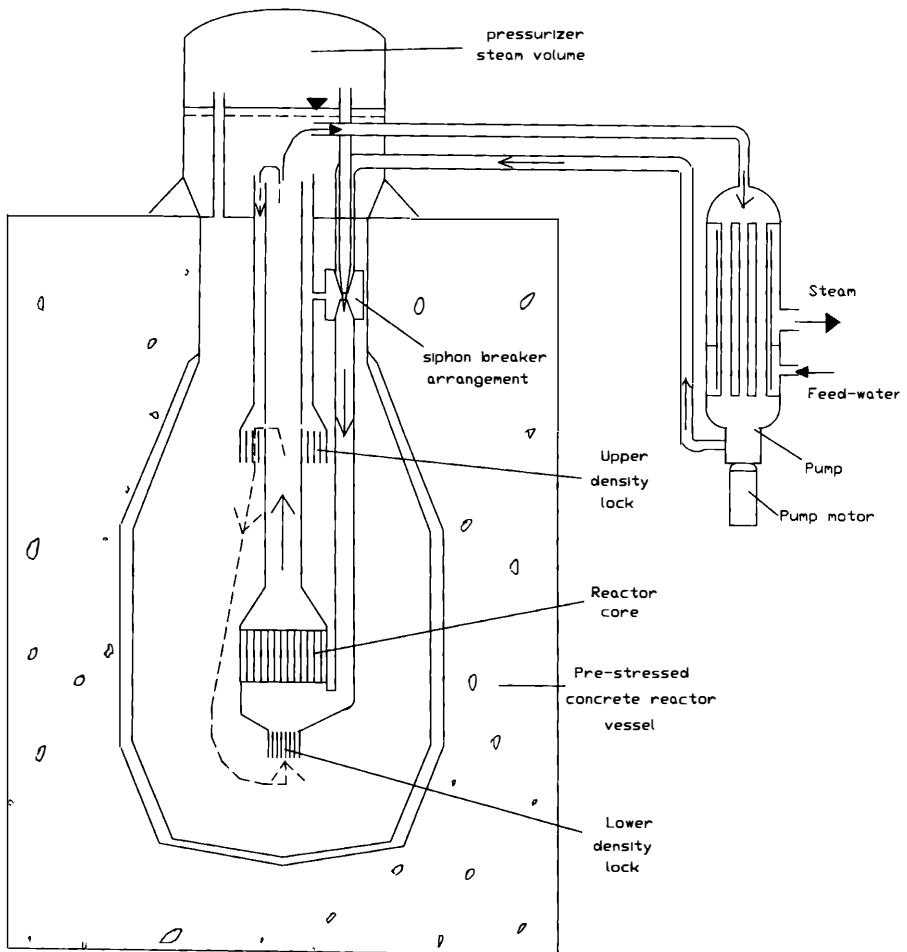


Fig. 1.17 PIUS reactor configuration [based on information from *Nucl. News*, p. 85 (Sep. 1992)].

the standpipe is also connected to the pool at the upper end, the pressure at the upper end of the standpipe equals the pressure in the pool. Therefore, pool water does not enter the standpipe during normal operation. In the case of a loss-of-coolant flow, a natural-circulation loop would be set up with pool water entering at the bottom of the standpipe and exiting at the top.

The core, riser, pressurizer, and water pool are all located within a large (12 to 13 m. i.d.) prestressed concrete pressure vessel. A double stainless-steel liner protects the concrete from the borated pool water and prevents leakage. Natural circulation of the pool water through air-cooled heat exchangers provides enough heat removal to keep the pool water cooled under both normal and accident conditions. In the event that air access to the heat exchangers is curtailed, there is enough pool water to keep the core covered and cool for at least one week after shutdown.

The Japanese System Integrated Pressurized Water Reactor (SPWR) concept is similar to PIUS in a number of respects. The SPWR has a core located at the bottom of a long riser in a large vessel. However, in this case the vessel is made of steel and is considerably smaller than the vessel proposed for PIUS. In the SPWR, both pumps and steam generators are placed within the reactor vessel.

Advanced designs for CANDU-type reactors have also been proposed.²⁹ In these designs, the pressure tubes have the unique characteristic of becoming highly conductive to heat (instead of acting as an insulator) once a given temperature has been exceeded. The fuel decay heat can then be transmitted to the water in the low-pressure calandria. Heat pipes or other mechanisms then conduct this heat to the environment.

1.1.4 Status of Various Concepts

Although a variety of pressurized water concepts for generation of central station power have been examined, many of these are no longer actively considered. For large central stations, only three design types were in operation in 1994: (1) the vessel-type heavy-water moderated design, (2) the CANDU heavy-water-moderated design, and (3) the vessel-type light-water-moderated system at subcritical pressure (conventional PWR). The vessel-type PHWR design has been used for only two reactors in Argentina (1994). The CANDU design is used extensively in Canada and is also used in Pakistan, India, Korea, and Argentina (worldwide total of 30 units in 1994). However, the light-water-moderated conventional design, using slightly enriched fuel, has dominated PWR plant construction in the United States, Western Europe, Japan, the former Soviet Union, and Eastern Europe. More than 240 plants were deployed worldwide in 1994. Since the conventional PWR dominates PWR practice, this monograph concentrates on its analysis.

Some of the advanced concepts described in Section 1.1.3 are likely to be important in the future. However, it is too early to determine which of these will be widely employed.

1.2 STEADY-STATE POWER GENERATION AND DISTRIBUTION IN CONVENTIONAL PWR CORES

1.2.1 Power Distribution in Unperturbed, Uniformly Loaded Cores

Since light-water-moderated reactors cannot go critical using entirely natural uranium fuel, the designers of the early PWRs had three choices:

1. To utilize fuel elements that all contain slightly enriched fuel
2. To provide a *spiked core*, on which highly enriched fuel elements are uniformly dispersed in a matrix of natural uranium elements
3. To provide a region of highly enriched fuel surrounded by low-enrichment or natural uranium fuel.

The last alternative formed the basis of the seed-blanket design. Dispersion of enriched elements among natural uranium fuel results in power outputs for the enriched rods substantially greater than for the others. It is difficult to provide additional flow to the enriched elements and they could limit power output. Further, significantly lower fuel costs can be obtained by using slightly enriched fuel in all elements. This is the common practice for large, central station power plants. No practical reactor design has used a spiked core.

If the fuel elements were dispersed through the core in a uniform manner and if fuel enrichment were uniform throughout, we can roughly approximate overall neutron behavior by considering the core to be homogeneous. Most light water designs are essentially unreflected. Thus, we consider the unperturbed core as a bare homogeneous reactor. For such a core, an approximation of the neutron flux distribution can be obtained by solving the wave equation³⁴

$$\nabla^2\phi + B^2\phi = 0 \quad (1.1)$$

where ϕ is thermal neutron flux and B^2 is geometric buckling. Our boundary conditions for the solution of this equation are that flux goes to zero at the extrapolated boundaries of the reactor and that it is finite everywhere except at the extrapolated boundaries. The extrapolated boundaries are approximated by adding $0.71\lambda_{tr}$, where λ_{tr} is the transport mean-free-path (mfp) of a neutron, to the distance from the centerline. Thus, for a cylindrical reactor, extrapolated length L' is

$$L' = L + 1.42\lambda_{tr} , \quad (1.2)$$

TABLE 1.I
Thermal Flux Distribution in Bare Homogeneous Reactors

Geometry	Coordinate	Distribution Function
Infinite slab		$\cos\left(\frac{\pi x}{L'}\right)$
Rectangular parallelepiped	$x, y, \text{ or } z$	$\cos\left(\frac{\pi x}{L'}\right), \cos\left(\frac{\pi y}{L'}\right), \cos\left(\frac{\pi z}{L'}\right)$
Sphere		$\frac{\sin(\pi r / R')}{\pi r / R'}$
Finite cylinder	$\begin{cases} r \\ z \end{cases}$	$J_0(2.405 \, r / R')$ $\cos\left(\frac{\pi z}{L'}\right)$

where L is the actual length of the reactor and R' the extrapolated radius, is given by

$$R' = R + 0.71 \lambda_{tr} \quad (1.3)$$

where R is the actual radius. For large reactors, λ_{tr} is negligible with respect to R and L and can be ignored. The solution for several of the most common geometries is listed in Table 1.I (Ref. 35). Under the conditions we have postulated, and if the effects of burnup are not significant, power generation is proportional to thermal neutron flux. Thus, the functions indicated in Table 1.I provide an overall description of power distribution.

Power distribution within a reactor free of control rods is not only of academic interest; two control systems lead to such a situation for base-loaded plants. In the chemical shim control system, boric acid is added to the reactor to compensate for excess reactivity at the beginning of life (BOL). However, it is gradually withdrawn as core reactivity declines with lifetime. A similar procedure is used in spectral shift control,* where the ratio of D_2O to H_2O in the moderator is decreased in response to the change in reactivity with lifetime. Decreasing the amount of D_2O present increases the number of thermal neutrons present in the core, thus increasing reactivity and compensating for the decrease in reactivity due to fuel burnup. At full power, such reactors can be operated with all control rods essentially withdrawn.

*The original spectral shift control system is no longer of commercial interest. However, similar approaches are used. The calandrias of some D_2O -moderated pressure tube reactors are pierced by vertical hollow tubes, which can be filled with H_2O . By varying the H_2O level, the neutron spectrum can be varied. A similar idea is employed in some advanced vessel-type PWRs, in which a portion of the control element clusters are fitted with water displacement rods. Movement of these nonabsorbing rods changes the moderator-to-fuel ratio and thus varies the neutron spectrum.

Power distribution at BOL, in the uniform, unrodded cylindrical reactor, is closely approximated by a J_0 radial distribution and an axial cosine distribution. Thus, we write

$$q''' = J_0 (2.405r / R') \cos\left(\frac{\pi z}{L'}\right) q'''_{\max} \quad (1.4)$$

where

q''' = rate of volumetric heat generation

$q'''_{\max}$ = maximum volumetric heat generation at the center of the core

r = radial location

z = axial location (from centerline).

Most PWRs operating in 1992 utilize fuel assemblies that are uniformly enriched in the axial direction. In the absence of part length burnable absorbers, a cosinusoidal flux and power distribution in the axial direction is thus a good approximation at BOL. As burnup proceeds, the flux and power distribution are flattened both axially and radially. This results from the more rapid depletion of burnable isotopes in the high-power regions. Since power peaking is generally reduced as the cycle proceeds, the limiting thermal-hydraulic conditions generally exist at the beginning of the cycle.

1.2.2 Effect of Fuel Loading

1.2.2.1 Standard Core Arrangement

Most of the early power PWRs were uniformly loaded. In very small plants, where simplicity of refueling is important, this loading scheme must still be useful. For large, central station plants, one disadvantage is that it leads to high power peaking in the central region of the core. In addition, relatively low-average burnups are obtained since even at the end of life (EOL) the outermost fuel elements have been only moderately burned down. One means of alleviating these difficulties is to use a zone loading scheme, where the core is loaded with fuel elements of three or more different enrichments. The most highly enriched fuel elements are placed in the outermost position; the least enriched are placed in the central region. Since power is approximately proportional to the product of thermal neutron flux and fissionable fuel concentration, the power level in the inner region of the core is lowered and that of the outer region is raised. This power flattening can significantly increase the total power capability of the core. When criticality is no longer possible, the entire core is replaced.

Replacement of the complete core at each refueling leads to a low-average core burnup. This is increased in most fuel cycling schemes by replacing only a portion of the fuel at each refueling. In the simplest of these schemes, the initial core is zone loaded as described above. At the

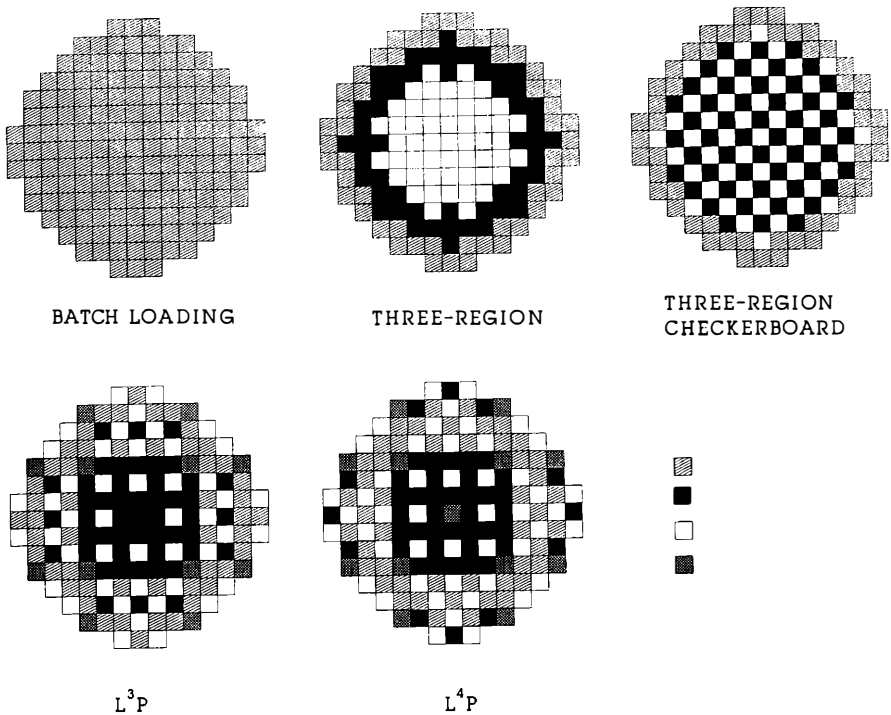


Fig. 1.18 Fuel loading patterns (based on information from Westinghouse Electric Corp. *Fuel Technology Update* No. 11).

end of the first core life, the most burned fuel from the central core region is removed, the outer regions are moved inward, and fresh fuel is placed in the vacated spaces in the outermost region. The same procedure is followed in all subsequent core loading so that at equilibrium all the removed fuel has been through three or more cycles and has a high burnup. Batch and three-region zone loading are illustrated in Fig. 1.18.

Several variations of this fuel cycling scheme are available. An early improvement was the so-called "Roundelay" method. In the equilibrium cycle, fresh elements are inserted in uniform distribution throughout the core at each refueling. For example, with Roundelay 3 batch, every third element is replaced while the other elements are left in place. In Fig. 1.19, the fuel elements are numbered in the order in which they might be replaced in the reference core; i.e., during one refueling all No. 1 fuel assemblies would be replaced, and so on. Mixing fresh and burned assemblies produces a strong coupling between them and permits improved power production from the burned elements.

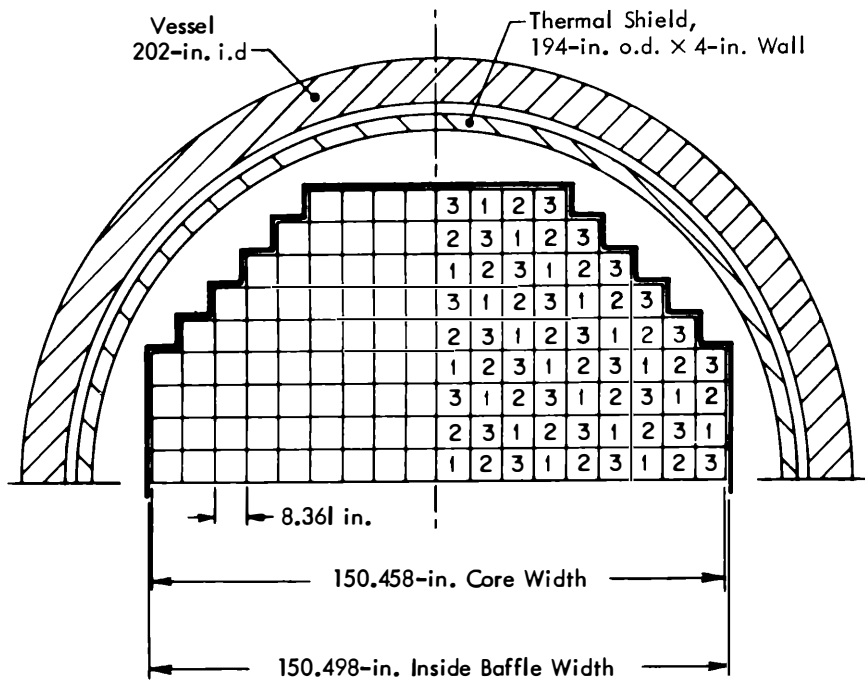


Fig. 1.19 Roundelay fuel cycling scheme (from Ref. 36).

Figure 1.20 shows a typical radial power distribution in equilibrium cores after refueling for three-region and Roundelay-loaded reactors. The power distributions shown are the ratio of the power at the given point to the average power across the core at that axial location. The marked "ripple" in the Roundelay power distribution results from significantly different power production from adjacent fuel assemblies with different burnups. In general, for both refueling schemes, as burnup proceeds, peak power decreases from that shown and power in the lower power region increases. Note that small changes in fuel assembly placement can significantly affect peak power.

The three-region checkerboard scheme illustrated in Fig. 1.18 was somewhat more widely used than the Roundelay approach. In this refueling procedure, the outer region is fresh fuel, but the innermost region is a checkerboard pattern of second- and third-cycle fuel. The actual determination of the location of the specific assembly locations in the central region is fairly sophisticated. The final arrangement resulting from these studies may not strictly conform to the checkerboard pattern shown in Fig. 1.18. This is certain to be the case if fuel failures require the insertion of extra

.361 (1.284)	.311 (1.282)	.312 (1.322)	.210 (1.512)						
.777 (1.143)	.691 (1.167)	.554 (1.158)	.441 (1.303)						
.879 (1.106)	1.021 (1.097)	.891 (1.137)	.661 (1.162)	.522 (1.286)	.298 (1.492)				
1.088 (1.070)	1.029 (1.111)	1.198 (1.082)	1.012 (1.130)	.746 (1.137)	.549 (1.287)				
1.490 (1.074)	1.315 (1.084)	1.150 (1.080)	1.299 (1.075)	1.066 (1.129)	.745 (1.157)				
1.368 (1.101)	1.583 (1.075)	1.396 (1.074)	1.202 (1.073)	1.329 (1.075)					
1.463 (1.040)	1.394 (1.127)	1.641 (1.075)	1.424 (1.075)						
1.836 (1.061)	1.622 (1.066)	1.430 (1.121)							
1.604 (1.126)	1.851 (1.074)	$F_R = 1.99$							
1.561 (1.023)									

FOR EACH ASSEMBLY

Upper number is (average power in assembly)/(average power in core).

Lower number is (maximum power in assembly)/(average power in assembly).

Fig. 1.20 Typical radial power distribution in Roundelay-loaded reactor (from Ref. 36).

fresh fuel or the reuse of the third-cycle fuel. A detailed study of possible placement schemes is made; the *scatter-loading* scheme, which provides the best economics, is selected. In view of the very large number of placement policies that can be followed, a methodical procedure limiting the interchanges considered is required. Rothleder^{37a} lists a series of heuristic rules that can be used to limit the number of policies considered.

More recently, low leakage fuel loading patterns, such as the pattern indicated by L³P in Fig. 1.18, have been introduced. In these fuel patterns, fresh fuel is no longer in the peripheral positions. This has the advantage of reducing neutron leakage from the core and allowing fresh fuel of a lower enrichment to be used. In addition, the lower neutron flux at the core edges means lower rates of reactor vessel embrittlement and hence leads to longer reactor vessel lifetimes.

Ultra-low-leakage fuel pattern schemes such as that indicated by L⁴P in Fig. 1.18 have also been used. Such schemes provide additional improve-

ments in neutron economy and some additional reduction in the fast neutron flux at the reactor vessel wall. If the presence of impurities such as copper have reduced the ductility of reactor vessel welds, it may be desirable to further reduce the flux at the vessel wall by insertion of absorber rods (e.g., stainless steel or hafnium) in the assemblies closest to the welds.

Since power is shifted from the core periphery to interior locations by low-leakage fuel management, higher peaking results. Acceptable peaking factors are obtained by insertion of burnable poison rods in those fresh assemblies with high peak powers (Fig. 1.21).

Burnable poison materials, such as those containing gadolinium, erbium, or boron, provide a significant reduction in reactivity at BOL. As the cycle progresses, the poison is substantially depleted by interaction with the thermal neutrons present. At the end of the first cycle, the poison is almost entirely consumed and hence it causes only a slight decrease in end-of-cycle reactivity. If the poison were contained in rods placed in the rod cluster control (RCC) tube, the poison rods would be removed from the assembly at the end of the first cycle. When the poison is mixed with fuel, removal at the end of the first cycle is, of course, not possible.

Because addition of burnable poison significantly reduces beginning-of-cycle (BOC) reactivity, less boric acid must be dissolved in the moderator

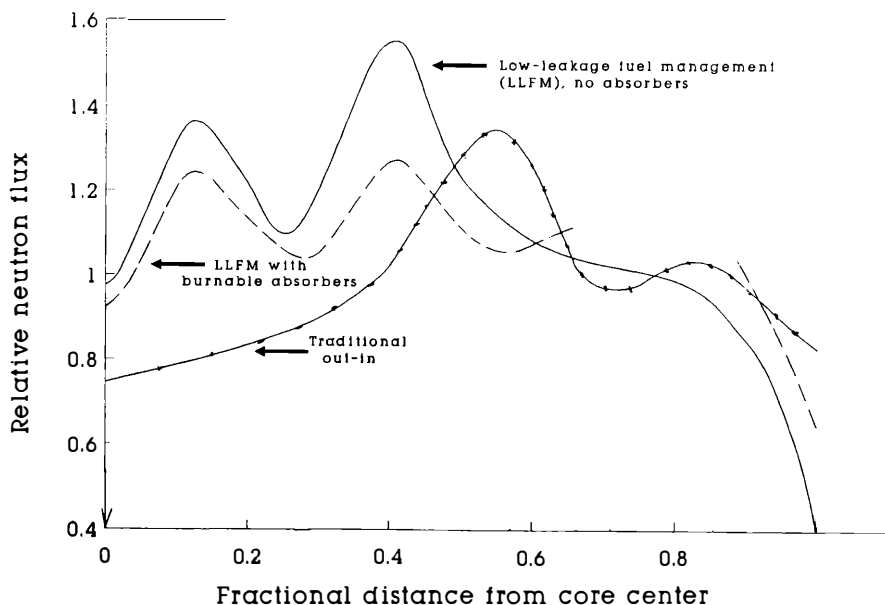


Fig. 1.21 Effect of burnable absorbers on radial flux flattening (based on information provided by Westinghouse Electric Corp.).

to produce a multiplication factor of 1.0. This is highly desirable when long cycle lengths (i.e., 18 or 24 months) are used. If dissolved boric acid alone were used to reduce BOC reactivity, the high boron concentration required is likely to lead to a positive moderator temperature coefficient. Since a negative moderator temperature coefficient is required by U.S. Nuclear Regulatory Commission (NRC) regulations, long refueling cycles are not feasible without the addition of burnable poison.

Although all PWR fuel designed prior to about 1990 had an axially uniform enrichment, some later designs have included axial blankets.^{37b} These blankets, which are about 6 in. long and placed at both ends of the fuel assembly, contain pellets produced from natural uranium. The axial blankets serve the same purpose as the low-leakage x - y positioning of the fuel assemblies, reducing neutron leakage. The enrichment of the center portion of the fuel stack is increased to compensate for the natural fuel at the ends. However, the average enrichment of the entire assembly is reduced slightly resulting in an overall savings of 1 to 2% in fuel cycle costs. Since the power production in the end regions is reduced, the power in the central region is correspondingly increased. Without burnable absorbers, this can result in a 6 to 8% increase in axial power peaking. This peaking can be counteracted by burnable absorber rods. When axial blankets are present, the neutron flux and power drop sharply in the blanket region. A cosine function no longer provides an adequate description of the axial flux variation.

1.2.2.2 Use of Mixed-Oxide (MOX) Fuel

The availability of reprocessed plutonium in Europe has led to the use of plutonium-fueled (UO_2 enriched with PuO_2) assemblies in some PWRs. As of 1992, when fuel containing plutonium has been used, it has made up only a portion of the fresh fuel added to the core (e.g., 30%).³⁸ Since the nuclear characteristics of plutonium and ^{235}U differ significantly, the power peaking in the region of a plutonium-enriched assembly adjacent to a ^{235}U -enriched assembly poses a problem. Both the fission and absorption cross sections of ^{239}Pu are higher than those for ^{235}U . Therefore, to obtain the same reactivity worth of a fresh assembly, the Pu assemblies have a higher average enrichment than those with ^{235}U . Since the flux must be continuous across an assembly boundary, the higher enrichment would produce an unacceptable high heat flux if it were used in the fuel rods on the periphery of the assembly. This difficulty can be avoided by using three concentric rings with three different enrichments in the same assembly. The outermost zone contains fuel with the lowest enrichment, whereas the innermost zone contains fuel with the highest enrichment. A typical arrangement would have fuel with approximately 4% Pu in the outer zone, 6% in the middle zone, and 8% in the innermost region.

An alternative way of dealing with Pu-enriched fuel assemblies is to use reconstitutable assemblies and use three burnup regions within the assem-

bly.³⁸ A kind of in/out reloading scheme would be used. At equilibrium, the following rearrangement would occur at the end of a cycle:

1. Thrice-burned fuel pins of the outermost zone are unloaded and removed from the core.
2. Twice-burned fuel pins are unloaded from the middle zone and loaded into the outer zone.
3. Once-burned fuel from the central zone is moved to the outer zone.
4. Fresh fuel is added to the central zone.

1.2.2.3 High-Conversion-Ratio Core Arrangements

The earliest efforts to obtain a core with a high conversion ratio were based on the *seed-and-blanket* design of the Shippingport reactor. This reactor was the first PWR to be built for civilian power production.

A seed-and-blanket core is an alternative to a design using slightly enriched uranium throughout the core. Such a core consists of one or more relatively small seed regions containing highly enriched fuel assemblies, and blanket regions of natural or low-enrichment fuel surrounding the seed regions. The blanket is composed of material with a multiplication factor less than unity and cannot be made critical by itself. Reactivity of the core is primarily determined by the properties of the seed; thus, control rods are required only in the seed.

The geometry initially considered was that of a cylindrical annular seed with blanket regions both inside and outside. This arrangement (see Fig. 1.22 and Refs. 39 and 40) was used in Shippingport core 1. The blanket elements were bundles of zirconium-alloy-clad cylindrical rods containing UO_2 pellets fabricated from natural uranium. The seed elements consisted of a series of fuel plates held by Zircaloy alloy boxes, each segmented into four quadrants. The space between the quadrants was used for a control rod passage. The fuel plates themselves consisted of an enriched fuel alloy strip sandwiched between two zirconium alloy cover plates and four side strips. In later seeds, a dispersion of UO_2 in Zircaloy replaced the U-Zr alloy fuel.

The power distribution within a seed-and-blanket reactor is appreciably different from the distribution heretofore considered. In reactors of this type, infinite multiplication of the annular seed region is greater than unity, whereas that of the blanket is less than one. The higher fuel content and multiplication factor of the seed result in a higher fast neutron flux in the seed. In addition, the highly enriched fuel in the seed has a higher thermal absorption cross section than the blanket. This results in a leakage of fast neutrons from the seed into the blanket and a leakage of thermal neutrons in the opposite direction.

In the blanket, ^{238}U fast fission produces the power at BOL. As burnup proceeds, formation of plutonium in the blanket creates new power-

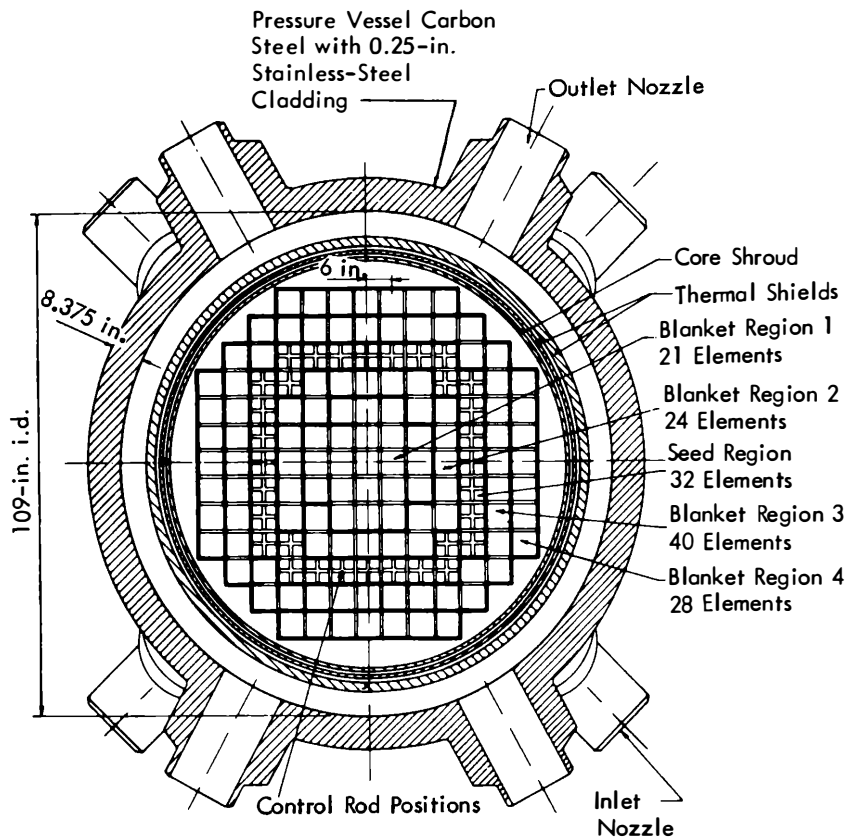


Fig. 1.22 Cross section of Shippingport reactor core [from *Directory of Nuclear Reactors, Vol. IV Power Reactors*, pp. 21–32 (1962)].

producing fuel. Leakage of fast neutrons from the seed thus results in power production in a region that cannot by itself maintain a chain reaction. It is economically desirable for as much power as possible to be produced in the relatively low-cost natural uranium blanket.

The power division between blanket and seed is one of the important factors in determining the thermal design of a reactor of this type. Radkowsky and Bayard⁴¹ discuss the procedures used for calculating the power distribution in Shippingport core 1. At BOL, the ratio of blanket power to total power was calculated as ~ 0.52 .

Seed power does not decrease as rapidly as seed volume and, therefore, a decrease in seed volume results in increasing seed power density. Hence, power sharing tends to be determined by the maximum power density that can be extracted from the seed. Thus, the average power density in the seed can be more than four times that of the blanket.⁴¹ Since resonance capture

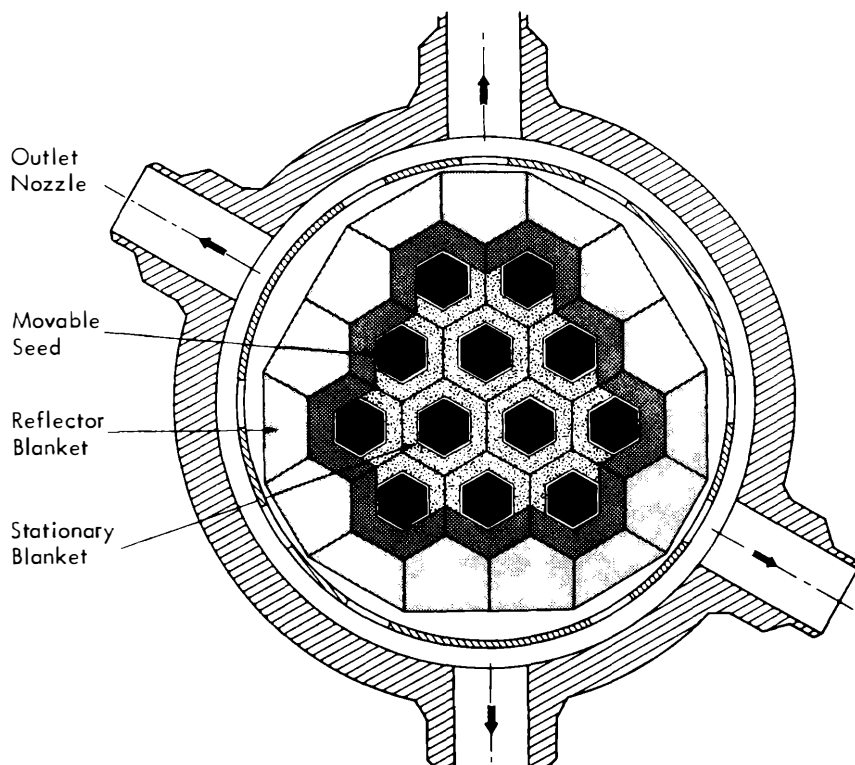


Fig. 1.23 Cross-sectional view of the first Shippingport LWBR core.

is of no consequence in the seed, it is feasible to use thin fuel elements with a high surface area per core volume to attain the high power removal rates required. Inlet orifices can be used to provide considerably higher coolant mass fluxes to the seed than to the blanket.

Later studies with the seed and blanket concept were directed toward developing a light water breeder reactor (LWBR). The concept was based on the ^{232}Th - ^{233}U cycle. Both the seed and blanket consisted of ThO_2 fuel rods enriched with varying amounts of $^{233}\text{UO}_2$. (Seed rods contain up to 6 wt% UO_2 ; blanket rods contain up to 3 wt% UO_2 .) From the cross section of the first Shippingport LWBR reactor (Fig. 1.23),⁴² we see that the core consisted of a symmetrical array of hexagonal modules surrounded by a reflector blanket. Each core module contained an axially movable seed region ($k > 1$) and a stationary annular blanket ($k < 1$). Both seed and blanket regions were made up of arrays of tightly packed cylindrical fuel rods containing oxide pellets.

Control of the LWBR was accomplished by moving the seed. As the seed elements were moved out of the core, leakage was increased and the core

became subcritical. This control procedure, which avoids losing neutrons to poisons, was necessary since the calculated breeding ratio was only slightly greater than unity.

The results obtained from operation of the LWBR appear to indicate that, when all reprocessing and fabrication losses are considered, true breeding is probably not possible. This conclusion, and the obvious complexity of seed-and-blanket cores with movable seeds, has led to the virtual abandonment of this approach.

While the seed-and-blanket approach continues to have an occasional advocate,⁴³ most recent interest has been focused on homogeneous tight-lattice cores⁴⁴⁻⁴⁶ using the uranium-plutonium cycle. In this approach, originally suggested by Edlund,⁴⁷ the standard PWR core is replaced with a core formed by hexagonal fuel assemblies containing fuel rods on a triangular pitch. By using a tight lattice, the water volume fraction is reduced, thus leading to a harder neutron spectrum. When such a lattice is fueled with plutonium discharged from LWRs, the hard spectrum and the significant percentage of ^{241}Pu in the fuel allows conversion ratios of 0.8 to 0.9 to be achieved. This compares with conversion ratios of 0.5 to 0.6 found in conventional PWRs. With a conversion ratio of 0.9, approximately 4% of the uranium fuel can be utilized in contrast with a 1% utilization when Pu is recycled in a conventional LWR.

These reactors are homogeneous in the sense that all fuel rods have identical shapes. However, the plutonium concentration may vary with position in order to reduce power peaking. Both axial and radial blankets would be used. In most designs, the outer row of assemblies serves as the radial blanket and these assemblies contain no Pu at the beginning of life.

Both Zircaloy and stainless steel⁴⁴ have been proposed for cladding materials. Because of the epithermal neutron spectrum, stainless steel produces much less of a poisoning effect than it would in a conventional PWR. Further, the higher strength of stainless steel results in considerably less distortion if the fuel is exposed to the high temperatures possible during a LOCA. This is important in maintaining a coolable geometry with the small clearances available in a tight lattice. Another advantage to the use of stainless steel is that it can be formed with integral spacer ribs (Fig. 1.24). These avoid the difficulty of fabricating spacer grids for tight lattices and, in addition, such spiral ribs improve coolant heat transfer characteristics. Table 1.II compares the geometrical characteristics of a proposed tight lattice design to those of conventional PWR designs.

1.2.3 Effect of Control Rods, Water Slots, Voids, and Burnable Absorbers

1.2.3.1 Effects of Control Rods on Power Distribution

In some reactor designs, all excess reactivity is compensated for by inserting control rods. The presence of the strongly absorbing material perturbs the flux both axially and radially. However, this is not entirely a disadvantage

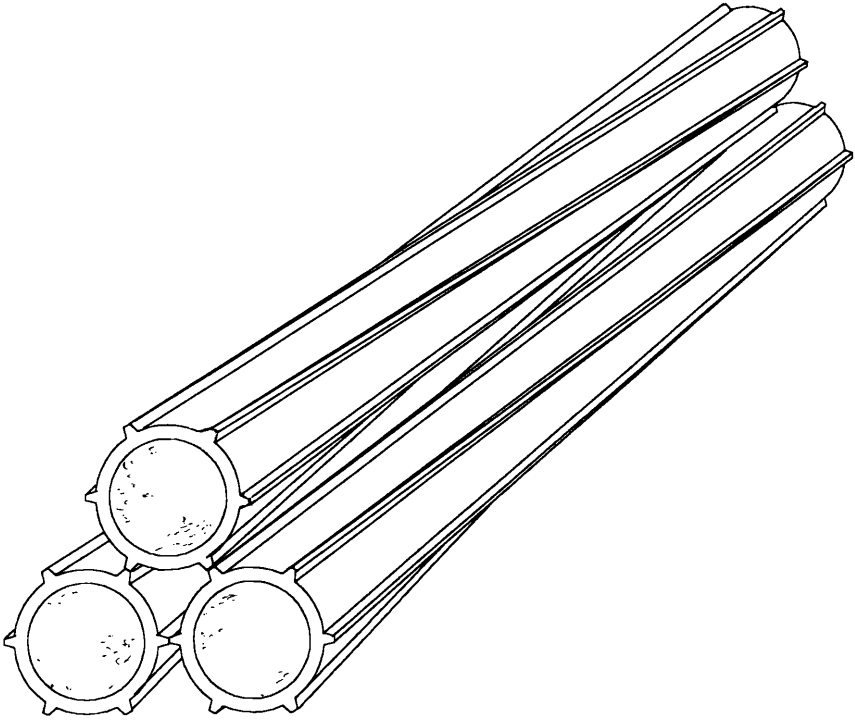


Fig. 1.24 Fuel rods with integral spiral spacer ribs for tight-lattice core [Broeders and Dalle-Donne, *Nucl. Technol.*, **71**, 85 (1985)].

TABLE 1.II
Geometrical Characteristics of PWR Fuel Assemblies

Reactor Type	Lattice Type	Typical Fuel Rod Diameter (mm)	Typical Minimum Clearance Between Fuel Rods (mm)	Fuel Rods per Assembly	Fuel Rod Length (cm)
Standard PWR, U.S. or West European design	Square	9.5–10.8	3.5	204–300	330–440
Standard PWR, Russian design	Triangular	9.1	3.1	126	255
Tight-lattice PWR	Triangular	9.5	1.9	313	220

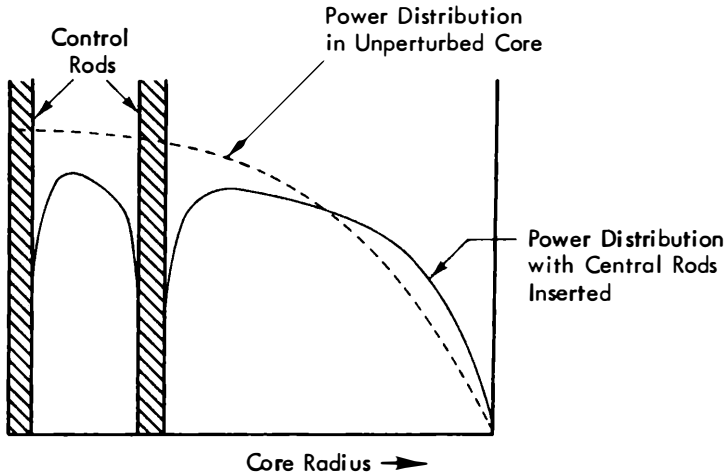


Fig. 1.25 Radial power distribution in a cylindrical reactor with and without control rods.

since the rods can be used to improve radial flux distribution. For example, some, or all, of the rods in the center of the core can be partially inserted at BOL, thereby reducing flux peaking. This effect is illustrated in Fig. 1.25, where we observe that flux essentially goes to zero at the boundaries of the control rods, as well as at the extrapolated boundary of the reactor. The flux and, hence, the power level in the outer region are increased concurrently with the decrease in the central region.

The improvement in radial distribution attainable by inserting rods is often more than negated by the change in axial distribution. The usual situation in a PWR where the rods are top entering is shown in Fig. 1.26. At BOL, the partially inserted rods skew the flux toward the bottom. As the rods are withdrawn, the less burned fuel at the top of the core causes the flux to be skewed toward the top at EOL. As observed in Fig. 1.26, the ratio peak-to-average power can be higher than that of the unperturbed core.

Power distribution of the type shown in Fig. 1.26 generally can be represented by modifying the usual cosine distribution to the form

$$\phi(l) = (A + Bl) \cos(\alpha_0 l) \quad (1.5)$$

where A , B , and α_0 are constants and l is a dimensionless length coordinate varying from -1 to $+1$ along the heated length of the core. Expressions of this form are applicable only if the ratio peak-to-average flux is < 1.892 . Establishing constants and using expressions of this form are discussed in Sec. 4.1.1.

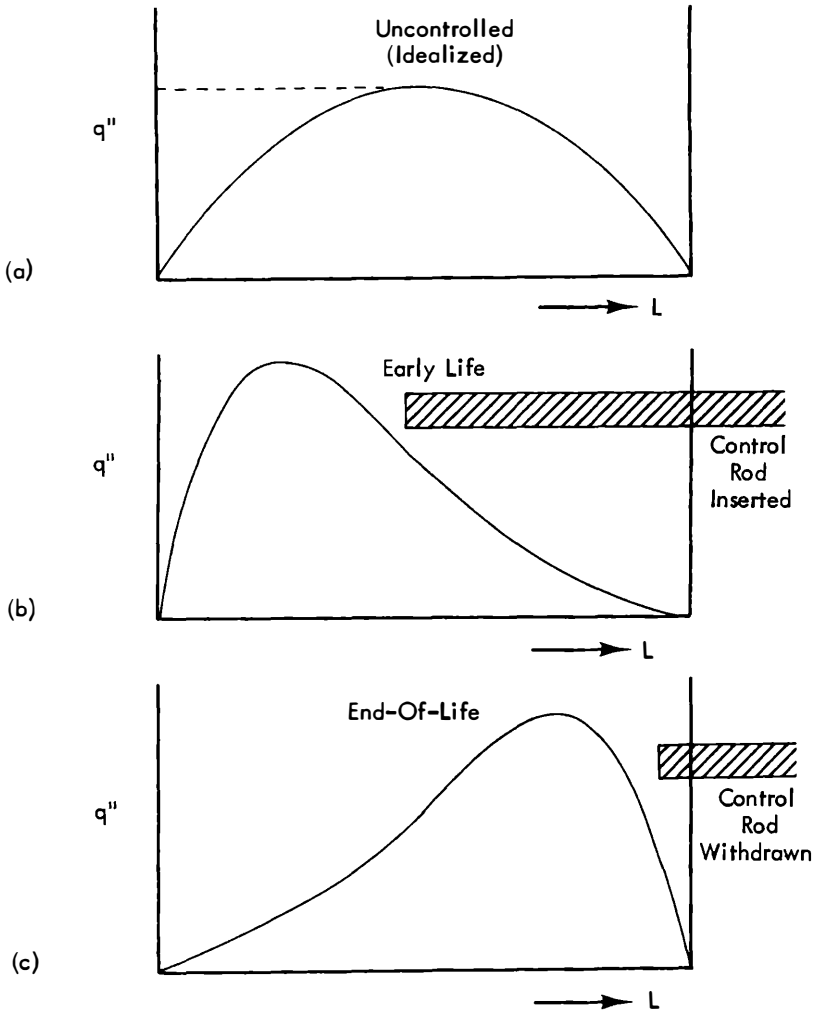


Fig. 1.26 Axial power distribution.

Chemical shim control, via addition of boric acid to the coolant, has been almost universally adopted for currently operating (1994) conventional PWRs designed for central power. This means that, if the plant is base loaded, these reactors operate with control rods almost entirely withdrawn at full power throughout core life. Further, axial flux distortion due to rod motion is minimized. Note that addition of boric acid is also used to compensate for the reactivity increase in going from hot-zero power conditions to cold-zero power conditions.

When the shutdown rods are maintained in exactly the same all-rods-out (ARO) position throughout core life, excessive wear can occur in the upper internals area where the guide tubes interface with the rodlets of the control elements. To avoid this problem, it is desirable to reposition the rods slightly during the cycle. It has been suggested⁴⁸ that at the beginning of the cycle the control elements be inserted to a position slightly below the top of the outer core. The rods are then periodically withdrawn, one step at a time, so that at the end of the cycle they are slightly above the active core. This approach avoids undesirable axial flux peaking as well as maximizing EOC shutdown margin.

Although base load operation of nuclear power plants is possible in power systems where most of the power generation is non-nuclear, a portion of the nuclear plants needs to be load following when a substantial fraction of the system generation is nuclear. Some power variations will always have to be accommodated even in plants that are primarily base loaded. The procedures used to minimize flux distortion following a load change are discussed in Sec. 1.5.

In some advanced core designs, only some of the control rods are absorber rods. Most of the rods used are water displacement rods of ZrO_2 encapsulated in a zirconium alloy. These water displacement rods are fully inserted at the beginning of the cycle and then sequentially removed as the cycle progresses. Much of the change in reactivity with lifetime is thus compensated for by these rods through a shift in the neutron energy spectrum. The conversion ratio is improved because of the reduced amount of boric acid needed in the coolant and because of the harder neutron spectrum. The combination of burnable poison added to some fuel rods plus water displacement rods can allow the complete elimination of chemical shim control.

Advanced cores with water displacement rods require that all the RCC guide tubes be used rather than only the tubes directly under a drive mechanism. Control rod clusters are redesigned so that one spider controls water displacement rods in four assemblies. To limit the number of rods in a cluster, the number of guide tubes per assembly is reduced, but each guide tube replaces four fuel rods instead of one.

Control Rod Behavior. In those load-follow designs in which some absorber rods are inserted for an appreciable fraction of the operating time, the behavior of the absorber material at high neutron fluences is of concern. Experience has shown that the Ag-In-Cd alloy (80 wt% Ag, 15 wt% In, 5 wt% Cd) used in many designs has good irradiation stability. Since the corrosion resistance of the alloy in high-temperature water is unsatisfactory, the alloy must be encapsulated in stainless steel or Inconel. Rods using this material have performed well for more than 10 years.

Although somewhat less desirable than Ag-In-Cd, boron carbide absorbers have often been used because of their lower cost. In addition to deple-

tion of the boron, long-term operation leads to helium production and swelling of the B_4C . These phenomena can stress the cladding and lead to cladding failures. Although such failures have been a problem in BWRs where the rods receive neutron fluences generally above those seen by PWR rods, PWR operation appears to be satisfactory.

Hafnium is an excellent absorber and has both good corrosion resistance and dimensional stability. Although it has been used in naval reactors and in the Shippingport reactor, it has not been used in subsequent central station PWR power plants.

1.2.3.2 *Power Perturbations Due to Structural Material, Water Slots, and Voids*

No fuel-water lattice is entirely uniform; additional structural material is present at all grid locations, in fuel assembly cans, in the support structure required at core edges, etc. Since such material is a neutron absorber without being a moderator or source, it causes a local decrease in flux and, hence, in power. The effect can be small if the material is of low cross section (i.e., Zircaloy), but it can be substantial if a high cross-section material, such as stainless steel, is used.

Perhaps of even more concern to the designer is local peaking due to additional water slots in a light-water-moderated reactor. Such spaces result from clearance between fuel assemblies, variations in lattice dimensions, and slots left by the withdrawal of control rods. Since water provides additional moderation, the local thermal flux and, hence, power, increases. This effect is illustrated in Fig. 1.27 for a circular water hole in a low-enrichment, stainless-steel-clad core.⁴⁹ Slots of a size that could cause substantially more peaking than illustrated can easily arise; care is taken to avoid these. For example, where cruciform or Y-shaped control rods are used, followers of a low cross-section material are provided to prevent large water slots from being introduced when the rods are withdrawn. Such followers are not required with RCC and, as previously noted, the RCC concept allows the use of shorter reactor vessels.

Creation of a steam void at the exit of the warmest core region is an additional factor that distorts normal flux patterns. In general, replacement of water moderator by steam reduces core reactivity and, hence, flux and power, in the region of the voids. The effect of such voids is particularly significant during transient and accident situations where coolant enthalpy rises appreciably above its normal value. Power reduction due to void creation can decrease the severity of some situations.

1.2.3.3 *Burnable Absorbers*

We have noted previously that the addition of burnable poisons or absorber rods is useful in (1) suppressing BOC power peaking and (2) allowing a negative moderator temperature coefficient to be maintained by reducing

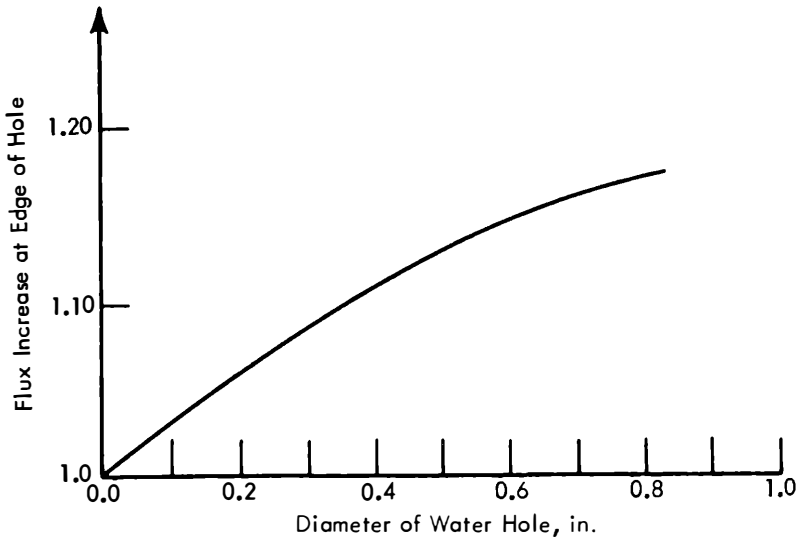


Fig. 1.27 Flux peaking at the edge of a circular water hole [from *Nucl. Eng. Des.*, 6, 301 (1967)].

BOC soluble boron concentrations. Originally, the burnable absorber was added to the core by placing burnable poison rods in some of the vacant RCC positions. The first of these rods consisted of an annulus of borosilicate (Pyrex) glass enclosed in a stainless-steel tube. Subsequently, some reactors used the wet annular burnable absorber (WABA) design. The design consists of annular pellets of a mixture of aluminum oxide and boron carbide (3.5 to 7 vol% B_4C) contained within two concentric Zircaloy tubes. Since coolant (moderator) flows through the central tube, the additional neutron moderation in the central hole reduces the flux variation across the absorber. This provides a more even burnup of the boron poison and less of a reactivity penalty at the end of the cycle. Solid pellets of aluminum oxide and boron carbide have also been used as burnable poison rods. These poison rods are removed from the fuel assembly at the end of the first cycle.

The burnable absorber rods will, of course, reduce the flux in the fuel assemblies in which they are inserted. In addition to this assembly-wide flux reduction, there is a significant local flux dip in the vicinity of each of the absorber rods. A further disadvantage is that burnable poison rods cannot be placed in any assembly directly under a control rod drive position.

More recently, burnable absorber material has been added directly to some of the fuel rods. The absorbers chosen have been gadolinium oxide (Gd_2O_3) or erbium oxide (Er_2O_3) mixed directly with the UO_2 and formed into pellets. Burnable absorbers of this type have been used to suppress

corner rod peaking in BWRs for some time. The number of fuel rods containing burnable poison can be considerably greater than the number of RCC positions in a fuel assembly and, therefore, lower poison concentrations may be used. This leads to less of a flux dip in the vicinity of the absorber. Because lower absorber concentrations can be used, a more even radial burnout of the absorber can be achieved than with absorber rods placed in RCC thimbles. Assemblies with the burnable poison in the fuel rods can be located at any position in the core, thus providing greater flexibility in refueling sequences. The poison rods remain with the assembly throughout its life in the core.

An alternative to mixing the poison material intimately with the fuel is the use of a thin coating on the outside of the pellets. These integral fuel absorbers have used zirconium diboride (ZrB_2) as the poison. Since the poison is on the outside of the pellet, it burns out uniformly. Properly designed absorber rods of this type lead to a very low residual reactivity penalty.

Absorber rods can also provide some improvement in axial peaking factors. By eliminating absorber material from the regions at either ends of the assembly, some axial flux flattening is achieved. In any assembly containing an axial blanket, absorber material is always confined to a region that is somewhat shorter than the enriched fuel length.

1.2.4 Nuclear Hot-Channel Factors

The thermal designer can readily determine average core parameters from a knowledge of total heat output, core heat transfer data, and flow to the core. However, core performance is not limited by average conditions, but by the most severe conditions. It is convenient and useful to define the hot channel of the core as that coolant channel or flow path where core heat flux and enthalpy rise is a maximum. Conditions in the hot channel are defined by several ratios of local-to-average conditions. These ratios are referred to as *hot-channel factors*. Those aspects of the complex nuclear data that are most significant to the thermal designer can be conveyed by three nuclear hot-channel factors. We define

$$F_R^N = \text{Radial nuclear factor} \\ = \frac{\text{Mean heat flux in hot channel}}{\text{mean heat flux in average channel of core}} \quad (1.6)$$

$$F_Z^N = \text{Axial nuclear factor} \\ = \frac{\text{Maximum heat flux in hot channel}}{\text{Mean heat flux in hot channel}}, \quad (1.7)$$

$$F_Q^N = \text{Nuclear heat flux factor}$$

$$= \frac{\text{Maximum heat flux in the core}}{\text{Mean heat flux in the core}} \quad (1.8)$$

From the above definitions, it immediately follows that

$$F_Q^N = F_Z^N F_R^N \quad (1.9)$$

Some authors define F_Z^N as the highest value that the ratio (maximum channel flux-to-average flux of given channel) can have and, when so defined, the maximum of the ratio may not occur in the hot channel.* When using reported values of F_Z^N , we should determine the basis for their definition.

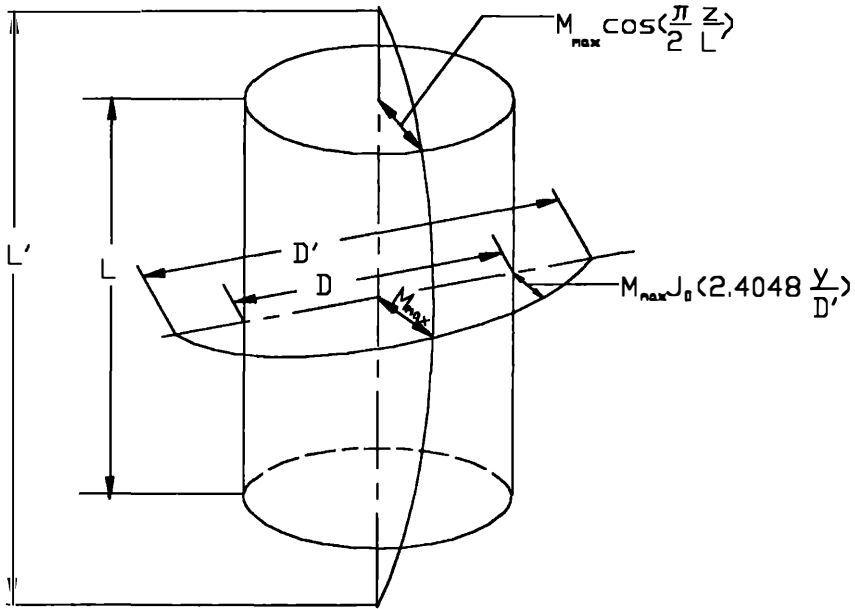
The above nuclear factors are defined on the basis of nominal channel dimensions. In computing limiting heat fluxes and enthalpy rises, consideration must also be given to deviation from nominal dimensions and flow. The procedure used to account for these deviations is discussed in Sec.5.1.4.

An estimate of the magnitude of the nuclear hot-channel factors can be obtained by considering a large, bare, homogeneous reactor of cylindrical shape. For a uniformly enriched, unpoisoned, unperturbed reactor, we see that the heat flux follows a J_0 radial distribution and a cosine axial distribution. Figure 1.28 shows axial and radial nuclear hot-channel factors for such a reactor as a function of the ratios of (actual length)/(actual length + extrapolation length). For large reactors of conventional design, the extrapolation length becomes negligible, and the values for L/L' and D/D' approach unity. Thus, we find that under these idealized conditions $F_Q^N = 1.5708 \times 2.3161 = 3.638$. In an actual uniformly loaded core, the overall hot-channel factor would be higher due to such effects as lattice nonuniformities, control rods, changes in fuel and fission product concentration, and coolant property variation. These effects can often be more than compensated for by improvements in F_R^N that can be obtained through non-uniform loading.

Although the extrapolation length is negligible in most large cores, its effect should be considered in compact military cores. In some advanced core designs, a reflector or ZrO_2 encapsulated in zirconium alloy tubes is placed between the core baffle and core barrel. As will be noted in Section 1.3, such a reflector effectively increases the distance beyond the core edge at which the flux extrapolates to zero. The effective (L/L') for the core is then below one, resulting in a somewhat flatter radial power distribution.

For a seed-and-blanket core, the overall core nuclear hot-channel factor has little meaning in view of markedly different power generations in the

*If it is then desired that the hot channel contain both the highest heat flux and enthalpy rise, it will not correspond to a real channel. In that case it should be considered as a hypothetical channel subject to the most severe thermal-hydraulic conditions possible.



L/L'	F_{Ax}^N	D/D'	F_R^N
0.8	1.3213	0.8	1.6571
0.9	1.4313	0.9	1.9297
1.0	1.5708	1.0	2.3161

Fig. 1.28 Axial and radial nuclear hot-channel factors for unperturbed cylindrical reactors.

seed and blanket. It is useful to extend the concept to define hot-channel factors for each region of the core. We can define separate seed-and-blanket hot-channel factors by using Eqs. (1.6), (1.7), and (1.8) with the word *core* replaced by *seed* or *blanket*.

The entire radial power distribution in any core can be described by defining a local radial factor, F_{Rl}^N , such that

$$F_{Rl}^N = \frac{\text{mean heat flux in given channel}}{\text{Mean heat flux in average channel of core}} \quad (1.10)$$

For a typical core, Fig. 1.29 illustrates the relative number of fuel elements with radial nuclear hot-channel factors in a given range. The peak radial flux is reached in a very small number of elements (often just one); the majority of the elements have radial factors < 1.0 .

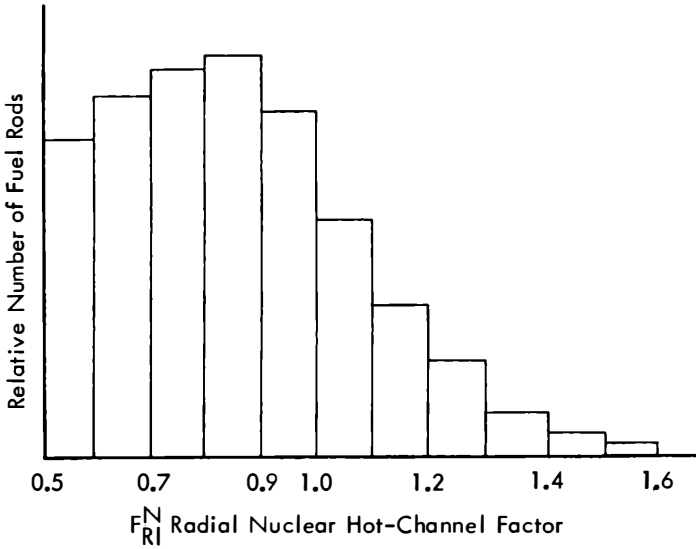


Fig. 1.29 Power census in a typical rodged core.

An accurate value of F_Q^N for a real core requires a three-dimensional nuclear calculation for evaluating power throughout the core. The nuclear heat flux factor can then be calculated from

$$F_Q^N = \frac{P'(x_0, y_0, z_0)}{(1/V) \iiint P'(x, y, z) \, dx \, dy \, dz} \quad (1.11)$$

where

$P'(x, y, z)$ = local power density at (x, y, z)

(x_0, y_0, z_0) = location of peak local power density

V = core volume.

Accurately determining the highest possible F_Q^N that could occur during life would require three-dimensional power calculations to be conducted for a very large number of conditions in a load-following plant. To avoid the expense of these calculations, a simpler synthesis procedure is often used.

The synthesis approach defines two additional factors:

$F_{xy}(z)$ = Peak-to-average power density at axial elevation z

$P'(z)$ = Relative core power at axial elevation $\left[\int_0^L P'(z) \, dz = 1 \right]$

With these definitions, the value of F_Q^N is then given by

$$F_Q^N = \max[P'(z) \cdot F_{xy}(z)] \quad (1.12)$$

In the synthesis approach, the values of $P'(z)$ are obtained from separate calculations. The values of $P'(z)$ are estimated from a one-dimensional axial calculation; values of $F_{xy}(z)$ are obtained by two-dimensional calculations at a number of planes. By appropriate modeling, it is possible to obtain

1. A one-dimensional axial distribution model that is consistent with three-dimensional average axial power distribution behavior during reactor operation
2. Two-dimensional values for $F_{xy}(z)$ that are always equal to or greater than the value of $F_{xy}(z)$ obtained for a three-dimensional calculation.⁵⁰

The synthesis approach yields conservative values for F_Q^N under these conditions. However, the more recent availability of rapidly running three-dimensional nodal codes (see Sec. 1.3.1.2) makes it possible to evaluate F_Q^N directly from a number of three-dimensional calculations if desired.

In addition to using the synthesis approach to define an overall F_Q^N , we can also define a heat flux factor at each elevation. That is,

$$F_Q^N(z) = [P'(z) F_{xy}(z)] \quad (1.13)$$

The values of $F_Q^N(z)$ and $F_{xy}(z)$ obtained at the various planes examined may be connected to provide continuous $F_Q^N(z)$ and $F_{xy}(z)$ functions. These curves then allow the calculation of the maximum linear power versus a function of elevation at full power reactor operation. The values so obtained are compared against the limiting curve of linear power versus elevation determined on the basis of LOCA and critical heat flux (CHF) considerations. If the computed linear powers exceed the allowable, then reactor operation must be restricted so that the control rod arrangements leading to the unacceptable peaking are never used.

1.2.5 Heat Generation Within Fuel Elements

It is often assumed that the rate of power generation is essentially constant across the fuel rod. While this is often satisfactory as a first approximation, it is not strictly true. As previously noted, heat generation in a volume element of fuel of a thermal reactor is essentially proportional to the thermal neutron flux at that point. Since thermal neutron flux decreases toward the center of the element due to neutron absorption in the fuel, power generation decreases similarly.

Following the explanation in Ref. 51, we consider a unit cell of a uniform rectangular lattice containing cylindrical fuel rods of radius R_0 . The problem is simplified by supposing that the square unit cell is replaced by a circle, radius R_1 , with the equivalent area (Fig. 1.30). Further, we assume

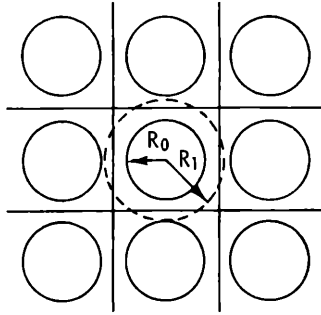


Fig. 1.30 Equivalent cell of fuel rod lattice.

that the neutron slowing down is constant in the moderator and zero in the fuel. The thermal diffusion equation for monoenergetic neutrons in the fuel rod is then

$$D_1 \nabla^2 \phi - \Sigma_a \phi = 0 \quad (1.14)$$

where ϕ is neutron flux at any point in the rod, D_1 is the diffusion coefficient, and Σ_a is the macroscopic absorption cross section. After dividing by D_1 and replacing Σ_a/D_1 by κ_0^2 , we obtain

$$\nabla^2 \phi - \kappa_0^2 \phi = 0 \quad (1.15)$$

In cylindrical coordinates, this becomes

$$\frac{d\phi^2}{dr^2} + \frac{1}{r} \frac{d\phi}{dr} - \kappa_0^2 \phi = 0 \quad (1.16)$$

Since κ_0^2 is positive, Eq. (1.16) is a modified Bessel equation with the general solution

$$\phi = AI_0(\kappa_0 r) + A'K_0(\kappa_0 r) \quad (1.17)$$

where I_0 and K_0 are zero-order, modified Bessel functions of the first and second kind, respectively. The second term can be eliminated since K_0 would require the neutron flux to go to infinity at the axis of the fuel rod. Thus, for the flux in the rod we obtain

$$\phi = AI_0(\kappa_0 r) \quad (1.18)$$

The value A can be determined from the boundary condition that ϕ is equal to ϕ_{surface} at $r = R_0$.

It is also useful to relate neutron flux at the surface of the fuel rod to average flux in the fuel with the definition

$$F = \frac{\text{Thermal neutron flux at fuel rod surface}}{\text{Mean thermal flux in interior of rod}} \quad (1.19)$$

With the previous assumptions, for a cylindrical rod we can show that

$$F = \frac{\kappa_0 R_0}{2} \frac{I_0(\kappa_0 R_0)}{I_1(\kappa_0 R_0)} \quad (1.20)$$

For an infinite slab fuel element, we obtain

$$\phi = \frac{\phi_0 \cosh(\kappa_0 x)}{\cosh(\kappa_0 a)} \quad (1.21)$$

$$F = \kappa_0 a \coth(\kappa_0 a) \quad (1.22)$$

where

x = distance from the centerline

a = half thickness of the fuel element

ϕ_0 = surface flux.

The computation of κ_0 requires a value of D_1 for the fuel. Because diffusion coefficients are normally not tabulated for fuel materials, it is necessary to obtain D_1 from

$$\frac{1}{D_1} = 3\Sigma_{tr} = 3\Sigma_t(1 - \bar{\mu}) \left[1 - \frac{4}{5} \frac{\Sigma_a}{\Sigma_t} + \frac{\Sigma_a \bar{\mu}}{\Sigma_t(1 - \bar{\mu})} \right] + \quad (1.23)$$

where

Σ_a = macroscopic absorption cross section of fuel

Σ_{tr} = macroscopic transport cross section of fuel

Σ_t = total macroscopic cross section of fuel

$\bar{\mu}$ = mean value of cosine of scattering angle $\approx 2/(3A)$

A = atomic mass number of fuel.

The previous assumption (that power distribution across the fuel rod is directly proportional to thermal flux) is not entirely correct because, as burnup proceeds, the fuel and poison distribution are no longer uniform. In view of the usual uncertainties in flux levels and in the physical properties of the fuel elements, consideration of these nonuniformities is rarely required. Perhaps more important are the limitations on simple diffusion theory. More exact calculations show that diffusion theory tends to underestimate flux depression in the fuel.

Most modern computer calculations of flux depression are based on escape probabilities. In particular, the method of Amouyal *et al.*⁵² is widely used. This method is carried out only for the equivalent cell; the neutron current is taken to be zero at the outer surface of this cell. However, diffusion theory is not assumed to hold within the fuel. Amouyal *et al.* calculate p_F as the probability that neutrons produced uniformly within the

fuel ultimately escape from the fuel. The expression obtained for a cylindrical fuel rod of radius R_0 (cm) is

$$\frac{1}{p_F} = 1 + (\Sigma_{aF}/\Sigma_{tF}) \{ A[1 + \alpha(\Sigma_{sF}/\Sigma_{tF}) + \beta(\Sigma_{aF}/\Sigma_{tF})^2] + R_0 \Sigma_{tF} \} \quad (1.24)$$

where

$$A = \frac{1 - p_{F0}}{p_{F0}} - R_0 \Sigma_{tF}$$

p_{F0} = probability that a neutron escapes without collision from an infinite cylinder of radius R and macroscopic cross section Σ_{tF} (given by Fig. 1.31)

Σ_{aF} = macroscopic absorption cross section of the fuel, cm^{-1}

Σ_{tF} = total macroscopic cross section of the fuel, cm^{-1}

Σ_{sF} = macroscopic scattering cross section of the fuel, cm^{-1}

α, β = functions of $R_0 \Sigma_{tF}$ plotted in Fig. 1.32.

It may be shown that $1/p_F$ is equivalent to F . Further details on this method are available from Lamarsh.⁵³

1.2.6 Distribution of Power Among Fuel, Moderator, and Structure

In addition to axial and radial power distributions, the thermal designer must also be concerned with the fraction of heat generated within the fuel elements. Fission energy is released in the form of kinetic energy of fission fragments, neutron kinetic energy, and gamma and beta rays. The very short range of fission fragments and beta particles ensures that this heat release will take place within the fuel elements.* The energy released during neutron thermalization is released to the moderator. The energy released on neutron capture is largely released within the fuel, but a portion of it is released within the reactor structure. The large mass of fuel produces a substantial self shielding, thus most gammas are captured within the fuel. Table 1.III quantitatively describes the distribution.

A total of 5 MeV of neutron kinetic energy is absorbed during thermalization. Hence, $\sim 2.5\%$ of the fission energy is definitely absorbed by the moderator. This can be taken advantage of in computing the heat that must be transferred by the fuel.

The portion of gamma and neutron energy absorbed by the moderator and core structure depends on the specific design of the reactor. In a typical

*Approximately 10% of the beta energy is converted to gamma energy as bremsstrahlung and should be treated as gamma energy in the division.

PWR design, somewhat less than 1% of the total power is absorbed within the thermal shield; most of that energy is released by the outer fuel elements. In the central region of the core, the gamma energy is roughly distributed among the core constituents proportionally to their mass. In a uniform lattice core, this leads to the conclusion that essentially all gammas in the central core region are captured by the fuel elements. In a pressure

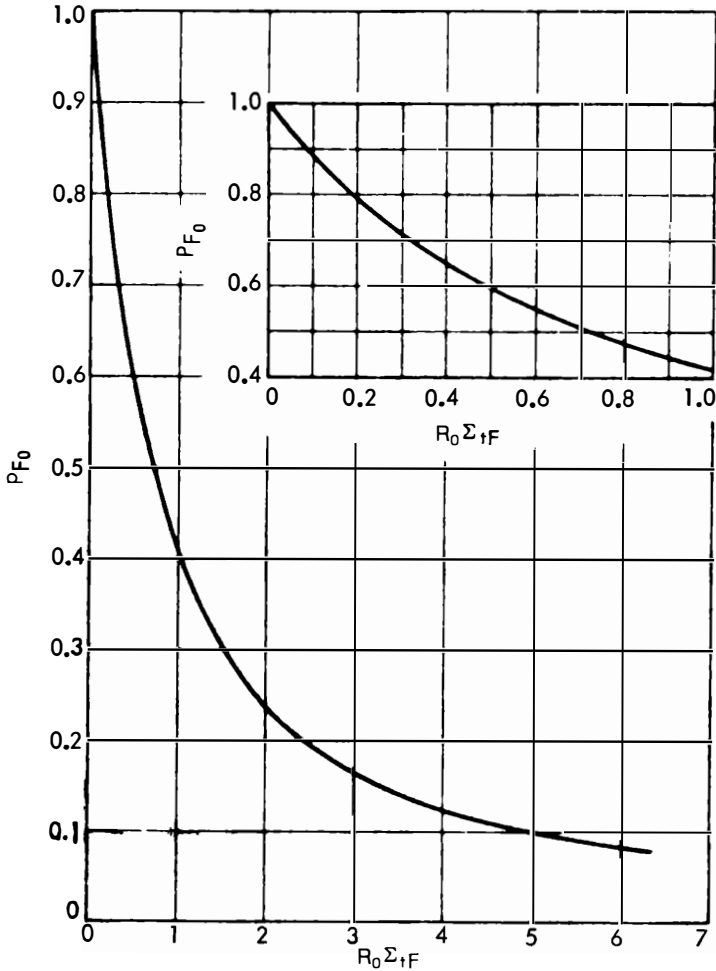


Fig. 1.31 The probability, p_F , that a neutron escapes without collision in an infinite cylinder of radius R_0 and macroscopic cross-section Σ_{tF} [from *J. Nucl. Energy*, 6, 79 (1957)].

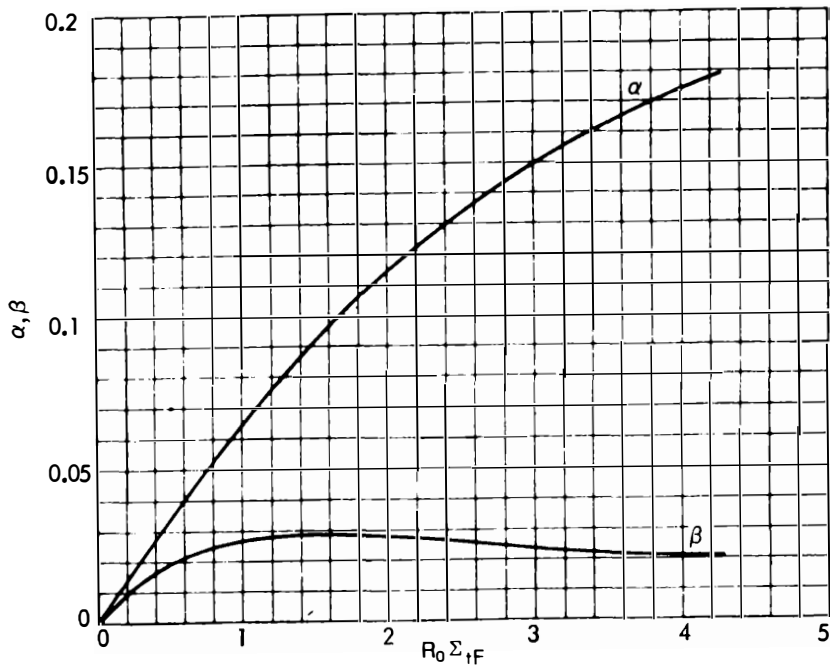


Fig. 1.32 Parameters α and β as a function of $R_0 \Sigma_{tF}$ [from *J. Nucl. Energy*, 6, 79 (1957)].

tube reactor, which has a larger mass of moderator and internal structure, appropriate adjustment should be made.

The energy released per fission somewhat depends on fissile material as well as on control material and reactor structure. Walker⁵⁴ measured the total energy release for various fissile materials in pressure tube reactors of the CANDU design. His results (Table 1.IV) indicate a value for ^{235}U fission that is $\sim 2\%$ higher than that listed in Table 1.III.

TABLE 1.III
Energy Breakdown (MeV/Fission) in a Water-Cooled Reactor

Receiver	Time		
	Prompt	Delayed	Total
Fuel element	Fission fragment KE 168	β 7	175
Dispersed among fuel,	Neutron KE 5	Neutron capture 4	
moderator, and structure	γ 7.5	γ 6	22.5
TOTAL	180.5	17	197.5

TABLE 1.IV
Total Energy* Released per Fission
in Reactors of CANDU Design

^{232}Th	$196.2 \pm 1.1 \text{ MeV}$
^{233}U	$199.7 \pm 1.1 \text{ MeV}$
^{235}U	$201.7 \pm 0.6 \text{ MeV}$
^{238}U	$208.5 \pm 1.1 \text{ MeV}$
^{239}Pu	$210.7 \pm 1.2 \text{ MeV}$
^{241}Pu	$213.8 \pm 1.0 \text{ MeV}$

*Includes decay of absorption products but not energy of neutrinos.

1.3 METHODS FOR ESTIMATION OF STEADY-STATE CORE POWER IN CONVENTIONAL PWRs

1.3.1 Pin-by-Pin Diffusion Computations

To analyze core behavior properly, an accurate prediction of the three-dimensional pin-by-pin distribution is needed. While neutron transport calculations provide the most accurate prediction, no practical methods are available for performing routine three-dimensional pin-by-pin transport computations. Most often, neutron transport methods are used to analyze the behavior of a small region of the core (e.g., a single fuel assembly) in two dimensions. The results of such computations are used as benchmarks from which correction factors for less precise methods can be determined.

PWR analyses rely on diffusion theory approximations. However, core-wide pin-by-pin analyses in three dimensions are still prohibitively expensive even when diffusion theory is used. The usual approach uses a synthesis of a two-dimensional pin-by-pin diffusion calculation in the x - y (horizontal) plane and a less detailed one-dimensional axial calculation. This approach assumes separability of the radial and axial flux shapes and hence that three-dimensional peaking can be predicted by a synthesis of two-dimensional models.

PWR diffusion calculations usually only consider two-neutron energy groups: fast and thermal. The first step in such calculations is the determination of the diffusion constants and cross sections for those energy groups by appropriately collapsing the flux weighted properties based on the neutron spectrum present. The group constants are calculated by such pin-cell codes as GAM⁵⁵ and THERMOS.⁵⁶

The second stage of the computations is the use of a two-dimensional diffusion code, such as PDQ,⁵⁷ to solve the two-group diffusion equation. For the fast neutron groups we have at an interior node

$$\begin{aligned}
& -\nabla D_1(x,y) \nabla \phi_1(x,y) + \Sigma_1(x,y)\phi_1(x,y) \\
& = \frac{1}{k_{\text{eff}}} [\nu \Sigma_{f_1}(x,y)\phi_1(x,y) + \nu \Sigma_{f_2}(x,y)\phi_2(x,y)]
\end{aligned} \tag{1.25}$$

For the thermal neutron group we have

$$\begin{aligned}
& -\nabla D_2(x,y)\nabla \phi_2(x,y) + \Sigma_2(x,y)\phi_2(x,y) \\
& = \Sigma_{s_1}(x,y)\phi_1(x,y)
\end{aligned} \tag{1.26}$$

where

k_{eff} = effective multiplication factor of reactor

D_1, D_2 = diffusion coefficients for fast and thermal neutrons, respectively

Σ_1, Σ_2 = macroscopic neutron absorption cross sections for fast and thermal groups, respectively

Σ_{s_1} = macroscopic cross section for scattering of fast neutrons into thermal energy group

$\Sigma_{f_1}, \Sigma_{f_2}$ = macroscopic fast and thermal fission cross sections, respectively

ν = average number of neutrons emitted per fission.

Since neutron diffusion is considered only in the x and y directions by Eqs. (1.25) and (1.26), a correction for leakage in the z direction is made by addition of a term depending on the z direction buckling.

Equations (1.25) and (1.26) are rewritten in finite difference form for each mesh point considered. The resulting set of linear equations is then usually solved by an inner and outer loop iteration scheme.^{58,59} The equations are first solved iteratively for the flux distribution by initially assuming a value for k_{eff} and solving for the fluxes on the right-hand sides of Eqs. (1.25) and (1.26). An improved value for k_{eff} can then be obtained using the definition of k_{eff} as the ratio of fission neutrons generated in two successive generations or, in this case, iterations. The revised estimate of k_{eff} is then:

$$k_{\text{eff}}^{(n+1)} = \frac{\int S_f^{(n)}(x,y) dx dy}{\int S_f^{(n-1)}(x,y) dx dy / (k_{\text{eff}}^n)} \tag{1.27}$$

where

$$S_f(x,y) = \nu \Sigma_{f_1}(x,y)\phi_1(x,y) + \nu \Sigma_{f_2}(x,y)\phi_2(x,y)$$

and $(n+1)$ and (n) refer to the $(n+1)$ and (n) iterations. The improved value of k_{eff} and initial set of fluxes is then used to generate a new flux distri-

bution. The process continues iteratively until successive values of k_{eff} converge within some preset limit.

The most important deficiency of pin-by-pin diffusion methods is that these methods fail to predict the flux correctly in the immediate vicinity of water holes or strong absorbers. To correct for this, pin-cell cross sections are usually adjusted by multiplicative terms called *g factors*. The factors that are used for water holes, burnable absorber rods, and pins immediately surrounding these locations adjust the diffusion theory results so that they provide a reasonable approximation of transport theory results for small core regions.

Most pin-by-pin diffusion models do not incorporate any thermal-hydraulic feedback since only a two-dimensional representation is used and, in contrast to BWRs, vapor volume fractions during normal operation are small. However, there are some modified versions of PDQ⁶⁰ that allow for thermal-hydraulic effects through tabular input, which supplies approximate corrections that depend on local power levels. It is possible to link thermal calculations directly to the detailed reactor physics of a pin-by-pin code, as in the THUNDER code,⁶¹ but this procedure has been considered too time-consuming for general use. This assessment may change as computer capabilities increase.

If the variation of power distribution over core life is desired, it is necessary to couple the diffusion calculations to burnup calculations that determine the changes in fuel composition. The power distribution and burnup calculations are then repeated in a stepwise manner over the length of the operating cycle. Due allowance must be made for changes in control rod position and in the concentration of dissolved boron.

1.3.2 Nodal Methods

1.3.2.1 Simple Nodal Methods

The necessity for incorporating thermal-hydraulic feedback into BWR neutronic calculations leads to the use of three-dimensional calculations for this reactor type. Since pin-by-pin calculations in three dimensions are, as we have seen, prohibitively time consuming, it is customary to obtain the overall core flux distribution based on a coarser subdivision of the core. In the radial direction, the BWR core is usually considered on an assembly-by-assembly basis. Axial segments on the order of 0.5 to 1 ft are common. Each three-dimensional segment is considered a node.

When such a coarse division of the core is used, the basic assumptions used for the diffusion of thermal neutrons are questionable since the diffusion length for a thermal neutron is considerably less than the distance between assembly centers. Further, the finite difference approximation of flux gradients is no longer valid. Accordingly, the basic equation for determination of the neutron flux distribution is based on the probability that a

neutron born in one location or node will be absorbed in another location or node.

In FLARE, the oldest nodal code,⁶² the neutron conservation equation is written for each node as

$$\phi_i = (k_{\infty i} / \lambda) \left[\sum_{m=1}^j W_{mi} \phi_m + \left(1 - \sum_{m=1}^j W_{mi} \right) \phi_i + \alpha_i \phi_i \right] \quad (1.28)$$

where

W_{mi} = probability that neutron born at node m is absorbed at node i
(transit probability)

$k_{\infty i}$ = infinite lattice multiplication factor at spatial node i

α_i = reflection parameter at node i

λ = eigenvalue

ϕ_i = thermal neutron flux at node i .

If three dimensions are being considered, the summation is taken over the six nearest neighbors. In a two-dimensional problem, the summation is taken over the assemblies at the four sides of node i . The reflection coefficient α_i has a nonzero value only when the assembly is at the core edge, and one or more adjacent nodes must then be omitted from the summation.

In revised versions of FLARE,⁶³ the transit probability, W_{mi} , which is composed of transport and diffusion parameters, is given by

$$W_{mi} = \frac{(1 - g')(M_m^2)^{1/2}}{2\Delta l_{mi}} + \frac{g'[(M_m^2)/k_{\infty}]}{(\Delta l_{mi})^2} \quad (1.29)$$

where

M_m^2 = migration area

Δl_{mi} = node interval

g' = mixing factor (input parameter).

Accurate results from a nodal code require that input parameters g' (different g' can be used for horizontal and vertical directions) and α be fitted so initial agreement is obtained between the nodal code and few-group calculations. If an appropriate regression formula for the effect of voids, burnup, etc., on k_{∞} is obtained, the nodal code can then be used to evaluate the effect of thermal conditions on flux and power. Somewhat better agreement can be obtained by considering coupling with diagonally adjacent nodes as is done in the TRILUX code.⁶⁴

Although both FLARE and TRILUX were originally devised for BWRs where void effects are of major importance, simple nodal codes were also adapted for PWR use where rapid power estimations were needed.

Chitkara and Weisman⁶⁵ used a similar approach for the rapid power estimation needed in fuel shuffling studies. These authors considered g' to be unity in Eq. (1.29). They also included the effect of diagonal assemblies by extending the summation of Eq. (1.28) to include these assemblies. In evaluating W_{mi} for diagonally placed assemblies, Eq. (1.29) was multiplied by an obliqueness parameter to which they assigned a value of 0.7

Chitkara and Weisman developed simplified relationships for P' the thermal power

$$P' = \phi_{th} [\sigma_5^{th} N_5 E_5 + \sigma_9^{th} (N_9 E_9 + N_1 E_1)] + r \phi_{th} [\sigma_5^f N_5 E_5 + \sigma_9^f (N_9 E_9 + N_1 E_1) + \sigma_8^f N_8 E_8] \quad (1.30)$$

where

ϕ_{th}	thermal flux at given node ratio of fast to thermal flux, assumed constant for purposes of power estimation
$\sigma_5^{th}, \sigma_9^{th}$	thermal fission cross sections of ^{235}U and ^{239}Pu , respectively
$\sigma_5^f, \sigma_9^f, \sigma_8^f$	fast fission cross sections of ^{235}U , ^{239}Pu , and ^{238}U , respectively
N_1, N_5, N_8, N_9	number densities of ^{241}Pu , ^{235}U , ^{238}U , and ^{239}Pu , respectively
E_1, E_5, E_8, E_9	fission energies liberated by ^{241}Pu , ^{235}U , ^{238}U , and ^{239}Pu , respectively.

For simplicity, we assume that $E_8 = E_5$ and $E_9 = 1.04 E_5$. The number densities of the various fissile isotopes are described in terms of N , the number density of ^{238}U and b , the degree of burnup. The degree of burnup is defined by

$$db = \phi_{th} \sigma_{a5}^{th} dt \quad (1.31)$$

where t equals time and σ_{a5}^{th} equals the thermal absorption cross section of ^{235}U .

At the end of a cycle, b_{fb} , the final degree of burnup is

$$b_{fb} = b_{in} + \sigma_{a5}^{th} \int_0^T \phi_{th} dt \quad (1.32)$$

where T = cycle length and b_{in} = degree of burnup at BOC. Chitkara and Weisman⁶⁵ concluded that b_{fb} could be approximated by

$$b_{fb} = b_{in} + T \sigma_{a5}^{th} \bar{\phi} \quad (1.33)$$

where

$$\bar{\phi} = \text{effective average thermal flux} \approx \phi_{in} \left(1 + \frac{1 + \phi_{in}}{3.0} \right) \phi_{avg}$$

ϕ_{in} = thermal flux at BOC

ϕ_{avg} = average thermal flux across entire core, or radial slice if only a two-dimensional calculation.

The number density of ^{235}U is then obtained from

$$N_5/N_8 = (N_5/N_8)^0 \exp(-b) \quad (1.34)$$

where $(N_5/N_8)^0$ is the initial concentration of ^{235}U .

The fissile plutonium concentration is given by

$$N_{9,1}/N_8 = C_R \{ (N_5/N_8)^0 [1 - \exp(-b)] (1 - 0.25b) \} \quad (1.35)$$

where C_R is the conversion parameter, which was found to be 0.43 for the San Onofre core.

To obtain accurate flux distributions with a nodal code, the user must carefully fit mixing parameters and reflection coefficients to achieve initial agreement with a multigroup calculation for some base loading of a core with the same configuration and power level. Some investigators⁶³ have used formal optimization procedures for this parameter determination.

The very rapid simple nodal codes are particularly well adapted to fuel management studies where a large number of fuel loading arrangements need to be examined. An interesting variant of this approach is the so-called "backward diffusion" approach of Chao and Hu.⁶⁶ Since the assembly power distribution is primarily determined by the reactivity, Chao and Hu begin with a desired power distribution. They then solve the diffusion equation backwardly to determine the corresponding core reactivity distribution. A loading pattern search subsequently matches the available assemblies to the predicted reactivity distribution.

When using a simple nodal code for global power distribution, the pin-by-pin power in the hot assembly may be obtained by superimposing the hot assembly flux distribution on the global power. This can be done by writing the finite difference approximation for the thermal or two-group neutron diffusion equations for each of the pins in the hot assembly. The boundary conditions can be either a zero flux gradient at the assembly edge or estimated neutron fluxes along the assembly edges obtained by interpolation between the values at the centers of neighboring nodes.

1.3.2.2 Advanced Nodal Diffusion Computations

Although simple nodal methods were originally used in PWR analysis only to obtain approximate power distributions rapidly, advanced nodal meth-

ods that are considerably more accurate are now available. These advanced methods, which directly and rapidly supply a three-dimensional flux distribution without the necessity of a radial and axial synthesis, have largely displaced the pin-by-pin diffusion model as the basic neutronic computation procedure for PWR design.

In common with the simple nodal coupling methods, the advanced nodal diffusion methods factor the core-wide pin power distribution into (1) a global (homogeneous) power distribution and (2) a local (heterogeneous) power distribution. The relative local power distribution is determined from multigroup transport theory calculations for a single assembly. These calculations, which generally assume a zero-leakage boundary condition, determine a set of pin factors that are subsequently superimposed on the global flux distribution. The transport calculations also provide the homogeneous assembly cross sections required as input by the global power calculations. CASMO^{67,68} and PHOENIX⁶⁹ are two of the well-known transport-based computer programs that provide appropriate group constants and pin factors. Since local power peaking around water slots or burnable absorbers is obtained from assembly transport calculations, the previously noted diffusion theory errors are not encountered.

To obtain the global power distribution, the core is divided into a series of nodes each of which is considered to be homogeneous. In the usual PWR practice, each assembly is broken into four subsections in the x - y plane and into 6- to 12-in. segments in the z (axial) dimension. Diffusion equations are written for both the thermal and fast group neutrons. However, unlike the simple nodal methods where only the average nodal flux is determined, the advanced nodal methods determine the parameters enabling both the average flux and distribution of the flux within the node to be calculated. This is done by determining the parameters of the equations representing the flux distributions in the individual nodes. When the static diffusion equation for every group G is integrated over node K of volume $\Delta X \Delta Y \Delta Z$, we obtain⁷⁰

$$\begin{aligned} \frac{1}{\Delta X} \left[J_{GX+}^K - J_{GX-}^K \right] + \frac{1}{\Delta Y} \left(J_{GY+}^K - J_{GY-}^K \right) + \frac{1}{\Delta Z} \left(J_{GZ+}^K - J_{GZ-}^K \right) \\ + \sum_G^{R,K} \bar{\Phi}_G^K = \bar{Q}_G^K \end{aligned} \quad (1.36)$$

where

- J_{GX+}^K the current on the right (plus x) face of the node, etc.
- $\bar{\Phi}_G^K$ the average flux in the node
- $\bar{Q}_G^K$ the average neutron source in the node
- $\Sigma_G^{R,K}$ = macroscopic absorption cross section for node.

The source term includes fission and scattering in, and therefore depends on the node flux. To establish additional relationships between the nodal

flux and currents on the nodal faces, the nodal diffusion equation is integrated over two dimensions at a time. This results in three one-dimensional equations, one for each coordinate. Thus, integrating over $\Delta Y \Delta Z$ results in the following equation dependent on the X coordinate:

$$\begin{aligned} \frac{d}{dx} \bar{J}_{GX}^K(X) + \sum_G^{R,K} \phi_{GX}^K(X) = \bar{Q}_{GX}^K(X) \\ - \frac{1}{\Delta Y} L_{GY}^K(X) - \frac{1}{\Delta Z} L_{GZ}^K(X) \end{aligned} \quad (1.37)$$

where the transverse averaged one-dimensional flux is given by

$$\phi_{GX}^K(X) = \frac{1}{\Delta Y \Delta Z} \int_{-\Delta Z/2}^{\Delta Z/2} dZ \int_{-\Delta Y/2}^{\Delta Y/2} dY \phi_G^K(XYZ) \quad (1.38)$$

The transverse leakages, $L_{GZ}^K(X)$ and $L_{GY}^K(X)$, are then

$$L_{GY}^K(X) = \frac{1}{\Delta Z} \int_{-\Delta Z/2}^{\Delta Z/2} dZ - D_G^K \frac{\partial}{\partial Y} \phi_G^K(X, Y, Z) \Big|_{Y=-\Delta Y/2}^{Y=\Delta Y/2} \quad (1.39)$$

$$L_{GZ}^K(X) = \frac{1}{\Delta Y} \int_{-\Delta Y/2}^{\Delta Y/2} dY - D_G^K \frac{\partial}{\partial Z} \phi_G^K(X, Y, Z) \Big|_{Z=-\Delta Z/2}^{Z=\Delta Z/2} \quad (1.40)$$

Integration of the one-dimensional flux and transverse leakages over the dependent dimension, and dividing by the dependent dimension, converts the transverse quantities to node-averaged values.

Since the foregoing equations involve the transverse leakages, a solution cannot be obtained without an analytical function providing the flux as a function of position. In the so-called "analytic nodal" method,⁷¹ each of the one-dimensional equations are solved analytically. In the "nodal expansion" method, typified by computer programs such as ANC,⁷² the one-dimensional solutions are represented by a polynomial expansion.

The single-node solutions must be connected through interface boundary conditions. The boundary conditions imposed can either require interface neutron flux continuity or interface neutron current continuity. The resulting equations are generally termed *nodal coupling equations*.

The simultaneous solution of the nodal equations provides the overall flux and power distribution in the reactor core. To recover the pin powers, the overall distribution equations for the assembly of interest are superimposed on the pin factors determined by the transport calculations for a single assembly. The resulting powers are found to have an accuracy that is comparable or superior to the results from pin-by-pin diffusion models.⁷³ The major advantage lies in the fact that they are 30 to 100 times more computationally efficient⁷³ and hence can easily provide three-dimensional results. This ability is particularly important for the newer fuel assembly designs containing axial blankets and/or part length burnable absorber rods.

1.3.3 Effective Fast Group Methods

In its simple form, the effective fast group approach, or *1.5-group coarse-mesh approach*, is an alternative to the simple nodal methods. The 1.5-group method has the advantage that it does not require empirical mixing parameters. Although much of the initial impetus came from BWR design⁷⁴ the approach applies equally well to the PWR.⁷⁵

In the simple effective fast group approach the fast group diffusion equation is solved using finite difference equations. The thermal flux is then calculated assuming zero buckling in the thermal group. We assume that thermal group difference equations are not applicable since the 16- to 24-cm nodes are several thermal mean-free-paths apart. Therefore, thermal flux is calculated as if the mesh size were infinite for each node, and the thermal leakage from each node is taken as zero. In a two-dimensional calculation where only four adjacent cells are considered, fast flux ϕ_0^f at cell 0 is obtained from

$$\phi_0^f = \frac{S_0 V_0 + \sum_{k=1}^4 \frac{\bar{D}_k A_k}{l_k} \phi_k^f}{\sum_0^R V_0 + \sum_{k=1}^4 \frac{\bar{D}_k A_k}{l_k}} \quad (1.41)$$

where

S_0 = neutron source in cell 0

V_0 = volume associated with cell 0

l_k = distance between mesh point k and mesh point 0

A_k = area of the boundary separating mesh point k and mesh point 0

ϕ_k^f = fast flux at mesh point k

Σ_0^R = fast group removal cross section at 0.

Here, $\bar{D}_k$ is an effective diffusion coefficient defined by

$$\bar{D}_k = \frac{D_0 D_k (\delta_0 + \delta_k)}{D_0 \delta_k + D_k \delta_0} \quad (1.42)$$

where D_i = diffusion coefficient for fast group at i and δ_i = width of cell i .

Fast flux is obtained from Eq. (1.41) by using standard inner and outer iteration methods with fission source updates after each iteration. Fission source S_0 is calculated by

$$S_0 = (1 / k_{\text{eff}}) (\nu \Sigma_{f0} \phi_0^f + \nu \Sigma_{\text{th}0} \phi_0^{\text{th}}) \quad (1.43)$$

where $\nu \Sigma_{f0}$, $\nu \Sigma_{\text{th}0}$ = fast and thermal fission source cross sections, respectively, and ϕ_0^{th} = thermal flux at cell 0. Thermal flux is obtained from

$$\Sigma_{th0}\phi_0^{th} = \Sigma_{s0}\phi_0^f \quad (1.44)$$

where Σ_{s0} is the transfer scattering cross section from fast to thermal group at location 0.

Because of large mesh size spaces, the fast and thermal fluxes calculated from Eqs. (1.41) and (1.44) must be corrected to account for increases or decreases near the edge of the assembly. The average fast flux $\bar{\phi}$ is expressed as

$$\bar{\phi} = A^i \phi_0^f + \frac{1 - A^i}{4} \sum_{j=1}^4 \phi_j^i \quad (1.45)$$

where ϕ_j^i = flux at the interface between nodes i and j , and A^i = relative weight factor on the point flux of node i . Borreson⁷⁴ found A^i to be in the range of 0.7 to 0.76.

In the 1.5-group approach, the outside boundary condition may be treated by including reflector nodes at the core boundary. The flux is set equal to zero at an appropriate distance into the reflector. The determination of reflection coefficients can thus be avoided.

Stout and Robinson⁷⁵ compared 1.5-group calculations of the assembly average power distribution to that obtained from a multigroup fine-mesh calculation. They found a maximum error slightly in excess of 6%. In this calculation, all number densities were known. They also found that, by combining the 1.5-group approach with mathematical programming techniques, optimal fuel management policies were readily determined.

More recently, the effective fast group approach has been combined with some of the concepts from the advanced nodal methods. In the NOVA program,⁷⁶ the source term in Eq. (1.41) is rewritten in terms of k^* , which differs from the normal definition of the multiplication factor, k_{∞} , in that it includes the thermal leakage effect to the first order. After the fast neutron flux has been obtained, the thermal flux obtained from Eq. (1.44) is corrected on a node-by-node basis for higher order effects. An analytical function is then derived from the three-dimensional flux distribution in each node by imposing the requirements that (1) the volume average of the distribution function equal the average nodal flux from the diffusion equation and (2) fluxes at the nodal interfaces must be continuous. This approach is considerably simpler than the usual advanced nodal code and is substantially faster.

Chao and Penkrot⁷⁷ have noted that much more rapidly running versions of the effective fast group approach can be obtained by making use of what they call *diffusive homogeneity*. For PWRs, the variation of the fast neutron diffusion coefficient is small and it may be assumed to be homogeneous throughout the core. They, therefore, rewrite the basic diffusion equation for a given node as

$$-\bar{D}\nabla^2\phi_1 + \bar{\Sigma}_a\phi_1 = \Sigma_f^*\phi_1 \quad (1.46)$$

where

$\bar{D}$ = reference diffusion coefficient for fast flux

$\bar{\Sigma}_a$ = reference absorption cross section

Σ_f^* = effective fission operator.

The effective fission operator provides the source term and absorbs the differences due to the deviation of D and Σ_a from $\bar{D}$ and $\bar{\Sigma}_a$. Since $\bar{D}$ and $\bar{\Sigma}_a$ are considered invariant, the diffusion equation can be written as

$$\phi_1 = [\mathbf{G}](\Sigma_f^* \phi_1) \quad (1.47)$$

where $[\mathbf{G}]$ is a square matrix relating each node to the other nodes. This matrix depends only on $\bar{D}$ and the core geometry and the boundary conditions. Hence, the matrix $[\mathbf{G}]$ can be precalculated for a given core geometry and stored. That being the case, the conventional inner and outer scheme required for the solution of the diffusion equation is replaced by a single iteration loop. Computer programs based on this approach are sufficiently fast running so that they can be used for on-line core monitoring.

1.4 POWER GENERATION AND DISTRIBUTION IN PRESSURE TUBE-TYPE CORES

1.4.1 Overall Power Distribution

In contrast to light-water-moderated reactors, the fuel in heavy water or graphite-moderated reactors is generally not homogeneously distributed throughout the moderator. As described in Sec. 1.1.1, the fuel elements in most designs are grouped together in clusters within pressure-containing tubes dispersed throughout the moderator. Nevertheless, an approximation of the overall distribution of thermal neutron flux, which is adequate for thermal design purposes, can be obtained by considering the moderator and fuel as homogeneous. However, we cannot normally consider these reactors to be bare since they are usually surrounded by effective radial reflectors. The effects of such a reflector can be approximately evaluated by considering two neutron energy groups—thermal and fast.

For a cylinder of infinite length, thermal flux within the core of a uniformly enriched reactor is given by⁷⁸

$$\phi_t = AJ_0(\mu'r) + CI_0(\gamma r)$$

where A and C are constants, and μ' and γ are properties of the reactor core materials (see Ref. 50 for further discussion). For thin reflectors, the first term is dominant and flux distribution differs only slightly from that

of an unreflected core. The change in flux pattern produced by a reflector around an infinite cylinder is illustrated in Fig. 1.33.

A somewhat cruder approximation of the behavior of a reflected core can be obtained by considering all neutrons to be thermal. When this is done, the resulting flux distribution equations have the same form as those for a bare core, but the overall dimensions are replaced by effective dimensions. For example, the flux of an unperturbed cylindrical core is approximated by

$$\phi = \phi_{\max} J_0(2.405r/R_e) \left(\cos \frac{\pi z}{L_e} \right) \quad (1.48)$$

where R_e = effective radius of core allowing for radial reflector and L_e = effective length of core allowing for axial reflectors. Equation (1.48) does not predict the slight rise in core flux that occurs adjacent to the reflector. However, it can provide (Fig. 1.31) a reasonable estimate of flux throughout most of the core.

In the common cylindrical core, the large amount of structural material at the ends limits the effectiveness of any axial reflector. A cosine distribution with $L_e \approx L$ is thus often a very good approximation of the axial power variation in a uniformly loaded, unperturbed core.

The large diffusion length of neutrons in heavy water permits an accurate determination of core power and overall flux shape using coarse-mesh

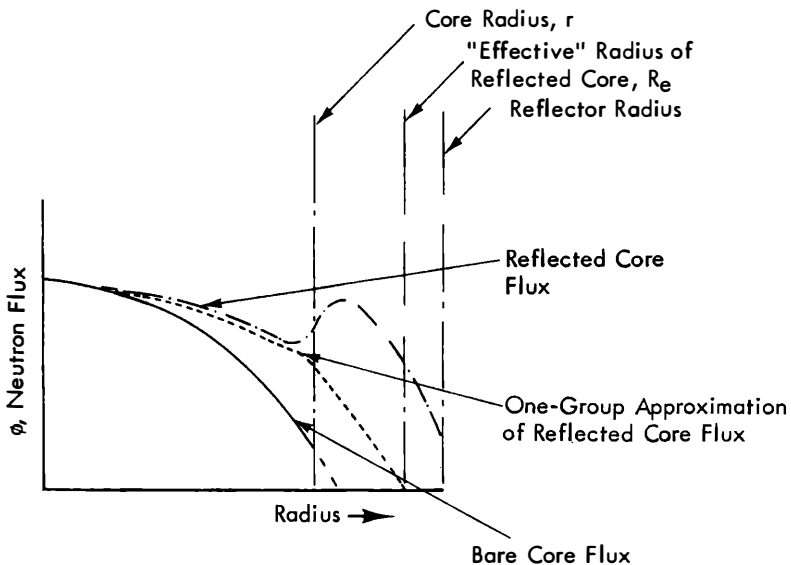


Fig. 1.33 Thermal neutron flux in bare and reflected cylindrical reactors.

finite difference diffusion computations.⁷⁹ The core is modeled by dividing it into a series of nodes by means of a grid of mesh planes. Cross sections for each mesh volume are obtained from lattice cell cross sections of the fuel bundle and moderator contained within the cell at an appropriate irradiation. Incremental cross sections are added to represent any other material that may be present in the mesh volume. Sufficiently accurate results are usually obtained with two-neutron energy group calculations using mesh spacings that are typically one lattice pitch by one lattice pitch by one bundle length. The large spacing allows three-dimensional calculations to be readily obtained. Some computer codes allow variable lattice spacing so that smaller mesh sizes can be used where desired.

The overall core reactivity and spatial flux distribution in many CANDU reactors are regulated by zone control units. These units are composed of vertical tubes containing light water that run through the calandria perpendicular to the fuel channels. Core reactivity and local flux are controlled by varying the level of light water in the individual tubes.

The control rods for CANDU reactors are usually divided into two groups. The *shutoff rods*, often a cadmium annulus between two stainless-steel tubes, are usually fully withdrawn from the core. They are inserted only for reactor shutdown. The *adjuster rods* provide additional power shaping and compensate for relatively short-term reactivity changes such as those due to xenon transients. In some designs, the adjuster rods are composed of a number of bundles containing Zircaloy tubes filled with cobalt.

1.4.2 Power Generation Within Fuel Clusters

The distance between rods within the pressure tube is generally orders of magnitude smaller than that between pressure tubes, resulting in negligible moderation within the pressure tube in comparison to that obtained between the tubes. As a first approximation, we therefore can ignore the area within the pressure tube as a source of thermal neutrons and approximate the thermal diffusion equation for thermal neutrons within the pressure tube by a modification of Eq. (1.14). Here we have

$$D_{av}\nabla^2\phi - \Sigma_{av}\phi = 0 \quad (1.49)$$

where D_{av} and Σ_{av} , respectively, represent the diffusion coefficient and macroscopic absorption coefficient based on homogenizing material within the pressure tube. From the development of Sec. 1.2.5, we see that this leads to an $I_0(R)$ flux distribution across the fuel cluster. Figure 1.34 illustrates this behavior in fuel clusters of the heavy-water-moderated and -cooled CVTR.⁸⁰

Since the diffusion coefficient, D_{av} , equals the reciprocal of $3\Sigma_{tr}$, the value of D_{av} for use in Eq. (1.49) is best obtained via the sum of the macroscopic transport cross sections for the elements within the fuel cluster.

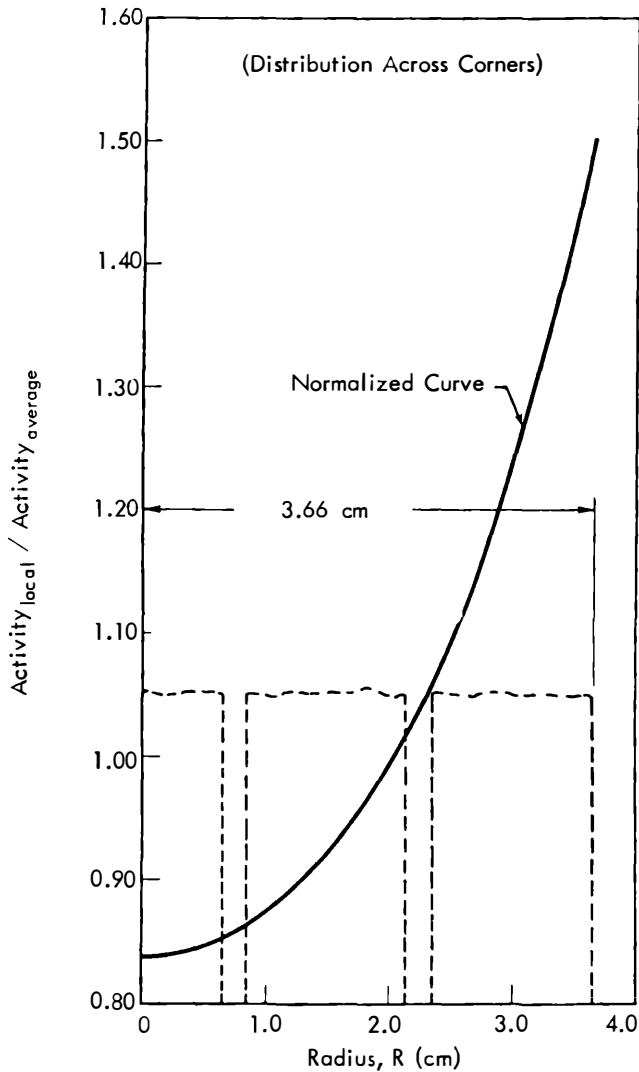


Fig. 1.34 Flux distribution across radial axis of CVTR assembly (from Ref. 80).

The $I_0(R)$ flux distribution within the fuel cluster is clearly only a rough approximation. Highly accurate values of the flux and power within the cluster may be obtained with fine-mesh computations based on neutron transport theory. Less accurate, but acceptable, flux and power distributions can be obtained by fine-mesh multigroup diffusion calculations.

Flux depression within the fuel assembly, often called the *fine flux dip*, is superimposed on the normal radial flux variation across the reactor as a whole. We can then write

$$F_R^N = F_G^N \times F_{R_0}^N \quad (1.50)$$

where

$$F_G^N = \frac{\text{Mean flux in the hot fuel rod}}{\text{Mean flux in the hot fuel cluster}}$$

$$F_{R_0}^N = \frac{\text{Mean flux in the hot fuel cluster}}{\text{Mean flux in the average fuel cluster}}$$

In designing the CVTR reactor, F_G^N was assigned a value⁸⁰ of 1.25 (Fig. 1.34).

Thermal flux within a particular fuel rod will vary with the position of the rods in the cluster and, as we proceed around a fuel rod, the value of R or the radial distance from the pressure tube center varies. In addition, for any tube not in the center of the core, there is an overall flux pattern effect on flux distribution within the cluster. This can be significant in a small core such as that used in the CVTR.⁸⁰ Thus, for any rod, we can show that

$$\phi = f(r, \theta) \quad (1.51)$$

To simplify temperature distribution calculation, distribution for a given rod is often approximated by a function of the form

$$\phi = C_1 + C_2 r \cos \theta \quad (1.52)$$

The approximate function is made to fit actual distribution at the center of the rod and at the point on the circumference with the peak heat flux.⁸⁰ In a large core (CANDU designs), variation of the overall flux across a single pressure tube is generally small and angular power variation is of lesser importance.

1.4.3 Effect of Refueling Procedures

As previously noted, the CANDU reactor is the major PWR pressure tube reactor type of commercial interest. In this design, the large cylindrical calandria is usually placed with its axis horizontal and is pierced by a large number of once-through horizontal pressure tubes. Since these reactors are fueled with natural uranium, maximum burnups are low (below 10,000 MWd/1000 kgU) and frequent refueling is necessary. Excessive downtime can be avoided by using on-power refueling.

Short fuel assemblies are used so that each channel contains a number of assemblies. When it is desired to add fuel to a channel, specially designed refueling machines are connected to each end of the channel. Fresh fuel assemblies are added at one end of the channel, while the refueling machine removes burned assemblies at the other end. By always adding fuel at the same end of a given channel, the fuel assemblies gradually move across the

reactor. Thus, there is a significant burnup gradient along the axis of a given channel and, hence, axial power generation will not follow a simple cosine distribution. To prevent a power tilt in one direction, half the pressure tubes are loaded from one end of the reactor, while the others are loaded from the opposite end. Adjacent tubes are always loaded in opposite directions.

At BOL, a fuel assembly will be at the edge of the core and generate a low power. Later in life, it will see the high flux in the center of the reactor and operate at its maximum power. At EOL, it will again be near the edge of the core and at low power. A typical fuel assembly operating history is shown in Fig. 1.35.

A realistic estimate of axial power distribution requires that variation in fuel burnup be considered. Since the burnup distribution is not axially continuous, axial power in a given channel will show a stepwise variation.

The objectives of the usual CANDU refueling procedures⁸¹ are to achieve an optimal radial power distribution and to minimize refueling machine relocations. To achieve these results using natural uranium as the fuel, the core is divided into radial zones (usually two), each of which is characterized by an exit fuel burnup and a bundle shift scheme. The bundle shift scheme is designed to insert the maximum number of fresh fuel assemblies into a given channel during refueling without causing excessive power peaking or reactor control problems. In the central region of a large core with channels containing 12 bundles, the typical exit burnup would be set slightly in excess of 7000 MWd/tonne and 8 bundles would be added at a time. The size of the inner region is set so as to keep the maximum flux-to-average flux ratio within a prescribed limit. In the outer region, exit burnup would be of the order of 6000 MWd/tonne and 4 fresh bundles would be added at a time.

If a CANDU reactor is to be fueled using fuel slightly enriched with ^{235}U or Pu, a revised refueling procedure is required to prevent axial flux distortions and reductions in the reactivity worth of control and safety rods. The method proposed for slightly enriched fuel⁸¹ is the so-called “check-

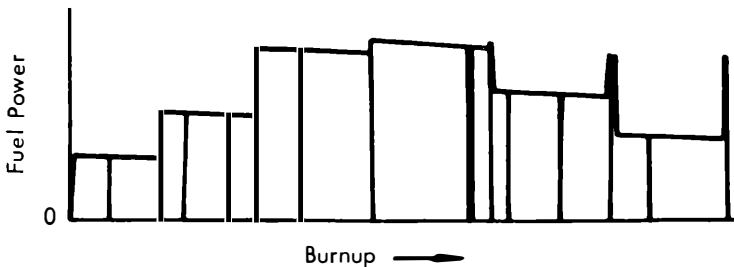


Fig. 1.35 Typical operating history of CANDU fuel assembly [from *J. Nucl. Appl. Tech.*, 9, 145 (1970)].

erboard" fuel management scheme (CFS). In CFS, the core is divided into three radial zones. In the central zone, a checkerboard arrangement of bundle shifts is used. For example, in a group of four adjacent channels, two would undergo a two-bundle shift and two would undergo a six-bundle shift. The two outer core zones would have a constant bundle shift throughout each zone.

1.5 TRANSIENT POWER GENERATION AND DISTRIBUTION

Reactor power distributions during transients are computed via an elaboration of steady-state diffusion theory. In the most rigorous approach, the neutron diffusion equations [Eqs. (1.25) and (1.26)] for each control volume are placed in the transient form by the addition of terms for neutron accumulation and emission of delayed neutrons. These equations must be coupled to computations of xenon, samarium, and delayed neutron precursor concentration changes. A short time step is assumed and the equations are solved for the neutron flux, nuclide concentrations, and power distribution at the end of the time interval. Alternatively, simplified approaches, such as those based on the point kinetics equation, can be used. These approaches are discussed in greater detail in Sec. 6.2.2 where accident analysis techniques are described. In the present section, the discussion is confined to the problems introduced by load changes and heat generation after shutdown.

1.5.1 Power Distribution Following a Load Change

As previously noted, in a chemical-shim-controlled core, the control rods are nearly fully withdrawn at full power. However, if the plant is to be load following, rods would normally be inserted during part-load operation. Improper positioning of these rods for part-load operation can lead to an unacceptably high axial power peaking on return to full power.

When control rods are partially inserted, there is a spatial change in fission product distribution. The distribution of ^{135}Xe is of particular importance because of its high yield and cross section (3×10^6 b). Randall and St. John⁸² recognized this problem and pointed out that changes in xenon distribution following load changes can result in significant power shifts. More recent studies confirm this viewpoint.⁵⁰

If all the reactivity change required for low-power operation is compensated for by rod motion, a deep insertion of the rods is required. This causes the flux to peak in the bottom half of the core and leads to reduced concentrations of ^{135}Xe and its precursor, ^{135}I , in the top half of the core. On return to full power by rod withdrawal, the initial ^{135}Xe concentration would largely be determined by the concentration of ^{135}I set at low-power

operation. The power now peaks in the upper portion of the core because of the lower ^{135}Xe concentration there. This behavior, illustrated in Fig. 1.36, results in a higher axial peaking factor than that obtained under steady-state conditions.

The concept of *constant axial offset control* has been used to avoid increased axial peaking due to xenon transients.⁴⁹ The axial offset (AO) can be defined as

$$\text{AO} = \frac{P'_T - P'_B}{P'_T + P'_B} \quad (1.53)$$

where P'_T = total power generated in the upper half of the core and P'_B = total power generated in the bottom half of the core.

When a core operates at full power with all rods withdrawn, there is generally an axial offset of somewhere between 0.0 and -0.1 (between 0.0 and -0.05 on all but the first cycle). Axial offset varies slowly with lifetime. The equilibrium value with rods withdrawn at any given point in life is called the "target" axial offset. It has been found that if AO were required to remain at the target value through all load changes, then on return to full power the axial peaking factor is not significantly changed.

If all the changes in reactivity occasioned by a load reduction were compensated for by boron addition, there would be an upward shift in core

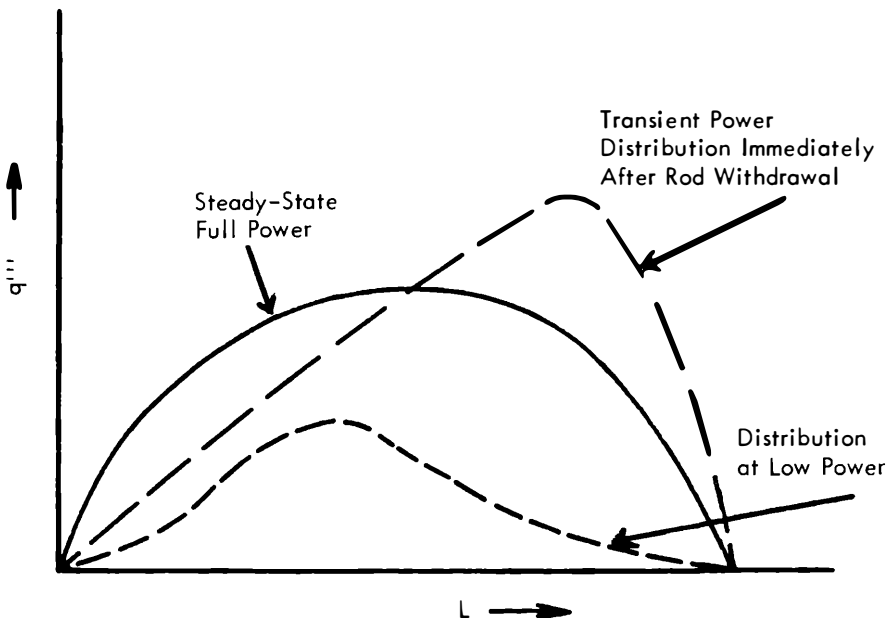


Fig. 1.36 Axial power shifts after control rod motion.

power; i.e., AO would become positive. This shift arises because of the local negative moderator coefficient effect and the larger temperature change near the top of the core compared to that in the bottom portion. Control rods are inserted at a level sufficient to prevent this power shift, and any additional reactivity reduction is achieved by the addition of boric acid. By using a combination of rod motion and boric acid concentration changes, load changes on the order of 0.3%/min can be accommodated. This rate of power change is sufficient to handle the daily load cycle to which a plant may need to respond.

Axial offset can be readily monitored by the reactor operator. In one scheme, ionization chambers that run the full length of the core are placed at four locations around the core circumference. Each chamber is split into an upper and lower portion and separate signals obtained from each section. Electronic comparison of these signals allows the axial offset to be read directly by the operator.

The concept of constant axial offset, which minimizes changes in power shape during core lifetime, is similar in some respects to the Haling principle.⁸³ This principle has been widely used for setting the optimum axial flux shape in BWRs. Haling states that rods compensating for reactivity changes with lifetime should be programmed to keep the axial power shape invariant over the in-core life of the fuel.

Although constant axial offset control, which will typically restrict axial offsets to between -12 and $+3\%$, allows peaking limits to be satisfied, operation outside these limits can be beneficial. Following a reactor trip or shutdown of short duration the rate at which full power can be attained is significantly restricted by tight offset limits. To alleviate this problem, most plants now use *relaxed axial offset control*.⁸⁴ In this approach, it is recognized that although F_Q^N increases with decreasing power, the power decreases faster than F_Q^N . This fact, together with additional margin during transients disclosed by more sophisticated analytical methods, means that wider AO limits are permissible at reduced power levels. In most cases, a target AO is established but the actual axial offset limits are now specified by a figure similar to Fig. 1.37. Since much wider axial offset limits are allowed at low power, operational problems are simplified. However, good operational practice calls for the return to the target axial offset as soon as practicable.

The AO limits are determined by considering the power distributions resulting from rod configurations producing a given axial offset. Values of $F_Q^N(z)$ are computed for various power levels. An axial offset is unacceptable at a particular power level if it produces a peak power at any elevation that is in excess of the limiting peak power versus elevation curve. As seen in Fig. 1.34, positive axial offsets (flux skewed to top of core) are more tightly restricted than negative offsets. This is caused by the CHF limit, which is more difficult to meet in the upper portion of the core where coolant enthalpies are higher.

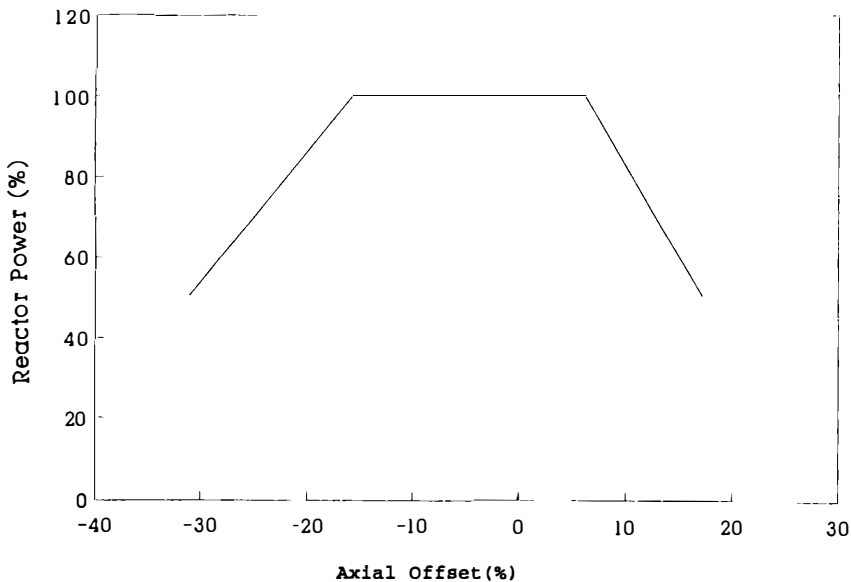


Fig. 1.37 Typical relationship between maximum allowable power and axial offset.

Limitations on the rate at which boric acid can be added make it difficult for axial offset limitations to be maintained using full-length rods if a rapid (e.g., 5%/min) load change must be accommodated. Some plants have been equipped with “part-length” control rods in an effort to achieve the rapid load following required when a plant is part of the system’s spinning reserve. A part-length rod contains poison material only in the lower portion of the control elements and an inert material, such as Al_2O_3 , in the upper portion. If these rods are deeply inserted, they can depress the flux in the lower portion of the core and, thus, produce a more acceptable axial offset. Although it has been possible to meet the peak axial requirements for steady-state operation, such operation can lead to high local stresses on fuel cladding. Such stresses can be calculated as being in excess of those stress levels capable of causing cladding failure by stress corrosion cracking in the presence of iodine.

Little use is now made of part-length rods and rapid load follow is more often accomplished by means of “gray” control rods.^{85,86} A standard or “black” control rod cluster in a 17×17 rod assembly will typically contain 24 rods of highly absorbing material (i.e., Ag-In-Cd, boron carbide). In the “Mode G” approach,^{85,86} a gray cluster would typically contain only 8 highly absorbing rods but also 16 stainless-steel rods. Since the reactivity worth of such a cluster is substantially reduced, its motion will have a smaller impact on the power distribution. A large (1300-MWth) core would

typically have 28 gray clusters arranged in four banks. Typically, an additional 25 highly absorbing rod clusters would also be present. Seventeen of these rod clusters would usually be used for shut down and the other 8 used as a reactivity control bank (bank R).

To assure that the gray rods are available for unexpected load increases from a scheduled power level, their position is determined solely by the power level. These banks do not react to reactivity changes such as those due to xenon poisoning. These changes are compensated for by the motion of bank R. However, the maneuvering range of this bank is restricted to a narrow band at the top of the core. At this elevation small motions of bank R have little effect on the power distribution.

The gray banks are inserted sequentially in the core as the power level is reduced. However, the insertion pattern calls for the overlapping of banks so that the next bank is partially inserted before the previous bank is fully inserted. Because the gray banks produce less radial distortion than insertion of a highly absorbing bank, wider axial offset limits may be used and rapid load follow is possible.

In some new plants, the ex-core two-region ionization chambers are replaced by units with a number of axial regions. The readings of these units, together with accurate rod position information, allow the power distribution at a given time to be reconstructed. The allowable operating range then depends on the actual power distribution existing in the core and is wider than that allowed by the offset limits set on the basis of the most adverse conditions.

Load-follow operation is possible using all highly absorbing rods providing the control rod banks are properly configured. In the so-called "MSHIM" approach⁸⁷ a fraction of the usual control rods is regrouped to create a set of three relatively low worth banks (M banks) and one high-worth bank (A/O bank). The remainder of the rods are designated as shut-down rods. The M banks, which are 75% inserted at full power, are inserted or withdrawn to compensate for the reactivity change with power. If the M bank movement produces an axial offset outside of the allowable range, the A/O bank (normally in the upper 15% of the core) is adjusted to compensate for this. Additional M bank movement may then be required to counteract the reactivity change due to the A/O bank movement.

Both the MSHIM and Mode G load-following approaches require the continuous insertion of a high worth control rod bank. This causes local burnup shadowing and some asymmetry in the steady-state power shape. In addition, withdrawal of this bank to meet a rapid increase in power may distort the power distribution. Park and Christenson⁸⁸ have proposed a revised approach that avoids this difficulty. In their "LG" approach a light-gray bank and moderator temperature are used to control negative axial offset, while a high-worth bank is used to control positive AO. Insertion of the high worth bank is not required at steady-state full power. This bank

is needed only when there is a pronounced skew of the power toward the upper portion of the core.

Load following complicates plant safeguards studies. A reliable accident analysis begins with power distribution in the core at the onset of the accident. Since an accident could occur at any time and since the exact power distribution is a function of the power-time history of the core, there are a very large number of power distributions possible at the onset of an accident in a load-following plant. It becomes impractical to analyze each possible accident for all possible power distributions. One way of avoiding repetitive accident analyses is to construct an envelope of peak local power density (kW/ft) versus core height.⁵⁰ An attempt is made to determine peak power levels under all anticipated conditions by analytically simulating all anticipated load-following behavior over core lifetime. From these many power distribution studies, the worst peak power at each axial position is selected. The synthesis method, described in Sec. 1.2.4, is often used for this determination. These powers, which form an upper envelope of all possible powers, are then used to describe core condition at the beginning of the accident. When accident analysis shows that the core design is safe for these conditions, then the core is clearly safe for the less severe conditions that would actually exist. This approach is particularly useful in LOCA analyses.

1.5.2 Power Generation During Shutdown

1.5.2.1 Fission Power Generation

The rate of power decrease during a reactor shutdown is of concern to the thermal-hydraulic designer, since it affects fuel temperature during accident situations. We consider the situation where the reactor is shut down after operating at steady state for an appreciable period. During steady-state operation, the effective multiplication factor, k_{eff} , must be unity. For startup, it must be greater than unity. For shutdown, it must be less than unity. It is useful to define an excess multiplication factor k_{ex} , where

$$k_{\text{ex}} = k_{\text{eff}} - 1 \quad (1.54)$$

Thus, k_{ex} is positive for a supercritical reactor and negative for a subcritical reactor. If all neutrons can be considered emitted promptly at the time of fission, the time behavior of the bare thermal reactor is given by

$$\phi(\bar{r}, t) = \phi_0 \exp(k_{\text{ex}} t / l) \quad (1.55)$$

where

ϕ_0 = steady-state neutron flux at $\bar{r}$

t = time since the power was suddenly changed by k_{ex}

l = mean lifetime of thermal neutrons.

Equation (1.55) can be considered valid immediately after the change. For longer periods, the effects of delayed neutron groups must be consid-

ered. If the delayed neutrons are approximated by a single group with a decay constant λ equal to the weighted average for the five groups, for small reactivity changes we obtain

$$\phi = \phi_0 \left\{ \frac{\beta}{\beta - \rho'} \exp\left(\frac{\lambda \rho' t}{\beta - \rho'}\right) - \frac{\rho'}{\beta - \rho'} \exp\left[-\frac{(\beta - \rho')t}{l}\right] \right\} \quad (1.56)$$

where β is the total fraction of delayed neutrons and ρ' the reactivity, is defined as

$$\rho' = \frac{k_{\text{ex}}}{k_{\text{eff}}} \quad (1.57)$$

The equation is valid only when $(\beta - \rho')$ is positive. Since l is on the order of magnitude of 10^{-3} s and λ is $\sim 0.08 \text{ s}^{-1}$, the flux during shutdown is the sum of two positive terms (ρ' negative), one of which decreases much more rapidly than the other. The flux and, hence, power initially decrease rapidly and then fall off slowly in accordance with the first term within the bracket; subsequent slow decay is due to the effect of delayed neutrons.

The usual reactor shutdown is brought about by large reductions in reactivity. For this condition and considering all five groups of delayed neutrons, the neutron flux is closely approximated by

$$\begin{aligned} \phi = \phi_0 [& A_1 \exp(-t\lambda_1) + A_2 \exp(-t\lambda_2) + \dots + A_5 \\ & \times \exp(-t\lambda_5) + A_6 \exp(-t/l)] \end{aligned} \quad (1.58)$$

Here, λ are the decay constants for the five groups of delayed neutrons. For large reactivity decreases, the rate of flux decay is independent of the reactivity change. Noting the very small value of l and examining the values of λ in Table 1.V (Ref. 89), we see that initially there is a rapid decrease in flux due to the last term, but after a short time, all terms become negligible except those with the smallest value of λ . Flux then decays exponentially with a period corresponding to the maximum $1/\lambda$, or ~ 80 s.

Based on the foregoing, a good approximation of the fission energy power following a large reactivity decrease is a sudden decrease (prompt jump) followed by a relatively slow exponential decay. The residual power of a light-water-moderated reactor shutdown, following sustained opera-

TABLE 1.V
Delayed Neutrons in Thermal Fission of ^{235}U

Mean Life	Decay Constant [λ (s^{-1})]	Fraction (β_i)
0.62	1.61	0.00084
2.19	0.456	0.0024
6.50	0.151	0.0021
31.7	0.0315	0.0017
80.2	0.0124	0.00026

tion at constant power, can be approximated over the time period during which decay heat is important by

$$P'/P'_0 = 0.15 \exp(-0.1 t) \quad (1.59)$$

where t is in seconds.

Since the neutron lifetime on which the decay constant depends is largely determined by the reactor moderator, Eq. (1.59) should only be used for LWRs. For heavy-water-moderated reactors, a decay constant of 0.06 should replace the 0.1. Also note that Eq. (1.59) does not apply to a plutonium-fueled core. The delayed neutron fraction for ^{239}Pu is only $\sim 0.21\%$ and, hence, the residual power will be about one-third that of ^{235}U fuel.

1.5.2.2 Decay Heat

In addition to residual fission energy, there are two other major heat sources: (1) fission product radioactive decay and (2) neutron capture product radioactive decay. Each of these sources must be considered.

Perkins and King,⁹⁰ Smith,⁹¹ and Shure,⁹² among others, considered fission product decay heat generation. The 1973 design standard⁹³ was based on Shure's work.⁹² Decay heat generation can be determined from the curves of Fig. 1.38 for reactors with an infinitely long operating time. The

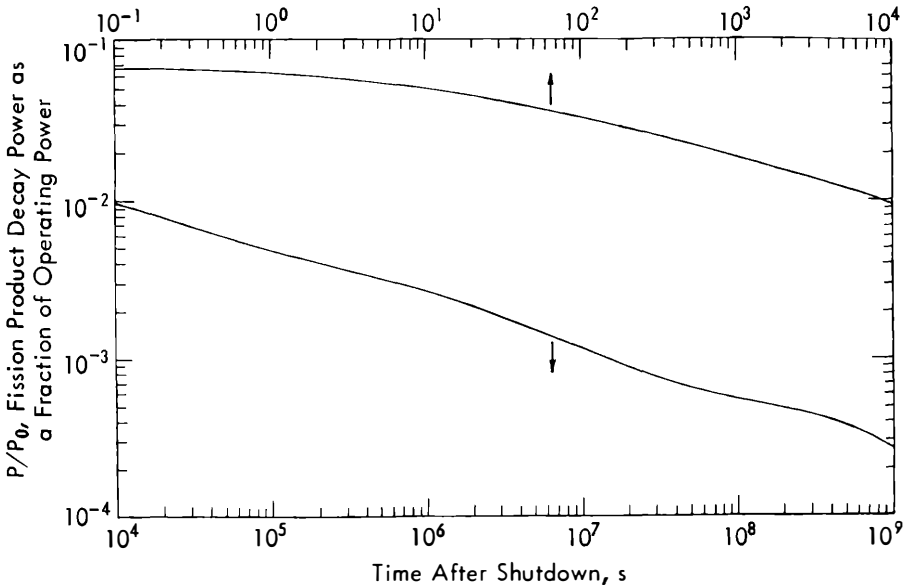


Fig. 1.38 Standard fission product decay heat curve for uranium-fueled thermal reactors (infinite operating time assumed).

ordinate of this figure, P'/P'_0 , is the ratio of decay power to operating power. An upper design limit is obtained from these curves since they were obtained by adding an additional 20% to Shure's values to account for uncertainties. Schenter and England⁹⁴ compared Shure's curve to more recent decay heat computations and experiments. They concluded that the Shure + 20% curve is conservative. Nevertheless, the NRC regulations for evaluation of emergency core cooling systems⁹⁵ require that the curve of Fig. 1.38 be used. This requirement remained in force at the time this text was prepared (1994). As a safety evaluation requirement, the curve has the advantage of being conservative and easy to use.

Schenter and Schmittroth⁹⁶ point out that Shure's decay heat evaluation is based on ^{235}U fission and decay heat depends on the fissionable isotope used. They observe that decay heat from ^{241}Pu fission products is very close to that for ^{235}U fission products for short cooling times and slightly below at long cooling times. Fission products of ^{239}Pu produce heats that are ~15% below the ^{235}U curve at zero time and the same at ~1000 s. Decay heat resulting from fast fission of ^{238}U is ~20% above the ^{235}U at zero time and slightly below at 1000 s. However, ^{238}U fast fission is only a very small fraction of the total fissions. The 20% uncertainty included in the design curve would appear to be very conservative and encompass any effects due to fissioning isotopes other than ^{235}U .

If the residual heat generation after several days of operation were of interest, the fact that the reactor has operated for a finite time rather than an infinite time becomes important. Obenshain and Foderaro⁹⁷ state that residual heat for a finite time of operation (T_0) can be found by subtracting the value of $(t + T_0)$ from the value for infinite operation with a true decay time of t . That is,

$$\frac{P'}{P'_0}(t_0, t_s) = \frac{P'}{P'_0}(\infty, t_s) - \frac{P'}{P'_0}(\infty, t_0 + t_s) \quad (1.60)$$

where

$$\begin{aligned} \frac{P'}{P'_0}(t_0, t_s) &= \text{decay heat ratio for a reactor that has operated for time } t \text{ and been shut down for time } t_s \\ \frac{P'}{P'_0}(\infty, t_s) &= \text{decay heat ratio for a reactor operated for an infinite time and shut down for time } t_s \\ \frac{P'}{P'_0}(\infty, t_0 + t_s) &= \text{decay heat ratio for a reactor operated for an infinite time and shut down for time } (t_0 + t_s). \end{aligned}$$

The 1993 decay heat standard⁹⁸ prescribes a more accurate and elaborate procedure in place of Eq. (1.60). Also note that for decay times in excess of 1 min, the contribution of residual fission energy generally can be ignored.

The American standard was revised in 1979 and again in 1993⁹⁹ to reflect the more recent information. The most important of these revisions was the

inclusion of separate estimates of decay power from thermal fissions of ^{235}U , ^{239}Pu , ^{241}Pu , and fast fission of ^{238}U . The values of $F_i(t, \infty)$, the decay power per fission (MeV/fission), after an infinite reactor operating time and cooling time t , which are contained in the 1993 standard, are summarized in Table 1.VI. Note that when this table is used, the decay power depends on the isotopic composition of the core fuel and the fraction of the total power generated by each isotope. To obtain the decay power (MeV/s) from Table 1.VI, the tabulated values must be multiplied by the reactor fission rate (fission/s) prior to shutdown.

The values from the revised standards presented in Table 1.VI are based on the assumption that there is no neutron capture in the fission products. To account for this effect, the values from the table are multiplied by a factor, designated $G(t)$, which corrects for this capture. The values of $G(t)$ presented in Table 1.VII are the maximum values believed to apply. This is done to assure that conservative decay powers are obtained.

In addition to accounting for the decay heat from fission products, the revised American standard requires that the heating due to actinide decay be included. This contribution, for reactors that have been operating for long periods, is given by:

$$F_{239\text{U}}(t) = E_{239\text{U}} R \exp(-\lambda_1 t) \quad (1.61)$$

$$F_{239\text{Np}}(t) = E_{239\text{Np}} R \left[\left(\frac{\lambda_1}{\lambda_1 - \lambda_2} \right) \exp(-\lambda_2 t) - \frac{\lambda_2}{\lambda_1 - \lambda_2} \exp(\lambda_1 t) \right] \quad (1.62)$$

where

t = time after shutdown, s

$E_{239\text{U}}$ = average energy from the decay of one ^{239}U atom = 0.474 MeV

$E_{239\text{Np}}$ = average from the decay of one ^{239}Np atom = 0.419 MeV

R = atoms of ^{239}U produced per second per fission per second
evaluated for reactor composition at the time of shutdown

λ_1 = decay constant for ^{239}U ($= 4.91 \times 10^{-4} \text{ s}^{-1}$)

λ_2 = decay constant for ^{239}Np ($= 3.41 \times 10^{-6} \text{ s}^{-1}$)

Table 1.VI
Fission Product Decay Power*

Isotope	10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7
^{235}U	12.38	9.50	6.20	3.80	1.91	9.73×10^{-1}	5.55×10^{-1}	2.50×10^{-1}
^{239}Pu	10.27	8.25	5.69	3.52	1.73	9.47×10^{-1}	5.14×10^{-1}	2.34×10^{-1}
^{238}U	14.76	10.36	6.38	3.71	1.80	9.33×10^{-1}	5.10×10^{-1}	2.23×10^{-1}
^{241}Pu	12.10	9.13	5.78	3.44	1.65	9.105×10^{-1}	5.02×10^{-1}	2.35×10^{-1}

*In MeV/fission at t seconds after an infinite reactor operating time at thermal fission in the absence of neutron capture in fission products (1993 ANS standard).

TABLE 1.VII
Factor Accounting for Effect of Neutron Capture by Fission
Products on Decay Heat

$G(t)$	$t(s)$	$G(t)$	$t(s)$
1.020	10^0	1.064	10^4
1.022	10^1	1.124	10^5
1.023	10^2	1.124	10^6
1.033	10^3	1.181	10^7

and F has units of (MeV/s) per (fission/s). The decay heat power contributed by these heavy elements, $P_{dHE}(t)$, is then calculated from

$$P_{dHE}(t) = \frac{P}{Q} [F_{239U}(t) + F_{239Np}(t)] \quad (1.63)$$

where P is taken as the maximum reactor power during the operating history and Q is the effective energy release per fission evaluated for the reactor composition at the time of shutdown.

Besides the American standard on decay heat, a German standard has been adopted¹⁰⁰ and standards have been proposed (1992) by the Japan Atomic Energy Research Institute¹⁰¹ and the International Standards Organization (ISO).¹⁰² The latter is for use in countries desiring a standard and not having one of their own. Dickens *et al.*¹⁰³ have compared these standards and find a substantial degree of similarity between them, but minor differences at a number of points.

An alternative approach to computation of decay heat is the use of one of the computer programs that determines the concentration of the individual fission products and their heat generation as a function of operating and shutdown times. The ORIGEN code¹⁰⁴ is, perhaps, the best known of these programs.

Decay heat due to capture in control material varies greatly depending on the nature of the absorber present in the core during power operation. Decay heat due to silver-indium-cadmium control rods would be more significant than that due to boron-stainless-steel rods. In any case, the capture product decay heat is substantially less than that due to capture in ^{238}U , but no general expression for this heat generation is available.

The contribution from capture in the structure is small. It is generally satisfactory to ignore this.

By combining the estimates from Fig. 1.38 with those of Eqs. (1.59) and (1.63), we estimate the total power generation at time t after a reactor shutdown. Figure 1.39 shows the relative power generation for a typical light-water-moderated reactor, where the heat released to the coolant is appreciably greater than the heat generated, due to the release of heat stored in the fuel. Means for taking this into account are discussed in Sec. 2.6.

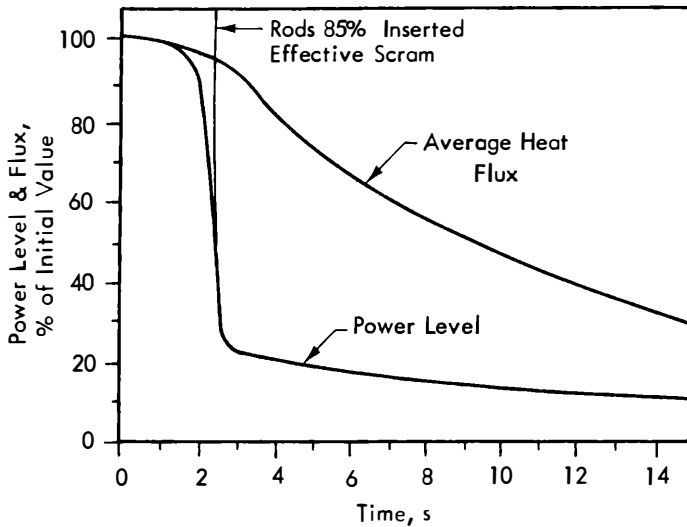


Fig. 1.39 Relative power level and heat flux after a scram [from *Westinghouse Engineer*, 25, 5, 145 (1965)].

1.6 HEAT GENERATION IN REACTOR STRUCTURE AND MODERATOR

1.6.1 Structure

Heat is produced in the fuel by absorption of beta rays and the kinetic energy of the fission fragments, while outside the fuel, heat is generated primarily by gamma-ray absorption. Gamma rays come from gammas released in fission, those released by decaying fission products, or those formed by absorption of neutrons in the core materials.

The variation of the uncollided flux of gamma rays ϕ^0 of a given energy through a slab with a source on one side is given by

$$\phi_i^0 = S_{a_i} \exp(-\mu_i x) \quad (1.64)$$

where

S_{a_i} = flux at the surface, MeV/s cm²

x = distance from surface, cm

μ_i = attenuation coefficient, cm

The associated heat production is then

$$q_i^0 = \mu_{a_i} S_{a_i} \exp(-\mu_i x) \quad (1.65)$$

where μ_{a_i} = energy absorption coefficient (cm⁻¹) and q_i^0 = heat production (MeV/s cm³). In a reactor where we have a source emitting rays of a num-

ber of different energies, the total heat production due to uncollided flux is the sum of heat production due to each energy level.

$$q_t^0 = \mu_{a1} S_{a1} \exp(-\mu_1 x) + \mu_{a2} S_{a2} \exp(-\mu_2 x) + \quad (1.66)$$

The actual heat production is greater than indicated by Eq. (1.66) due to absorption of secondary gammas from Compton scattering. Additional heat production is computed by buildup factors B_{a_i} such that total heat production q is given by

$$q = q_1^0 B_{a1} + q_2^0 B_{a2} + q_3^0 B_{a3} + \quad (1.67)$$

The buildup factors differ for different materials and are a function of μx and the energy level of the gamma rays. Rockwell¹⁰⁵ gives a graphic presentation of these.

In a reactor core that has been operating for 100 hr or more, the gamma-ray energy from both fission and fission products is assumed to be directly proportional to the local power density. Values for gamma-ray energy production from fission and fission products, as well as energy attenuation coefficients, are summarized by McLain and Martens.¹⁰⁶ Where gamma flux leaving the core must be determined, power density near the surface of the core can be used as an approximation, since high self-shielding of the core minimizes contributions from other regions.

The variation of neutron flux through a thin slab with a source on one side is of the same form as that for gamma rays

$$\phi = \phi_0 \exp(-\Sigma x) \quad (1.68)$$

where

ϕ = local neutron flux at x

ϕ_0 = neutron flux at surface of slab

Σ = neutron absorption coefficient, cm^{-1}

In a structural material consisting of heavy elements, neutron heat production is primarily due to absorption of gammas produced by neutron interaction with the structure. Heat production q''' , due to capture gamma rays of a given energy level, is therefore directly proportional to the neutron flux and a function of μ_a , x , and total slab thickness. If the neutron flux is given by Eq. (1.56), then

$$q_E''' = KEf(E)\mu_a(\Sigma/\mu)\phi_0 \exp(-\Sigma x)\{F_1(\mu x, \Sigma/\mu) + F_1[\mu(L-x), -\Sigma/\mu]\} , \quad (1.69)$$

where

q_E''' = heat production due to capture gamma-ray absorption, W/cm^3

$f(E)$ = fraction of captures producing gamma rays of energy E

L = total slab thickness

$$K = 0.8 \times 10^{-13} \text{ Ws/MeV}$$

μ = linear attenuation coefficient for gammas of energy E , cm^{-1}

μ_a = energy absorption coefficient for gammas of energy E , cm^{-1}

and

$$F_1(b, a) = \int_0^b \exp(at) E_1(t) dt; \quad E_1(b) = \int_b^\infty \frac{\exp(-t)}{t} dt \quad (1.70)$$

where b and a are the appropriate arguments required by Eq. (1.69). Function F_1 can be evaluated from Rockwell's graphs.¹⁰⁷ Equation (1.69) neglects buildup of gamma rays due to Compton scattering; therefore, these results should be modified by application of the appropriate buildup factor for thick elements. Total heat generation is then the sum of these values and those obtained from Eq. (1.67).

For steel slab thicknesses of <1.5 cm, it is usually satisfactory to assume a constant heat production due to neutron absorption. In this range, buildup factors approximately compensate for neutron flux attenuation. Furthermore, in the slabs with thicknesses between 1.5 and 6 cm, absorption of capture gammas account for $<10\%$ of total heat production. This heat production can be approximated as a constant fraction of total heat production. Note that in a pressure vessel wall adjacent to a shield tank, the neutron flux begins to rise near the shield tank due to the reflected flux. Here, a better approximation would be constant heat production due to neutron absorption.

1.6.2 Moderator

While heat generation in the reactor structure is primarily from gamma-ray absorption, heat generation within the moderator is primarily from neutron thermalization through elastic scattering. For elastic scattering $q'''(x)$, the volumetric heat generation rate at some location x can be approximated by¹⁰⁸

$$q'''(x) \approx \Sigma_s \phi_f(x) \xi \quad (1.71)$$

where Σ_s = macroscopic elastic scattering cross section of the fast neutron flux $\phi_f(x)$ and ξ = average energy decrement per collision.

1.7 THERMAL DESIGN BASIS

In view of the many variables that affect power generation within the reactor fuel, it is clear that an accurate computation of the relative power distribution in the reactor is quite complicated. The simple approximations presented in this chapter for uniform, unperturbed cores can be regarded as useful in providing very rough estimates for preliminary design. How-

ever, any actual design must be based on realistic power distributions obtained from the nuclear designer. The major results of the nuclear calculations can be relayed to the thermal designer in terms of nuclear hot-channel factors. These factors, with an estimate of the axial power-distribution shape, can serve as a basis for initial plant thermal design. Because of the wide use of chemical shim control, simple relationships are often adequate to describe axial power shape.

Refined analyses of plant performance may wish to consider thermal and nuclear effects simultaneously. This can be accomplished by linking the approximate methods for power estimation from Secs. 1.3.2 and 1.3.3 with the appropriate thermal design procedures.

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CHAPTER TWO

Fuel Elements

The thermal design of a reactor can be considered to begin with the design of the heat generating elements. As its first step, prediction of fuel element behavior in a reactor requires a knowledge of fuel temperatures during steady-state and transient conditions. Therefore, for the fuels of interest, we first review the properties necessary for temperature estimation and then examine some of the computational methods involved. We then consider the overall performance of reactor fuel elements.

2.1 FUEL ELEMENT CHARACTERISTICS

2.1.1 Fuels and Their Thermal Properties

2.1.1.1 *Metallic Alloy Fuels*

Radiation damage to metallic fuel elements is caused largely by highly energetic fission fragments. The damage mechanisms fall into four categories:

1. Displacement of individual atoms (the primary knock-on atoms) from the normal lattice
2. Displacement of additional atoms by the primary knock-on atoms to form displacement spikes
3. The presence of fission products as impurities
4. Cavity formation at points of mechanical weakness.

These changes result in both axial growth and radial swelling of the fuel.

Pure uranium metal is not a satisfactory fuel for PWRs designed for power production alone, since it exhibits very poor dimensional stability

under prolonged irradiation and its high corrosion rate would result in rapid fuel element rupture following any cladding failure. Exposure of the failed fuel to 550°F water for a few hours would completely destroy the fuel element. On the other hand, when reactor operation is for both plutonium production and power, uranium metal is acceptable because its radiation stability is adequate during the short reactor exposure designed to limit higher isotope production. Coolant temperatures can be significantly reduced so that the consequences of any failures will be less severe.

Much of uranium's irradiation instability is traceable to the anisotropy of its alpha phase, which leads to expansion in the direction of the (010) crystal plane and contraction in the (100) plane during low-temperature irradiation. A substantial reduction in anisotropic behavior can be obtained by beta heat treating processes using oil quenching.¹ Further improvement in uranium properties can be obtained by adding a sufficient amount of an alloying element so the gamma phase can be retained at low temperatures. The gamma phase has a cubic structure that does not exhibit anisotropic properties.

Considerable effort was expended in attempts to develop alloys fully suitable for PWR service. The most promising were binary alloys of uranium and molybdenum.² Molybdenum is capable of delaying the beta-alpha transformation when present as a dilute (<5%) alloying addition. In moderate amounts (>5%), it can retain the metastable gamma phase at room temperature. Gamma-stabilized alloys were found to have significantly better aqueous corrosion resistance than pure uranium; alloys containing 10 to 13.5% molybdenum appear to exhibit adequate corrosion resistance for power reactor use when covered by cladding. Their dimensional stability is better than that of unalloyed uranium; however, while considerably improved, swelling remained a problem.³ Interest was also shown in gamma-phase alloys of uranium, niobium, and zirconium. A uranium-molybdenum alloy fuel was initially considered for the blanket of the first Shippingport core, but the superior performance of uranium dioxide led to the use of ceramic fuel instead.

Uranium-molybdenum alloys have been successfully used as a fuel for pulsed test reactors.⁴ The fuel for such reactors must be able to withstand intense shock loading and high dynamic stresses. Total burnup tends to be low; fuel swelling is not expected to prove a problem. The gamma-phase U-10 wt%-Mo alloy has been the most widely used alloy for this purpose.⁴

Attention has also been given to developing very-low-alloy uranium fuels for a heavy-water-moderated PWR. The greater neutron economy achievable with low-alloy uranium is highly important in such reactor systems. Furthermore, the consequences of fuel element failure are less severe. Since heavy water units are pressure tube reactors, refueling is easier and severe element failures will not propagate throughout the reactor as they might in the close-packed lattice of a light water reactor.

The concept employed has been to form alloys using small amounts of suitable metals that tend to stabilize the gamma or beta phase. When transformation to the alpha phase takes place, the resulting material consists of small, randomly oriented crystals. The alloys have better dimensional stability and corrosion resistance than cast alpha-phase uranium. The alloys examined in depth have included U-2 to 2.5 wt%-Zr, U-0.3 wt%-Cr, U-1.5 wt%-Mo, and U-Fe alloys.^{5,6} The so-called "adjusted uranium" alloy that contains ~500-ppm iron, 250-ppm aluminum, 600-ppm carbon, and 20-ppm nitrogen has also been studied.

An extensive study of the swelling of low alloys containing varying amounts of iron, silicon, aluminum, and molybdenum was conducted during the development of an improved Savannah River reactor fuel.¹ Alloys containing 250- to 350-ppm silicon alone, or in conjunction with other alloying elements—especially molybdenum—were most effective for exposures up to 5000 MWd/tonne. Alloys containing 350-ppm iron, 350- and 800-ppm aluminum had satisfactory swelling behavior at temperatures below 400°C and exposures below 13,000 MWd/tonne.

While it has been possible to reduce fuel swelling substantially through alloy addition, the problem has not been completely eliminated. Below ~640°C, the swelling obtained varies greatly with alloy content; this effect is consistent with the concept that at these temperatures swelling occurs by mechanical processes. The benefits obtained from alloying additions appear to be due to the influence of the additions on mechanical properties. A minimum swelling rate is reached in the 640 to 700°C range. At temperatures above this, the rates again increase, but are only slightly different for the various alloys.⁷

The Canadians investigated the use of U₃Si (U-3.8 wt% Si) for PWR service. While swelling is considerably lower than that of unalloyed uranium, it is higher than desired. However, the fuel is sufficiently plastic so the addition of a central void appears to offer a way of accommodating a major portion of the swelling.⁸

The water corrosion resistance of uranium is significantly improved by the addition of small amounts of molybdenum, zirconium, Nb + Zr, or silicon. Indeed, U₃Si has a corrosion resistance perhaps 500 times that of unalloyed uranium. However, attack of a fuel element of these alloys with a cladding hole leak would be severe enough to require fairly prompt removal of the element from the reactor. Small aluminum additions, e.g., 1.5% aluminum, substantially improve the corrosion resistance of U₃Si (Ref. 9).

A major inhibition to the use of uranium silicide is that U₃Si undergoes rapid and gross swelling at fuel temperatures in excess of 900°C (Ref. 9). Such temperatures can be encountered during a loss-of-coolant accident (LOCA). Expected external pressures are not adequate to prevent severe ballooning. Mathews and Swanson⁹ suggest that such high-temperature swelling may be typical of all metallic fuels with fission gases in irradiation-

induced pores and may pose a severe limitation on the use of metallic fuels in reactors where accident temperatures approach the fuel melting point.

High-alloy uraniums with more satisfactory properties are available. Such alloys provide a means for dispersing highly enriched uranium in a structure with a sufficiently large surface area to attain the high heat removal rates required.

Uranium-aluminum alloys have been widely used in research reactors and in some production reactors. However, the poor corrosion resistance of aluminum in high-temperature water limits use of these alloys to low-temperature applications. The only high-alloy uranium used for power producing PWRs are alloys of uranium and zirconium. Zirconium's low neutron absorption cross section, high melting point, and corrosion resistance, and uranium's high solubility in zirconium make it a natural alloying material. Zirconium alloys have adequate aqueous corrosion resistance and adequate strength when they contain a small amount of tin. In actual practice, uranium is alloyed with Zircaloy-2, which contains 1.5% tin plus traces of iron, nickel, and chromium. Dimensional stability under irradiation is good. Alloys containing up to 14 wt% U have satisfactorily withstood burn-ups of up to 3.3 at. %. Maintenance of dimensional stability seems to require that the fuel not be cycled above the phase transition temperature ($\approx 600^{\circ}\text{C}$) (Ref. 10). An alloy of uranium and Zircaloy was used for the meat of plate-type sandwiches in the first Shippingport reactor seeds.

The thermal design properties of U-Zr alloys were investigated by Battelle Memorial Institute. Etherington¹¹ tabulates the thermal conductivity of the alloy and pure components as a function of alloy composition and temperature. Density of the alloy at room temperature is approximately a linear function of the atomic percentages of the constituents.¹² Similarly, specific heats can be estimated from that of the components. Further properties can be obtained from Ref. 13. A summary of significant properties is given in Table 2.I.

The thermal properties of U-Mo alloys have also been extensively investigated. Room-temperature thermal conductivity varies from $0.148 \text{ W/cm}^{\circ}\text{C}$ for 8 wt% Mo to $0.138 \text{ W/cm}^{\circ}\text{C}$ for 12 wt% Mo (Ref. 13). The thermal conductivity of 8 wt% Mo fuel increases linearly with temperature to $0.39 \text{ W/cm}^{\circ}\text{C}$ at 790°C (Ref. 14). Thermal expansion is not linear with temperature, but shows an inflection at 595°C . A total length change of 1.8% is seen at 790°C (Ref. 14). Additional properties can be obtained from Ref. 15.

Properties of the ternary uranium, 7.5 wt% Mo, 2.5 wt% Nb, alloy are reviewed by Weiss.¹⁵ Properties of a number of other alloys of interest are available in Refs. 13 and 16. Thermal properties for the most important alloys are summarized in Table 2.I.

Metallic thorium containing small amounts of enriched uranium has been proposed as a PWR fuel.¹⁷ Thorium has a face-centered cubic (fcc) structure and does not exhibit the anisotropic behavior of uranium. The irradiation experiments available (1978) give no indication of instability and

TABLE 2.1
Thermal Properties of Uranium Alloys

Fuel	Uranium	U-2 wt% Zr	U ₃ Si (U-3.8 wt% Si)	U-12 wt% Mo	Zr-14 wt% U
Fuel density (g/cm ³)	19.05 (200°F) 18.87 (400°F) 18.33 (1200°F)	18.3 (room temperature)	15.57 ± 0.02 (room temperature)	16.9 (room temperature)	7.16 (room temperature)
Crystal structure	α, room temperature—1200°F, orth. β, 1200 to 1238°F, tet. γ, 1400°F-mp, body-centered cubic (bcc)	Orth.	U + U ₃ Si ₂ > 1706°F ε < 1706°F tet. (ε formed by holding cast alloy at 1472°F)	bcc	α Zr (hex.) ε Zr (hex.)
Melting temperature (°F)	2070	2060	1805	2102	3240
Conductivity [Btu/(h ft °F)]	15.8 (200°F) 17.5 (600°F) 20.25 (1000°F) 22.00 (1400°F)	12.7 (95°F) 15.6 (572°F) 21.4 (1112°F) 27.8 (1652°F)	8.66 (77°F) 10.1 (149°F)	7.97 (room temperature)	6.36 (68°F) 6.71 (212°F) 7.12 (392°F) 7.52 (572°F) 10.4 (1292°F)
Thermal expansion [in./(in. °F)]	20.2 × 10 ⁻⁶ for (100) plane -5.17 × 10 ⁻⁶ for (010) plane 19.0 × 10 ⁻⁶ for (001) plane Volumetric 34.2 × 10 ⁻⁶	8.0 × 10 ⁻⁶ (105 to 930°F)	7.67 ± 0.28 × 10 ⁻⁶ (212 to 752°F)	7.23 × 10 ⁻⁶ (212 to 752°F)	3.78 × 10 ⁻⁶ (221 to 626°F) 3.84 × 10 ⁻⁶ (662 to 1022°F)
Heat capacity [Btu/(lb °F)]	0.0278 (200°F) 0.0410 (1000°F) 0.0464 (1200°F)	0.0287 (200°F)		0.032 to 0.036 (572 to 752°F)	0.0674 (200°F)

restrained swelling is 1 to 2% for 10,000 MWd/tonne exposure at 650°C (Ref. 17). The corrosion rate of pure thorium in 300°C water is two orders of magnitude lower than that of uranium. Addition of up to 6% uranium does not appear to affect corrosion rate. A further reduction in corrosion rate is achieved by small additions (0.03 to 0.12 wt%) of carbon. The carbon addition also leads to a more than threefold increase in yield stress.

Thorium is a semirefractory metal (melting point of 1845°C) and has a good thermal conductivity (~ 0.1 cal/cm s°C). Hence, fuel temperatures during a LOCA should be far below the melting point and disastrous swelling would not be expected. However, no direct tests of this appear to be available (1992). Use of thorium metal instead of ThO₂ in an LWBR is predicted to lead to a slight improvement in breeding ratio and a reduction in seed enrichment.¹⁷

At present (1992) there is essentially no industrial interest in using metallic thorium or low-alloy metallic fuels for lightwater reactors (LWRs). The technology of ceramic fuels is satisfactory and so well established that there is no real incentive for departure from ceramics for central station power generation. There is interest in using an intermediate metal alloy (62.5% U, 15% Pu, 22.5% Zr) for fast reactor fuel elements. Fuel rods with a large, sodium-filled pellet-to-cladding gap can accommodate significant radiation damage. This alloy shows sufficient thermal expansion so that a sodium-cooled fast reactor with this fuel can survive an unprotected loss-of-flow accident without core damage. Such behavior is not achievable with ceramic fuel rods.

Based on the renewed fast reactor interest in alloy fuel, Lancaster *et al.*¹⁸ have more recently (1992) proposed the use of a uranium metal alloy containing 10% zirconium as a fuel for water-cooled central station reactors. They suggest that the low centerline temperatures of metallic fuels, which lead to substantially less stored energy, are a significant safety advantage during LOCAs. Lancaster *et al.* believe high burnups can be achieved by using annular pellets. However, it is not at all clear that the corrosion resistance of this alloy would be sufficient for PWR use.

High-alloy fuels are not of interest for central station use because of the high costs associated with the use of the highly enriched uranium they require. However, these fuels would obviously be of interest for naval reactor cores where they can provide a means for obtaining both high specific power and long core life.

2.1.1.2 Dispersion Fuels (Cermets)

An alternative way to provide a structural base with the necessary high surface for highly enriched fuel is to physically disperse the fuel in a metallic matrix. Dispersions of UO₂ in aluminum, zirconium alloys, and stainless steels have been investigated, but only stainless-steel and zirconium alloy dispersions have been used in PWRs.

Stainless-steel- UO_2 dispersions are normally fabricated by powder-metallurgy techniques. Uranium dioxide and stainless-steel powder are blended, pressed, and sintered. Final dimensions and further densification are achieved by cold or hot working. The physical and mechanical properties of the dispersions are influenced by many variables including UO_2 particle size, type and amount of UO_2 , type of stainless steel, geometry of the element with respect to the property being measured, and fabrication techniques. Reference 19 provides a compilation of mechanical properties.

Zircaloy- UO_2 dispersions have been used as a replacement for the U-Zr alloy used in the first seed of the Shippingport reactor. Such dispersions have greater resistance to radiation damage than the alloy, since damage would be confined to the area around the UO_2 particles rather than being dispersed throughout the alloy.

There has been no interest in the use of dispersion fuels for central station cores since the virtual abandonment of the seed-and-blanket concept. However, dispersion fuels would clearly be of major importance for naval reactor cores. These fuels could lead to cores with high specific powers and lifetimes that could exceed those obtained from zircaloy-uranium alloy fuels.

Normally a dispersion fuel element offers good corrosion resistance regardless of the resistance of fuel particles. Exposure of small amounts of fuel to the coolant due to cladding failure means only a few particles are exposed. If there is stringering of the fuel particles, corrosion can proceed along the interconnected particles. Since UO_2 has good corrosion resistance in high-temperature water, the aqueous corrosion resistance of these dispersions is good even when some stringering occurs; however, the radiation damage resistance of such a fuel would be poor.

Resistance to radiation damage of stainless UO_2 dispersions is good, but as with all fuels, radiation damage places some limits on their performance. In a properly fabricated dispersion fuel, the particles are uniformly dispersed throughout the nonfissile matrix that predominates in volume. Fission products released from the fuel are then confined to narrow regions around the fuel particles; a continuous web of undamaged matrix remains. Radiation-induced swelling arises because of growth of the UO_2 particles due to an accumulation of fission products and the partial escape of fission gases from the UO_2 (Ref. 20). The latter effect is the most significant. A gas-filled void is created around the particles; this pressurizes the matrix shell causing it to expand as a thick-walled vessel under pressure. Thus, the matrix swelling that can be allowed will set the maximum burnup. Swelling limits can be determined on the basis of allowable dimensional changes or a maximum strain determined on the basis of the reduced ductility limit of the neutron-embrittled matrix.

The specific heat of a dispersion can be obtained by combining those of its constituents linearly in accordance with dispersion composition. Densi-

ties can be obtained in a similar manner providing an allowance is made for the fact that the UO_2 particles are porous and, therefore, densities of the dispersion range from 92 to 98% theoretical. To obtain the thermal conductivity of a dispersion of small particles of conductivity, k_p uniformly dispersed in a continuous substance of conductivity k_s , Jakob²¹ recommends an equation originally derived by Maxwell and used for the present purposes by Eucken:

$$k_d = k_s \frac{1 - (1 - ak_p/k_s)b}{1 + (a - 1)b} \quad (2.1)$$

where

k_d = thermal conductivity of dispersion

$a = 3k_s/(2k_s + k_p)$

$b = V_p/(V_s + V_p)$

V_s = total volume of continuous substance

V_p = total volume of distributed particles.

Equation (2.1) was derived for small values of b , but it holds approximately for $b \leq 0.5$. We suggest that use above $b \leq 0.25$ should be with caution. Stora²² suggests that for a mixture in which neither phase completely surrounds the other, Bruggeman's equation²³ should be used:

$$\frac{k_s - k_d}{k_s - 2k_d} = \left(\frac{b}{1 - b} \right) \left(\frac{k_d - k_p}{2k_d + k_p} \right) \quad (2.2)$$

If there is appreciable stringing of the UO_2 , thermal conductivity is anisotropic and Eq. (2.1) should not be applied. Under these conditions, Stora²² suggests that the Fricke equation²⁴ for dispersion of ellipsoidal particles can be used. He considers that all particles can be reduced to ellipsoids by appropriately varying the relative size of the A , B , and C axes. The anisotropic effect is simulated by changing the size and orientation of the axes of the ellipsoids with respect to the direction of heat flow. Fricke's equation for a dispersion is

$$k_d = k_s \left[\frac{Xk_s - k_p + Xb(k_s - k_p)}{Xk_s + k_p + b(k_s - k_p)} \right] \quad (2.3)$$

where

$$X = \frac{(k_p/k_s)(\beta - 1) + 1}{(k_p/k_s) - (\beta + 1)}$$

When axis A is parallel to the heat flow,

$$\beta = \frac{(k_p/k_s) - 1}{1 + [(k_p/k_s) - 1](1 - M)} \quad (2.4)$$

and when axis A is perpendicular to the heat flow,

$$\beta = \frac{(k_p/k_s) - 1}{1 + (k_p/k_s) - (M/2)} \quad (2.5)$$

Here, M is a trigonometric function of the A/B length ratio. By using the X parameter, Stora²² extended Bruggeman's equation for mixtures in which neither phase completely surrounds the other to ellipsoidal particles.

$$\frac{k_s - k_d}{k_s + Xk_d} = \left(\frac{b}{1-b} \right) \left(\frac{k_d - k_p}{Xk_d + k_p} \right) \quad (2.6)$$

However, his application of this equation to UO_2 -metal mixtures, in which the metal fraction ≤ 0.3 , yielded k values that differed from measured values by as much as 40%.

2.1.1.3 Uranium Dioxide

This is by far the most popular fuel material for LWRs; uranium dioxide, mixed uranium and plutonium dioxides, and mixed thorium and uranium dioxides are the only ceramic fuels that have received serious attention. Although the carbides can be satisfactory from a radiation damage viewpoint, their reaction with water eliminates them from consideration for LWR service. Uranium dioxide has been found to exhibit excellent dimensional stability to high burnup. It shows appreciable radial cracking,²⁵ but when properly restrained by fuel element cladding, cracking does not lead to dimensional instability. In addition, UO_2 has a high melting point, is relatively inert chemically, and has good pellet fabrication characteristics.

The generation of fission products leads to a swelling of the UO_2 crystals that is roughly linear with burnup. However, at low burnups, the fuel porosity can accommodate much of the crystal swelling and external dimensional changes are small. When the available porosity has been exhausted, swelling rates increase sharply. The burnup at which the swelling rate increased sharply used to be referred to as the *critical burnup*. The so-called critical burnup for unrestrained UO_2 at normal PWR operating conditions is shown in Fig. 2.1 as a function of the void within the fuel. Since the swelling rate also depends on fuel temperature, the critical burnup actually observed depends on the operating power level.

An illustration of the effect of cladding restraint is provided by Dayton,⁵ who noted that for platelet-type fuel elements, the increased cladding restraint of $1/4$ -in. platelets postponed critical swelling to a burnup over 130% that observed for $1/2$ -in.-wide platelets. Sophisticated design calculations now recognize that UO_2 at high temperatures has some plasticity; deformation of the fuel is affected by elastic, plastic, and creep properties of fuel and cladding materials.²⁶

While Fig. 2.1 was formerly used as a rough guide for determining design burnup limits, present design approaches are considerably more so-

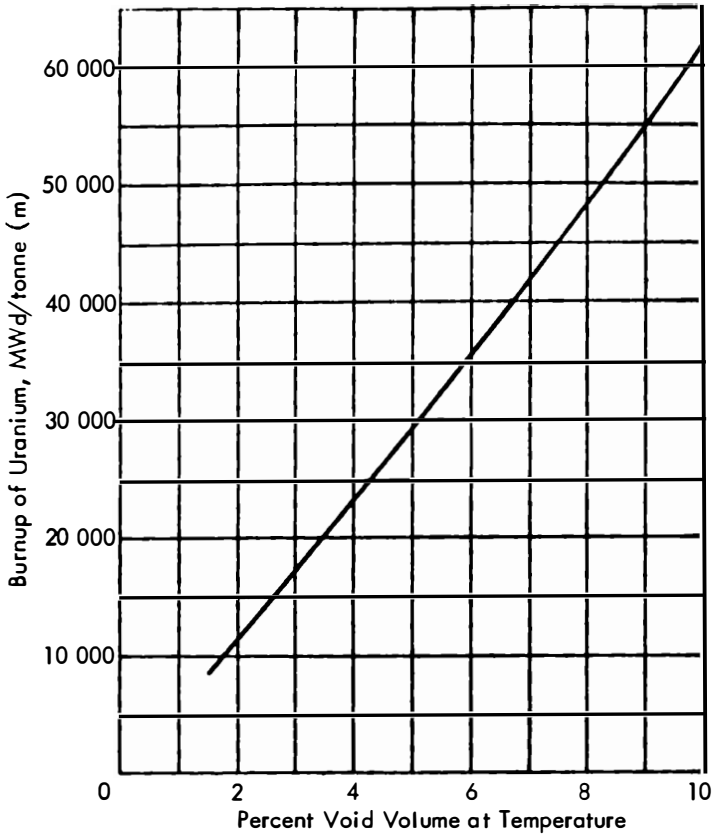


Fig. 2.1 Critical burnup for rapid swelling of unrestricted UO_2 [from *Nucl. Eng. Des.*, **6**, 301 (1967)].

phisticated. The achievement of low fuel costs requires burnups in excess of the critical burnup and fuel rods are now designed to accommodate the increased swelling obtained.

At high flux irradiation, appreciable UO_2 sublimation occurs; this leads to the formation of a central void surrounded by UO_2 of essentially theoretical density. Void migration has been theoretically investigated by De Hales and Horn²⁷ and Nichols,²⁸ who derived equations describing the rate of void migration for central temperatures above 2750°F.

At very high power, a molten core is obtained. Although some experimental data indicate such a core is not deleterious,²⁹ the avoidance of melting was generally accepted as a design criterion. The volumetric expansion of UO_2 on melting can lead to severe swelling of the cladding and, subsequently, to its destruction over a portion of the fuel rod. Failures of this

nature were observed by Eichenberg *et al.*³⁰ Successful operation with gross central melting was satisfactorily accomplished by Lyons *et al.*³¹ with hollow pellets that provided the free volume necessary to accommodate UO_2 expansion on melting. Later in-reactor experiments by Lepescky *et al.*³² indicate satisfactory fuel performance with central melting can be obtained with solid pellets.

It has been postulated that under particular conditions of fuel relocation and cracking, molten fuel could come in contact with the cladding, which could be brought to its melting point and burned through. This mechanism may be one of the factors contributing to the 1966 failure in the Plutonium Recycle Test Reactor (PRTR) rupture test loop, where fuel element failure was followed by burn-through of the surrounding pressure tube. The question of whether central melting is a real limit is probably moot since accident analyses now generally lead to the establishment of linear heat ratings considerably below those at which melting occurs.

Current technology provides UO_2 compacts of close to theoretical density by pressed and sintered pellets. As the maximum burnups increased, lower density fuels were used to reduce swelling. This led to renewed interest in vibratory compaction, swaging, or combined compaction and swaging of UO_2 powder as production processes. However, use of low-density fuels can lead to in-reactor densification. In some cases, this led to cladding collapse into the gaps in the pellet stack created by the densification. The use of high-density pelletized fuel remains the current (1995) commercial practice since high-density pellets mean less risk of fuel densification during irradiation. Interest could conceivably revive in vibratory compaction (sphere-pac fuel) because of the ease with which this process could be adapted to remote fabrication of mixed-oxide (MOX) fuel.

Under usual PWR operating conditions, UO_2 does not react with the coolant and corrosion resistance is good. Therefore, pinhole cladding failures do not lead to washout of the fuel element. However, waterlogging of failed elements at low power and subsequently expelling steam at higher power does lead to the escape of some fission products into the coolant. Under some conditions, it is possible for the waterlogging effect to be severe enough to cause fuel element failure. Eichenberg *et al.*³³ postulated that this waterlogging failure could occur if the defect were blocked by a particle of UO_2 , thus restricting the escape of steam during startup. Only a very few such failures have been observed in a large number of in-pile tests³⁰ with sintered pellets; no such failure of actual PWR sintered-pellet fuel elements in reactor service has yet (1994) been observed, although defective fuel elements have been present. Waterlogging seems to be more of a problem with vibratory compacted or swaged fuel elements than with those using sintered pellets.

Because of the wide use of UO_2 as a reactor fuel, its properties have been the subject of numerous investigations. Specific heat can be computed as³⁴

$$c_p = \frac{K_1 \theta^2 \exp(\theta/T)}{T^2 [\exp(\theta/T) - 1]^2} + 2K_2 T + \frac{YK_3 E}{2RT^2} \exp(-E/RT) \quad (2.7)$$

where

T = temperature, K

θ = 535.285

E = 1.577×10^5

K_1 = 296.7

K_2 = 2.43×10^{-7}

K_3 = 8.745×10^7

c_p = specific heat at constant pressure, J/kg K

R = 8.3143, J/mol K

Y = oxygen-to-metal ratio (2 for stoichiometric UO_2).

The theoretical density of UO_2 at room temperature is 10.96 g/cm^3 . Its thermal expansion can be obtained from the simple polynomial expression shown in Fig. 2.2. The density of UO_2 just below and just above the melting point is reported as 9.65 and 8.80 g/cm^3 , respectively.³⁵ Christensen³⁶ gives the thermal expansion coefficient of molten UO_2 as $3.5 \times 10^{-5} \text{ in./in. } ^\circ\text{C}$.

Despite the many investigations of UO_2 thermal conductivity, considerable scatter in the data remains. Data of a number of investigators for unirradiated UO_2 are shown in Fig. 2.3, where all have been corrected to 95% theoretical density. The simplest way to correct for density variation is to use the relationship

$$k_{95\%} = k_{\text{measured}} \frac{0.95}{1 - \alpha} \quad (2.8)$$

where α is the void fraction of the sample. Godfrey *et al.*³⁷ and May *et al.*³⁸ noted that the conductivity of UO_2 increased as the oxygen-uranium ratio is reduced. This stoichiometric effect can account for some of the data scatter. Belle *et al.*³⁹ show that data scatter is reduced when a revised method is used for correcting the data to a standard porosity. By assuming that a pellet is a dispersion of nonconducting pores in a UO_2 matrix, they used a modified version of the Maxwell-Eucken relationship, Eq. (2.1), to obtain

$$k_p = \frac{1 - \alpha}{1 + \beta\alpha} (k_{100\%}) \quad (2.9)$$

where k_p is the conductivity of the porous sample in question and β is a constant that depends on the material. A reduced data spread can be obtained by using $\beta = 0.5$ for fuels of 90% theoretical density and above and $\beta = 0.7$ for fuels of lower density.

Stora²² notes that β corresponds to X in Fricke's equation, Eq. (2.3), if a pellet is considered to consist of a dispersion of pores in UO_2 . A β value

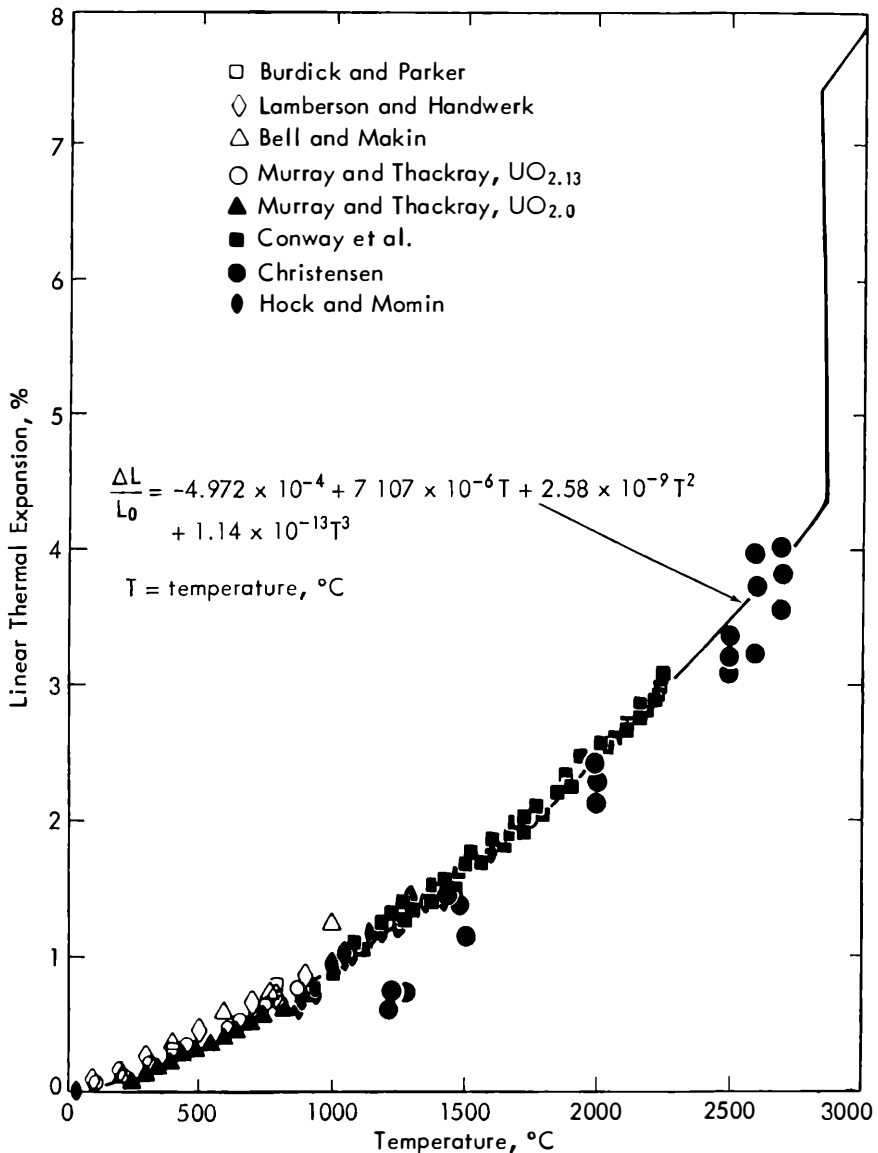


Fig. 2.2 Comparison of UO_2 thermal expansion data with prediction [from Ref. 53].

of 0.5 corresponds to spherical pores, while a larger value corresponds to ellipsoidal pores.

Thermal conductivity data below 2500°F generally follow the expected behavior for lattice conduction. In 1961, on the basis of postirradiation examinations, Bates⁴⁰ postulated an increase in conductivity at higher temperatures due to internal thermal radiation. This seems to be borne out by

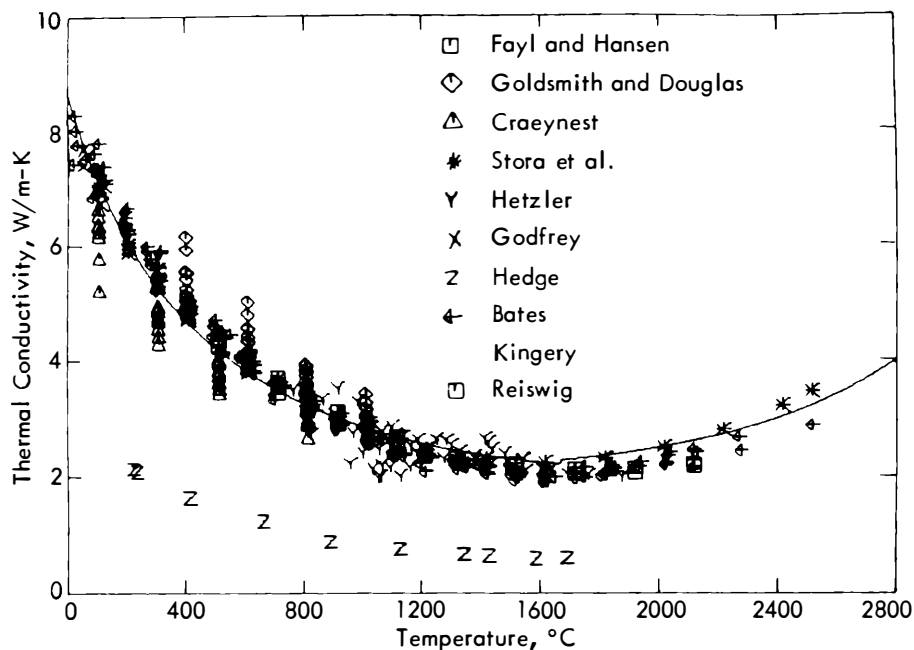


Fig. 2.3 Thermal conductivity of unirradiated UO_2 [from Ref. 53].

the more recent data of Godfrey *et al.*³⁷ and Nishijima⁴¹ for unirradiated UO_2 .

Recent advances in materials science have led to a considerably improved understanding of UO_2 thermal conductivity. It is now recognized that UO_2 conductivity is due to two mechanisms. At low temperatures, heat is primarily conducted by phonons (quantum mechanical description of vibration of atoms in crystal). At high temperatures, heat conduction is primarily via small polarons (quantum mechanical description of electron and hole motion). Since the phonon contribution decreases with increasing temperature while the polaron contribution increases as the temperature increases, the thermal conductivity curve shows a minimum value in the vicinity of 2000 K. Harding and Martin⁴² recommend on the basis of theoretical considerations and electrical conductivity measurements at high temperature that the thermal conductivity of 100% dense UO_2 between 773 and 3120 K be computed by:

$$k = (0.0375 + 2.165 \times 10^{-4} T)^{-1} + (4.715 \times 10^9 T^{-2}) \times \exp[-16361/T] \quad (2.10)$$

where k is the thermal conductivity of 100% dense UO_2 (W/m K) and T is the temperature in Kelvin. The first term in Eq. (2.10), which has the form

$(A + BT)^{-1}$, represents the phonon contribution; the second term represents the polaron contribution. Harding and Martin estimate that the uncertainty in k is about $\pm 7\%$ at 1800 K and increases to about $\pm 15\%$ at 3120 K.

A somewhat earlier and more elaborate expression for UO_2 thermal conductivity by Hagrman *et al.*³⁴ is also based on phonon and polaron contributions. It has the advantage of including correction factors for the effects of variation in the oxygen-to-metal ratio, theoretical density, plutonium content, and thermal expansion. In addition, it may be used at temperatures below 773 K. Hagrman *et al.* recommend:

$$k = \left[\frac{1 - \alpha}{1 + (6.5 - 0.00469T')(\alpha)} \right] \left[\frac{C_v}{(A + BT'')(1 + 3\epsilon_{th})} \right] + 5.2997 \times 10^{-3} T \left[\exp\left(-\frac{13358}{T}\right) \right] \left\{ 1 + \left[0.169 \left(\frac{13358}{T} \right) + 2 \right]^2 \right\} \quad (2.11)$$

where

k = thermal conductivity, W/m K

α = void fraction = 1 (fraction of theoretical density)

C_v = phonon contribution to the specific heat at constant volume, J/kg K. The first term of Eq. (2.7) is used for this factor.

ϵ_{th} = linear strain caused by thermal expansion when temperature is > 300 K, unitless.

T = fuel temperature, K

T' = fuel temperature if < 1364 K. [For temperatures > 1834 K, the porosity factor, first bracket on right-hand side of Eq. (2.11), equals 1.0. For temperatures in the range of 1364 to 1384 K, the factor is found by interpolation.]

T'' = fuel temperature if < 1800 K. For temperatures > 2300 K, T'' is equal to 2050 K, and for temperatures in the range 1800 to 2300 K, T'' is found by interpolation.

A = a factor proportional to the point of defect contribution to the phonon mean free path, m-s/kg-K. The correlation used for this factor is $0.339 + 11.1 \times |(2.0 - O/M \text{ ratio})|$.

B = a factor proportional to the phonon-phonon scattering contribution to the phonon mean free path, m-s/kg-K. The correlation used for this factor is $0.06867 \times (1 + 0.6238 \times \text{weight fraction PuO}_2 \text{ in fuel})$.

The terms prior to the plus sign in Eq. (2.11) represent the phonon contribution to thermal conductivity, while the terms following the plus sign represent the polaron (electron-hole) contribution. The expression is valid only in the range of 90 to 100% of theoretical density. When the fuel is molten, the terms preceding the plus sign are neglected.

Note that any departures from a stoichiometric UO_2 results in a reduction of the conductivity.⁴³ This is accounted for in Eq. (2.11) by the variation of A with oxygen-to-metal ratio.

Considerably more uncertainty is attached to the thermal conductivity of UO_2 after irradiation than to unirradiated UO_2 conductivity. In 1959, Runnals⁴⁴ noted a decrease in conductivity at low temperatures after an irradiation of 9×10^{17} n/cm² and no significant change afterward. Other investigations generally noted qualitatively similar behavior; Fig. 2.4 compares the suggested curve for unirradiated conductivity with the available data for irradiated UO_2 . It seems probable that there is some decrease in conductivity below 1500°F and little change between 2000 and 3000°F.

In 1967, Belle *et al.*³⁹ surveyed all the available data and concluded that conductivity was a function of both burnup and temperature. They proposed a correlation that included a significant burnup effect. Subsequently, Stora *et al.*⁴⁵ argued that the apparent irradiation-induced reduction in ther-

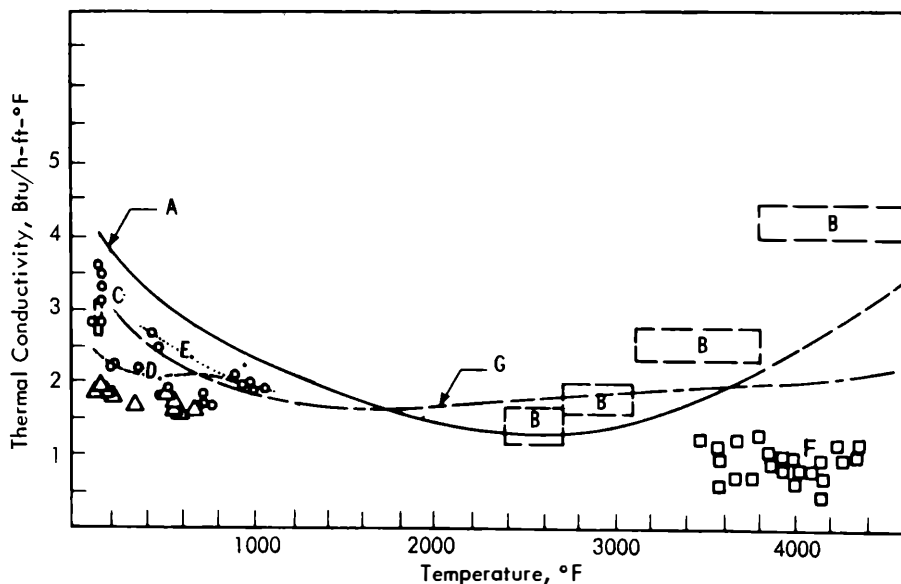


Fig. 2.4 Irradiated UO_2 thermal conductivity and out-of-pile data [from *Nucl. Eng. Des.*, 6, 301 (1967)]. Notation:

A = curve based on unirradiated data

B = from Ref. 40, postirradiation measurements

C = from Bain and Robertson, *J. Nucl. Mater.*, 1, 109 (1959), postirradiation data

D = Dayton and Tipton, BMI-1448 (Rev.), Battelle Memorial Institute (1960), postirradiation data (---heating, cooling)

E = from Dayton and Tipton, BMI-1448 (Rev.), Battelle Memorial Institute (1960), postirradiation data (Δ first-cycle heating, $\circ$ second-cycle heating, $\square$ third-cycle cooling)

F = Lyons *et al.*, *Trans. Am. Nucl. Soc.* 7, No. 1, 106 (1964), postirradiation data

G = from Robertson *et al.*, *J. Nucl. Mater.*, 7, 225 (1962), $\int_{2800}^{\infty} k dt = 97$ W/cm.

mal conductivity is, in reality, due to thermal stress cracking. They observed that the magnitude of conductivity reduction increased as the gap between cladding and pellet increased. When a zero gap existed, essentially no conductivity reduction was observed. They suggested that when cracking occurs, the pellet segments move apart to the degree allowed by the available fuel space; thus, the small gaps introduced into the pellet reduce the effective conductivity of the fuel. Since the amount of cracking gradually increases up to some maximum with operation, the effect could be irradiation related. MacDonald and Weisman⁴⁶ provided a quantitative analysis of this effect (Sec. 2.2.5).

There is now general agreement that most of the observed changes in apparent UO_2 conductivity with irradiation are due to cracking of the UO_2 and subsequent healing of the cracks with UO_2 swelling. However, current design approaches generally note that some changes do occur due to the buildup of fission products and plutonium in the fuel and the changes in fuel porosity with irradiation. The effect of plutonium content is considered in Eq. (2.11) by its effect on the parameter B and by inclusion of the phonon contributions to specific heat (C_v), which vary with fuel composition. The thermal conductivity decreases somewhat as Pu content increases.

Perhaps the most important effect of high burnup on the conductivity of uncracked UO_2 is the introduction of fission products in the lattice.* This results in a reduction of the phonon contribution to thermal conductivity. For thermal conductivity equations such as Eq. (2.11) where the phonon contribution has the form $(A + BT)^{-1}$, it has been proposed that A be written as $[1 + C (\text{burnup})]$. Unfortunately, there are now (1992) no values for C in the open literature.

Krammen and Freeburn⁴⁸ propose a more readily used burnup correction. They simply multiply the conductivity of unirradiated UO_2 by a correction factor. They propose

$$k_r = \begin{cases} k_0(1 - 5.0 \times 10^{-6} \text{Bu}) & \text{for } 0 \leq \text{Bu} \leq 2 \times 10^4 \\ k_0[0.9 - 4.0 \times 10^{-6}(\text{Bu} - 2 \times 10^4)] & \text{for } 2 \times 10^4 < \text{Bu} \leq 3 \times 10^4 \\ 0.86 k_0 & \text{for } 3 \times 10^4 < \text{Bu}_{13} \end{cases} \quad (2.12)$$

where Bu is the average burnup at given axial location (MWd/MTU) and k_0 and k_r are the unirradiated and irradiated thermal conductivity, respectively.

Verbeek and Hoppe⁴⁷ recommend that, when computing the thermal conductivity of irradiated UO_2 , the actual porosity of the irradiated fuel

*Bagger, Mogensen, and Walker [J. Nucl. Mater. **211**, 11 (1994)] find that there is also a marked decrease in the thermal conductivity of the fuel at the outer rim of the pellet. At burnups above about 40 GWd/tonne U, the microstructure of the fuel in the outer 2% of the pellet radius changes due to the presence of a high concentration of small gas-filled pores. This reduces the rim-region thermal conductivity to less than 1 W/m K.

matrix be used. Some designers suggest that the effect of irradiation on fuel matrix porosity be combined with the effect of fuel cracking. Krammen and Freeburn⁴⁸ suggest that both of these effects be accounted for through a porosity correction that has the form:

$$k = k_0(1 - 0.971\alpha - 6.06\alpha^2) \quad (2.13)$$

where k is the thermal conductivity of uncracked UO_2 and $\alpha = \alpha_p + \alpha_{\text{relo}}$; α_p is the fuel matrix porosity (void fraction) allowing for original fuel porosity and increase in fuel matrix porosity due to fission gas accumulation (gaseous component of fuel swelling); and α_{relo} is a user-specified fraction of the relocation crack volume addition to fuel porosity.

Early studies of the melting point of UO_2 by Christensen⁴⁹ indicated that the melting point of UO_2 is a function of the total irradiation received by the fuel (see Fig. 2.5). Christensen⁴⁹ found an initial increase in the melting point and then a gradual decrease with burnup. Later studies⁵⁰ have confirmed the decrease in melting point with extended burnup but there is some indication that the initial increase at low burnups may be spurious.

When less than the stoichiometric amount of oxygen is present, the melting point is reduced. The data given by Guinet *et al.*⁵¹ show that $\text{UO}_{1.8}$

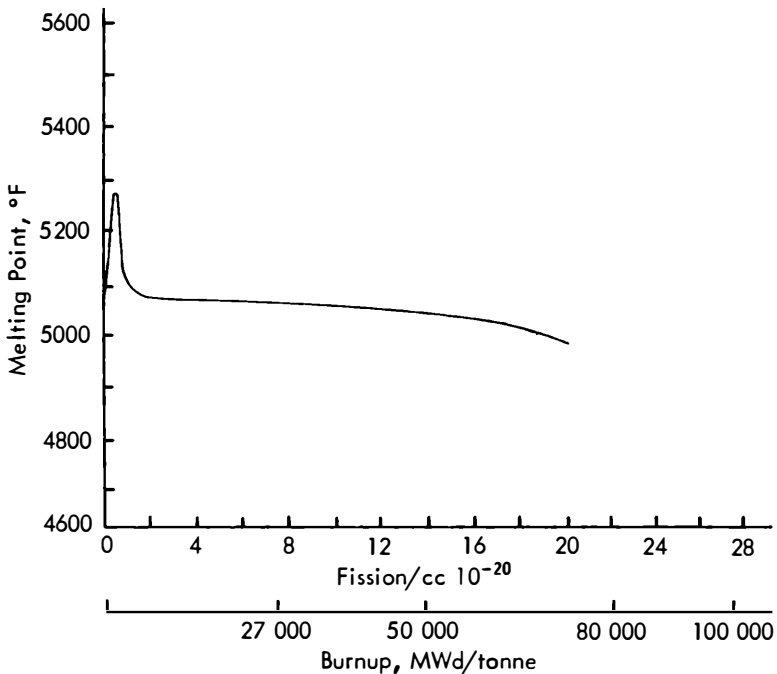


Fig. 2.5 Melting point of UO_2 as a function of burnup [from Ref. 49].

begins to melt at 2690°C in contrast to 2750°C for pure UO_2 . Komatsu *et al.*⁵⁰ have correlated the melting point of plutonium and uranium oxide mixtures as a function of plutonium content, metal-to-oxygen ratio, and burnup. They indicate that the temperature at which melting begins, T_m , can be correlated by

$$T_m = \left[\frac{1 - .2388(2 - \text{O/M})}{1 + 0.1811Y - 0.0111Y^2} \right] T_{m(\text{UO}_2)} \quad (2.14)$$

where

$T_{m(\text{UO}_2)}$ = melting point of pure unirradiated UO_2

$Y = x + 0.016\text{Bu}$

Bu = burnup, at. %

x = mole fraction of Pu in mixture

O/M = oxygen-to-metal mole ratio.

A brief summary of UO_2 properties is provided in Table 2.II and extensive information is provided by Belle,⁵² MacDonald and Thompson,⁵³ and Hagrman *et al.*³⁴

Mixed thorium and uranium dioxides were produced for use as a reactor fuel with the spectral shift control concept. Subsequent interest centered

TABLE 2.II
Thermal Properties of Ceramic Fuels

Fuel	UO_2	UO_2 -80 at. %* PuO_2 -20 at. %	ThO_2
Fuel density (g/cm ³)	10.97	11.08	10.01
Melting temperature (°F)	4980 (see Fig. 2.5)	5036	5970
Thermal conductivity [Btu/(h ft °F)]	2.54 (930°F) 1.04 (3630°F) (see Fig. 2.3)	2.02 (930°F) 1.04 (3630°F)	7.29 (200°F) 5.34 (400°F) 3.59 (700°F) 2.68 (1000°F) 2.07 (1400°F) 1.68 (2400°F)
Thermal expansion (in./in. °F)	6.12×10^{-6} (75 to 5070°F) (see Fig. 2.2)	6.12×10^{-6} (75 to 5035°F)	
Heat capacity (Btu/lb °F)	0.057 (90°F) 0.076 (1350°F) 0.090 (3150°F) 0.12 (4050°F)	0.057 (90°F) 0.075 (1350°F) 0.088 (3150°F) 0.11 (4050°F)	0.055 (90°F) 0.070 (1350°F) 0.078 (3150°F) 0.082 (4050°F)

*The 20% PuO_2 mixture of Table 2.II is typical of that required for fast reactor fuels. LWR fuels would be in the range of 3 to 5% PuO_2 and their properties would be very close to those of UO_2 since the addition of even 20% PuO_2 causes little change in properties.

TABLE 2.III
Constants (Ref. 39) for Thermal Conductivity Correlation

Material	A_0	B_0
ThO ₂	0	2.21×10^{-4}
ThO ₂ + 10 wt% UO ₂	0.0225	1.78×10^{-4}

around the use of ThO₂ as a blanket in a seed-and-blanket thermal reactor. More recently, there has been some discussion of the use of mixed ThO₂-UO₂ fuel in CANDU reactors.⁵⁴

The behavior of ThO₂ under irradiation is very similar to that of UO₂. Thermal conductivity is affected by fuel porosity and thermal stress cracking in the same manner as UO₂. Belle *et al.*³⁹ were able to correlate the conductivity of ThO₂ and mixed ThO₂ + UO₂ by using almost the same polynomial as they used for UO₂. For unirradiated ThO₂ and ThO₂ + UO₂ they recommend

$$k = \left(\frac{1 - \alpha}{1 + \beta\alpha} \right) [A_0 + B_0 T + (6.23/T) + (49.82/T^2)] \quad (2.15)$$

where

k = thermal conductivity, Btu/(h ft °F)

T °R

β 0.5 for fuels of 90% theoretical density and above; $\beta = 0.7$ for lower densities

α = fuel void fraction.

The constants A_0 and B_0 are given in Table 2.III.

In-pile thermal conductivities of ThO₂-PuO₂ mixtures containing 1.1 to 2.7 wt% PuO₂ were measured by Jeffs,⁵⁵ who observed conductivities ~25% below those obtained by laboratory measurements of pure ThO₂. Within the scatter of the data, he saw no difference between the compositions tested.

Godfrey *et al.*⁵⁶ measured the specific heat of ThO₂ over a wide range. These and other thermal properties are summarized in Table 2.II.

2.1.1.4 Mixed Plutonium and Uranium Oxides

Interest in UO₂-PuO₂ fuels originally stemmed from their application to fast breeder technology. Renewed interest in the behavior of MOX fuels arose with their use in some European PWRs. As expected on the basis of the similarity of the two compounds, there is little difference in the physical behavior of mixed oxides and UO₂.^{*} In view of the small amount of PuO₂

^{*}Although PuO₂ and UO₂ are similar physically, there are significant neutronic differences that require some modification in core design (see Sec. 1.2.2.2).

present in PWR fuel (<7%), use of UO_2 properties for those of the mixture does not lead to major errors. However, properties of PuO_2 are now available and more accurate estimates of mixture properties can be made.

The specific heat of PuO_2 may be correlated by the same expression used for UO_2 [Eq. (2.7)] if the following constants are substituted³⁴

$$\begin{aligned} K_1 &= 347.4 \text{ (J/kg K)} & K_2 &= 3.95 \times 10^{-4} \text{ (J/kg K}^2\text{)} \\ K_3 &= 3.86 \times 10^7 \text{ (J/kg)} & \theta &= 571.0 \text{ (K)} \\ E &= 1.967 \times 10^{-5} \text{ (J/mol)} \end{aligned}$$

For MOX fuel, Hagrman *et al.*³⁴ recommend

$$C_p = (C_p)_{\text{UO}_2} (1 - F) + (C_p)_{\text{PuO}_2} F \quad (2.16)$$

where $(C_p)_{\text{UO}_2}$ and $(C_p)_{\text{PuO}_2}$ are the specific heats of UO_2 and PuO_2 , respectively, and F is the weight fraction of PuO_2 . The specific heat of molten UO_2 or PuO_2 is given³⁴ as 503 J/kg K.

Equation (2.11) for UO_2 thermal conductivity contains provision for obtaining the thermal conductivity of MOX fuel. The parameter B in this equation is a function of plutonium content. Further, the function C_v would be taken as the composition weighted average of the first term of the specific heat equation. The effect of the oxygen-to-metal ratio on mixed oxides is also included in Eq. (2.11).

The thermal expansion of PuO_2 for $0^\circ\text{C} < T < T_{\text{melt}}$, may be obtained from⁵³

$$\frac{\Delta L}{L} = -3.974 \times 10^{-4} + 8.496 \times 10^{-6} T + 2.1513 \times 10^{-9} T^2 + 3.714 \times 10^{-16} T^3 \quad (2.17)$$

where T is the temperature ($^\circ\text{C}$). The thermal expansion of MOX fuel can be estimated by a weighted average of the expansion of UO_2 and PuO_2 in the same fashion as used for specific heat in Eq. (2.16).

When the plutonium is introduced into the UO_2 matrix via neutron irradiation, one would expect a fairly uniform distribution of the plutonium in the UO_2 matrix (with proper allowance for flux depression in the fuel center). The property averaging approaches indicated earlier should clearly apply. However, when MOX fuel is fabricated, the resulting pellets have a heterogenous distribution of plutonium on a microscopic scale. The microstructure of such pellets is characterized by isolated highly Pu enriched particles embedded in a matrix of UO_2 (Ref. 57). After irradiation, this microstructure is preserved in the cooler regions of the pellet but a homogeneous structure is seen in the high-temperature regions. However, autoradiography shows that the Pu is still concentrated at local spots. One would expect this microscale segregation to lead to a situation where the Pu particles are slightly hotter than the surrounding UO_2 matrix.

In view of the segregation of PuO_2 MOX fuels, it might be preferable to treat the fuel as a dispersion for the purpose of computing a thermal conductivity of the mixture.

Epstein⁵⁸ measured the melting points of UO_2 - PuO_2 mixtures and observed melting points of $2840 \pm 20^\circ\text{C}$ for UO_2 and $2390 \pm 20^\circ\text{C}$ for PuO_2 . His data show a typical binary pair without a maximum or minimum in the liquidus or solidus curves and, for small additions of PuO_2 , his results can be closely approximated by assuming a linear decrease in melting point per atom percent of PuO_2 addition. Krankota and Craig⁵⁹ observed that the melting point of mixed oxides decreases slightly with burnup. After 85,000 MWd/tonne burnup, the melting point of $(\text{Pu}_{0.25}\text{U}_{0.75})\text{O}_2$ decreased 50°C .

Significant thermal design properties for $(\text{U}_{0.8}\text{Pu}_{0.2})\text{O}_2$ and UO_2 are summarized in Table 2.II. Properties of other mixtures can be estimated by interpolation. More accurate values of thermal conductivity, specific heat, and thermal expansion are available from the equations given in this section.

2.1.2 Fuel Element Cladding and Assembly Designs

2.1.2.1 Cladding Properties

Although the most common PWR fuels have good corrosion resistance, continued exposure of any of these to the coolant would result in coolant activity levels far beyond those consistent with direct maintenance. Therefore, it is the universal practice to clad various fuels by a thin layer of corrosion-resistant metal. Ideally, the cladding should be inert to the coolant, have a high strength and ductility that are unaffected by radiation, have a low-neutron-absorption cross section, and be fabricated easily and economically. Since no such material is known, one must sacrifice some attributes to gain those believed most important.

Modern PWRs use stainless-steel-clad or Zircaloy-alloy-clad fuel exclusively. Low neutron cross section, excellent corrosion resistance to low-temperature water, and low cost made aluminum an early candidate for this use. Unfortunately, common aluminum alloys do not have the necessary high-temperature strength or corrosion resistance in high-temperature water to allow them to be considered. Argonne National Laboratory (ANL) conducted extensive investigations on various aluminum alloys containing small amounts of nickel (i.e., 1%) and lesser amounts of iron and other constituents. Although these alloys have significantly reduced corrosion rates, ANL found the rates are still excessive at PWR conditions.⁶⁰

Stainless (Austenitic) Steel. With excellent corrosion resistance and high strength, the 300-series stainless steels possess many of the properties of an ideal cladding. Although their properties are slightly affected by radiation (yield point increases and ductility decreases appreciably), their radiation resistance is good, and they are readily fabricated without special tech-

niques. Their major drawback is a relatively high neutron cross section, which requires an increase in initial enrichment. These alloys are not suitable for reactors where natural uranium is the fuel.

Stainless steel is known to be subject to stress-assisted corrosion and stress corrosion cracking in high-temperature water when oxygen and halogens are present. Duncan *et al.*⁶¹ reported stress-assisted corrosion failures of this nature in the cold-worked stainless cladding of BWR fuel. No such cracking has been reported in PWRs, perhaps because of the lower oxygen content of the coolant. However, it presents a severe problem in superheat systems, where experience has been poor, and would be troublesome in designing a supercritical PWR.

The stainless alloy chosen for a particular case depends on the fabrication techniques used. All of the 300-series alloys have very nearly the same thermal design properties. Typical values are presented in Table 2.IV

The cladding stress levels reached during irradiation depend strongly on the thermal expansion and creep of the cladding, and highly accurate values

TABLE 2.IV
Properties of Cladding Materials

Material	Zircaloy-4	347 Stainless Steel
Composition	1.3 to 1.5% tin 0.2 to 0.22% iron 0.1% chromium 0.007% max nickel balance zirconium	17 to 19% chromium 9 to 12% nickel 0.8% columbium 0.2% max manganese 0.08% max carbon 1% max silicon
Density* (g/cm ³)	6.57 (room temperature)	8.03 (room temperature)
Melting point	3320°F	2550 to 2600°F
Thermal conductivity [Btu/(h ft °F)]	7.71 (100°F) 8.23 (200°F) 8.76 (400°F) 9.59 (600°F) 10.43 (800°F) 11.9 (1200°F)	8.6 (100°F) 9.0 (200°F) 9.8 (400°F) 10.6 (600°F) 11.5 (800°F) 12.4 (1000°F)
Mean coefficient* of thermal expansion (in./in. °F)	3.21 × 10 ⁻⁶ (25 to 800°C) (rolling direction) 5.17 × 10 ⁻⁶ (25 to 800°C) (transverse direction)	9.05 × 10 ⁻⁶ (68 to 100°F) 9.25 × 10 ⁻⁶ (68 to 200°F) 9.55 × 10 ⁻⁶ (68 to 400°F) 9.8 × 10 ⁻⁶ (68 to 600°F) 10.0 × 10 ⁻⁶ (68 to 800°F) 10.25 × 10 ⁻⁶ (68 to 1000°F)
Heat capacity* [Btu/(lb °F)]	0.073 (200°F) 0.076 (400°F) 0.079 (600°F) 0.081 (800°F) 0.083 (1000°F) 0.085 (1200°F)	0.12 (32 to 212°F)

*No distinction is made between properties of Zircaloy-2 and Zircaloy-4

of these parameters are desired. Smith and Was⁶² give the circumferential thermal expansion of 304 stainless as

$$\frac{\Delta D}{D_0} = -4.75 \times 10^{-4} + 1.90 \times 10^{-5} T \quad (2.18)$$

where $\Delta D/D_0$ is the thermal strain and T is the temperature ($^{\circ}\text{C}$). At the temperatures of PWR operation, 300 series (austenitic) stainless steel will not creep if not subjected to fast neutron irradiation. However, in the presence of such irradiation, significant creep can be observed. This irradiation creep can be correlated by an equation originally due to McElroy *et al.*⁶³ and slightly modified by Smith and Was.⁶² For 304 stainless steel under uniaxial tensile test conditions,

$$\begin{aligned} \epsilon_T = & 8.70 \times 10^{-7} \sigma_T \{1 - \exp[\phi t (2.17 \times 10^{20})]\} \\ & + 36.12 \times 10^{-28} \bar{E} \sigma_T \exp(1.44 - 0.0027T) \end{aligned} \quad (2.19)$$

where

ϵ_t = tensile strain (length/length)

ϕ_t = total fluence of fast neutrons ($E > 1$ MeV), n/cm^2

$\bar{E}$ = mean neutron energy of fast neutrons, MeV

σ_T = tensile stress, MPa

T = temperature, $^{\circ}\text{C}$.

Note that, under PWR conditions, stainless-steel creep is only a small fraction of that observed with Zircaloy. More elaborate descriptions of stainless-steel creep that combine stress-related (so-called "thermal creep") and irradiation creep, along with transient creep, are available.⁶⁴ However, while these are needed for fast reactor design, this level of complexity is generally not required for PWR analyses.

In the usual PWR configuration, use of stainless-steel cladding instead of Zircaloy would require that the enrichment be increased by approximately 1%. Because of the substantial cost increase this would occasion, only four PWRs currently (1993) use stainless-steel cladding. This situation could be changed by the development of tight-lattice cores where the epithermal neutron spectrum results in a much lower economic penalty for stainless steel.

Zirconium Alloys. Hafnium-free zirconium has a very low-absorption cross section (0.18 b compared to 2.43 b for iron and 4.5 for nickel) and good resistance to water corrosion at high temperatures. The addition of small amounts of tin and iron significantly improves the strength of zirconium. As a result of extensive investigations at the Bettis Atomic Power

Laboratory, three alloys have been developed that have the required strength and corrosion resistance—Zircaloy-2, -3, and -4.

Zircaloy has been almost universally used as the fuel element cladding in light-water-cooled reactors built for central power production. Zircaloy-4 (1.3% tin, 0.22% iron, 0.10% chromium, balance zirconium) has generally been used for PWR fuel cladding rather than Zircaloy-2 or -3. This is due to the greater resistance to hydrogen embrittlement shown by Zircaloy-4. However, the Russian VVER reactors have obtained generally satisfactory cladding behavior using a zirconium alloy containing 1% Nb. Fuel assemblies designed for very high burnup tend to be fabricated using newer proprietary alloys, such as Zirlo. These show significantly lower corrosion rates after long-term exposure to the reactor coolant.

Although the melting point of zirconium is high (1852°C), a phase change at 862°C affects the mechanical properties. At this temperature, pure zirconium goes from a close-packed hexagonal structure, α , to one that is body-centered cubic (bcc). Zircaloy, however, transforms over a range of temperatures. Pure Zircaloy begins to transform at 821°C and completes the transformation at 960°C (Ref. 34). Transformation temperatures are significantly increased with increasing oxygen concentration. At an oxygen concentration of 2.5 wt%, the transformation begins at 1283°C and is completed at 2026°C (Ref. 34). This effect of oxygen is generally significant only during accident situations since, under typical operating conditions, the oxygen concentration in Zircaloy is only about 0.12 wt%. At operating conditions, the Zircaloy cladding is in the α phase.

Zircaloy tubing or plate is more expensive than stainless steel, and fabrication must be more carefully controlled. Oxygen must be excluded from all welds; welding must be done in an inert atmosphere, in a glove box or similar enclosure. Since zirconium alloys creep at relatively low stresses at PWR temperatures, allowance must be made in the mechanical design of the fuel elements. However, creep can be beneficial since it acts to relieve any high-cladding stress.

Zirconium and its alloys react significantly with steam at high temperatures [$\text{Zr} + 2\text{H}_2\text{O}(\text{g}) \rightarrow \text{ZrO}_2 + 2\text{H}_2(\text{g})$]. The energy release (6.45×10^6 J/kg Zr) due to this reaction must be considered when accidents that can give rise to high-cladding temperatures are analyzed. At temperatures above 500°C, the hydrogen produced and the oxygen of the oxide layer diffuse into the metallic phase, thereby reducing the ductility of the metal. Stainless-steel cladding at high temperatures also reacts chemically with steam. However, there the reaction rate does not become appreciable until temperatures in excess of 1000°C are reached.

If interfacial temperatures rise above 675°C, Zircaloy cladding can react with UO_2 . Since such temperatures are $< 400^\circ\text{C}$ in normal operation, the reaction needs to be considered only for accident situations.

At normal operating temperatures, zirconium reacts slowly with water to form zirconium oxide and hydrogen. At operating conditions, oxygen from the oxide layer does not diffuse into the metal, but hydrogen produced partly diffuses through the oxide layer into the metal. The quantity of hydrogen (given as a fraction of the total hydrogen formed) that diffuses into the metal is called the "pickup fraction." Zircaloy-4 has a pickup fraction between 0.005 and 0.2 (Ref. 65).

In-reactor corrosion of Zircaloy is significantly increased by fast neutron flux under oxygenated coolant conditions. However, under low-oxygen conditions, fast neutron flux increases the corrosion rate only slightly. This latter situation generally holds in a PWR since hydrogen overpressure in the pressurizer inhibits radiolysis. In the Saxton core,⁶⁶ weight gains on the order of 200 mg/dm² were observed after 500 full-power days at 339°C.

Present modeling of Zircaloy-4 corrosion generally considers it to occur in two stages. The model described by the Electric Power Research Institute provides for an initial nonlinear time dependence (pretransition) followed by a linear time dependence (post-transition).⁶⁷ The equations comprising the model are:

$$\text{Pretransition: } dS/dt = (A/S^2) \exp(-Q_1/RT_1) \quad (2.20)$$

$$\text{Post-transition: } dS/dT = [C_0 + U(M\phi)^P] \exp(-Q_2/RT_1) \quad (2.21)$$

$$\text{Thickness at transition: } S_t = D \exp(-Q_3/RT_1 - ET_1) \quad (2.22)$$

where

dS/dt = corrosion rate, $\mu\text{m/day}$

S = oxide layer thickness, μm

T_1 = metal-to-oxide interface temperature, K

ϕ = fast neutron flux, ($E > 1$ MeV), $\text{n}/(\text{cm}^2 \text{ s})$

The constant values are listed below:

Constant	Value	Constant	Value
A	$6.3 \times 10^9 \mu\text{m}^3/\text{day}$	Q_2	27,354 cal/mol
Q_1	32,289 cal/mol	D	$2.14 \times 10^7 \mu\text{m}$
C_0	$8.04 \times 10^7 \mu\text{m}/\text{day}$	Q_3	10,763 cal/mol
M	$1.91 \times 10^{-15} \text{cm}^2 \text{ s}/\text{n}$	U	$2.38 \times 10^8 \mu\text{m}/\text{day}$
P	0.24	E	$1.17 \times 10^{-2} \text{K}^{-1}$
		R	1.98 cal/(mol K)

Calculation of the metal-oxide interface temperature should allow for the temperature drop across the oxide ($k_{\text{oxide}} = 0.0168 \text{ W}/\text{cm K}$).

It has generally been established⁶⁸ that external oxidation of the cladding should not exceed about 100 μm . Tests indicate that this limit is reached at about 50 GWd/tonne burnup with Zircaloy-4. This burnup can be exceeded

by the use of newer zirconium alloy claddings. The Zirlo alloy, which contains 1% Sn and 1% Nb, has in-pile corrosion rates 30 to 50% below Zircaloy-4.⁶⁹ Alternatively, a duplex cladding having an outer layer ($\sim 100\ \mu\text{m}$) of Zircaloy with extra-low Sn (ELS) bonded to Zircaloy-4 may be used. The low-tin outer layer is more resistant to corrosion than the standard Zircaloy-4 cladding. Since the outer layer is so thin, the mechanical properties of the duplex cladding would be essentially those of Zircaloy-4.

The total corrosion can be related to the average hydrogen concentration, C_H , in Zircaloy by using

$$C_H [\text{ppm}] = 1.25 \times 10^{-2} \alpha' \frac{\Delta w}{c \rho_{\text{Zr}}} \quad (2.23)$$

where

α' = pickup fraction

Δw = total weight gain, mg/dm^2

c = cladding wall thickness, cm

ρ_{Zr} = zirconium density, g/cm^3

Hydriding of Zircaloy cladding can also originate from the internal cladding surface. In the presence of water contamination of UO_2 pellets, internal hydriding can lead to bulges or blisters on the cladding surface due to the 13% volume change in the $\text{Zr} \rightarrow \text{ZrH}_{1.6}$ reaction. Internal hydriding has been one of the most persistent sources of fuel rod failure. The attack is localized and seems to occur at local hot spots or regions where the zirconium oxide film has been damaged. To avoid such attack, concentration of water vapor in the gas inside a fuel rod must be kept below $2\ \text{mg}/\text{cm}^3$ (Ref. 70). Manufacturers generally specify that the water contents of UO_2 pellets be held at a few ppm so this critical concentration does not arise. Hydrogen trapped in pellet pores is sometimes removed by getters.

Fission products released from UO_2 can react chemically with zirconium. Iodine-induced stress corrosion cracking can occur when locally high stresses arise, such as during power ramps.⁷¹ The possibility of such brittle cracking means that considerable care must be exercised in the rate at which the power of any fuel assembly is increased. High local stresses can be generated in the cladding of CANDU reactor fuel elements when power ramping accompanies axial fuel shuffling. High-burnup fuel rods in vessel-type reactors can also experience high-cladding stresses during rapid load changes or on startup after refueling. A number of fuel failures have been attributed to this.⁷² This phenomenon is generally referred to as *pellet-cladding interaction* and fuel failures attributed to this are designated PCI failures.

Irradiation of zirconium and its alloys increases their hardness and decreases the ductility. This decrease in ductility must be considered in de-

signing reactor components such as pressure tubes. In fuel element design, it often leads to a limitation on the maximum allowable cladding strain.

Thermal and mechanical properties of Zircaloy-4 are presented in Table 2.IV A comprehensive property compilation is available in MATPRO.³⁴

High Nickel Alloys. Sensitivity of the 300-series stainless steels to stress-corrosion cracking under boiling and superheat conditions, as well as the requirement for strength at high temperatures, has led to the consideration of high-nickel alloys for supercritical reactors. Various Inconel and Incoloy compositions (up to 72% nickel as contrasted to 8% nickel for 300-series stainless) have been proposed. These alloys are resistant to stress-corrosion cracking and have good high-temperature strength. Allio and Thomas⁷² reported that Westinghouse 16–20 alloys (16% chromium and 20% nickel with low carbon and nitrogen) behaved satisfactorily at high fluxes in an environment of supercritical water. These alloys have a macroscopic neutron absorption cross section of 0.266/cm as compared to 0.300/cm for Incoloy. Reference 53 presents the design properties of a number of high-nickel alloys. There is no longer (1995) any significant interest in these materials for water-cooled reactor use.

2.1.2.2 Fuel Assembly Designs

Rods. Metallic, low-enrichment uranium fuel has almost always been used in the form of clad, solid or annular, cylindrical rods. Tubing for the cladding is commercially available, fabrication is relatively simple, and the necessary surface area is obtained by proper selection of rod size. When aluminum is used as the cladding, a diffusion bond between the cladding and fuel is usually formed by a thin layer of aluminum-silicon alloy. Zircaloy-clad elements were formed for the Hanford New Production Reactor (NPR) by coextrusion of a copper-zirconium-uranium billet, with the copper stripped away after extrusion.

Rod elements are also most commonly used for uranium dioxide fuel; they are generally formed by loading pressed and sintered UO_2 pellets into Zircaloy or stainless steel* tubes to which end plugs are welded. Individual rods are assembled into bundles and end plates, and fittings are affixed. Figure 2.6 shows one of the earlier assemblies for a pressurized water design where brazed ferrules hold the rods together. In more recent designs (Fig. 1.4), the rods have been positioned by several grids containing spring fingers. This design allows higher strength cold-worked cladding to be used, which reduces the amount of absorber in the core and allows the fuel rods to expand, which reduces thermal stresses.

*All recent designs have used Zircaloy, or other zirconium-based alloys, as cladding in order to minimize neutron absorption.

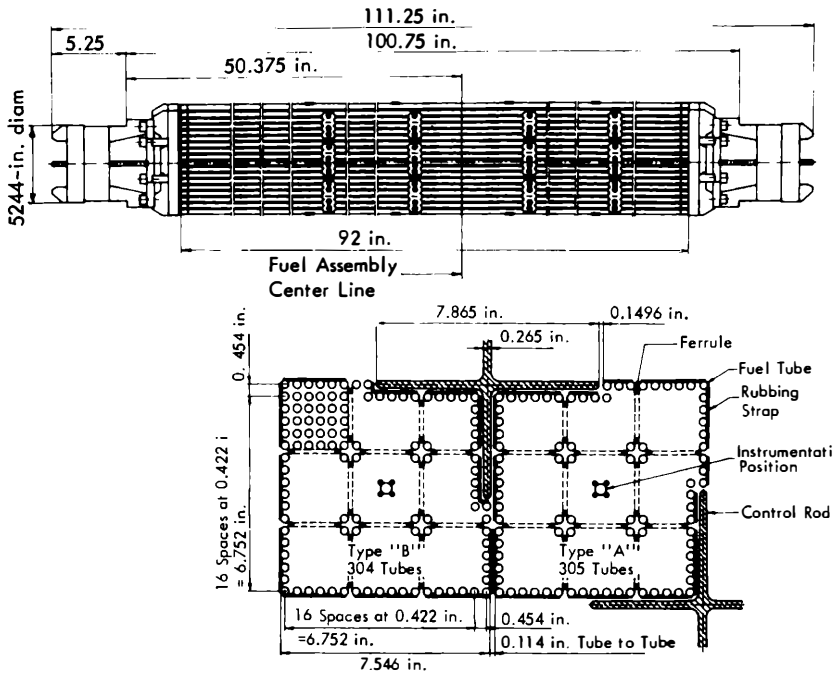


Fig. 2.6 Original Yankee fuel assembly [from *Directory of Nuclear Reactors*, Vol. IV, p. 34, International Atomic Energy Agency, Vienna (1962)].

As previously noted, the newer assembly designs also eliminate the cut-outs for cruciform control rods. Instead, a typical 15×15 rod assembly would contain 20 hollow tubes serving as guides for rod cluster control (RCC) elements.

Each fuel rod contains a stack of short (e.g., 0.7-in.-long) pellets. Prior to being loaded into the cladding, the pellets are centerless ground to an outside diameter, which is about 0.007 to 0.008 in. less than the cladding inside diameter. In addition, the pellets are usually ground to provide dished ends. During reactor operation the center region of the pellet is hotter and expands more than the outer region. The dishing is designed to produce nearly flat pellet ends at power.

A free gas volume (plenum) is provided in the upper few inches of each fuel rod. The plenum volume is sized so that the fission gases released during operation do not produce excessive internal pressure at the end of life. A hold-down spring placed in the plenum ensures that handling of the fuel rod will not result in movement of the pellet stack.

The rods are generally initially pressurized with helium. This improves the heat transfer across the pellet-to-cladding gap and reduces cladding creep-down due to the external pressure.

Solid pellets have been used in the fuel rods of the vast majority of operating reactors (1995). However, hollow annular pellets have been used in the Russian VVER reactor fuel elements.

Vibratory compacted UO_2 has been considered as a substitute for sintered pellets. Because of the difficulty of producing very-high-density compacts and the greater tendency of vibratory compacted material to wash out through cladding pinholes, vibratory compacted fuel has not been used (1994) for operating plants. The need to fabricate $\text{PuO}_2\text{-UO}_2$ fuel elements remotely has, however, led to renewed interest in this approach.

Table 2.V provides typical assembly and fuel rod dimensions for a 15×15 assembly designed for a large (800- to 1200-MWe) PWR. In the later 17×17 assembly designs, rod pitch and rod and pellet diameters are reduced by about 10% from those indicated in Table 2.V

Fuel assemblies designed for the CANDU pressure tube reactor are considerably different from those used in vessel-type PWRs. Since such an assembly must fit within a pressure tube, it contains only a small number—19 to 37—of fuel rods. These assemblies have all been short with bundle lengths no more than six times the diameter of the rod cluster. In one design, a completely integral structure with end plates welded to the ends of all the pins is provided.⁷¹ Spacing between the fuel rods is maintained by metal warts brazed obliquely to the fuel pins. In other designs for pressure tube assemblies, wire wraps, ferrules, and spacer pads have been used to maintain rod spacing. A drawing of a typical CANDU fuel assembly is shown in Fig. 2.7

Plate-Type Elements. Designs for highly enriched uranium-Zircaloy elements and UO_2 -stainless-steel dispersion elements are similar. In both cases, the so-called “picture frame” method is used for fabrication, which means

TABLE 2.V
Typical Assembly and Fuel Rod Dimensions for Large PWRs

Rod array	15×15
Fuel rods per assembly (20 RCC thimbles, 1 in-core flux detector thimble)	204
Assembly length	159.7 in.
Rod pitch	0.563 in.
Rod o.d.	0.422 in.
Pellet o.d. (cold)	0.366 in.
Cladding thickness	0.024 in.
Cladding material	Zircaloy-4
Overall fuel rod length	151.8 in.
Active fuel length	144 in.
Number of support grids	7 (6 in active core)
Length of gas plenum	~5 in.

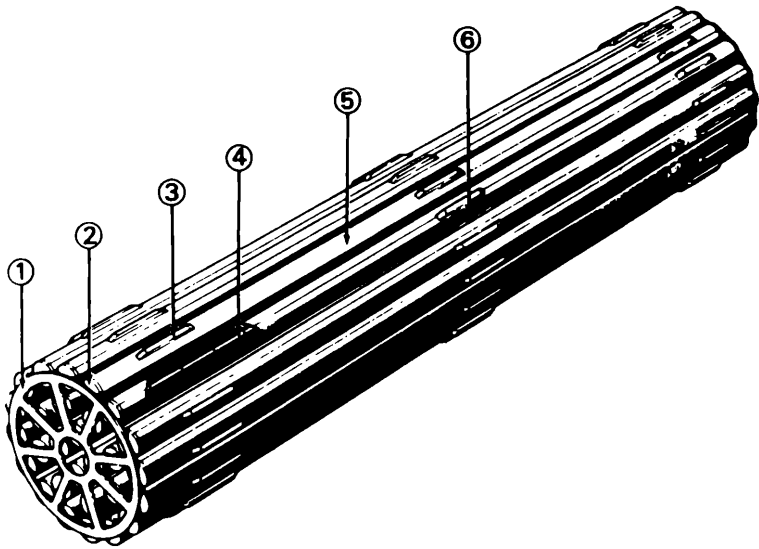


Fig. 2.7 CANDU fuel element: (1) Zircaloy structural end plate, (2) Zircaloy and cap, (3) Zircaloy bearing pads, (4) uranium dioxide pellets, (5) Zircaloy fuel sheath, (6) Zircaloy spacers [from J. A. L. Robertson, AECL-4520, Atomic Energy of Canada Ltd. (1973)].

that a sheet of the fuel alloy, or dispersion, is rimmed by a frame of cladding. Upper and lower sheets of cladding are then welded to the frame and the pack is rolled to final dimensions. A group of fuel plates are then placed in a box-like enclosure made of cladding material. This maintains plate spacing and provides an assembly that can be handled. Plate-type elements containing uranium-Zircaloy fuel used in Shippingport Core I are illustrated in Fig. 2.8. Plate-type elements using a UO_2 -stainless-steel dispersion as fuel have been used in the Army Package Power Reactor (APPR).

Plate-type elements for UO_2 fuel have been used in Shippingport Core II. Rectangular platelets of UO_2 fuel are placed in a slotted Zircaloy plate, which is sandwiched between two Zircaloy cover plates. The fairly substantial Zircaloy plates served to restrain the UO_2 platelets under high burn-up. Such plates can be fabricated by the gas-pressure bonding process⁵ in which the fuel element can be assembled from plates and strips that are subsequently bonded together at high pressures and temperatures.

Although plate-type fuel elements are no longer considered for use in central power station cores, they are obviously of interest for naval reactors. Plate-type fuel elements can provide the high power density and compact cores needed in this application.

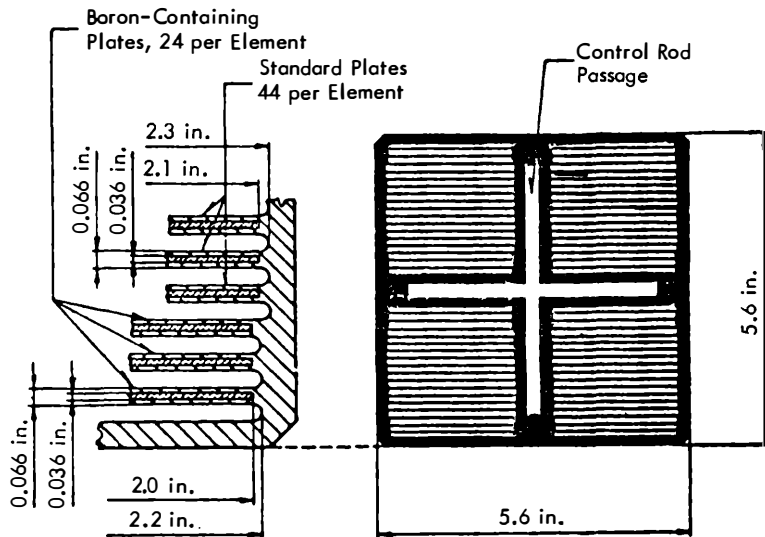


Fig. 2.8 Shippingport Core 1 seed assembly [from *Directory of Nuclear Reactors*, Vol. IV, p. 21, International Atomic Energy Agency, Vienna (1962)].

Tubular Fuel Elements. Tubular fuel elements can be fabricated from sandwiches of dispersion fuel and cladding. Standard stainless-steel forming operations can be used to make the sandwich sheets.⁷⁴ Elements of this type were used in a portable reactors (PM series) designed for the U.S. Army nuclear power program.

Annular fuel elements, such as those of the first USSR power station, can be formed from hollow UO_2 pellets. Various vibratory compaction techniques can also be used for annular fuel element fabrication. Multipass, concentric ring, annular elements in individual pressure tubes have been proposed for supercritical pressure reactors.⁷⁵

2.1.2.3 Advanced Fuel Assembly Designs Using Cylindrical Fuel Rods

The newer assembly designs for conventional PWRs incorporate a number of significant improvements. These changes have been made in order to reduce fuel failure frequency, allow extended burnup, increase thermal margins, and improve neutron economy. A typical advanced fuel assembly is shown in Fig. 2.9.

All of the advanced designs have improved bottom nozzles, which are designed to block debris entry and thus reduce debris-caused failures. In many cases, this is simply accomplished by using small flow holes in the nozzle instead of the large size openings previously used. In some designs, the debris removal is enhanced by incorporating an additional antidebris filter in the form of a screen under the nozzle. Alternatively, the lower

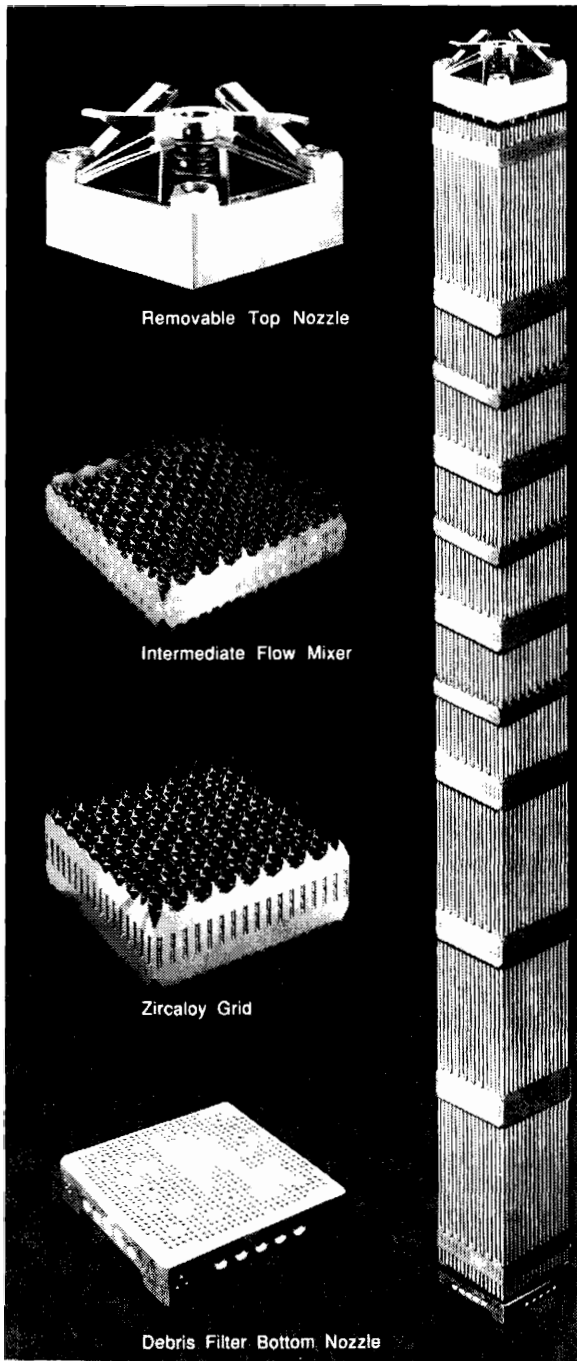


Fig. 2.9 Typical advanced fuel assembly design (courtesy Westinghouse Electric Corp.).

nozzle plate may consist of a parallel array of closely spaced curved blades configured so as to preclude debris entry.

In most advanced assembly designs, the stainless or Inconel spacer grids have been replaced by all-Zircaloy grids. In one design, the grids have been replaced by Zircaloy but the springs holding the rods in place are Inconel. There is some evidence that Zircaloy springs tend to relax during irradiation and the use of Inconel for the springs avoids this problem. Additional absorber is removed from the assembly by using Zircaloy for the control element guide tubes. These changes significantly reduce the parasitic neutron absorption and allow lower fuel enrichments to be used.

In essentially all of the advanced assembly designs, the spacer grids contain small mixing vanes that enhance interchannel mixing. Most advanced designs also contain intermediate flow mixer grids in the high-flux region of the upper portion of the assembly. These low-pressure drop grids do not provide rod support but do contain mixing vanes that increase interchannel mixing in the region where the critical heat flux is of concern. They therefore provide additional thermal margin.

A removable upper nozzle is used in all of the advanced assembly designs. If an assembly contains failed fuel elements and has a fixed upper nozzle, the entire assembly must be removed during refueling. Ultrasonic testing techniques, which compare the time required for a signal to proceed around the cladding of failed and unfailed elements, now allow individual failed rods to be located within an assembly. Therefore, with a removable top nozzle, only those rods that have failed need to be removed. The assembly may then be reconstructed, using replacement rods only for the failed fuel elements, and then returned to the core.

The top nozzle is now provided with flexible clamps and/or spring assemblies that hold the assemblies firmly in position against the upper core plate. In a few assembly designs, springs are used to hold the individual fuel rods in position against the top nozzle.

The fuel rods that constitute the assembly have also been modified to allow for higher burnup and greater neutron economy. The early spring-finger assembly designs generally used a 15×15 array. In response to LOCA considerations, the array was subsequently increased to 17×17 and rod sizes were reduced to about 0.375 in. More recently, some designs have reduced rod sizes to about 0.36 in. The increased moderator-to-uranium ratio that this produces slightly reduces fuel costs. A further reduction in fuel costs is achieved by reducing neutron leakage through the use of axial blankets of natural or depleted uranium oxide pellets.

In earlier designs, control of reactivity in excess of that provided by the control rods was provided by burnable absorber rods placed in unused control element guide tubes. Excess reactivity control requirements are increased in the advanced designs because of the high burnup desired. In all of the advanced assembly designs, excess reactivity control is provided by

burnable poisons that coat or are mixed with the UO_2 of some fuel rods (see Chapter 1, Sec. 1.2.3.3).

High fuel burnup leads to increased fission gas release and increased axial growth of the fuel rods. The increased fission gas release is accommodated by increasing the length of the gas plenum. Increased rod growth is accommodated by increasing the clearance between the top nozzle and the rods.

The assemblies described in this section have been designed for use in conventional vessel-type PWRs. As indicated in Chapter 1 and Sec. 2.1.2.2, fuel assemblies for pressure-tube cores and for tight-lattice, high-conversion cores differ substantially from the designs of this section.

2.2 BEHAVIOR OF UO_2 FUEL ELEMENTS

Since fuel elements consisting of UO_2 pellets in cylindrical Zircaloy tubes are used in nearly all power reactors, the special design properties of these elements are examined.*

2.2.1 Mechanical Properties of UO_2

At temperatures below $\sim 1000^\circ\text{C}$, UO_2 is a brittle material. Below the brittle to ductile transition temperature, T_c , fracture stress is smaller than yield stress and brittle fracture occurs. Hence, at relatively modest thermal stresses, cracking of a UO_2 pellet occurs. Below T_c , the stress-strain curve is linear and Young's modulus, E , can be obtained from⁷⁶

$$E = 22.9 \times 10^4 - 20.1T - 58.7 \times 10^4 \alpha \quad (2.24)$$

where E is in N/mm^2 , T is in K, and porosity α is given as a fraction. The value of E depends slightly on the oxygen-uranium ratio. If the oxygen-uranium ratio is increased from 2.0 to 2.16, an $\sim 22\%$ decrease in E can be expected.⁷⁶

Since the fuel is much stiffer than the cladding, fuel design computations are insensitive to the value of E used for the fuel. Some computational procedures, therefore, take E as a constant (e.g., $2 \times 10^{11} \text{ N/m}^2$).

The typical value of Poisson's ratio, γ , is reported to be 0.316 (Ref. 76). No data are available on temperature dependence, but other ceramic materials show little effect of temperature on γ . The dependence on porosity is very small.

*Interest in burning plutonium removed from nuclear warheads could lead to some use of ceramic fuels that are not primarily UO_2 . To avoid production of additional Pu during irradiation, the Pu would have to be dispersed in an inert material. Akie *et al.* [*Nucl. Technol.* 107, 182 (1994)] indicate that a mixture of PuO_2 - Al_2O_3 and stabilized zirconia (ZrO_2 plus small amounts of YO_2) should be suitable for once-through burning of weapons plutonium.

Fracture stress σ_f of UO_2 depends on porosity, grain size, and temperature. The effect of porosity is small⁷⁶ and can be neglected. Both temperature and grain size have significant effects, although there seems to be some disagreement as to the magnitude of these effects. Stehle *et al.*⁷⁶ suggest that fracture stress can be expressed by

$$\sigma_f = \sigma_{f_0} + AT \quad (2.25)$$

where σ_f is in N/mm^2 and T is in K. Evans and Davidge⁷⁷ and Canon *et al.*⁷⁸ indicate that σ_{f_0} varies from ~ 98.5 to 151 and A from 0.025 to 0.028.

The brittle to ductile transition temperature, T_c , is approximately equal to about half the absolute melting temperature. The value of T_c also varies with porosity, grain size, and strain rate. At zero or low strain rates, T_c varies from ~ 1000 to 1100°C . Increasing the porosity of the pellet slightly decreases T_c .

A second transition temperature, T_t , is defined as the temperature that shows a marked decrease in the ultimate tensile stress measured in bending tests. In the intermediate range, $T_c < T < T_t$, some deformation occurs before fracture (semibrittle fracture). For $T > T_t$, the fracture mode is completely ductile. The value of T_t depends on strain rate and porosity. At low strain rates, T_t is on the order of 1400°C , while at very high strain rates it can increase to 1800°C (Ref. 78).

An alternative and simpler way of representing the brittle-plastic behavior of UO_2 is to assume that below a temperature-dependent yield stress, σ_y , the material is elastic and stress and strain are proportional. Above σ_y , the fuel is taken as perfectly plastic. Krammen and Freeburn⁴⁸ correlate σ_y by

$$\sigma_{y'} = \begin{cases} \infty & \text{for } T \leq T_{Pa} \\ D/(T - T_{Pa}) & \text{for } T > T_{Pa} \end{cases}$$

where

$\sigma_{y'}$ = yield strength UO_2 , Pa

T = fuel temperature, $^\circ\text{C}$

T_{Pa} = plasticity onset temperature = 1200°C

D = yield coefficient = 1.6×10^{10} , Pa $^\circ\text{C}$.

At high temperatures, UO_2 exhibits significant thermal creep. However, fissioning oxide fuels exhibit significantly enhanced plasticity and show fission-induced creep at temperatures where thermal creep does not occur. Olsen⁷⁹ presented an empirical correlation of the data on thermal creep and combined thermal and fission-induced or enhanced creep. This correlation, as revised by Hagrman *et al.*³⁴, is given by

$$\dot{\epsilon} = \frac{\sigma(A_1 + A_2 \dot{F}) \exp(-Q_1/RT)}{(A_3 + d)G^2} + \frac{(A_4)\sigma^{4.5} \exp(-Q_2/RT)}{(A_6 + d)} + A_7 \sigma \dot{F} \exp(-Q_3/RT) \quad (2.26)$$

where

$\dot{\epsilon}$ = creep rate, m/m·s	$A_7 = 3.72 \times 10^{-35}$
σ = compressive stress, Pa	$A_1 = 0.3919$
$\dot{F}$ = fission rate, fissions/(m ³ s)	$A_2 = 1.31 \times 10^{-}$
$Q_3 = 2.617 \times 10^3$, J/mol	$A_3 = -87.7$
G = grain size, μm	$A_4 = 2.039 \times 10^{-}$
R = gas constant, J/mol K	$A_6 = -90.5$
T = temperature, K	d = density (fraction of theoretical).
x = oxygen-to-metal mol ratio	

Also

$$Q_1 = 17,885 \left\{ \exp \left[\frac{-20}{\log(x-2)} - 8 \right] + 1 \right\} + 72,124$$

$$Q_2 = 19,872 \left\{ \exp \left[\frac{-20}{\log(x-2)} - 8 \right] + 1 \right\} + 111,544$$

When stressed, plastic flow of UO_2 can lead to densification of fuel pellets through "hot hydrostatic pressing." Stresses on the UO_2 can arise when fuel swelling brings the outer diameter of the fuel pellet in contact with the cladding. Such stresses can also arise if pellet jamming at some elevation prevents free axial expansion of the stack. In contrast to creep, which is governed by deviatoric stress, hot pressing is governed by hydrostatic pressure.

Although several mechanisms contribute to pressure sintering (hot hydrostatic pressing), it would appear that volume creep due to lattice diffusion is the dominant mechanism for UO_2 and MOX fuel. The available data have been represented by Hagrman *et al.*³⁴ as

$$\frac{1}{\rho_1} \frac{d\rho_1}{dt} = A \left(\frac{1-\rho_1}{\rho_1} \right)^a \frac{P}{Tg^2} \exp[Q/(RT)] , \quad (2.27a)$$

where

$a = 2.7$ for UO_2 , 2.55 for MOX fuel

t = time

g = grain size, μm

ρ_1 = fraction of theoretical density

P = hydrostatic pressure, Pa

$A = 48940$ for UO_2 ; 1.8×10^7 for MOX fuel

T = temperature, K

$R = 8.314$, J/mol K.

For UO_2 , the pressure sintering activation energy, Q , is given by

$$Q = R \left\{ 9000 \exp \left[\frac{20 - 8|\log(x - 1.999)|}{|\log(x - 1.999)|} + 1.0 \right]^{-1} + 36,294 \right\} \quad (2.27b)$$

where x is the oxygen-to-metal ratio. For MOX fuel, $Q = -45,000$.

Since reactor fuels are close to the theoretical density of UO_2 , the left-hand side of Eq. (2.27a) equals the volumetric strain rate ($d\epsilon_{\text{hp}}/dt$). Further, the volumetric strain is readily reduced to linear strain by noting that the hot pressing strain components are all equal

$$[(\epsilon_{\text{hp}})_\theta = (\epsilon_{\text{hp}})_r = (\epsilon_{\text{hp}})_z] = (1/3)(\epsilon_{\text{hp}})$$

2.2.2 Fuel Densification and Restructuring

Hot pressing is not the only mechanism that can lead to in-pile densification of UO_2 . Pellet densification and shrinkage can be caused by irradiation-induced dissolution of pellet porosity. Many investigators believe densification results from dispersing fine pores as vacancies by fission fragments. These vacancies subsequently diffuse to a grain boundary leaving a densified fuel grain.^{80,81} The process is illustrated in Fig. 2.10.

Irradiation-induced fuel densification is of greatest importance in low-density (e.g., $\leq 92\%$ theoretical) fuels. When low-density fuels were first used, irradiation-induced densification caused pellet shrinkage that led to gaps in the pellet stack and cladding collapse in the space between pellets. The problem is now understood; reactor vendors control densification by limiting the initial volume of fine pores in the fuel.⁸¹ Increased sintering temperature and sintering time lead to a significant decrease in densification.

Several approaches have been suggested for estimating the rate of densification. Rolstad *et al.*⁸² suggested a relationship between percent densification, sintering temperature (a measure of initial pore and grain size), initial density, and burnup. The graphs required for the use of this model are reproduced in Ref. 83.

Marlowe⁸⁴ explains irradiation-induced densification by an irradiation-enhanced diffusion model. It follows Coble's⁸⁵ theory for sintering, which is based on the assumption that the driving force for pore shrinkage is the effort to reduce free energy by lowering pore surface area. According to Marlowe, in-pile pellet density, ρ , is obtained from

$$\rho = \rho_0 \exp(-S\dot{F}t) + \frac{M}{A} \exp \left[-S \left(\frac{G_0^3}{AD_{\text{irr}}} + \dot{F}t \right) \right] \ln \left(1 + \frac{AD_{\text{irr}}}{G_0^3} \dot{F}t \right), \quad (2.28)$$

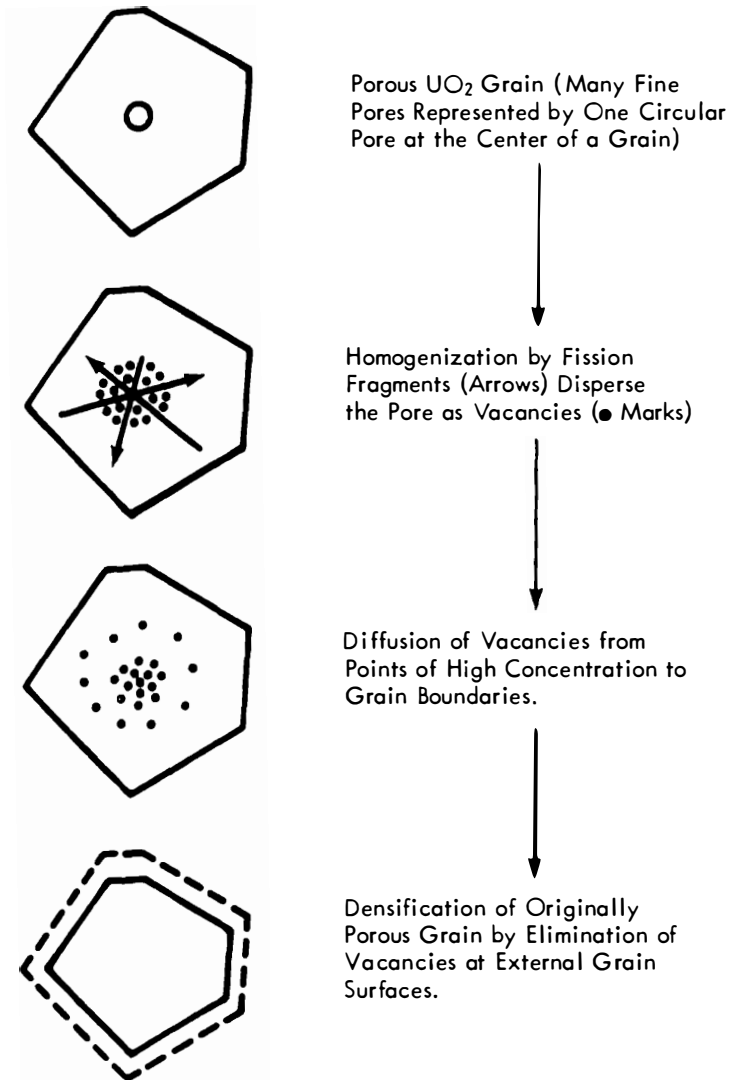


Fig. 2.10 Schematic illustration of in-reactor pore removal [from Ref. 81].

where

ρ_0 = initial density

A = grain growth constant

$\dot{F}$ = fission rate

M = densification rate constant

D_{irr} = constant relating irradiation-induced diffusivity to fission rate

S = volumetric swelling rate for 100% dense fuel

t = irradiation time

G_0 = initial grain size.

Quantities M and A can be determined from experiments on the thermal sintering of the pellets under study. Buescher and Horn⁸⁶ suggest that D_{irr} can also be calculated from thermal sintering data by using $D_{irr}/D' = 1.15 \times 10^{-15} \text{ cm}^3 \text{ s/fission}$, where D' is the thermally activated bulk diffusion rate.

Stehle⁸⁷ points out that the net densification observed is the algebraic sum of fuel swelling and densification. Further, experimental studies show that the fine pores shrink very much faster than the larger pores. He concludes that the net densification can be expressed by a semiempirical equation

$$\Delta V/V_0 = S \text{ Bu} - \sum_i (p_0)_i \left\{ 1 - \left[1 - \frac{\alpha' \text{ Bu}}{(r_0)_i} \right]^3 \right\} \quad (2.29)$$

where

$\Delta V/V_0$ = fractional change in volume

Bu = burnup (GWd/tonne)

$(p_0)_i$ = initial pore volume in pore class i

$(r_0)_i$ = average radius of pores in class i

α' = constant.

The foregoing equation leads to a more rapid disappearance of the finer pores. A pore class disappears when the bracket in the equation becomes unity.

In view of the improved behavior of current fuels, some more recent approaches tend to use simpler, more empirical approaches. In-pile data are used to establish (1) the total amount of densification possible with a given fuel and (2) the rate at which this densification occurs. This latter quantity can be expressed as a rate constant⁴⁷ or in terms of a burnup at which the densification is complete.⁴⁸ With the latter approach we have:

$$\frac{\Delta V}{V_0} = (\Delta \rho)_D \{ \exp[\text{Bu} \ln(0.01)/(\text{Bu}_D)] - 1 \} \quad (2.30)$$

where $(\Delta \rho)_D$ is the total densification that can occur as a percent change in fuel density and Bu_D is the burnup at which densification is complete. Densification occurs fairly quickly and Bu_D is about 5000 MWd/tonne for $T \geq 750^\circ\text{C}$. The value of Bu_D appears to increase at low temperatures.

Thermal effects also lead to in-pile restructuring of UO_2 . In Sec. 2.1.1, we noted that porous oxide fuel, irradiated at high heat fluxes, shows large columnar grains distributed around a cylindrical central void. In out-of-pile

experiments, MacEwan and Lawson⁸⁸ showed that columnar grains would be produced at temperatures above 1700°C if a sufficiently high-temperature gradient were maintained. Columnar grains resulted from the migration of voids, transversely oriented with respect to newly formed grains, toward high-temperature regions. These grains therefore have their long axes parallel to the radial temperature gradient.

The observed grain growth is attributed to a vaporization-condensation process that results in a net movement of the pore at high temperatures. We assume that fuel vaporization occurs at the high-temperature end of the pore and condensation occurs at the low-temperature end. Nichols⁸⁹ developed a theoretical model for this process. More recently, considerably simpler correlations for the rate of pore movement have been proposed. Hagrman *et al.*³⁴ correlate the pore movement by

$$u = \frac{49.22(\nabla T) \exp(-44980/T)}{T^2} \quad (2.31)$$

where

u = rate of pore movement, m/s

∇T = temperature gradient, K/m

T = temperature, K.

Nichols has taken the region in which all pores have had sufficient time to migrate to the central void as the columnar grain growth region. By using Nichols' model or Eq. (2.31), we can compute the radial boundary of the grain growth region, r_{cg} . Figure 2.11 presents the results of Nichols' calculations for a typical UO_2 pellet. He assumed the surface temperature of the pellet was 800 K and that a parabolic temperature gradient existed across the fuel. Helium was assumed to be the primary vapor constituent. Observe that r_{cg} can become a significant fraction of r_0 , the total pellet radius, at high temperatures. The radius of the central void, r_v , is computed by

$$(r_{cg}/r_0)\alpha^{1/2} = r_v/r_0 \quad (2.32)$$

The columnar grain region is surrounded by an outer region of equiaxial grains. These are enlarged fuel grains having all sides of approximately the same length. The rate of equiaxial grain growth during any time interval can be calculated from³⁴

$$G' = \left[\frac{1.0269 \times 10^{-13} t \exp(-35873/T)}{(1.0 - 5.746 \times 10^{-6} \text{ Bu})^2 T} + G_0^4 \right]^{1/4} \quad (2.33)$$

where

G' = grain size at end of time interval, m

G_0 = grain size at beginning of time interval, m

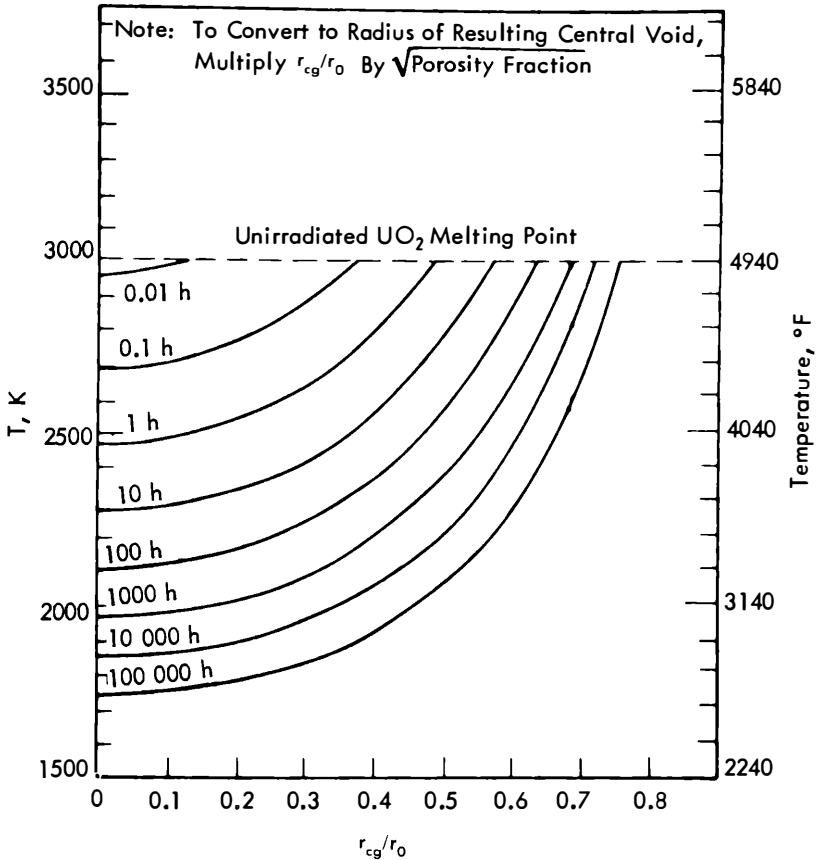


Fig. 2.11 Columnar grain growth radius in UO_2 rods [from Ref. 28].

T = temperature, K

Bu = burnup, MW·s/kg

t = time interval, s.

The outermost region of the pellet is as fabricated unstructured fuel. This region will, however, show significant thermal-stress cracking.

At the relatively low linear power at which current PWRs operate, no columnar grains or central void are formed. Central temperatures at steady-state operation are now (1995) generally no higher than 1800°C. Columnar grains may grow during short-term transients but they do not strongly influence rod behavior. Hot-cell examinations of irradiated PWR fuel subjected only to normal operation show only an outer unstructured (but cracked) region and moderate growth of equiaxial grains in the center of

the rod. Since only equiaxial grains are normally seen, Eq. (2.33) may be used to compute the grain size needed for densification and hot pressing estimates. At low fuel temperature, Eq. (2.33) will simply indicate a grain size close to the fabricated size. If the rod has been subjected to a high-power transient, a region of shattered or desintered grains, consisting of fuel grains that are fractured free of bonds to other grains, may be seen.

2.2.3 Fuel Swelling

Previously we observed that uranium dioxide swells during irradiation. The data indicate that the rate of fuel swelling is a function of temperature. The swelling initially fills a portion of the fuel porosity. When the portion of the porosity that is effective in accommodating swelling has been consumed, net dimensional changes occur. Table 2.VI indicates approximate swelling rates and effective porosities for several temperature ranges.⁹⁰ The burnup at which the effective porosity is consumed is similar in concept to the critical burnup for rapid swelling (see Fig. 2.1).

Godesar *et al.*⁹¹ suggest an analytical expression for total swelling, which was calibrated to a series of in-reactor experiments. They propose

$$\Delta V/V_0 = f\alpha \left[\exp\left(-F \frac{\Delta V}{V_0}\right) - 1 \right] + \beta(1-\alpha)b \quad (2.34)$$

where

$\Delta V/V_0$ = relative volume expansion by swelling

b = burnup, MWd/1000 kg U

α = initial porosity

β = swelling rate of 100% dense oxide

= 1.6×10^{-6} (MWd/1000 kg U)⁻¹ for usual PWR fuel temperatures

f = density correction factor = $0.149(\alpha)^{-0.746}$

F = geometrical factor = 100.

This model takes into account the influence of porosity on swelling rate. As the volume of pores decreases with time, due to inner swelling, the

TABLE 2.VI
Approximate UO₂ Swelling Rates and Effective Porosities

Temperature Range	Fuel Swelling % ($\Delta V/V_0$) per 10 ⁴ MWd/tonne	Effective Component of Initial Voidage (%)
<1300°C	1.6	30
1300 to 1700°C	1.7	50
>1700°C	0.7	80

apparent outer swelling rate increases toward an asymptotic value. Although Eq. (2.34) has generally been supplanted by more complex models, it does provide a simple expression that is moderately accurate and describes the major trends observed experimentally at modest burnup.

Recent approaches to fuel swelling have been more mechanistic. Fuel swelling is generally attributed to production of bubbles by gaseous fission products and the accumulation of solid fission products. Solid fission products produce swelling of between 0.5 and 1% per at.% burnup.^{92,93} The contribution of solid fission products to swelling is most important at low temperatures.

At high temperatures, gaseous fission products are the major cause of fuel swelling. By calculating the volume of gas bubbles within the fuel, the increase in fuel volume and, hence, swelling due to fission gas, can be estimated. Several models have been proposed for this purpose. The first fairly comprehensive model was Nichols and Warner's BUBL-1 code,^{94,95} which calculates the nucleation of gas bubbles, their coalescence, interaction with grain boundaries and dislocations, and migration of bubbles due to the fuel temperature gradient. The model assumes the following:

1. Bubble sizes are determined by gas pressure balancing surface tension and hydrostatic pressure.
2. There is a critical force required to pull a bubble away from a dislocation or grain boundary.
3. Bubble migration along the temperature gradient is due to a surface diffusion mechanism.

Fission gas behavior is followed by a Monte Carlo technique. When two bubbles overlap over a dislocation, they are assumed to coalesce. When a bubble grows sufficiently, the driving force due to the temperature gradient pulls it off the dislocation and the bubble migrates to a grain boundary by surface diffusion. The bubble remains at the grain boundary until further coalescence allows it to grow large enough to be pulled away. The bubble then migrates along the temperature gradient. Release of the gas is assumed to occur after the bubble has gone a distance approximating the distance to a crack or surface.

Yu-Li *et al.*⁹⁶ and Rest^{97,98} describe a somewhat similar model. However, their approach, which is embodied in the GRASS code, is somewhat more general. The general approach of GRASS is basically the same as BUBL in that coalescence, release from defects and boundaries, and migration along the temperature gradient are followed. However, the model allows for radiation-induced resolution of the bubbles as a negative term in bubble growth. The GRASS code also allows for random motion of bubbles and coalescence during migration. Surface diffusion is not the sole method for bubble diffusion. Diffusion components are calculated on the basis of surface diffusion, volume diffusion, and evaporation-condensation. In each

fuel region and for each bubble size, the mechanism giving the highest bubble velocity is chosen. Recent models of GRASS also account from the fact that the capacity for storage of gas bubbles at the grain boundary is limited.

The GRASS program is slow running since the code uses a bubble size distribution, which changes through bubble coalescence and resolution. These changes are determined by solution of a large number of coupled nonlinear differential equations. To avoid this problem, the FASTGRASS program⁹⁸ uses an average bubble size and characterizes intergranular, grain face and grain edge bubbles by number density and an average number of atoms per bubble. Gas release from the grain boundaries occurs only when a specified fraction of the grain boundary area is covered by bubbles. This leads to running times that are only a fraction of those of the original GRASS program. The GRASS series of codes is considered to be the (1993) state-of-the-art programming for fission gas swelling and release from LWR fuel by the U.S. Nuclear Regulatory Commission.

The detailed models of fission gas behavior determine the fission gas released, the fission gas dissolved in the crystal lattice, and the fission gas contained within the bubbles located at dislocation and grain boundaries. Fuel swelling is determined by the latter quantity. By assuming that the trapped gases are located primarily on the grain boundary, MacDonald *et al.*⁹⁹ computed the fission-gas-induced swelling $(\Delta V/V)_g$, due to grain-boundary gas accumulation, as:

$$\left(\frac{\Delta V}{V}\right)_g = \frac{2\pi r^3 \alpha N_b}{R} (1 - X) \quad (2.35)$$

where

N_b = grain face bubble density, m^{-2}

X = fuel surface area to grain boundary area

R = average grain radius, m

r = grain face bubble radius, m

$\alpha = [1 - (3 \cos \theta - \cos^3 \theta)/2] \sin^3 \theta$

θ = semidihedral angle of lenticular bubbles $\approx \pi/3$.

Despite the improvements in running time offered by FASTGRASS and other recent codes, such as OGRES,¹⁰⁰ many analysts consider such models to be too time consuming for routine design use. Considerably simpler swelling models are therefore often employed despite the present availability of well-benchmarked mechanistic approaches.

The ESCORE code⁴⁸ is typical of many current design programs. ESCORE computes the swelling as a sum of solid and gaseous swelling less a portion accommodated by the fuel porosity. The solid-fission product

swelling is taken as directly proportional to burnup and independent of temperature.

$$(\Delta V / V)_s = 7 \times 10^{-7} (1 - \alpha) \text{Bu} \quad (2.36a)$$

where α is the porosity fraction and Bu is the burnup (MWd/tonne). The fractional volume change due to gaseous fission product swelling is taken as:

$$(\Delta V / V)_g = G (1 - \alpha) (\Delta \text{Bu}) \quad (2.36b)$$

where G is the swelling rate for fuel in specified axial location for given time step and

$$G = \begin{cases} 0 & \text{for } 650^\circ\text{C} \geq T_{\text{avg}} > 2100^\circ\text{C} \\ 2 \times 10^{-6} (T_{\text{avg}} - 650) / 450 & \text{for } 650^\circ\text{C} \leq T_{\text{avg}} < 1100^\circ\text{C} \\ 2 \times 10^{-6} [1 - (T_{\text{avg}} - 1100) / 1100] & \text{for } 1100^\circ\text{C} \leq T_{\text{avg}} \leq 2100^\circ\text{C} \end{cases}$$

The average fuel temperature, T_{avg} , is for a given axial segment ($^\circ\text{C}$). Note that the foregoing gas-induced swelling approach is very similar to that of Table 2.VI.

In using the previous two equations, it is assumed that a portion of the swelling computed may be accommodated within the fuel pores, fuel pellet dishes, and radial pellet cracks. The volume so accommodated, $(\Delta V / V)_A$, corresponds to 10% of the solid fission swelling and 50% of the gas induced swelling. That is,

$$(\Delta V / V)_N = -0.1 (\Delta V / V)_s - 0.5 (\Delta V / V)_g \quad (2.36c)$$

where $(\Delta V / V)_N$ is the net or observable swelling. When the absolute value of $(\Delta V / V)_N$ equals the total volume available for swelling accommodation, no further accommodation occurs. The use of pellet dishes and fuel crack volumes for swelling accommodation differs from the usual approach, which accommodates swelling only in fuel pores.

The ESCORE model of swelling is conservative in that the fission-gas-induced swelling is always directly proportional to the burnup. Many of the current mechanistic fission gas release and swelling models postulate that, after the concentration of fission gas bubbles on the given boundaries reaches a limiting value, the grain boundaries are all interlinked. Any fission gas subsequently arriving at a grain boundary is therefore released. The bubble concentration at the grain boundary would then reach a limit at high burnup. Since gas-induced swelling depends on this concentration, the swelling would reach a maximum value. This idea is embodied in the swelling computations of COMETHE III⁴⁷ and SPEAR.¹⁰¹ The relationship used to determine the increase in pellet porosity due to gas bubbles is:

$$(\Delta V / V)_g = a \{ 1 - \exp[- \text{Bu} / (\text{Bu})_L] \} , \quad (2.37)$$

where

$$a = (\alpha_L - \alpha_F) / (1 - \alpha_L)$$

$(\text{Bu})_L$ = burnup constant

Bu = current burnup

α_F = as fabricated porosity (pellet void fraction)

α_L = limiting porosity (function of applied hydrostatic stress).

Equation 2.37 predicts that the total gas-induced swelling will asymptotically approach a . This is in contrast with Eq. (2.34), used in earlier COMETHE versions, which indicates an asymptotic "swelling rate." The foregoing model computes the externally observed swelling since the fabricated porosity is subtracted from the limiting porosity.

An approximate value of the limiting fuel matrix swelling in the absence of applied stress may be obtained from Eq. (2.35) by making use of values for bubble size, r , and grain bubble density when interlinkage of grain boundary bubble occurs. MacDonald *et al.*⁹⁹ suggests that under these conditions

$$N_b \approx 2 \times 10^{-7} \text{ m}^2 \quad \text{and} \quad r = 2 \times 10^{-7} \text{ m}$$

The limiting externally observed swelling would then be determined from the computed $(\Delta V / V)_g$ by making an appropriate allowance for swelling accommodation within the fuel pores.

The idea that grain boundary bubble interlinkage leads to a maximum swelling value would also appear to apply to metallic fuels. In-pile experimental observations of metallic Pu-U fuel elements designed for fast reactor use show that swelling rates are initially higher than for oxide fuels. However, a maximum swelling is reached and further increases of burnup do not cause a further increase in fuel dimensions. By designing the fuel rod to accommodate the maximum swelling, metallic fuel elements capable of withstanding very high burnups can be fabricated.

Net swelling is seen as the algebraic sum of irradiation-induced densification and fission product swelling

$$(\Delta V / V_0)_{\text{net}} = (\Delta V / V_0)_{\text{swell}} - (\Delta V / V_0)_{\text{densification}} \quad (2.38)$$

Therefore, a small amount of fuel densification may not be injurious since it may partly counteract the effects of swelling.

2.2.4 Fission Gas Release and Internal Pressure

2.2.4.1 Fission Gas Release Models for Normal Operation

Gaseous fission products are generated at a rate of approximately 0.3 atoms/fission. A significant fraction of the gases so produced is released by UO_2 . It is generally agreed that below ~ 600 to 800°C , the rate of fission gas

release is controlled by recoil and knockout.^{102,103} Release rates are low and not temperature dependent.

In the region between 800 and 1800°C, fission gas release rates are higher and markedly temperature dependent.* For a number of years, the rate of gas release in this region was believed to be determined by the rate at which gas diffused out through solid UO₂ particles. Available data were correlated using a fission gas diffusivity, D , which followed an Arrhenius rate equation

$$D = D_0 \exp(-E/RT) \quad (2.39)$$

where

D_0 = a constant

E = activation energy, cal/g mole

R = perfect gas law constant, 1.986 cal/g mol, K

T = absolute temperature, K.

According to Booth,¹⁰⁴ activation energy is essentially constant, but D_0 varies with the nature of the compact. By using this theory, he developed a method for computing fission gas release based on the diffusion equation for a spherical particle of radius a . He expressed his results in terms of F , the fraction of total gas produced that has diffused out of the sphere in time t . From $t = 0$ to $t = 1/(\pi^2 D')$ (at which time $F = 0.57$), fractional release can be approximated by

$$F = 4 \left(\frac{D't}{\pi} \right)^{1/2} - \left(\frac{3D't}{2} \right) \quad (2.40)$$

where $D' = D/a^2$. To account for later data showing that fission gas release is burnup dependent, some subsequent diffusion models have multiplied the right-hand side of Eq. (2.39) by a factor that depends on burnup and grain size.

In most instances, the designer must consider conditions in which the fuel temperature varies with time and can be considered constant for only short periods. For a short period of constant temperature, the gas release may be determined by considering the change in the average concentration of the fission gas in the particle. Verbeek and Hoppe⁴⁷ approximate the solution for the average gas concentration in the Booth particle at the end of time step (Δt) as:

*Bagger, Mogensen, and Walker [*J. Nucl. Mater.* **211**, 11 (1994)] found that, at actual reactor operating conditions, fission gas release by UO₂ pellets is negligible under about 1200°C.

$$C_{t+\Delta t} = \frac{\beta}{15D'} R(A) + C_{(t)} \left[\frac{R(A_{t+\Delta t} + A_t) - R(A_{t+\Delta t})}{R(A_t)} \right] \quad (2.41)$$

where

$$A_{(t+\Delta t)} = A_t + \pi^2 D' (\Delta t)$$

$$R(A) = (90/\pi^4) \sum_{n=1}^{\infty} (1 - e^{-An^2})/n^4$$

and C_t is the average gas concentration in particles (mol/cm^3). The fission gas release, G , during the time step is then

$$G = \beta(\Delta t) - (C_{t+\Delta t} - C_t) \quad (2.42)$$

where β is the fission gas production rate ($\text{mol}/\text{cm}^3 \text{ s}$) and G is the fission gas release rate per unit volume ($\text{mol}/\text{cm}^3 \text{ fuel s}$).

Later studies indicated that the fission gas release process is much more complicated than originally believed. It has been suggested that a coupled diffusion-trapping process is the rate controlling mechanism.⁶² In such a model, fission gas is assumed to be trapped in lattice defects; its release is partially controlled by its rate of escape from these traps. More recent models have been of the bubble migration type, which have been used for predicting fuel swelling. Thus, the previously described BUBL (Ref. 94) and GRASS (Ref. 97) codes can be used for predicting both fission gas release and swelling. However, a critical survey by Ronchi and Matzke¹⁰⁵ estimates that most of the fission gas is kept in the matrix by resolution and that atomic diffusion accounts for a large fraction of the release. This view is now generally agreed on, so models like BUBL-1, which describe fission gas release solely in terms of bubble migration, are considered inappropriate. Models like GRASS, which account for trapping resolution and atomic diffusion, do appropriately describe the release process when calibrated against experimental data.

Although the GRASS codes were initially developed to provide a detailed phenomenological model for fast reactor fuel, they have been incorporated into such fuel design codes as the LIFE code for fast reactor fuel and FRAP for LWR fuel design.

While the GRASS codes are considered (1993) by the U.S. NRC as providing the most satisfactory "best-estimate" fission gas release model,¹⁰⁶ the OGRES steady-state computer model appears to have an equivalent position in Great Britain. The OGRES-I model determines the gas arrival at the grain boundaries as primarily due to single gas atom diffusion. The trapping of gas atoms by intragranular gas bubble and resolution of the gas by fission fragment destruction of the bubbles are in essential equilibrium. However, migration of large bubbles to the grain boundaries is considered. Release of the gas at the grain boundaries occurs primarily by gas

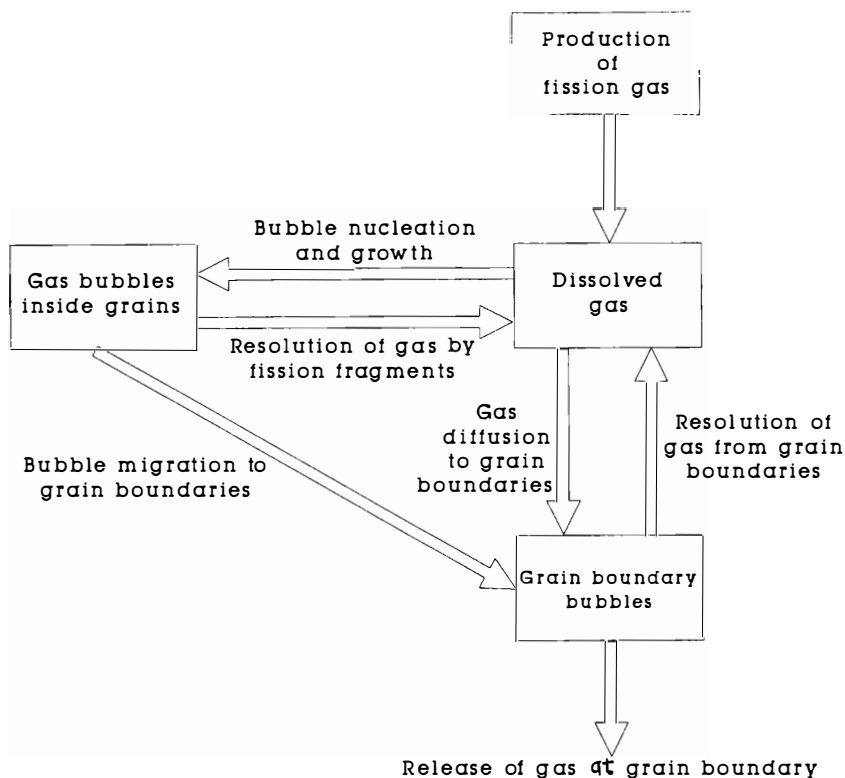


Fig. 2.12 Fission gas release flow diagram of OGRES and TRAFFIC codes [from R. Cameron *et. al*, *Nucl. Energy*, **29**, 2, 147 (1990)].

bubble interlinkage. As can be seen from the overview of the mechanisms considered, shown in Fig. 2.12, the OGRES approach is somewhat simpler than that taken by GRASS.

The FUTURE program¹⁰⁷ is another well-known fuel performance code containing a fully mechanistic swelling and fission-gas release model. The fission gas release model may be said to follow the same general approach shown in the OGRES flow diagram (Fig. 2.12). However, the modeling of the grain boundary release process is somewhat more complex than that of OGRES.

The complexity and long running time of fully mechanistic models, as well as the belief by some designers that simpler models are capable of adequate precision, has led to the use of simple fission gas release approaches in a number of fuel design programs.

The simplest approaches are those based on empirical or semiempirical correlations. Some of the correlations proposed are as follows:

1. Lewis¹⁰⁸ simple temperature zone model. The gas release fraction, F , is given by

$$F = (0.005V_1 + 0.1V_2 + 0.6V_3)\sqrt{t/3} + 0.95V_4 \quad (2.43)$$

where V_1, V_2, V_3, V_4 are fractional volumes of fuel in the temperature regions $< 1000^\circ\text{C}$, 1000 to 1300°C , 1300 to 1600°C , and $> 1600^\circ\text{C}$, respectively, and t is irradiation time in years.

2. Hoffman and Coplin's¹⁰⁹ correlation between release fraction and average fuel temperature during maximum power.
3. Beyer and Hann¹¹⁰ temperature zone model. In this correlation, the gas release fraction, F , is given by

$$F = 0.05X_1 + 0.141X_2 + 0.807X_3 \quad (2.44)$$

where X_1, X_2, X_3 are fractional amounts of gas produced between 1200 to 1400°C , 1400 to 1700°C , and $> 1700^\circ\text{C}$, respectively.

As an alternative to the simple correlation approach, some modelers have chosen to develop simple models, which are deemed to cover the most significant of the gas release mechanisms. The probabilistic approach developed by Weisman *et al.*¹¹¹ assumes that (a) a proportion, K_1 , of the fission gas escapes directly from the fuel matrix without being trapped and (b) fraction K_2 of the gas trapped at grain boundary areas and dislocations is released per unit time. The moles of gas released, r , at time t are given by

$$r = p' \left\{ t - \frac{1 - K_1}{K_1 K_2} [1 - \exp(-K_1 K_2 t)] \right\} \quad (2.45)$$

where p' is the gas production rate and t is the time.

By assuming that reactor power operation can be described by a series of constant power steps, total gas release can be determined. The number of moles released, Δr_i , during the i 'th time interval, Δt_i , is then given by

$$\begin{aligned} \Delta r_i = r_i - r_{i-1} = p'_i \left\{ \Delta t_i - \frac{1 - K_1}{K_1 K_2} [1 - \exp(-K_1 K_2 \Delta t_i)] \right\} \\ + C_{i-1} [1 - \exp(-K_1 K_2 \Delta t_i)] \end{aligned} \quad (2.46)$$

where C is gas concentration in fuel equal to $p't - r$. Since the total gas release from time zero is $\Sigma \Delta r_i$, the fraction of released gas produced is given by

$$F = \frac{\sum_i \Delta r_i}{\sum_i p_i \Delta t_i}$$

MacDonald and Wang¹¹² give constants K_1 and $K_1 K_2$ for 95% dense fuels as

$$K_1 = \exp(-12450/T + 1.84) \quad (2.47)$$

$$K_1 K_2 = 0.25 \exp(-21410/T)$$

where T is in $^{\circ}\text{R}$ and t is in hours.

The simple model of Verbeek and Hoppe⁴⁷ approaches the problem by modifying the Booth diffusion equation¹⁰⁴ to consider both bubble resolution and gas trapped at defects or precipitated in bubbles. If capture and resolution rates are taken as constant, then the original Booth diffusion model can be used providing, D' the diffusion constant, which was originally D/a^2 , is redefined as:

$$D' = D''b/[a^2(b+g)] \quad (2.48)$$

where

$$D'' = D + K_b R_f$$

D = original diffusion coefficient

a = sphere radius

b = resolution rate = $K_a R_f$, s^{-1}

R_f = fission rate density

K_a, K_b = empirical constants

g = capture rate, s^{-1}

To account for the enhanced fission gas release once bubble interlinkage begins, the sphere radius, a , is given one of two values. At low burnups, the sphere radius is considered to represent the actual size of the average particle making up the sintered compact. At high burnups, it is calculated as $3/(s/v)$ where (s/v) is the measured fuel surface to volume ratio. Above the critical burnup, gas needs only to diffuse to the grain boundaries where interlinking bubbles are assumed to provide a path for its immediate release. The sphere radius is then taken as equal to the crystal grain radius. Since this is very much smaller than the particle size, release rates are considerably enhanced.

The critical burnup is related to the limiting porosity used in the swelling model of Eq. (2.37). The sphere radius is decreased when

$$\frac{\alpha - \alpha_F}{\alpha_L - \alpha_F} < 0.1 \quad (2.49)$$

where α is the existing fuel porosity and other symbols have the same meaning as in Eq. (2.37).

The models of SPEAR-BETA¹¹³ and ESCORE⁴⁸ are a step closer to the fully mechanistic view. A small portion of the fission gas produced is released directly due to recoil. An additional direct release occurs in the grain growth region as small grains coalesce into the boundaries of larger grow-

ing grains. The bulk of the gas release is considered as indirect and determined by two steps. The first step is the release of the gas from within the grain to the grain boundary. The total fission gas concentration within a grain is determined by a balance between the production rate and losses due to recoil and transport to the grain boundary. Only gas that is mobile can be released. The ratio of mobile to trapped gas within the grain is determined by an Arrhenius-type reaction rate equation. The release rate can then be determined as the mobile gas concentration divided by a release rate time constant.

The second step of the process is the release of gas from grain boundaries to the free volume. The rate depends on both the grain boundary concentration and a second time constant.

The differential equations⁴⁸ are readily integrated and the integral equations are solved simultaneously for the changes in the concentration within the fuel grains and on the grain boundaries. For a given time step, Δt , the ESCORE model represents these by:

$$\Delta C_g = [\beta \tau_g (1 - p_d) - C_{g,0}] [1 - e^{-\Delta t / \tau_g}] [1 - B_g] - C_{g,0} B_g \quad (2.50)$$

$$\begin{aligned} \Delta C_b = & \{ [\beta (1 - p_d) \tau_b - C_{b,0}] [1 - e^{-\Delta t / \tau_b}] + \frac{\tau_b}{\tau_g - \tau_b} [C_{g,0} - \beta \tau_g (1 - p_d)] \\ & \times [e^{-\Delta t / \tau_g} - e^{-\Delta t / \tau_b}] \} \{ 1 - B_b \} - C_{b,0} B_b + B_g (C_{g,0} + \Delta C_g) (1 - B_b) \end{aligned} \quad (2.51)$$

where

$C_{g,0}$ = concentration in grain interiors at beginning of time step, moles per m³ fuel volume

$C_{b,0}$ = concentration in grain boundaries at beginning of time step, moles per m³ fuel volume

β = fission gas production rate, mol/cm³ s

p_d = direct release fraction (0.004)

B_g = fraction of fission gas, from grain interiors immediately released by grain growth

B_b = fraction of fission gas, from grain boundary immediately released by grain growth

τ_g = grain interior fractional release time constant, hr

τ_b = grain boundary fractional release time constant, hr.

The fraction of fission gas from within fuel grains that is transferred to the grain boundaries immediately upon grain growth, due to a fraction of the new grain volume being swept free of fission gas, is given by:

$$B_g = [1 - (d_{g,0} / d_g)^{B_0}] [1 - \exp(-Bu/5000)] , \quad (2.52)$$

where

$d_{g,0}$ = average grain diameter at the beginning of the timestep, μm

d_g = average grain diameter at the end of the timestep, μm

B_0 = born-free exponent for the grain interior = 2

Bu = the burnup (MWd/MTU), with Bu set equal to 5000 for the range of 0 to 5000 MWd/MTU.

Similarly the fraction of fission gas on grain boundaries that is released immediately upon grain growth is determined from

$$B_b = [1 - (d_{g,0}/d_g)^{F_0}][1 - \exp(-\text{Bu}/5000)] \quad (2.53)$$

where F_0 is the born-free exponent for the grain boundaries; it is equal to 3.

In the equations for B_g and B_b above, the exponential involving burnup provides an incubation period that retards gas release at low burnups.

The probable rate of fission gas release to the grain boundaries is assumed to be directly proportional to C_g , mobile gas concentration within the grains, and inversely proportional to the time constant τ_g . Since the release rate is highly temperature and grain size dependent, the time constant governing this release is given by an Arrhenius type equation

$$\tau_g = 0.895\bar{g}[\exp(Q_{gi}/RT) - 45] \quad (2.54)$$

where

τ_g = time constant for release from grain interiors, hr

$\bar{g}$ = the mean grain size for each timestep, μm

R = gas constant = $1.987 \text{ cal/mol K}^{-1}$

T = temperature, K

Q_{gi}/R = the grain-interior activation energy (cal/mol)/ R
 $= \max\{100 \text{ or } 7950[1 + 9 \exp(-\text{Bu}/5000)]\}$

and Bu is defined and limited as for Eq. (2.52).

The release from grain boundaries is considered to be proportional to the grain boundary concentration, C_b , and inversely proportional to the grain boundary time constant, τ_b . An Arrhenius rate constant equation is also used to compute τ_b .

$$\tau_b = 5.09 \times 10^{-17} [\exp(Q_{gb}/RT)] \quad (2.55)$$

where

τ_b = time constant for release from grain boundaries, hr

R = gas constant = $1.987 \text{ cal/mol K}^{-1}$

T = temperature, K

Q_{gb} = grain-boundary activation energy (cal/mol)/ R
 $= \max\{100 \text{ or } 6700[1 + 0.9 \exp(-\text{Bu}/5000)]\}$.

The burnup, Bu , is again defined as for Eq. (2.52). The burnup term, appearing in the equations for the activation energies, models the decreased fission gas release seen at low burnups.

The moles of gas released to the free volume, ΔC_{rel} (mol/m³ fuel), is determined by the balance between the production of fission gas and the changes in C_g and C_b during the timestep; that is,

$$\Delta C_{rel} = \Delta C_p - \Delta C_g - \Delta C_b \quad (2.56)$$

where ΔC_p is the fission gas produced in timestep (mol/m³ fuel).

The ESCORE fission gas release model has been shown to provide good agreement with a wide range of in-pile test data.

The moderately sophisticated fission-gas release model by Rausch and Panisko¹¹⁴ has been used as a conservative standard against which conservative fuel rod design codes could be benchmarked. The modeling is divided into high- and low-temperature submodels for both stable and radioactive isotopes. In the low-temperature model, the release of stable fission gas depends only on fuel burnup. In the high-temperature model, power and temperature are assumed to have remained constant for several half lives. The fractional release is taken as a function of time, fuel temperature, and burnup. The fractional fission gas release for a particular ring of a given axial segment is computed as the larger of the values computed from the high- and low-temperature models.

Since Pu is a major fraction of the fissile material in any fuel rod at the end of life, one would expect that models which are satisfactory at high burnup should apply to MOX fuel. However, release fractions from MOX fuel can be higher than those predicted by the usual models if there is substantial inhomogeneity. In an inhomogeneous fuel, the small granules of PuO₂ experience a high local burnup. Since the fraction of fission gas release increases with burnup, fission gas releases will be higher than those expected on the basis of the average fuel burnup.

2.2.4.2 Pressure Limits

Once the fission gas release has been obtained, the internal gas pressure, P_i , may be determined from

$$P_i = \frac{RM}{\sum_i (V_i/T_i)} \quad (2.57)$$

where M is the total moles of released fission gas and initial fill gas, $\sum_i (V_i/T_i)$ is the sum of ratio of each free volume in rod to average absolute temperature of the volume, and R has its previous meaning of the ideal gas constant.

The volumes available for gas containment include the open fuel porosity, pellet-to-cladding gap, pellet cracks, dishes, and gas plenum. The com-

putation of these volumes should be based on the hot fuel and cladding dimensions with appropriate allowances for creep and elastic deformation.

In early rod-type fuel element designs, internal fission gas pressure at the end of life (EOL) was generally required to be no greater than the normal external system pressure. More recently, this criterion has been relaxed and EOL gas pressures are set so that there is no cladding "lift-off." That is, once the cladding is finally in contact with the pellet, pellet-to-cladding gap is not allowed to develop. This assures that excessive fuel temperatures due to a large gas-filled gap cannot occur. The no lift-off requirement can be easily implemented in fuel rod design computations by requiring that the contact pressure be above a specified minimum value at full power at EOL.

In fuel plates, unacceptable deformation of the plates limits fission gas release. Lustman,¹¹⁵ who assumed that plate compartments deform into trapezoidal prisms, obtained the following expressions for maximum deformation y and internal pressure P_i

$$y = \frac{0.0138a^4}{Ec^3} \left(\frac{N^*RT}{V_0 - 2ya^2/3} - P_0 \right) \quad (2.58)$$

$$P_i = (N^*RT)/(V_0 + 2ya^2/3) \quad (2.59)$$

where N^* is the moles of gas released, P_0 is the external pressure, c is the cladding thickness, V_0 is the initial free volume in plate element (cold condition) (in.³) and a is the edge length of plate compartment square (in.). Other symbols have their previous meanings. Creep has again been omitted.

2.2.4.3 Status of Fission Gas Release Models for Normal Operation

To obtain an appreciation of the magnitude of fission gas release from UO_2 pellets, we may make use of the approximation that the fractional fission gas release may be related to the volumetric average temperature of the fuel pellet at full power.¹⁰⁹ Experimental measurements¹⁰⁹ indicate that moderately burned UO_2 with average temperatures at or below 1000°C releases 5% or less of the fission gas produced.¹⁰⁹ Release rates increase rapidly as the average temperature rises above 1000°C. At a volumetric average temperature of 1400°C, moderately burned fuel will release about 30% of the gas produced. At a volumetric average temperature of 1800°C, release rates of about 40% are seen at moderate burnups.¹⁰⁹

During the late 1970s and beginning of the 1980s, there was concern that available fission gas release models severely underpredicted gas releases at high burnup.¹¹⁶ Some modelers moved to incorporate this. For example, earlier versions of the ESCORE model included a burnup enhancement factor. However, considerably more high burnup data has become available

since these earlier observations. These data indicate that the rate of fission gas release does not accelerate at burnups up to 54,000 MWd/tonne U.¹¹⁷ The current modeling approaches, without additional burnup enhancement terms, are now considered as being applicable to high-burnup fuel (e.g., the ESCORE model has been shown to apply up to 60 GWd/tonne).

The specific details of the fission gas release process are likely to remain a matter of debate for a considerable period. Models that differ significantly in their details can often provide equally adequate data correlations. It is, however, generally recognized that (1) a small portion of the fission gas is released directly and (2) the bulk of the fission gas is released indirectly through a diffusion and trapping process. Most mechanistic models also assume saturation of the traps at moderate burnup.

Some early researchers concluded that fission gas release rates were enhanced during normal shutdown and startup periods. This effect does not seem to be specifically included in current models.

The ability of various computer programs to predict fission gas release has been examined on several occasions. In a 1983 study¹⁰⁷ of 16 European programs, most performed adequately for steady-state conditions but only the modified diffusion models of COMETHE III and FEMAX I-III¹¹⁸ gave satisfactory results for transients. Subsequently, the URANUS model was improved¹⁰⁷ and all three of the models gave good results (e.g., prediction of ~17% fractional release versus a 15% experimental value). Less satisfactory models have sometimes predicted release rates that deviate from experiment by a factor of 2. However, the performance of the models cannot really be assessed independently since the other facets of the program strongly affect calculated releases. Totev and Dickson¹¹⁹ avoided this problem by examining the ability of the Rausch and Panisko,¹¹⁴ Beyer and Hann,¹¹⁰ and Weisman *et al.*¹¹¹ models using the FRAPCON program¹²⁰ with the deformable pellet option for all the fission gas models. They found the Weisman *et al.* model¹¹¹ to provide the best results with the discrepancy between observed and measured internal pressures being on the order of 3%. However, only six rods were considered and somewhat greater discrepancies might have been seen if a larger sample had been used.

2.2.4.4 Fission Gas Release During Transients

In accident situations of very short duration (e.g., some design basis LOCAs where core uncovering may last only a few seconds), gas release during the transient is often ignored. Fission gas pressures are often computed using the gas inventory at the beginning of the accident and the fuel temperatures of the accident period. However, for transients of longer duration the additional fission gas release during the accident could be significant.

In the early 1980s, most of the computational models available at that time underpredicted transient fission gas release. The reasons for this underprediction are not entirely clear. In some cases, the grain boundary

sweeping of fission products, caused by rapid growth of equiaxed UO_2 grains at high temperature, may have been ignored by the computations. Macinnes and Gittus¹²¹ suggest that the tiny gas bubbles within the fuel grains are not infinite sinks for gas atoms colliding with them during transients. By allowing only some of the colliding gas atoms to be trapped by the bubbles, Macinnes and Gittus were able to develop a revised gas release model that agreed with transient data.

The fully mechanistic models of GRASS-ST, FASTGRASS, and FUTURE have been revised to take into account the available transient data. FUTURE has been shown to agree with the transient data that had previously given most programs difficulties.¹⁰⁷ The ESCORE fission gas release model [Eqs. (2.50) to (2.56)] can also be applied to transient release. The model provides for direct release of fission gas via boundary sweeping [Eqs. (2.52) and (2.53)]. These release equations, coupled to an equiaxed grain growth model, would provide for rapid gas release at high temperatures.

Leech *et al.*¹²² feel that the most accurate results for transient release are obtained with an empirical approach. They suggest that the additional fractional release obtained during a transient can be estimated from

$$F = 1 - \exp(-Kt) \quad (2.60)$$

where

T temperature, K

$K = 48.3 \exp(-23,450/T)$

time at elevated temperature (s).

2.2.5 Thermal Resistance of Fuel and Fuel-Cladding Gap

Since pressed and sintered oxide pellets are now (1994) used exclusively for cylindrical fuel rods, this section considers only this fuel fabrication approach.

2.2.5.1 Behavior with Open Gap at BOL

With pressed and sintered pellets, there is usually an appreciable resistance to heat transfer between the pellet surface and cladding. Since bonding agents are not used in PWR design, interfacial resistance can be that of a gas-filled gap or UO_2 in actual contact with the cladding surface. It has often been assumed that at the beginning of life (BOL) the pellet remains nearly centered within the rod and that gap conductance can be obtained from the conductance of an annular gas gap. If this assumption is correct, then gap conductance h_f can be estimated from

$$h_f = \frac{k_g}{l + l_0} \quad (2.61)$$

where k_g is the thermal conductivity of gas in gap (Btu/h°F ft), l is the gap width at operating conditions, and l_0 is the constant specified to provide a conductance consistent with that computed at zero contact pressure.

Cohen *et al.*¹²³ investigated gap conductance for UO₂ stainless-steel-clad fuel elements under operating fluxes. The operating gap was obtained by correcting for the thermal expansion of cladding and fuel and by assuming the fuel remains centered within the cladding. Figure 2.13 shows that data can be fitted by an equation like Eq. (2.61).

Although radiation from the pellet to the cladding would very slightly increase the gap conductance, this effect is not normally considered. If a 3-mil radial gap filled with helium is assumed for a standard fuel rod operating at 16 kW/ft, radiation would increase the gap conductance by only about 2.5%. With smaller size BOL gaps (which are usual) or pellet-cladding contact, the effect of radiation heat transfer would be even smaller.

Proper evaluation of the gap conductance, prior to the establishment of contact between the cladding and fuel pellet, depends on determining appropriate values for the gas thermal conductivity, k_g , and the parameter l_0 .

Gas thermal conductivity can be determined from the data of Von Ubisch *et al.*,¹²⁴ who examined the thermal conductivity of binary and ternary mixtures of xenon, krypton, and helium. They concluded that a mixture of xenon and krypton, which approximates fission gas (15.3% Kr, 84.7% Xe), had a thermal conductivity, k_g , represented by

$$k_g = k_0 (T/302)^s \quad (2.62)$$

where

T = gas temperature, K

$k_0 = 147$

$s = 0.86$

k_g = gas thermal conductivity, 10^{-7} cal/cm² K s.

Since helium is used to pressurize the fuel rods initially, its conductivity is required. The thermal conductivity of helium can be represented by Eq. (2.62) with $k_0 = 3670$ and $s = 0.72$. To estimate the thermal conductivity of mixtures of fission gas and helium, Von Ubisch *et al.*¹²⁴ suggest

$$k_m = (k_{\text{He}})^x (k_{\text{FG}})^{1-x} \quad (2.63)$$

where

k_m = thermal conductivity of mixture

k_{He} = thermal conductivity of helium

k_{FG} = thermal conductivity of fission gas

x = mole fraction helium.

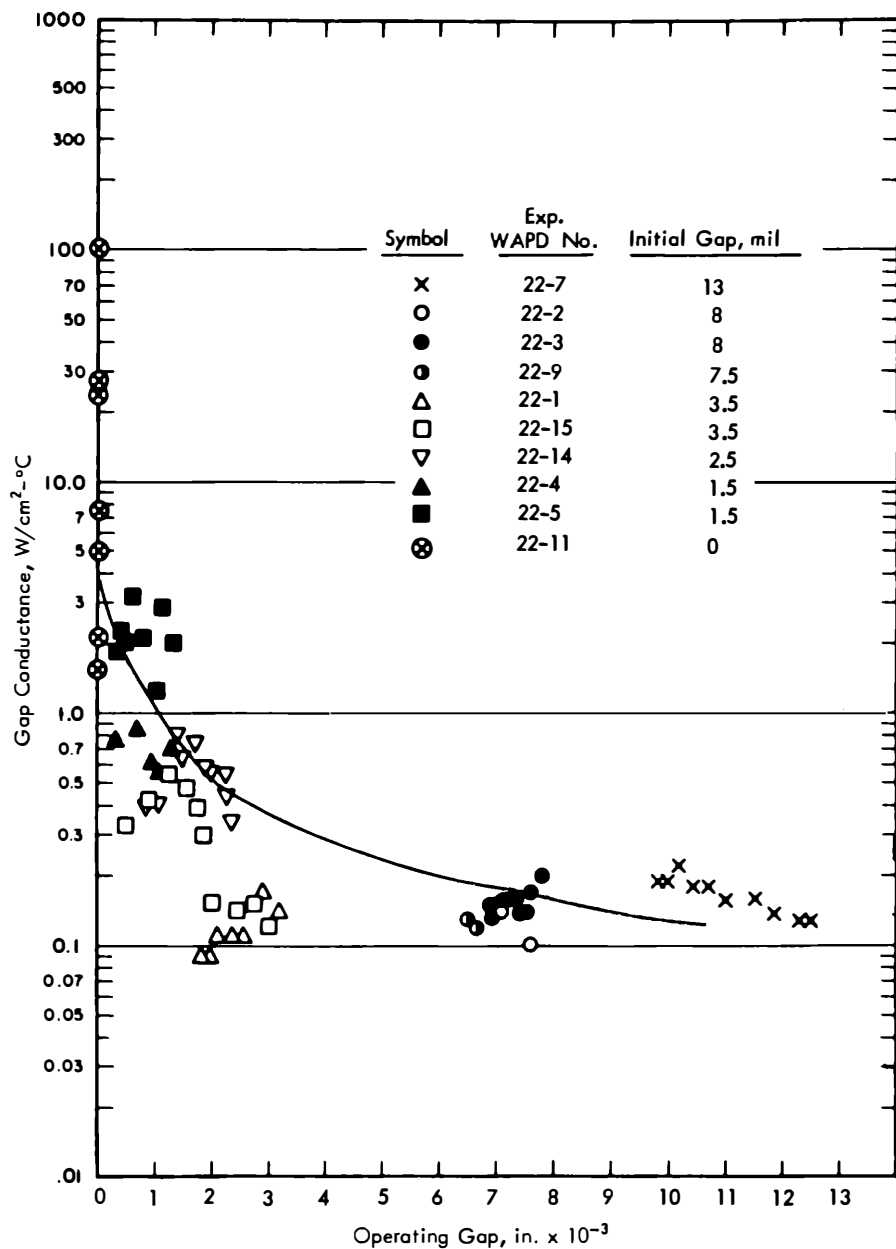


Fig. 2.13 Gap thermal conductance versus operating gap; fill gas-air at atmospheric pressure [from Ref. 123].

Although a more complex approach, based on the kinetic theory of gases, is more commonly used for evaluation of mixture conductivities, it is not clear that this is warranted in view of the other uncertainties in determining the gap conductance.

As long as the effective gap thickness is relatively large with respect to the mean free path of the gas molecules (from about 2×10^{-4} to 5×10^{-5} cm), the gas thermal conductivity may be taken as independent of pressure.

Standard PWRs are designed so that the cladding is "free standing" and, hence, the cladding will not collapse on the fuel pellet. The tolerance required to add pellets to a metal tube without problems are such that there is a pellet to cladding gap at initial operating conditions. However, pressurized heavy water reactors, such as the CANDU design, using natural uranium have little excess reactivity. It is therefore desirable to minimize parasitic absorption by using thin cladding, which collapses onto the surface of the fuel. The pressure difference ΔP , which produces such a collapse, may be conservatively estimated from¹²⁵

$$\Delta P = \frac{E}{4(1 - \gamma^2)} \left(\frac{L_c}{R} \right)^3 \quad (2.64)$$

where

E = Young's modulus (evaluated at collapse stress)

L_c = cladding thickness

R = mean radius of cladding tube

γ = Poisson's ratio (evaluated at collapse stress).

When operating below the differential pressure indicated by Eq. (2.64), a gap may be expected between pellet and cladding at BOL operating conditions.

The determination of the nominal gap thickness at operating conditions requires the proper determination of the pellet and fuel diameters. Pellet diameters must be corrected for thermal expansion, net irradiation swelling, and if fuel temperatures are high, for creep and hot pressing. The cladding diameter must be determined by allowing for thermal expansion, creep, and for the elastic deformation of the cladding due to the differential between external and internal pressure. For an infinitely long cylinder, subject to internal and external pressure, elastic deformation, ΔD_i , is given by

$$\Delta D_i = \frac{D_i}{E(K^2 - 1)} \{ P_i [(1 - \gamma) + (1 + \gamma)K^2] - 2K^2 P_0 \} , \quad (2.65)$$

where

D_i = internal diameter, in.

D_0 = external diameter, in.

P_i = internal pressure, lb/in.²

P_0 = external pressure, lb/in.²

E Young's modulus for the cladding

K D_0/D_i

γ = Poisson's ratio for the cladding.

Once the gas thermal conductivity and nominal gas thickness are known, the gap conductance may be calculated if an estimate of the parameter l_0 in Eq. (2.61) is available. This parameter allows for the effects of surface roughness and the lack of thermal equilibrium between the gas molecules and solid surface in a confined space. The parameter may be obtained from the sum¹²⁶

$$l_0 = \beta'(g_1 + g_2) + g_3 + (l_1 + l_2) \quad (2.66)$$

where

β' = empirical constant (2.0 according to Ref. 120)

g_1, g_2 arithmetic mean roughness of cladding and fuel, respectively,
m

g_3 fragmented fuel roughness, m

l_1, l_2 = temperature jump distances at fuel and cladding, respectively.

The nominal gap may be thought of as the distance between the high points on the rough fuel and pellet surfaces. Since the surfaces are not flat, a portion of these surfaces is separated by distance greater than the nominal gap. The product of β' and the roughness factors, g_1 and g_2 , thus increases the effective distance between the two surfaces to account for this fact. Some investigators add the additional term g_3 to account for the additional fuel roughness introduced by in-pile operation. Values between 5×10^{-6} and 8×10^{-6} m have been suggested for g_3 (Ref. 48).

It is found¹²⁷ that at gap distances having an order of magnitude not far from that of the mean free path of the gas molecules, the energy interchange between gas molecules and a solid surface is incomplete. The accommodation coefficient, α , is defined as the ratio of the temperature difference between the incident and reflected gas molecules to the temperature difference between the solid and incident molecules. The temperature jump distance required by Eq. (2.66) may be computed from the accommodation coefficient by¹²⁷

$$(l_1 + l_2) = 0.782 \left(\frac{k\sqrt{T}}{P} \right) \left(\frac{1}{\sum_i \alpha_i x_i / \sqrt{M_i}} \right) \quad (2.67)$$

where

k = thermal conductivity of gas molecule, W/m K

P = gas pressure, Pa

T = average gas temperature, K

α_i = accommodation coefficient of gas component i

M_i = molecular weight of i 'th component, kg/mol

x_i = mole fraction of the i 'th component.

Lanning and Hann¹²⁸ suggest the accommodation coefficient can be taken as a property of fill gas only. Based on the data of Ullman *et al.*,¹²⁹ they suggest

$$\alpha'_{\text{He}} = 0.425 - 2.3 \times 10^{-4} T_s \quad (2.68)$$

$$\alpha'_{\text{Xe}} = 0.749 - 2.5 \times 10^{-4} T_s \quad (2.69)$$

where T_s is the surface temperature (K). Accommodation coefficients for other gases are obtained by linear interpolation between α'_{He} and α'_{Xe} based on molecular weight.

Temperature jump distances for a gas mixture may also be estimated directly from the temperature jump distances of pure gases. Ross and Stoute¹³⁰ estimated these quantities for helium, argon, and fission gases from their data. For temperatures between 150 and 300°C, $(l_1 + l_2)$ is $< 10 \times 10^{-4}$ cm for helium, 5×10^{-4} cm for argon, and $< 1 \times 10^{-4}$ cm for xenon. The distances are assumed proportional to the mfp of the gas molecules and, hence, inversely proportional to the temperature. Campbell and DesHaies¹³¹ verified Ross and Stoute's temperature jump distance estimates. To use these data for gas mixtures, Veckerman and Harris¹³² show that

$$l_m = \frac{\sum x_i l_i / (M_i)^{1/2}}{\sum x_i / (M_i)^{1/2}} \quad (2.70)$$

where

l_m = temperature jump distance of the gas mixture

l_i = temperature jump distance of constituent gases

x_i = mole fraction

M_i = molecular weight.

Wesley and Yovanovich¹³³ propose a substantially different approach for the determination of the gap conductance, h_f . They consider the stochastic variation in the gap size due to the variation of the heights of the projections on the two gap surfaces. The gap is described in terms of Y , the distance between the surface mean planes, and an effective surface roughness, σ . If it is assumed that the gas mixture in the gap can be treated as a single gas with appropriately averaged properties, then Wesley and Yovanovich's result may be approximated as

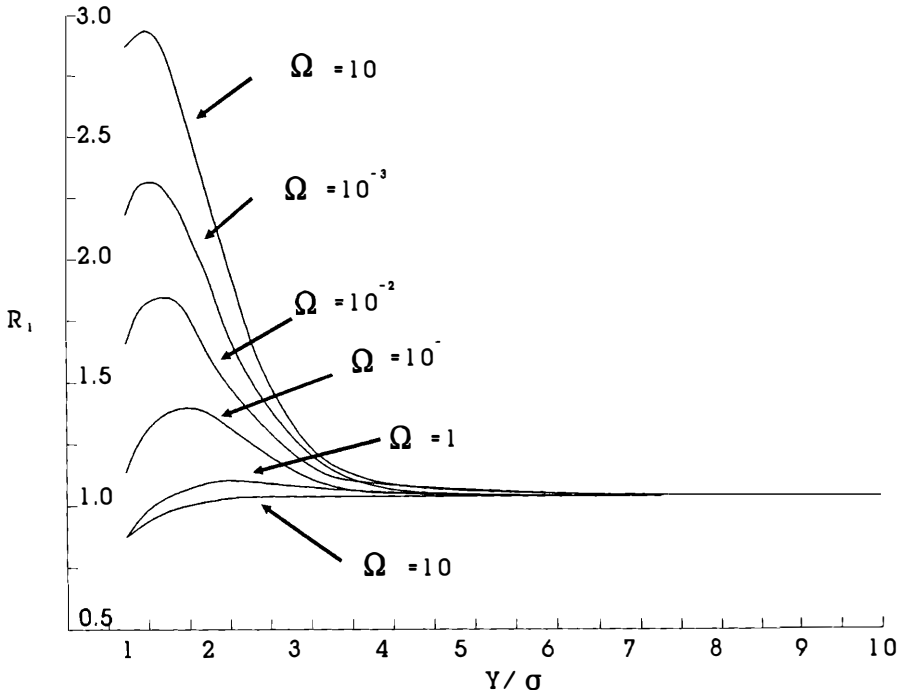


Fig. 2.14 Roughness factor versus dimensionless temperature jump lengths [from Ref. 133].

$$h_f = k_g \left(\frac{R_1}{Y + \Omega \sigma} \right) \quad (2.71)$$

where

$$\sigma = (\sigma_1^2 + \sigma_2^2)^{1/2}$$

σ_1, σ_2 = standard deviation of surface projection heights of cladding and fuel, respectively.

R_1 = roughness factor determined from Fig. (2.14)

$$\Omega = (l_1 + l_2)/\sigma.$$

Figure 2.14 shows that roughness effects die out rapidly with increasing gap spacing and that they can be ignored for $Y/\sigma > 5$. Equation (2.71) would seem to provide a more exact calculation of h_f when gap sizes are small. Here conductances are considerably above those calculated by the more conventional approach [Eqs. (2.61) and (2.66)]. This is due to contact between the larger projections even though there is still a nominal gap.

Postirradiation examination of fuel rods with an open gap indicates that the fuel temperature predictions are somewhat too high when gap conduc-

tance models are based on Eq. (2.61) and the assumption of a conventional concentric annular gap.¹³⁴ When a simple annular gap model is used, the operating gap thickness needs to be reduced by $\sim 25\%$ for helium-filled rods and about 75% for pure xenon to produce best estimate temperature predictions.⁴⁸ It has been postulated that the reduced temperatures were due to the fuel pellet columns being nonconcentric and in partial contact with the cladding. An alternative explanation is that pellet cracking produced partial pellet-cladding contact (see Section 2.2.5.2).

2.2.5.2 Performance with Closed Gap

When the pellet expands sufficiently to be in direct contact with the cladding, an improved gap conductance results. Several theoretical explanations of the phenomenon of contact conductance have been proposed and, in general, they consider a model in which the solid surfaces are in actual contact at only a few discrete points (Fig. 2.15). Since the thermal conductivity of the solids in contact is generally much greater than the thermal conductivity of gas filling with interstices, heat flow tends to channel through the points of contact. When contact pressure is increased, peaks in contact are deformed and the contact points increase in size and number. The number and size of the contact points also depend on the surface finishes. At low contact pressures, the primary mode of heat transfer can be by conduction across the gas gaps. The effective distance between the surfaces depends on the size and shape of surface irregularities.

Various analytical approaches have been proposed for computing contact conductance. Cetinkale and Fishenden¹³⁵ presented an equation for the contact conductance of two metal surfaces based on a calculation of the isotherms at the contact points of the two solids. A somewhat different approach was taken by Fenech and Rohsenow,¹³⁶ who found an approximate analytical solution for heat conduction at a contact point. The results obtained from these approaches have not been widely used by designers. A revised form of the Mikic and Todreas¹³⁷ model was used in fuel rod design but this is now superseded.

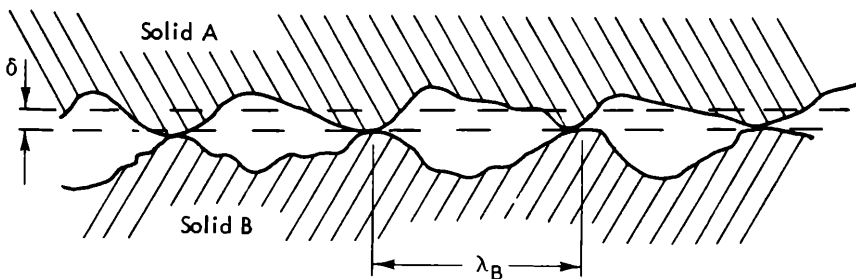


Fig. 2.15 Contact conductance model.

The total gap conductance, h_g , is often computed following the method suggested by Ross and Stoute.¹³⁰ The conductance is taken as the sum of h_s , the conductance due to solid contact regions, and h_f , the gas conductance

$$h_g = h_s + h_f \quad (2.72)$$

One approach to the evaluation of h_f would be the use of Eq. (2.61) with the assumption that the nominal gap, l , is zero and that l_o can be obtained by the procedures of the previous section. This, however, ignores the decrease in surface projection heights due to externally applied pressure. Lanning and Hann¹²⁸ conclude that the gas conductance can be obtained from

$$h_f = k_g / (\Delta L_{\text{eff}}) \quad (2.73)$$

where

$$\Delta L_{\text{eff}} = K_1[C(g_1 + g_2) + (l_1 + l_2)] + K_2$$

$$K_1 = 1.8$$

$$K_2 = -0.00012$$

$$C = 2 \exp(-0.000125 P)$$

$$P = \text{contact pressure, kg/cm}^2$$

$$g_1, g_2 = \text{mean surface roughness (projection height) of fuel and cladding, respectively, cm}$$

$$l_1, l_2 = \text{temperature jump distance, cm.}$$

The contact conductance, h_s , in Eq. (2.72) corresponds to the solid conductance that would be observed in a vacuum. It is usually evaluated by means of empirical correlations of data that have been corrected for the effect of gas conductance. Although some data on irradiated specimens is available,¹²³ these data tend to be clouded by the effects of irradiation on UO_2 properties. The correlations therefore are based on out-of-pile data. These data include the very early data of Wheeler¹³⁸ (see Fig. 2.16), which showed the importance of the yield strength in determining the conductance. More recent data include those of Dean,¹³⁹ Ross and Stoute,¹³⁰ Campbell,¹⁴⁰ Garnier and Begej,¹⁴¹ and Madhusudana.¹⁴² Although general approaches to contact conductance are available, most modelers have simply correlated the Zircaloy- UO_2 contact conductance data. The simplest approach that has been used is to consider the contact conductance as directly proportional to the contact pressure

$$h_s = aP \quad (2.74)$$

where P is the contact pressure (Pa) and a is the empirical constant. The ESCORE program⁴⁸ states a as $1.36 \times 10^{-3} \text{ W}/(\text{m}^2 \text{ } ^\circ\text{C Pa})$.

The most reliable (*in vacuo* data) of the available experimental data UO_2 -Zircaloy contact conductance indicate that a less than linear dependence is appropriate. Madhusudana and Fletcher¹⁴³ correlated these data by

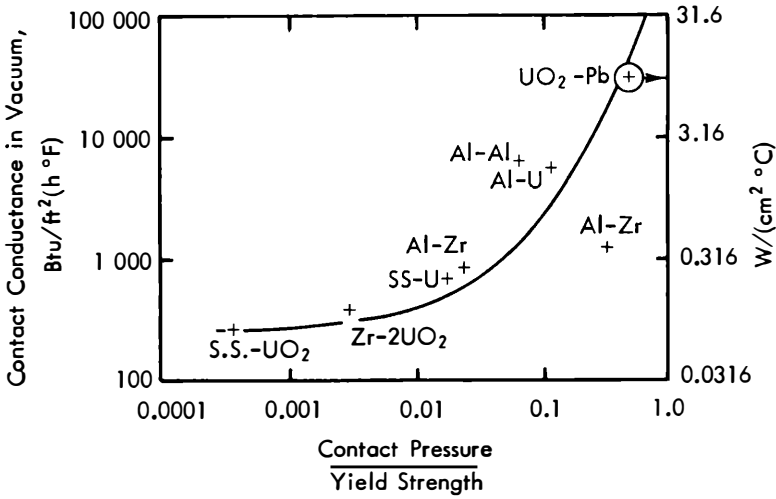


Fig. 2.16 Contact conductance measured by Wheeler [from *Reactor Handbook*, Vol. IV, Interscience Publishers, Inc., New York (1964)].

$$\frac{h_s L_o}{k_m} = 12.3 \times 10^{-3} \left(\frac{P}{H'} \right)^{0.66} \quad (2.75)$$

where

L_o = root mean surface roughness $(g_1^2 + g_2^2)^{1/2}$

g_1, g_2 = rms roughness of fuel and cladding, respectively

P = contact pressure

H' = flow pressure (Meyer hardness) of softer material

k_m = harmonic mean of thermal conductivities k_1 and k_2 of contacting surfaces $[(2k_1 k_2) / (k_1 + k_2)]$.

Typical values of H' are 13×10^4 psi for stainless steel and 14.2×10^4 psi for Zircaloy.

Lassman and Hohlefeld¹²⁶ correlate essentially the same set of data with an equation of the form

$$h_s = (a') \bar{g} \bar{k} \left[\frac{P}{3(\bar{g})^2 \sigma_y} \right]^{0.7}, \quad (2.76)$$

where

a' = empirical constant

$\bar{g}$ = arithmetic mean roughness

$$\bar{k} = (k_1 + k_2) / 2$$

σ_y = yield stress of softer material.

Both Eqs. (2.74) and (2.75) appear to model the reliable UO₂-Zircaloy data adequately. They both predict similar effects for contact pressure, hardness, or yield and thermal conductivity. However, the effect of surface roughness is more pronounced in Eq. (2.74). Equation (2.75) may be used for both stainless steel and Zircaloy but Eq. (2.74) was derived for Zircaloy only.

General correlations of contact conductance that can be used with any combination of surfaces are available. The best of these is probably that of Shylkov.¹⁴⁴ This correlation appears to give results that are somewhat above most of the UO₂-Zircaloy data.¹²⁸ There now appears to be no advantage in the use of generalized correlations for fuel rod calculations.

Contact pressure between the pellet and cladding can be estimated by computing their dimensions while ignoring any elastic deformation of the cladding. Once contact is established, the unrestrained pellet diameter will exceed the unrestrained internal cladding diameter. When the fuel operates at the relatively low linear power of current (1995) PWR designs, the pellet is essentially rigid. Pellet deformation occurs due to thermal expansion, swelling, and densification. The displacement of the fuel is then independent of the cladding deformation. With the rigid pellet assumption, the internal pressure required to produce an elastic cladding expansion that would provide a zero gap is given by Eq. (2.65). Since the cladding and pellet are in true contact over only a small fraction of the total area, the internal pressure, P_i , is given by

$$P_i = P_{\text{gas}} + P_{\text{contact}} \quad (2.77)$$

At high fuel temperatures (e.g., accident situations) the rigid pellet assumption is not entirely appropriate. Fuel deformation under stress (hot pressing) should be considered and deformation of fuel and cladding must be computed simultaneously.

If the fuel is operated with pellet contact and nearly constant power for an appreciable period, the cladding creep rate will be in equilibrium with the net swelling rate of the fuel. The cladding circumferential stress required to produce this creep rate can be determined from an appropriate creep rate equation. This stress can, in turn, be used to estimate the internal pressure and from that the contact pressure. By assuming that the thin wall approximation holds,

$$(P_i - P_o) = (2L_c \sigma_\theta) / D, \quad (2.78)$$

where

L_c = cladding thickness

D = average diameter of cladding

P_i, P_o = internal and external pressure, respectively
 σ_θ = circumferential cladding stress.

2.2.5.3 Performance With Open Gap and Cracked Pellet

Since UO_2 cracks so extensively at the heat generation levels at which a PWR operates, it has been suggested that it is unrealistic to assume that an annular gas gap can be retained after a few hundred hours of operation, even if calculations indicate there is a significant space between pellet and cladding. It is likely that cracked pellet pieces are lying against the cladding so gap conductance would approach the value computed from one of the contact conductance equations while assuming a zero contact pressure.

The data of Calza-Bini *et al.*¹⁴⁵ support this point of view. They observed that on the first rise to power, Eq. (2.61) provided reasonable estimates of the measured gap conductance. However, on subsequent power cycles, the measured gap conductance increased significantly. Stehle *et al.*⁷⁶ report that Brzoka and Wunderlich collected results showing somewhat similar behavior. They concluded that ~40% of the initial gap was eliminated during the first rise to power. After 500 hr of irradiation, they concluded that another ~20% of the gap was closed. These estimates apparently made no allowance for a decrease in UO_2 thermal conductivity due to cracking. It is likely that such an allowance would lead to estimates of nearly complete gap closure after 500 hr of operation.

Lanning *et al.*¹⁴⁶ avoided the assumption of an annular gap and simply correlated in-reactor data from 16 sources. For helium-filled rods, they suggest

$$h_g = 0.0321 + \frac{0.1167\sqrt{T_{\text{gap}}}}{2L'/D} - \frac{2.36}{2L'D} \quad (2.79)$$

For krypton- and xenon-filled rods,

$$h_g = 0.1129 + \frac{0.1401}{2L'/D} + \frac{0.0069q'}{D} \quad (2.80)$$

where

T_{gap} = gap temperature, K
 q' = linear heat rating, W/cm
 L' = gap thickness
 D = tube diameter

and h_g is given in $\text{W}/\text{cm}^2 \text{ } ^\circ\text{C}$.

MacDonald and Weisman⁴⁶ proposed a fairly complete model for cracked pellet behavior. They conclude that after a short period of initial operation at power, the pellet cracks and segments move outward, lying

against the cladding. This causes an increase in gap conductance, but a decrease in fuel conductivity. They indicate that when the uncracked pellet diameter is less than the cladding diameter, gap conductance can be represented by

$$h_g = \frac{f k_g}{L'_0} + (1-f) \frac{k_g}{L' + L'_0} \quad (2.81)$$

where

f = fraction of the pellet surface in contact with the cladding at zero contact pressure

L'_0 = root mean square of fuel and cladding surface projections

L' = gap thickness neglecting pellet cracking or eccentricity.

Based on Kjaerheim and Rolstaad's suggestion,¹⁴⁷ the portion of fuel in contact with the cladding is obtained from

$$f = C_1 + (1 - C_1) C_2^{(L' C_3 / D)} \quad (2.82)$$

where

C_1 = fraction of cracked fuel pellet assumed to be in contact with the cladding, which is independent of gap size

C_2, C_3 = constants representing the rate at which contact area fraction decreases with increasing gap

D = hot pellet diameter.

For the data examined, MacDonald and Weisman concluded $C_1 = 0.3$, $C_2 = 0.2$, and $C_3 = 50$. Once the gap closed ($L' < 0$), gap conductance was represented by a simplified Ross and Stoute¹³⁰ model.

By assuming that most of the annular gap is replaced by crack area in the pellet, MacDonald and Weisman⁴⁶ derived an effective thermal conductivity, k_e , for the cracked pellet. They obtained

$$k_e = k_{\text{UO}_2} \left\{ \frac{1}{\frac{2L' - A}{D[B(k_g/k_{\text{UO}_2}) + C]} + 1} \right\} \quad (2.83)$$

where

k_{UO_2} = thermal conductivity of uncracked UO_2

k_g = thermal conductivity of gas

D = diameter of fuel pellet.

With L' as gap thickness in mils and D expressed in mils, they found that the constants were $A = 2.5$, $B = 0.077$, and $C = 0.015$. For $2L' \leq 2.5$, $k_e \approx k_{\text{UO}_2}$.

MacDonald and Weisman⁴⁶ found that the centerline fuel temperatures, calculated using their cracked pellet model, compared very well with centerline temperature measurements in several instrumented fuel rod irradiations.

More recently MacDonald and Smith¹⁴⁸ analyzed a series of in-pile experiments in which they examined the effects of fuel cracking on fuel conductivity. They concluded that the cracking leads to an outward fuel relocation such that firm pellet-cladding contact occurs sooner than expected. They also concluded that the relocation for 1-cm-diameter fuel pellets operating at LWR power levels could be expressed as

$$R_p = 0.014 + 0.14L'_c \quad (2.84)$$

where R_p is the outward pellet relocation (mm) and L'_c is the initial cold gap (mm). They then expressed the effective thermal conductivity, k_e , of the fuel in terms of a hot gap corrected for the relocation. For low-burnup rods containing helium as the fuel gas,

$$k_e = k_{\text{UO}_2} - (0.0002189 - 0.050867\psi + 5.6578\psi^2) \quad (2.85)$$

where

$$\psi = (L' - R_p)(0.0545 / L'_c)$$

L' = hot gap without considering relocation, mm

k_{UO_2} = thermal conductivity of UO_2 , kW / m K.

Although Eq. (2.85) is based on helium fill gas, it should be applicable in the early portion of fuel life where helium is the main fuel gas constituent and cracking most significantly affects fuel properties. Note that according to Eq. (2.85), when $L' = R_p$ the fuel conductivity is essentially that of uncracked fuel.

Later studies⁴⁸ of irradiated rods with significant burnups indicated that the average outward relocation of the cracked fuel is a function of burnup, linear power rating, rod diameter, and gap size. Krammen and Freeburn⁴⁸ suggest that the average relocation, R_p , can be obtained from

$$R_p / D_0 = 0.80 [Q(L_i / D_0)(0.005\text{Bu}^{0.3} - 0.20 D_0 + 0.3)] \quad (2.86)$$

where

$$Q = \begin{cases} 0 & \text{for } q' \leq 6 \text{ kW/ft} \\ (q' - 6)^{1/3} & \text{for } 6 \text{ kW/ft} < q' \leq 14 \text{ kW/ft} \\ (q' - 10)/2 & \text{for } q' > 14 \text{ kW/ft} \end{cases}$$

and where D_0 the as-fabricated cold diameter of the pellet, in.

q' = the local linear heat rating, kW/ft

Bu = the local fuel burnup, MWd/MTU

$$L_t = L_0 - L_{\text{closure}}$$

L_0 = the as-fabricated cold diametral gap, in.

L_{closure} = the permanent diametral gap closure that has already occurred, in.

Krammen and Freeburn indicate that after pellet contact is established, most of the pellet relocation is recovered over a period of time. They conclude that the recoverable relocation is limited to a 0.1% decrease in the pellet diameter. Although no relationship for relocation recovery is available in the open literature (1992) it is clear that the contact pressure must remain low until all, or the maximum allowable, relocation is recovered. The assumption of nominal low-contact pressure, which allows continued clad creep-down until closure, would be expected to produce satisfactory fuel temperatures.

When Krammen and Freeburn estimate gap conductance, they add the value of R_p to the hot pellet diameter in order to calculate the gap thickness. Although they make no direct allowance for any pellet to cladding contact until the corrected pellet diameter equals the internal cladding diameter, they employed a further gap reduction which they state accounts for pellet eccentricity. It would appear that both the pellet relocation concept and the cracked pellet model are two somewhat different approaches to modeling of the same phenomenon. Acceptable fuel temperature estimates appear to be attainable by either approach.

2.2.6 Mechanical Behavior of Cladding

Calculation of the fuel-cladding interactions depends on an accurate description of both cladding and fuel behavior. Fuel pellets expand thermally and swell (possibly densify) and when subject to pressure by the cladding, creep. Cladding expands thermally, deforms elastically, and creeps inward or outward depending on the direction of pressure forces.

The hexagonal close-packed lattice of α -Zircaloy exhibits a preferred crystallographic orientation that confers a pronounced anisotropy in some mechanical properties. The thermal expansion of α -Zircaloy is one of the properties that differs along the various crystal planes. Further, the drawing operation to which Zircaloy tubing is subjected results in a preferred crystal orientation. A fully accurate determination of the thermal expansion requires (1) a determination of the thermal expansion of single crystals in the direction parallel and perpendicular to the basal crystal plane and (2) a knowledge of the crystal or grain orientations in the sample of interest. The crystal orientation in a sample is referred to as the sample *texture*. It is obtained by means of a *pole figure*.

To construct a pole figure, a sample of the material of interest is considered as the center of a sphere. Each of the crystal planes of the single crystals

making up the sample will have a projection on the surface of the sphere. The further projection of the sphere surface locations onto a plane tangent to the sphere is the pole figure. If the crystals are all aligned in one direction, a set of single dots will appear on the figure. In the real case, clusters of dots will appear. These clusters may be used to obtain the average orientations of the crystals. By an appropriate volume weighting over the sample, the average sample properties may be obtained from the single crystal properties. When this detailed texture information is available, highly accurate cladding thermal expansions can be computed (see Hagrman *et al.*³⁴).

When detailed texture information is lacking, the following correlations represent reasonable approximations of typical Zircaloy cladding behavior.³⁴ For diametral change

$$\Delta D / D = -2.344 \times 10^{-4} + 6.72 \times 10^{-6} T \quad (2.87a)$$

and for axial changes

$$\Delta L / L = -2.788 \times 10^{-5} + 4.44 \times 10^{-6} T \quad (2.87b)$$

when $(\Delta D / D)$ and $(\Delta L / L)$ represent the total change in length per unit length and T is in K. The foregoing holds for $(300 < T < 1073 \text{ K})$.

As previously noted, the Zircaloy alpha-beta phase transition starts at $\sim 1090 \text{ K}$ and ends at $\sim 1230 \text{ K}$ for oxygen-free cladding. Above 1230 K both axial and radial expansion of beta-Zircaloy can be taken as $\sim 9.7 \times 10^{-6} \text{ in./in. K}$.

At low loads, Zircaloy strain, ϵ , is proportional to the stress, σ , divided by E , Young's modulus. Hagrman *et al.*³⁴ give Young's modulus as

$$E = \begin{cases} (1.088 \times 10^{11} - 5.475 \times 10^7 T + K_1) / K_2 & \alpha \text{ phase} \\ 9.21 \times 10^{11} - 4.05 \times 10^7 T & \beta \text{ phase} \\ \text{linear interpolation} & \alpha + \beta \text{ phase} \end{cases} \quad (2.88)$$

where

E = Young's modulus for Zr-2 and -4 with random texture, Pa

T = cladding temperature, K

K_1 = cold work effect term = $-2.6 \times 10^{10} \text{ C}$, Pa

K_2 = fast neutron fluence effect term = $0.88 + 0.12(\phi t / 10^{25})$

C = cold work area ratio

ϕt = fast neutron fluence, n/m^2

The cold work area ratio is estimated from the yield strength by:

$$C = \begin{cases} 0 & \sigma_y \leq 140, \\ -0.269 + 0.001923\sigma_y, & 140 < \sigma_y < 660, \\ 1 & \sigma_y \geq 660, \end{cases} \quad (2.89)$$

where σ_y is the as-fabricated room-temperature axial yield strength (MPa). Hagerman *et al.* also include a term for the effect of as-fabricated oxygen concentration. However, these concentrations are so low that this effect is negligible and therefore this term has been omitted here.

Plastic deformation of Zircaloy can be estimated from

$$\epsilon = (\sigma / K)^{1/n} \quad (2.90)$$

where K is a strength coefficient and n is a dimensionless strain-hardening coefficient. Values for these quantities from Busby's¹⁴⁹ data are

$$n = -1.3498 \times 10^{-2} + 5.152 \times 10^{-4}T - 5.5947 \times 10^{-7} T^2 \quad (2.91a)$$

$$K = 1.398304 \times 10^{19} - 1.219965 \times 10^6 T - 96.6206 T^2 \text{ [N/m}^2\text{]} \quad (2.91b)$$

Temperature T is in K. More recent data show that the parameters K and n are functions of the cold work and irradiation the tube has received.³⁴ Transition from the elastic to plastic region occurs when plastic strain is greater than elastic strain.

Neutron irradiation has a substantial effect on the strength and ductility of Zircaloy. Irradiation leads to increases in the yield strength and decreases in ductility. Because of irradiation-induced ductility reduction and further possible reductions due to hydride formation, the maximum permissible strain must be set at a low level. Some irradiation experience¹⁵⁰ indicates that the maximum strain should not exceed $\sim 1\%$, since some test rods have failed at strains of this magnitude. Many investigators consider the strain range (maximum outward ϵ - and minimum inward ϵ), to be a better criterion for brittle failure.

It was previously observed that fast neutron irradiation also effects Zircaloy creep. These effects have been investigated by Ibrahim¹⁵¹ and others. Early correlations generally gave the creep rate as the product of neutron flux term and a stress term. For example,¹⁵¹

$$\dot{\epsilon} = K \phi [\sigma + B \exp(C\sigma)] \exp(-10000/RT) t^{-1} \quad (2.92)$$

where

$\dot{\epsilon}$ = transverse creep rate, m/m s

$K = 5.129 \times 10^{-29}$

$C = 4.697 \times 10^{-8}$

T = temperature, K

ϕ = fast neutron flux, n/m² s ($E > 1.0$ MeV)

$B = 7.52 \times 10^2$

$R = 1.987$ cal/mol K

t = time, s.

More recent representations of Zircaloy creep express the creep as a sum of thermal and irradiation creep terms. That is:

$$\epsilon_{\text{total}} = \epsilon_{\text{thermal}} + \epsilon_{\text{irradiation}} \quad (2.93)$$

Based on the thermal creep-model of Gorsuk and Pfennigworth¹⁵² and the irradiation-creep model of Franklin¹⁵³, Krammen and Freeburn⁴⁸ recommend:

$$\epsilon_{\text{thermal}} = A_1 [\sinh(A_2 \sigma_{\theta}^{A_3})]^{A_4} t^{A_5} \sigma_y^{A_6} \exp(-A_7 / T) \quad (2.94)$$

$$\epsilon_{\text{irradiation}} = B_1 t^{B_2} \phi^{B_3} \sigma_{\theta}^{B_4} [\exp(-B_5 / T)] \sigma_y^{B_6} [\cos \theta_{\text{max}}]^{B_7} \quad (2.95)$$

where

ϵ = total diametral creep strain, m/m

t = exposure time, hr

ϕ = fast neutron flux for $E > 1\text{MeV}$, $\text{n}/\text{cm}^2 \cdot \text{s}$

σ_{θ} = midwall hoop tangential stress, MPa

T = cladding average temperature, K

σ_y = cladding yield strength at room temperature, MPa

$\cos \theta_{\text{max}}$ = cosine of angle of maximum intensity of basal pole to radial direction, normal to axial direction (obtained from pole figure).

The constants in the creep equations, which were derived from a regression fit of LWR creep rate data, have the following values:

Constant	Value	Constant	Value
A_1	1.388×10^8	B_1	$2.35 \cdot 10^{-21}$
A_2	3.29×10^{-5}	B_2	0.811
A_3	2.28	B_3	0.595
A_4	0.997	B_4	1.352
A_5	0.770	B_5	22.91
A_6	0.956	B_6	1.58
A_7	23×10^3	B_7	2.228

Note that the Eq. (2.95) requires Zircaloy texture information. When this is not available, Eq. (2.92) can be used to provide a reasonable estimate of average cladding behavior.

The creep strain correlation of Eqs. (2.94) and (2.95) is based on data from both compressive stresses (seen in creep-down when external pressure exceeds internal) and tensile stress (seen in outward creep when internal pressure exceeds external). However, some investigators use separate correlations for compressive and tensile creep.³⁴

The predictions of total creep of Eqs. (2.94) and (2.95) and the creep rate of Eq. (2.92) are based on the data taken with the Zircaloy subjected to constant stress from zero time until time t . In any reactor situation, the stress will vary with time and it is not obvious how the correlations should be applied. Two approaches have been proposed: (1) *Time hardening law*: The equivalent creep rate is a function only of the time over which stress

has been applied. (2) *Strain hardening law*: The equivalent creep rate is only a function of the total strain. Current theory holds that the strain hardening law is the most appropriate. To determine the creep, over a given time interval with a specified stress and initial strain, with this approach, an equivalent time, t_{eq} , is required. This time is that needed to produce the specified strain if the current stress had been a constant from time zero. This equivalent time may then be used directly in Eq. (2.92) to determine a strain rate. Alternatively, the strain rate may be determined by substitution of t_{eq} into the partial derivatives of Eqs. (2.94) and (2.95) taken with respect to time.

Irradiation-included axial growth of Zircaloy can be of significant concern for high-burnup fuels. A fairly complex model is provided in the later version of MATPRO.³⁴ A simple correlation is provided by Hagrman¹⁵⁴ as follows:

$$\Delta L / L = A[\exp(T_0 / T)](\phi t)^{1/2}(1 - 3f_3)(1 + 2C_w) \quad (2.96)$$

where

$\Delta L / L$ = fractional change in length

A 1.407×10^{-6} (neutrons/m²)^{-1/2}

T_0 240.8 K

T = cladding temperature, K

ϕ = fast neutron flux, n/m²s ($E > 1.0$ MeV)

t = time, s

C_w = degree of cold work (fraction of cross-sectional area reduction)

f_3 texture factor (0.05 is a typical value).

It has been observed that axial growth depends on the tubing supplier. Some designers⁴⁸ therefore prefer to correlate these data by

$$\Delta L / L = A(\phi' t)^n \quad (2.97)$$

where ϕ' is in n/cm²s and A and n are determined for the given supplier. Values of A vary from 1.8×10^{-15} to 7×10^{-20} while n varies from about 0.55 to 1.0.

2.2.7 Accuracy of Fuel Element Property and Behavior Models

Those models or correlations describing simple fuel and cladding properties (e.g., coefficient of thermal expansion, specific heat, density) generally exhibit good accuracy. Unless noted to the contrary, the properties are assumed to be unaffected by fuel or cladding irradiation. When compared to experimental measurements at the high temperatures encountered at reactor conditions, most properties of unirradiated materials show root mean

square percentage errors (standard error) of about 10% (Ref. 34). The general difficulty of high-temperature measurements accounts for most of this error. Variation in sample preparation is another contributor to the error.

The effect of irradiation on fuel and cladding properties introduces additional uncertainties. For most properties, irradiation effects are entirely negligible or very small as noted earlier. Irradiation has a small effect on the melting point of oxide fuels.⁵⁰ The effect of irradiation on oxide fuel thermal conductivity is of considerably more importance to the designer. Because of the structural changes that oxide fuel undergoes during irradiation, it is difficult to disentangle the irradiation effect on the conductivity of the oxide itself from the effects of fuel cracking and changes in gap conductance. Estimates of the change in oxide conductivity due to irradiation may therefore show errors of $\pm 50\%$. However, since the irradiation effect reduces thermal conductivity by no more than about 15% (Ref. 48), the effect of this error on temperature measurements is modest.

Models and correlations for cladding and oxide fuel behavior (e.g., cladding creep rate, fuel hot pressing, fuel cracking, gap conductance) generally have a lower accuracy than those for properties. Cladding behavior can be significantly influenced by the exact fabrication method employed since this will vary the cladding texture. Oxide fuel behavior can be strongly influenced by the properties of the oxide powders used to produce the pellet. Information on cladding texture or oxide powder properties is often lacking and most general models therefore ignore these factors. In addition, irradiation effects are usually significant. Since determining the irradiation the sample has received is difficult, a further uncertainty is introduced. The problem is exacerbated by the possibility that the form of the model or correlation may not be entirely appropriate for the phenomena involved.

Fuel and cladding behavior models will generally compare reasonably well with the data set used to generate the models. However, when compared to other data sets, errors of up to 50% may sometimes be encountered.⁵³ Despite the uncertainties in property and behavior correlations, predictions of key thermal parameters of operating fuel rods generally show reasonable agreement with experimental data. For example, predictions⁵³ of centerline fuel temperatures as a function of power in a typical PWR fuel rod with low burnup tend to agree with actual measurements to within about 200°F.

2.3 STEADY-STATE CONDUCTION UNDER IDEALIZED CONDITIONS

2.3.1 Heat Generation Within a Fuel Plate

2.3.1.1 *Uniform Heat Generation*

Let us consider the temperature distribution within the fuel of a thin fuel plate. If we ignore axial conduction, a one-dimensional treatment can be

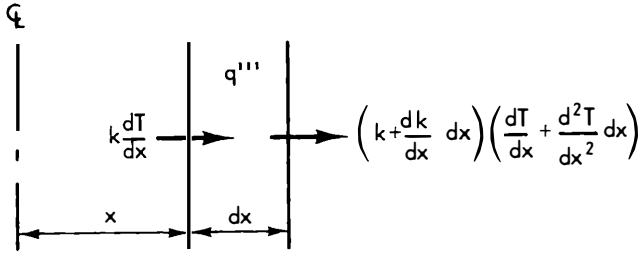


Fig. 2.17 Heat conduction in a fuel plate.

used. From a heat balance across a differential element (see Fig. 2.17), we have

$$\left(k + \frac{dk}{dx} dx\right) \left(\frac{dT}{dx} + \frac{d^2T}{dx^2} dx\right) - k \frac{dT}{dx} + q''' dx = 0 \quad (2.98)$$

where x is the distance from the fuel element centerline, q''' is the volumetric heat generation rate, and other symbols have their previous meanings.

After eliminating the second-order term, we simplify to

$$\frac{d}{dx} \left(k \frac{dT}{dx}\right) + q''' = 0 \quad (2.99)$$

By integrating on the assumption that q''' is constant and by imposing the boundary condition that dT/dx is zero at the centerline, we obtain

$$k \frac{dT}{dx} = -xq''' \quad (2.100)$$

and finally

$$\int_{T_c}^x k dT = -\frac{x^2}{2} q''' \quad (2.101)$$

where T_c is the center temperature. By substituting an appropriate expression for thermal conductivity, the temperature at any point can be determined. The assumption of negligible axial conduction can be justified for metallic fuel plates because of their very small thickness. For a UO_2 plate, the very low thermal conductivity of UO_2 provides the justification. The effect of axial conduction has been considered in detail by Fagan and Mingle,¹⁵⁵ who provide expressions for surface heat flux. They indicate that for most power reactor systems, the minimum heat flux will be in error by $<0.5\%$ if axial conduction is neglected.

2.3.1.2 Effect of Flux Depression

In a uniform lattice, diffusion theory indicates thermal neutron flux ϕ varies across the fuel plate in accordance with

$$\phi = \phi_0 \cosh(\kappa x) \quad (2.102)$$

where ϕ_0 is the flux at the centerline and κ is the reciprocal of the thermal diffusion length. Since q''' is proportional to ϕ in a thermal reactor,

$$q''' = q_0''' \cosh(\kappa x) \quad (2.103)$$

where q_0''' is the volumetric heat generation rate at the centerline. In actual practice, flux depression probably will be computed by the escape probability method (see Sec. 1.2.5). Although a simple, closed-form solution is not obtained, numerical results generally can be fitted quite well by equations like Eq. (2.102), provided κ is appropriately chosen.

If we proceed in the same manner as for a constant rate of heat generation, we obtain as the analog of Eq. (2.99)

$$\frac{d}{dx} \left(k \frac{dT}{dx} \right) + q_0''' \cosh(\kappa x) = 0 \quad (2.104)$$

Integration twice with $dT/dx = 0$ at the centerline yields

$$\int_{T_s}^x k \, dT = \frac{q_0'''}{\kappa^2} [\cosh(\kappa a) - \cosh(\kappa x)] \quad (2.105)$$

where a is the half thickness of the plate. We can also express our result in terms of surface heat flux per unit area q'' by noting that

$$q'' = \frac{q_0'''}{\kappa} \sinh(\kappa a) \quad (2.106)$$

hence,

$$\int_{T_s}^x k \, dT = q'' \left[\frac{\cosh(\kappa a) - \cosh(\kappa x)}{\kappa \sinh(\kappa a)} \right] \quad (2.107)$$

and

$$\int_{T_s}^{T_c} k \, dT = q'' \left[\frac{\cosh(\kappa a) - 1}{\kappa \sinh(\kappa a)} \right], \quad (2.108)$$

where T_c is the center temperature.

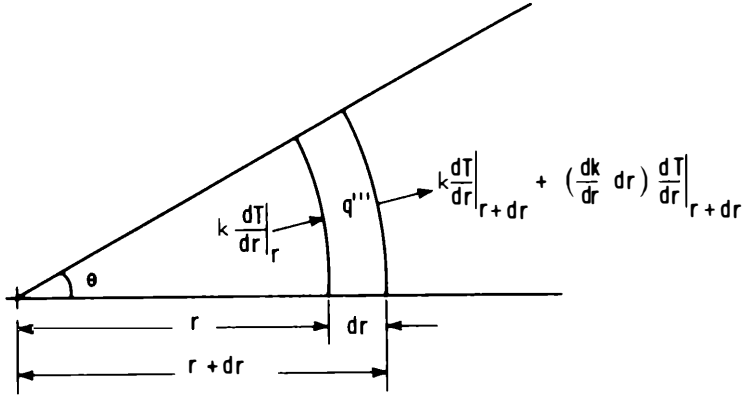


Fig. 2.18 Heat conduction in a fuel rod [from *Nucl. Eng. Des.*, **6**, 301 (1967)].

2.3.2 Heat Generation Within a Fuel Rod

2.3.2.1 Constant Heat Generation Across a Rod

From a heat balance across a differential annular ring (Fig. 2.18), we have

$$\theta \left(k + \frac{dk}{dr} dr \right) (r + dr) \left(\frac{dT}{dr} + \frac{d^2T}{dr^2} dr \right) - \theta k r \frac{dT}{dr} + q''' r \theta dr = 0 \quad (2.109)$$

By rearranging and omitting higher order terms, we obtain

$$\frac{1}{r} \frac{d}{dr} \left(k r \frac{dT}{dr} \right) + q''' = 0 \quad (2.110)$$

This can be integrated to yield

$$k r \frac{dT}{dr} = - \int_{r_c}^r r q''' dr = - \frac{q'''}{2} (r^2 - r_c^2) \quad (2.111)$$

$$\int_{T_r}^T k dT = \int_r \left[- \frac{q'''}{2} \left(r - \frac{r_c^2}{r} \right) \right] dr = \frac{q'''}{4} \left[(r^2 - r_c^2) - 2r_c^2 \left(\ln \frac{r}{r_c} \right) \right] \quad (2.112)$$

where T_c is the center temperature at central radius r_c and T_r is the temperature at radius r .

When we are dealing with solid cylinders (no central hole), we can simplify our last result to

$$\int_{T_s}^T k dT = \frac{q''' R_0^2}{4}, \quad (2.113)$$

where R_0 is outer radius and T_s is the surface temperature.

We can also express the volumetric heat generation in terms of the rate of heat generation per unit length of fuel rod q' as

$$q' = \pi R_0^2 q''' \quad (2.114)$$

then

$$q' = 4\pi \int_{T_s}^{T_c} k dT \quad (2.115)$$

For the approximation of a constant value of k , we obtain

$$(T_c - T_s) = \frac{q'}{4\pi k_{av}} \quad (2.116)$$

From Eqs. (2.115) and (2.116), we find that the temperature difference across a fuel pellet is independent of the radius of the fuel pellet. Specifying a given surface temperature and a given rate of power generation per unit length of fuel fixes the central fuel temperature.

2.3.2.2 Concept of $\int k dT$

Equation (2.115) states that q' the rate of heat generation per unit length of a cylindrical fuel rod, is directly proportional to the thermal conductivity integral,

$$\int_{T_s}^{T_c} k dT$$

Thus, if values of the integral are available, the power level necessary to produce a given center temperature is readily specified.¹⁵⁶ This is convenient for the designer since it avoids the necessity of performing the integration of thermal conductivity values with respect to temperature. It is also a convenient way to report experimental data from in-pile irradiations. Under those conditions, power generation, center, and surface temperature can be known. Hence, $\int_{T_s}^{T_c} k dT$ is known, but pointwise values of k are not available. Data are usually given in terms of $\int_{T_0}^{T_c} k dT$, where

$$\int_{T_s}^{T_c} k dT = \int_0^{T_c} k dT - \int_0^{T_s} k dT \quad (2.117)$$

Several curves of the integral as a function of its upper limit are shown in Fig. 2.19.

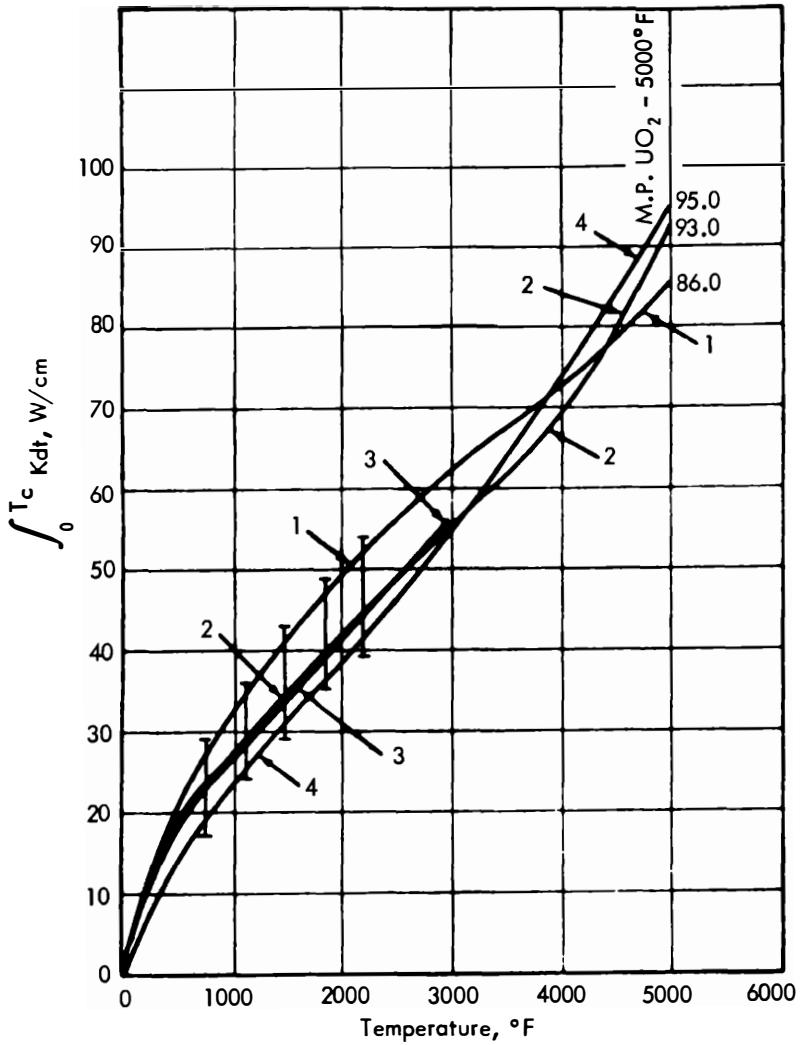


Fig. 2.19 Summary of $\int k dt$ for UO_2 . Notation:

1. Weidenbaum, GEAP-3771-8, General Electric Company (1963).
 2. Duncan, CVNA-142, Westinghouse Electric Corporation (1962).
 3. Reference 40.
 4. Robertson *et al.*, *J. Nucl. Mater.*, 7, 3, 225 (1962).
- [From McNelly, "Liquid Metal Fast Breeder Reactor Design Study," GEAP-4418, General Electric Company (1963).]

While the concept is of particular value for UO_2 fuel rods, it is also useful in describing UO_2 fuel plate behavior. As seen from Eq. (2.101), a knowledge of the integral and plate thickness is sufficient to establish the volumetric power generation required for a given temperature. Indeed, it is possible to relate $\int k dT$ to the heat output for any geometric shape. Thus, for constant heat generation

$$\int_{T_s} k dT = Cq' \quad (2.118)$$

where C depends on fuel geometry. Specification of the maximum allowable value of the integral fixes the power capability of the fuel in any given geometry. As previously noted, a common design criterion is the avoidance of center melting. Since this is an observable point, a series of tests at varying power levels can be used to determine the $\int k dT$ at which this first occurs. Observations are not dependent on a precise knowledge of fuel conductivity or temperature distribution within the fuel. Values, determined by several investigators, for the $\int k dT$ to produce melting are shown in the upper right of Fig. 2.19. More recently, Lyons *et al.*¹⁵⁷ evaluated this integral in a series of short-time, in-pile tests. From metallographic examinations they concluded that for 95% dense, sintered UO_2 pellets, the average value of the conductivity integral from 0°C to melting was 93.5 W/cm . Statistically determined 95% confidence limits were $\pm 3.5 \text{ W/cm}$. Direct measurement of the integral based on a gas-bulb sensor yielded a value of $\sim 90 \text{ W/cm}$. Vibratory compacted fuel was found to have a significantly lower integral even when the density correction was made.

Expressions for the thermal conductivity integral for sintered pellets have been provided by Lyons *et al.*¹⁵⁷ and MacDonald and Thompson,⁵³ among others. For 95% dense UO_2 with temperatures between 0 and 1650°C , MacDonald and Thompson suggest

$$\int_0^T k dT = 40.4 \ln(464 + T) + 0.027366 \exp(2.14 \times 10^{-3}T) - 248.02 \quad (2.119)$$

For temperatures between 1650°C and melting,

$$\int_{1650}^T k dT = 0.02(T - 1650) + 0.027366 \exp(2.14 \times 10^{-3}T) - 0.93477 \quad (2.120)$$

In these equations, T is in $^\circ\text{C}$ and $\int k dT$ is in W/cm . These equations yield 95.4 W/cm as the conductivity integral from 0°C to melting.

For temperatures above 773 K , accurate values of the thermal conductivity integral can be obtained by integrating the analytic equation of Harding

and Martin⁴² [Eq. (2.10)]. With the polaron contribution considered as negligible below 773 K, this integration yields

$$\int_{773}^T k \, dT = 4.619 \times 10^3 \ln[0.1830 + 1.0566 \times 10^{-3} T] - 4.715 \times 10^9 \exp(-16361/T) \sum_{i=0}^4 \frac{(-1)^i (4!) (1/T)^{(4-i)}}{(4-i)! (-16361)^{(i+1)}} \quad (2.121)$$

for 100% dense UO_2 . In Eq. (2.121), T is in K and k is in $[\text{W}/(\text{m K})]$.

Harding and Martin⁴² suggest that, when the integral is applied to less than 100% dense UO_2 , the integral be multiplied by $(1 - 2.5\alpha)$ where α is the void (porosity) fraction. The integral of Eq. (2.121) may be used to evaluate $\int_0^T$ for temperatures above 773 K with Eq. (2.119) providing values of the integral below 773 K.

The effect of long-time irradiation on the conductivity integral is uncertain. While most earlier observers concluded there is significant reduction due to irradiation, some data dispute this.^{47,158} Stora's previously cited explanation⁴⁵ that most apparent effects are simply due to thermal cracking and, thus, depend on the available gap, seems most reasonable. However, as noted in Sec. 2.1.1.3, there remains some irradiation effect after the effect of cracking is removed. The thermal conductivity correction factor of Eq. (2.12) can be used to correct the integral for the residual irradiation effect.

Soulhier¹⁵⁹ used the conductivity integral for correlation of fission gas release data. His results are presented as

$$\int_{400^\circ\text{C}}^{T_c} k \, dT(\text{max}) \text{ versus } F \int_{T_s}^{T_c} k \, dT$$

where F is the fractional gas release, T_c is the center temperature, and T_s is surface temperature.

The integral $\int_{400^\circ\text{C}}^{T_c} k \, dT$ is used to normalize all experiments performed at a given central fuel temperature to the same surface temperature of 400°C . The product $F \int_{T_s}^{T_c} k \, dT$ is used to normalize the fission gas released based on the volumetric average heating rate.

2.3.2.3 Rod with Flux Depression

We previously showed that diffusion theory indicates thermal flux distribution across a cylindrical fuel rod in a uniform lattice given by

$$\phi = A I_0(\kappa r) , \quad (2.122)$$

where

$A = \text{constant}$

$I_0 = \text{zero-order modified Bessel function of the first kind}$

$\kappa = \text{the reciprocal of the thermal neutron diffusion length in the fuel} = \sqrt{\Sigma_a / D_1}$.

When the escape probability method is used, which is more usually done, κ should be regarded as a constant selected to yield a closed-form approximation of the results. However, the experimental work of Gibson and Anno¹⁶⁰ indicates that measured values of κ differ by only about 10% from that obtained by simple diffusion theory. Since flux depression effects are small at PWR enrichments, the error in κ would cause only a very small error in the fuel temperature.

Since q''' is proportional to ϕ in a thermal reactor,

$$q''' = A' I_0(\kappa r) \quad (2.123)$$

Now let q_0''' be the value of q''' at the center of the pellet. Since $I_0(0) = 1$, our previous expression becomes

$$q''' = q_0''' I_0(\kappa r) \quad (2.124)$$

The average value of q''' over the entire rod cross section is given by

$$\bar{q}''' = \frac{\int_0^a q'''(2\pi r) dr}{\int_0^a 2\pi r dr} = \frac{2}{a^2} \int_0^a q''' r dr \quad (2.125)$$

where a is the outer radius of the pellet.

By substituting our expression for local flux in Eq. (2.125), we obtain

$$\bar{q}''' = \frac{2q_0''' [I_1(\kappa a)]}{(\kappa a)} \quad (2.126)$$

Substitution of the heat flux expression in our equation for the temperature gradient at any point [Eq. (2.110)] and integrating yield

$$\frac{q_0''' r [I_1(\kappa r)]}{(\kappa r)} + k \frac{dT}{dr} = 0 \quad (2.127)$$

On separating the variables and integrating from r to a , we obtain

$$\frac{q_0'''}{\kappa^2} [I_0(\kappa a) - I_0(\kappa r)] = \int_{T_s}^T k \, dT \quad (2.128)$$

If we rewrite the above in terms of the average heat generation rate, we get

$$\frac{\frac{\bar{q}''' a^2}{4} \{2[I_0(\kappa a) - I_0(\kappa r)]\}}{\kappa a [I_1(\kappa a)]} = \int_{T_s}^T k \, dT \quad (2.129)$$

At the center of the pellet, $r=0$ and $T=T_c$. Under these conditions, Eq. (2.129) becomes

$$\left(\frac{\bar{q}''' a^2}{4} \right) \frac{2[I_0(\kappa a) - 1]}{\kappa a [I_1(\kappa a)]} = \int_{T_s}^{T_c} k \, dT \quad (2.130)$$

If we let

$$f = \frac{2[I_0(\kappa a) - 1]}{\kappa a [I_1(\kappa a)]} \quad (2.131)$$

and replace $\bar{q}'''$ in terms of q' the rate of heat generation per unit length of rod [Eq. (2.114)], we obtain

$$q' f = 4 \pi \int_{T_s}^{T_c} k \, dT \quad (2.132)$$

The term f , which is the *flux depression factor*, becomes clear when we compare Eqs. (2.115) and (2.132). We see the thermal power associated with the given fuel surface and center temperature is greater for nonuniform than for uniform heat generation; the ratio of the two heat ratings is f .

Approximate values of f can be obtained from diffusion theory if values of κ are available. For a UO_2 of 95% theoretical density, κ varies from 2 to 3 cm^{-1} for enrichment values of 2.5 to 6% ^{235}U . Figure 2.20 shows f as a function of enrichment and fuel diameter for pellets of 95% dense UO_2 . For the enrichments used in PWRs the value of f is not far from unity. However, this effect should not be ignored since it can mean an increase of several percentages in reactor power output. The fact that f is close to unity can be used to justify the simple approximation that thermal flux inside the fuel follows a parabolic distribution. If this is done, values of F (the ratio of surface-to-average flux in the rod) obtained from escape probability theory can be used directly to compute f without assigning a value to κ . Conse-

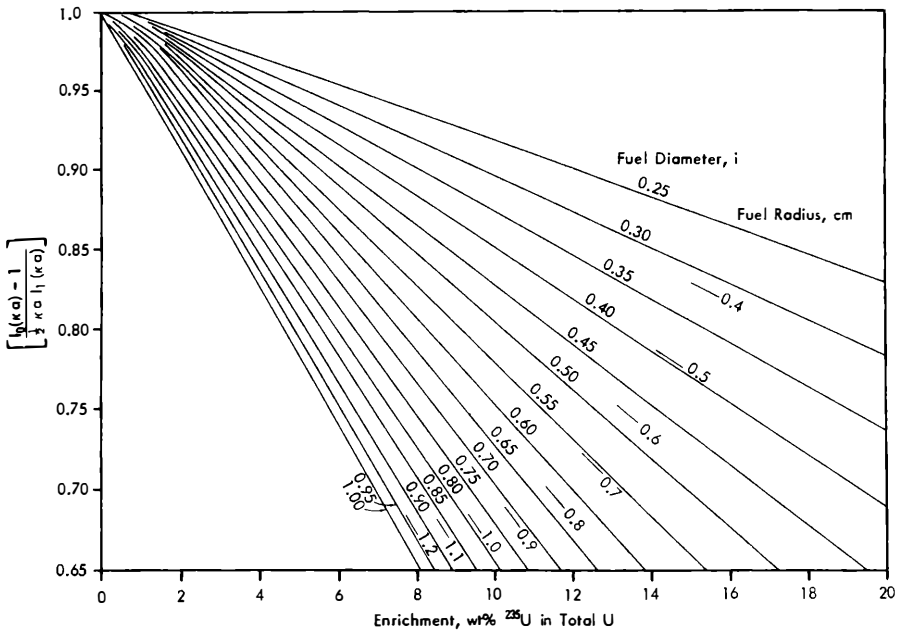


Fig. 2.20 Flux depression factors for UO_2 fuel rods of 95% theoretical density [from Ref. 156].

quently, for a parabolic flux distribution

$$f = (3 - F)/2 \quad (2.133)$$

2.3.2.4 Heat Generation with a Power Tilt or Eccentric Pellet

In Sec. 1.3.2, we indicated that flux distribution across a cluster of rods in a pressure tube of D_2O or a graphite-moderated reactor is approximated by the function $I_0(r)$. When a single rod is considered (Fig. 2.21), this appears as a flux tilt. A reasonable representation of the distribution has been found to be¹⁶¹

$$q''(r, \theta) = \bar{q}'''(1 + \epsilon' r/a \cos \theta) \quad (2.134)$$

where ϵ' represents half the difference between the minimum and the maximum heat generation rate divided by $\bar{q}'''$, the average volumetric heat generation rate

$$\epsilon' = (q'''_{\max} - q'''_{\min}) / 2\bar{q}''' \quad (2.135)$$

Since we now consider both radial and azimuthal variation in temperature, we must use the general form of the steady-state conduction equation

$$k\nabla^2 T = q''' = 0 \quad (2.136)$$

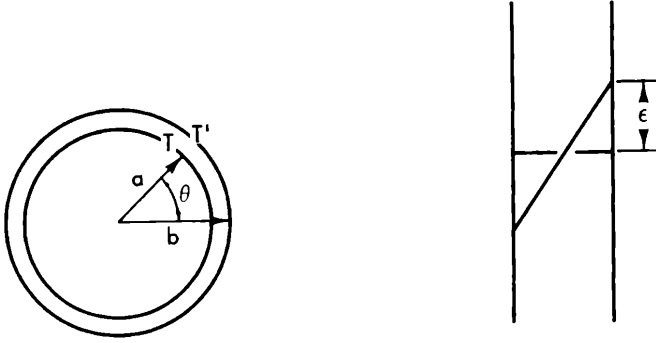


Fig. 2.21 Fuel rod with flux tilt.

In cylindrical coordinates with the present heat flux distribution, this becomes

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\bar{q}'''}{k} \left(1 + \epsilon' \frac{r}{a} \cos \theta \right) = 0 \quad (2.137)$$

in the pellet region, if we assume a constant value for k .^{*} Since we have no heat generation in the cladding region we obtain

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T'}{\partial r} \right) + \frac{1}{r^2} \left(\frac{\partial^2 T'}{\partial \theta^2} \right) = 0 \quad (2.138)$$

where T' represents the cladding temperature.

At the boundary of the pellet and cladding, there is normally a gap to which we assign a conductance h_g . Since heat transferred across the pellet, gap, and cladding must be equal to $r=a$, we have the boundary conditions at $r=a$:

$$h_g [T(a, \theta) - T'(a, \theta)] = -k \left(\frac{\partial T}{\partial r} \right) \Big|_{r=a} \quad (2.139)$$

$$k' \frac{\partial T'}{\partial r} \Big|_{r=a} = k \frac{\partial T}{\partial r} \Big|_{r=a} \quad (2.140)$$

where k and k' are the thermal conductivity of the fuel and cladding, respectively. If the heat transfer coefficient to the coolant is high, at $r=b$ we say

$$T'(b, \theta) \approx T_{\text{bulk water}} \quad (2.141)$$

^{*}Andrews and Dixmier¹⁶² provide a solution for the case of varying k .

Also note that at $r=0$, T is not infinite. With these boundary conditions, Eqs. (2.137) and (2.138) can be integrated to yield

$$T = \frac{\bar{q}'''}{4k}(a^2 - r^2) + \frac{1}{2h_g} a \bar{q}''' + \frac{\bar{q}''' a^2}{2k'} \ln \frac{b}{a} + \psi(r) \cos \theta + T_b \quad (2.142)$$

and

$$T' = \frac{\bar{q}''' a^2}{2k} \ln \frac{b}{r} + \psi'(r) \cos \theta + T_b \quad (2.143)$$

We must now evaluate $\psi(r)$ and $\psi'(r)$ by substituting Eq. (2.142) into Eq. (2.137) and solving the resulting differential equation to get

$$\psi(r) = Ar - \frac{\alpha}{8} r^3 \quad (2.144)$$

where $\alpha = \epsilon' \bar{q}''' / ak$. Similarly, if we substitute Eq. (2.143) into Eq. (2.138) we can solve to obtain

$$\psi'(r) = C(1/r - r/b^2) \quad (2.145)$$

Using these results in Eqs. (2.139) and (2.140), we can solve for constants A and C :

$$A = \frac{\alpha a^2}{8} \left[\frac{1 + 3 \left(\frac{k}{h_g a} + \frac{k}{k'} \frac{b^2 - a^2}{b^2 + a^2} \right)}{1 + \left(\frac{k}{h_g a} + \frac{k}{k'} \frac{b^2 - a^2}{b^2 + a^2} \right)} \right] \quad (2.146)$$

and

$$C = \frac{\alpha a^4}{4} \frac{k}{k'} \left[\frac{b^2 / (b^2 + a^2)}{1 + \left(\frac{k}{h_g a} + \frac{k}{k'} \frac{b^2 - a^2}{b^2 + a^2} \right)} \right] \quad (2.147)$$

If desired, the maximum fuel temperature at any radius can now be determined. This occurs at $\theta = 0$.

$$T_{\max} = \frac{\bar{q}'''}{4k}(a^2 - r_m^2) + \frac{1}{2h_g} a \bar{q}''' + \frac{\bar{q}''' a^2}{2k'} \ln \frac{b}{a} + T_b + \frac{\epsilon' \bar{q}'''}{8ak} \left(\frac{1 + 3\xi}{1 + \xi} a^2 r_m - r_m^3 \right) \quad (2.148)$$

where

$$\xi = \frac{k}{h_g a} + \frac{k}{k'} \frac{b^2 - a^2}{b^2 + a^2} \quad (2.149)$$

and r_m , the radial location of the maximum fuel temperature, is given by

$$m = \frac{2a}{3\epsilon'} \left\{ \left[1 + 3/4(\epsilon')^2 \left(\frac{1+3\xi}{1+\xi} \right) \right]^{1/2} - 1 \right\}$$

We can also determine the maximum heat flux [Btu/(h ft²)] that also occurs at $\theta=0$,

$$q''_{\max} = k' \left(\frac{\partial T'}{\partial r} \right)_{a=0} = \frac{\bar{q}''' a^2}{2b} + \frac{1}{2} \epsilon' \bar{q}''' a^3 [(1+\xi)(b^2+a^2)]^{-1} \quad (2.150)$$

Hence,

$$\frac{q''_{\max}}{q''_i} = 1 + \frac{ab\epsilon'}{(1+\xi)(b^2+a^2)} \quad (2.151)$$

For $b=a+\Delta r'$ where $a \gg \Delta r'$ the last equation simplifies to

$$\frac{q''_{\max}}{q''_{\text{mean}}} = 1 + \frac{\epsilon'}{2 \left[1 + \frac{k}{a} \left(\frac{1}{h_g} + \frac{\Delta r'}{k'} \right) \right]} \quad (2.152)$$

An example of what is obtained in a typical case is useful: Assume $\epsilon=0.2$, $k'=12$ Btu/h ft °F, $k=1.6$ Btu/h ft °F, $a=0.4$ in., $h_g=1000$, $\Delta r'=0.013$, then $q_{\max}/q_{\text{mean}}$ is ~ 1.09 .

Analytic approaches can also be used for solution of the eccentric pellet problem with a uniform pellet conductivity. The nonhomogeneous differential equation can be transferred into Laplace's equation by a change in variables.¹⁶³ The resulting partial differential equation may then be solved by the separation of variables technique. However numerical approaches using finite differences are more usually used. Sdouz and Dagbjartson¹⁶⁴ have shown that, with an open pellet to cladding gap, an eccentric pellet that touches the cladding can lead to noticeable circumferential variation in the outer fuel temperature [about 200 K when operating at 13 kW/ft] but almost no change in cladding temperature. The significant circumferential variations in gap conductance assumed here would only arise at the very beginning of fuel life. As noted earlier, significant fuel relocation occurs after the first rise to power and after 500 hr the gap is largely closed. Since cladding temperatures are unaffected and fuel relocation rapidly removes the azimuthal fuel temperature variations, pellet eccentricity does not appear to pose a real problem.

2.3.3 Annular Fuel Elements

For a hollow cylinder of outer radius a and inner radius b , diffusion theory calculations give the flux distribution as

$$\phi = C' \phi_0 \left[I_0(\kappa r) + \frac{I_1(\kappa b)}{K_1(\kappa b)} K_0(\kappa r) \right], \quad (2.153)$$

where

K_0 = zero-order modified Bessel function of second kind

K_1 = first-order modified Bessel function of second kind

C' = constant evaluated so that $\phi = \phi_0$ at $r = a$.

For heat removal at the outer surface only, we write

$$\int_{T_s}^{T_c} k \, dT = \frac{1}{4\pi} q' \left\{ \frac{\kappa b [I_1(\kappa b) K_0(\kappa a) + I_0(\kappa a) K_1(\kappa b)] - 1}{\frac{1}{2} \kappa^2 a b [I_1(\kappa a) K_1(\kappa b) - I_1(\kappa b) K_1(\kappa a)]} \right\} \quad (2.154)$$

When heat is removed at both surfaces, the situation is complicated if different boundary conditions hold at the two surfaces. When T_s is the same at both surfaces, we get

$$\int_{T_s}^{T_r} k \, dT = \frac{1}{2} q' b \left(\frac{[I_0(\kappa b) - I_0(\kappa r)] + [K_0(\kappa b) - K_0(\kappa r)] + \kappa R^0 \ln \frac{r}{b} [I_1(\kappa r) - \lambda K_1(\kappa R^0)]}{\frac{1}{2} \kappa b \left\{ \frac{R^0}{b} [I_1(\kappa R^0) - \lambda K_1(\kappa R^0)] - [I_1(\kappa b) - K_1(\kappa b)] \right\}} \right) \quad (2.155)$$

where $\lambda = I_1(\kappa b)/K_1(\kappa b)$ and R^0 is the radius at which maximum fuel temperature occurs. We obtain R^0 from

$$R_0 \ln \left(\frac{a}{b} \right) [I_1(\kappa R^0) - \lambda K_1(\kappa R^0)] = \frac{1}{\kappa} [I_0(\kappa a) - I_0(\kappa b)] + \frac{\lambda}{\kappa} [K_0(\kappa a) - K_0(\kappa b)] \quad (2.156)$$

2.4 REALISTIC EVALUATION OF STEADY-STATE TEMPERATURES

Fuel element temperature calculations of the previous section did not consider the slowly varying changes in fuel element properties that take place during reactor exposure. After the initial rise to power, cracking of the fuel pellet changes fuel conductivity and gap conductance. Gap thickness decreases with time as the Zircaloy cladding creeps down toward the fuel pellet. In most long irradiations, fuel swelling eventually brings the pellet into contact with the cladding and further increases gap conductance. In addition, at high heat ratings, a central void gradually forms. Because of these gradual changes, fuel element temperatures are a function of the

power-time history of the fuel rod. Realistic fuel temperature evaluations must include all of these time-varying effects.

In view of the complexity of the phenomena involved, satisfactory closed-form analytical solutions are not available*; numerical procedures are required. The fuel pellet is usually divided into a series of concentric annular rings. Using an assumed temperature distribution (or that from a previous time period), the response of each of these rings to the combined effects of thermal expansion, swelling, void migration, creep, and hot pressing during some short time period is determined. The interaction between pellet and cladding is then determined to establish gap conductance. Temperature distribution is then redetermined. If the new distribution is close to that originally chosen, the calculation proceeds to the next time interval. If agreement is poor, behavior of the fuel rings is redetermined and a revised temperature distribution is obtained. The process continues at a given time step until convergence is obtained.

2.4.1 Rigid Pellet, No Fuel-Pellet Friction

The simplest situation arises when the center temperature is below $\sim 1450^\circ\text{C}$. Under these conditions, fuel restructuring does not occur, no central void is formed, and fuel creep and hot pressing are minimal. Thus, the change in fuel dimensions is often taken as the sum of that due to thermal expansion and swelling. An initial temperature distribution is assumed; thermal expansion and swelling of each ring is evaluated. Summation of the ring dimensions provides the new fuel pellet diameter. This is compared to the cladding diameter and the gap width is then determined.

If a sufficiently large number of rings are chosen, properties are assumed constant across each annular ring. In one procedure,¹⁶⁶ it is assumed that heat flux q''_{av} and thermal conductivity k_{av} can be evaluated at the center of the annular ring and that the equation for the temperature drop across a slab can be used. Then the temperature drop across an annular ring, ΔT_r , is given by

$$\Delta T_r = \frac{q''_{av}(\Delta r)}{k_{av}} \quad (2.157)$$

where Δr is the thickness of the ring.

When fewer rings are chosen, more accurate results are obtained by modifying Eq. (2.112) to

$$\int_0^{r_1} k \, dT = \int_0^{T_{r_2}} k \, dT + \left\{ \frac{q''' }{4} [(r_2^2 - r_1^2) - 2r_1^2 (\ln r_2 / r_1)] \right\} \quad (2.158)$$

*Lamkin and Brehm¹⁶⁵ provide an exact closed-form solution for fuel temperatures and stresses, providing simplified creep and property formulation are accepted.

where T_{r_1} and T_{r_2} equal temperatures at the inner and outer radius of the ring. If the temperature of $r_2(T_{r_2})$ is known, $\int_0^{T_{r_2}} k dT$ is known for a given porosity. Then $\int_0^{T_{r_1}} k dT$ can be obtained and T_{r_1} is known.

Both of the above schemes are iterative. A surface temperature of the pellet must be assumed. The calculation then proceeds inward. The final temperature distribution is then used to compute a revised estimate of fuel dimensions and gap conductance, h_g . With a new h_g , a revised pellet surface temperature can be computed.

At low burnup, irradiation growth of the cladding is not significant. However this growth should be considered for high-burnup fuel. Thus, cladding strain is the sum of thermal expansion, irradiation growth, and elastic and creep strain. Circumferential cladding strain, ϵ_θ , is given by

$$\epsilon_\theta = \left(\frac{1}{E} \right) [\sigma_\theta - \gamma(\sigma_r + \sigma_z)] + \alpha_c(\Delta T) + \epsilon_c + \epsilon_{ir} \quad (2.159)$$

where

$\sigma_r, \sigma_\theta, \sigma_z$ = radial, tangential, and axial stresses, respectively

E = Young's modulus

α_c = coefficient of thermal expansion

ϵ_c = creep strain

ϵ_{ir} = strain due to irradiation growth

γ = Poisson's ratio

ΔT = increase in cladding temperature.

Given a cladding with outer radius b and inner radius a , and the absence of pellet-clad contact, the stresses are obtained from

$$\sigma_r = \frac{a^2}{b^2 - a^2} (P_i - P_o)(1 - b^2/r^2) - P_o \quad (2.160)$$

$$\sigma_\theta = \frac{a^2}{b^2 - a^2} (1 + b^2/r^2)(P_i - P_o) - P_o \quad (2.161)$$

$$\sigma_z = \frac{\lambda}{\lambda + G} \left(\frac{1}{b^2 - a^2} \right) (a^2 P_i - b^2 P_o) \quad (2.162)$$

where

P_i, P_o = inner and outer pressures, respectively

$G = E/[2(1 + \gamma)]$

r = radius at a given point

$\lambda = 2G/(1 - 2\gamma)$

E = Young's modulus.

Since a and b do not differ greatly, stresses at an average value of r can be used to estimate an approximate value of ϵ_r , the radial cladding strain.

The value of P_o is determined by external system pressure. When the fuel pellet is not in contact with the cladding, P_i is the pressure of the fission and fill gases inside the fuel rod and can be calculated via Eq. (2.57).

Evaluation of the quantity of total fission gas release requires an estimation of gas released from each radial node at a number of axial elevations. Once convergence on surface temperature at a given axial elevation is obtained, the process must be repeated at a number of other axial elevations. Fission gas release must then be determined. If significant changes in internal pressure or gas thermal conductivity are found, the process must then be repeated at all elevations.

Cladding creep will not occur when the cladding is in a hydrostatic stress state; i.e., $\sigma_\theta = \sigma_r = \sigma_z$. According to the Prandtl-Reuss assumption, creep is determined by deviatoric stress, s . In the circumferential direction,

$$s_\theta = \sigma_\theta - \sigma_m \quad (2.163)$$

where σ_m = mean stress = $(1/3)(\sigma_r + \sigma_z + \sigma_\theta)$. To relate the multiaxial situation to the uniaxial condition in which strain rates are normally measured, an equivalent stress, σ_{eq} , is defined as

$$\sigma_{eq} = (1/\sqrt{2})[(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_r - \sigma_z)^2]^{1/2} \quad (2.164)$$

The equivalent stress is proportional to the total shear stress, which gives a measurement of total plastic creep deformation in a polycrystalline material. The value of σ_{eq} is substituted in an appropriate uniaxial creep equation, e.g., Eq. (2.92), to determine an equivalent creep rate $\dot{\epsilon}_c^{eq}$. Creep rates in the principal axes are then obtained from

$$\dot{\epsilon}_c^r / s_r = \dot{\epsilon}_c^\theta / s_\theta = \dot{\epsilon}_c^z / s_z = \left(\frac{3}{2}\right)(\dot{\epsilon}_c^{eq} / \sigma_{eq}) \quad (2.165)$$

Hence, the tangential creep rate, $\dot{\epsilon}_c^\theta$, is obtained from

$$\dot{\epsilon}_c^\theta = \frac{3}{2} s_\theta (\dot{\epsilon}_c^{eq} / \sigma_{eq}) = (\dot{\epsilon}_c^{eq} / \sigma_{eq}) \left[\sigma_\theta - \frac{1}{2}(\sigma_r + \sigma_z) \right] \quad (2.166)$$

Total creep strain during the time interval can then be estimated from the product of $\dot{\epsilon}_c^\theta$ and Δt . The procedure is satisfactory as long as small creep strains are obtained. If large creep strains are encountered, the time step must be reduced.

If the total cladding strain computed from Eq. (2.159), using inner pressure as gas pressure, leads to the conclusion that cladding diameter is equal to or less than the uncracked diameter of the pellet, then the fuel and the cladding are in contact. The fuel will elastically strain the cladding so the pellet and cladding are just in contact. The elastic expansion, ΔD_i , of the cladding is

$$\Delta D_i = D_p - D_{i0}[\alpha_c(\Delta T) + \epsilon_c + \epsilon_{ir}] \quad (2.167)$$

where D_{i0} is the internal diameter of cladding at BOL conditions and D_p is the uncracked diameter of hot pellet, including swelling effects.

Internal pressure, P_i , required to obtain this expansion, can be computed from Eq. (2.65). As previously noted, contact pressure equals P_i less internal gas pressure. The contact pressure is then used to evaluate gap conductance.

If one makes the assumption that interaction with the pellet stack does not restrain the vertical movement of the cladding, then Eqs. (2.160) to (2.162) can be used to obtain the stresses from the external pressure by using an average value for r . Again an iterative procedure is required to obtain agreement between the assumed and calculated cladding and fuel movement.

In estimating the fission gas pressure, appropriate temperatures must be assigned to the various gas volumes. The gas in the fuel pores and dish volumes can be assumed to be at the average fuel temperature while the gas in the gap is obviously at the average gap temperature. The gas in the upper plenum is sometimes taken to be at the temperature of the exit coolant. However, this assumption underestimates the plenum gas temperature because it does not allow for (1) the heat supplied by the gamma heating, Q_s , of the hold-down spring located in the plenum and (2) transfer of heat from the uppermost pellet. This heat must be transferred to the cladding by natural convection. Since natural convection heat transfer resistances are high in comparison with the thermal resistance of the cladding and coolant film, we may write

$$Q_s + h_p(\pi r_p^2)(T_f - T_{pl}) = h_c(T_{pl} - T_c) \quad (2.168)$$

where

r_p = pellet radius

h_p = natural convection heat transfer coefficient between top pellet and plenum gas

h_c = natural convection heat transfer coefficient between cladding and plenum gas

T_c, T_f, T_{pl} = temperature of coolant, fuel, and plenum gas, respectively.

Plenum temperatures calculated from Eq. (2.168) are noticeably above the coolant temperature. Since most of the fission gas is in the plenum, this temperature has a significant effect on the calculated gas pressure.

2.4.2 Rigid Pellet, Fuel-Pellet Friction Considered

Under most circumstances, the assumption of no friction between the fuel and cladding is unwarranted. Since the fuel cracks shortly after the first rise to power, there is resistance to relative motion between fuel and clad-

ding even when there is a nominal open gap. The simplest assumption that can be made is that the friction between fuel and cladding always prevents significant axial strain of the cladding. If the axial strain is largely eliminated, then the axial stress is relatively small. Further, the radial stress is relatively small in comparison with the circumferential stress. If both σ_z and σ_r are considered negligible, then the equivalent stress, σ_{eq} , of Eq. (2.164) reduces to σ_θ . The circumferential creep rate from Eq. (2.166) then reduces to the creep rate from the appropriate uniaxial creep rate equation with σ_θ as the uniaxial stress. The average circumferential stress to be used may be computed from thin-wall theory [Eq. (2.78)].

The assumption of a negligible axial stress can be improved on even when thin-wall theory is used. If the axial strain of cladding is set equal to the axial strain of the fuel obtained from thermal expansion and net swelling, ϵ_z for the cladding is fixed.

The radial displacement at the inside diameter of the cladding, Δr_i , is determined by the radial movement of the outer radius of the pellet. If we follow the treatment of Berna et al.,¹²⁰ the displacement of the cladding, r , is related to the circumferential and radial strain by

$$\Delta r_i = \bar{r} \epsilon_\theta - \frac{C}{2} \epsilon_r \quad (2.169)$$

where C is the cladding thickness and $\bar{r}$ is the average radius of the cladding. By taking the average radial stress as negligible, the strains are then related to the stresses by

$$\epsilon_\theta = \frac{1}{E} (\sigma_\theta - \gamma \sigma_z) + \epsilon_\theta^p + \epsilon_\theta^{th} \quad (2.170)$$

$$\epsilon_z = \frac{1}{E} (\sigma_z - \gamma \sigma_\theta) + \epsilon_z^p + \epsilon_z^{th} \quad (2.171)$$

$$\epsilon_r = -\frac{\gamma}{E} (\sigma_\theta - \sigma_z) + \epsilon_r^p + \epsilon_r^{th} \quad (2.172)$$

where ϵ_θ^p , ϵ_z^p , ϵ_r^p are the permanent plastic and creep strains in the θ , z , and r directions; ϵ_θ^{th} , ϵ_z^{th} , ϵ_r^{th} are the thermal strains in the θ , z , and r directions; and γ is Poisson's ratio.

By substitution of Eqs. (2.169) and (2.172) in Eq. (2.170), we obtain

$$\frac{\Delta r_i}{\bar{r}} + \frac{C}{2\bar{r}} \left(\epsilon_r^p + \epsilon_r^{th} \right) - \left(\epsilon_\theta^p + \epsilon_\theta^{th} \right) = \frac{1}{E} \left[\left(1 + \frac{\gamma C}{2\bar{r}} \right) \sigma_\theta + \gamma \left(\frac{C}{2\bar{r}} - 1 \right) \sigma_z \right] \quad (2.173)$$

Simultaneous solution of Eqs. (173) and (171) yields

$$\sigma_\theta = \frac{B_1 A_{22} - B_2 A_{12}}{A_{11} A_{22} - A_{12} A_{21}} \quad \sigma_z = \frac{B_2 A_{11} - B_1 A_{21}}{A_{11} A_{22} - A_{12} A_{21}}$$

where

$$\begin{aligned}
 A_{11} &= 1 + \frac{\gamma C}{2\bar{r}}, \quad A_{12} = \gamma \left(\frac{C}{2\bar{r}} - 1 \right), \\
 A_{21} &= -\gamma, \quad A_{22} = 1, \\
 B_1 &= \frac{E(\Delta r_i)}{\bar{r}} + \frac{EC}{2\bar{r}} (\epsilon_r^p + \epsilon_r^{\text{th}}) - E(\epsilon_\theta^p + \epsilon_\theta^{\text{th}}), \\
 B_2 &= E[\epsilon_z - (\epsilon_z^p - \epsilon_z^{\text{th}})]
 \end{aligned}$$

The stresses thus obtained can then be used in Eq. (2.164) to determine the equivalent stress and from this stress the creep is determined. The internal pressure, P_i , is determined from

$$P_i = \frac{C\sigma_\theta - r_o P_o}{r_i} \quad (2.174)$$

where the subscript "o" indicates the values at the outer radius. As previously indicated the contact pressure is the difference between P_i and the gas pressure. An iterative process is required to obtain an agreement between the stresses and the cladding motion just as for the open gap case.

Berna *et al.*¹²⁰ assume that the cladding is free to move relative to the fuel when the gap is open. However, if the high-power pellets in the center of the rod are in firm contact with the cladding, those pellets in the lower portion of the rod are trapped. The FRAPCON and FRAP-T programs¹⁶⁷ assume that the axial expansion of the fuel will still be imparted to the cladding. The total change in the cladding length, ΔL , is computed from the sum of the pellet length changes. This ΔL must equal the sum of the length changes of the individual axial segments. That is,

$$\Delta L = \sum_{j=1}^n [(\epsilon_z)_j - (\epsilon_z^0)_j](\Delta z)_j \quad (2.175)$$

where $(\epsilon_z^0)_j$, $(\epsilon_z)_j$ are the axial cladding strains on the j 'th segment at the beginning and end of the current time step and $(\Delta z)_j$ is the length of axial segment j . The axial stress σ_z is taken as constant along the trapped pellet stack. By expressing the axial strains for the individual segments in terms of the stresses we obtain

$$\sigma_z = \left[\sum_{j=1}^n \frac{E_j}{(\Delta z)_j} \right] \left\{ \Delta L + \sum_{j=1}^n (\Delta z)_j \left[\frac{\gamma_i(\sigma_\theta)_j}{E_j} + (\epsilon_z^0)_j - (\epsilon_z^p)_j - (\epsilon_z^{\text{th}})_j \right] \right\} \quad (2.176)$$

where the subscript j again indicates the j 'th axial segment. Since the pellets are not in contact with the stack, σ_θ may be obtained from Eq. (2.174) using the gas pressure P_i and the internal and external radii appropriate to the given node. The elastic axial cladding strains are related to σ_z via the value of E for the particular segment.

An alternative way of dealing with the effect of pellet interference on cladding motion is through the assignment of a frictional force between pellet and cladding.¹⁶⁸ In the upper region of the rod, the axial force, F_j , on the cladding at an elevation j is then given by

$$F_o = F^1 - \pi r_p^2 P_g \quad (2.177)$$

and

$$F_j = F_{j-1} + F_f'' \quad (2.178)$$

where

F^1 = force due to plenum spring

P_g = internal gas pressure

r_p = pellet radius

F_f'' = frictional force.

The maximum frictional force, $(F_f'')_{\max}$, depends on the contact pressure, P_c :

$$(F_f'')_{\max} = \mu P_c (2\pi r_i) + K (2\pi r_i) \quad (2.179)$$

where μ is the coefficient of friction and K is the frictional force per unit area at zero contact pressure (often set at zero).

If the pellets are in contact, the axial stress, σ_z , obtained assuming the cladding strain and pellet strain are equal, is first determined. The value of σ_z so determined is used to compute an alternative F_f from

$$\sigma_z \pi (r_o^2 - r_i^2) = -F_j \quad (2.180)$$

and Eq. (2.178). If F_f'' so determined exceeds $(F_f'')_{\max}$, then pellet slippage is possible and $(F_f'')_{\max}$ is used to reevaluate σ_z . Assignment of a maximum frictional force depending on contact pressure means that when contact is first established with a low contact pressure there is not an abrupt change in σ_z .

2.4.3 Behavior Considering Pellet Plasticity

At the high linear power at which fast reactors operate, columnar grain growth with central void formation is a significant factor affecting fuel rod behavior. However, if we use Nichol's calculations of the columnar grain radius presented in Fig. (2.11), we find that a temperature of over 1800 K is required to produce any columnar grains after very prolonged exposure (30,000 hr). At PWR coolant conditions, this would correspond to a linear power rating of about 15 kW/ft. More recent experimental evidence⁸⁷ indicates that temperatures in excess of 2100 K are needed for significant columnar grain growth. This would require operation at about over 16 kW/ft. Current (1995) PWRs do not operate at peak powers this high (usual

range 12 to 15 kW/ft) and hence columnar grain growth and central void formation are not observed.⁸⁷

Although void formation is not a problem in PWR fuel, central temperatures in the hotter fuel rods are sufficiently high so that the central fuel region exhibits some plasticity. A complete model of fuel behavior would need to take this plasticity into consideration. To do so, a stress and strain analysis of the fuel pellet is required. The stresses so calculated are used, with an iterative process, to obtain the plastic and elastic strains within the fuel. The computation must include relationships expressing these strains as a function of applied stress.

The FRACAS¹⁶⁷ deformable pellet model calculates the pellet stresses and strains by an approach somewhat similar to that for cladding strain computations given by Eqs. (2.169) through (2.173). At each axial segment the pellet is divided into a series of concentric annular rings. A generalized plane strain deformation is assumed so that ϵ_z is a constant for all radii. The value of this strain is subsequently determined from the requirement for axial force equilibrium.

The governing equations of equilibrium and compatibility for each ring in the absence of any displacement discontinuities are:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (2.181)$$

$$\frac{d\epsilon_\theta}{dr} + \frac{\epsilon_\theta - \epsilon_r}{r} = 0 \quad (2.182)$$

The elastic-plastic stress-strain relations, which are similar to those of Eqs. (2.170) to (2.172), are

$$\epsilon_r = \frac{1}{E}[\sigma_r - \gamma(\sigma_\theta + \sigma_z)] + \epsilon_r^{\text{th}} + \epsilon_r^p + d\epsilon_r^p \quad (2.183)$$

$$\epsilon_\theta = \frac{1}{E}[\sigma_\theta - \gamma(\sigma_r + \sigma_z)] + \epsilon_\theta^{\text{th}} + \epsilon_\theta^p + d\epsilon_\theta^p \quad (2.184)$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - (\sigma_r + \sigma_\theta)] + \epsilon_z^{\text{th}} + \epsilon_z^p + d\epsilon_z^p \quad (2.185)$$

Substitution of Eqs. (2.183) and (2.184) into Eq. (2.182) yields

$$\begin{aligned} & \frac{d}{dr} \left[\frac{\sigma_\theta}{E} - \frac{\gamma}{E}(\sigma_r + \sigma_z) + \epsilon_\theta^{\text{th}} + \epsilon_\theta^p + d\epsilon_\theta^p \right] \\ & + \frac{1 + \gamma}{E} \frac{(\sigma_\theta - \sigma_r)}{r} + \frac{\epsilon_\theta^{\text{th}} - \epsilon_r^{\text{th}}}{r} + \frac{\epsilon_\theta^p - \epsilon_r^p}{r} + \frac{d\epsilon_\theta^p - d\epsilon_r^p}{r} = 0 . \end{aligned} \quad (2.186)$$

Equations (2.181), (2.186), and (2.185) with ϵ_z constant relate the stresses as they vary across the pellet. The stresses are evaluated at the midpoints of each ring after writing the foregoing equations in finite difference form. A set of recursion relations,

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix}_{i+1} = [L(i)] \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix} + \{M(i)\} \quad (2.187)$$

is then obtained. This matrix equation relates the stresses at node $i+1$ to those at node i . The matrices $[L(i)]$ and $\{M(i)\}$ depend on the material properties, geometry, and plastic strains only.

By successive application of Eq. (2.187), a relation between the stresses at any node and the stresses at node 1, the node at the inside of the cylinder, can be obtained. This relation takes the form

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix}_i + 1 = [A(i)] \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix} + \{B(i)\} \quad (2.188)$$

The matrices $[A(i)]$ and $\{B(i)\}$ can be determined from $[L(i)]$ and $\{M[i]\}$. By recursion, $[A(i)]$ and $\{B(i)\}$ across the cylinder wall can be obtained, with the result that

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix}_N = [A(N+1)] \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix}_1 + \{B(N+1)\} \quad (2.189)$$

where N is the index of the ring at the outside surface of the cylinder and "1" indicates the inner ring. At the outside surface, $\sigma_r = -P_o$, where P_o is the gap pressure if the gap is open or the contact pressure if the gap is closed. The following condition can then be obtained:

$$-P_o = A_{11}(N-1)\sigma_r(1) + A_{12}(N-1)\sigma_\theta(1) + A_{13}(N-1)\sigma_z(1) + B_1(N-1) \quad (2.190)$$

At the innermost pellet ring, σ_r is set equal to σ_θ .

The condition of axial equilibrium

$$\sum \sigma_z(j) dA(j) = F_z \quad (2.191)$$

is used to determine the stress at the midpoint of the inner ring. By use of the recursion matrices this becomes

$$\begin{aligned} \sum \sigma_z(j) dA_j = & \{[I] dA_1 + [A_1] dA_z + \dots + [A_{N-1}] dA_N\} \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix}_1 \\ & + \{0 + \{B_1\}dA_2 + \dots + \{B_{N-1}\}dA_N\} . \end{aligned} \quad (2.192)$$

After determination of the total axial force by essentially the same approach as shown by Eqs. (2.177) and (2.178), the stresses at the inner node are obtained by simultaneous solution of Eqs. (2.192) and (2.191) with the boundary condition that $\sigma_r(1) = \sigma_\theta(1)$. Once these quantities are known the stresses in the remainder of the cylinder are determined via Eq. (2.188).

The transfer matrices $[L(i)]$ and $\{M(i)\}$ only depend on the fuel properties, geometry, and plastic strain. They may be independently specified for each ring. In the outer rings, where the fuel is brittle and cracks, the pellet properties may be adjusted to account for their cracking. The outer boundary of the equiaxial grain growth region is sometimes taken as the inner boundary of the cracked pellet region.¹⁶⁸ However, more recent approaches¹⁶⁹ consider the criterion for development of a crack perpendicular to the principal axis as:

$$T < T_c^* \quad \text{and} \quad \sigma_j > \sigma_F (j = r, \theta, z) \quad (2.193)$$

where σ_j is the stress along the principal axis (r, θ, z), T_c^* is the brittle-ductile transition temperature, and σ_F is the lower fracture propagation stress for fuel. Sakagami *et al.*¹⁶⁸ consider that the crack will heal in the presence of a compressive stress and adequate temperature. That is,

$$T \geq T_c^* \quad \text{and} \quad \sigma_j < 0 (j = r, \theta, z) \quad (2.194)$$

The presence of cracks sharply reduce Young's modulus for the fuel. Some calculational schemes simply set Young's modulus to an arbitrarily low value in the cracked region. However, other approaches attempt to correlate E . Sakagami *et al.*¹⁶⁸ define E' as an effective value for Young's modulus and γ' as an effective value for Poisson's ratio. They conclude

$$E' = \left(\frac{2}{3}\right)^{n-1} \left(\frac{E}{1-\gamma}\right) \quad (2.195)$$

$$\gamma' = \frac{1}{2} \left[1 - \left(\frac{1}{2}\right)^n \left(\frac{1-2\gamma}{2\gamma}\right) \right] \quad (2.196)$$

where n is the number of principal directions in which there are cracks [determined from Eqs. (2.193) and (2.194)]. Willford,¹⁷⁰ however, concludes that E' is a function of stress and represents E' by

$$E' = E_1 \sigma^s \quad (2.197)$$

where s is a constant, σ is fuel hydrostatic stress, and E_1 is the low stress intercept. Values of E' can be estimated from Fig. 2.22. Willford indicates that E_1 depends on the fuel cladding gap and this may account for the wide error bands in Fig. 2.22.

Some approaches use a compromise between the one-dimensional stress analysis with the simple assumption of a rigid pellet and stress analysis considering the variation of properties with fuel radius. In one such approach the fuel is assumed to consist of a fully plastic core, an intermediate

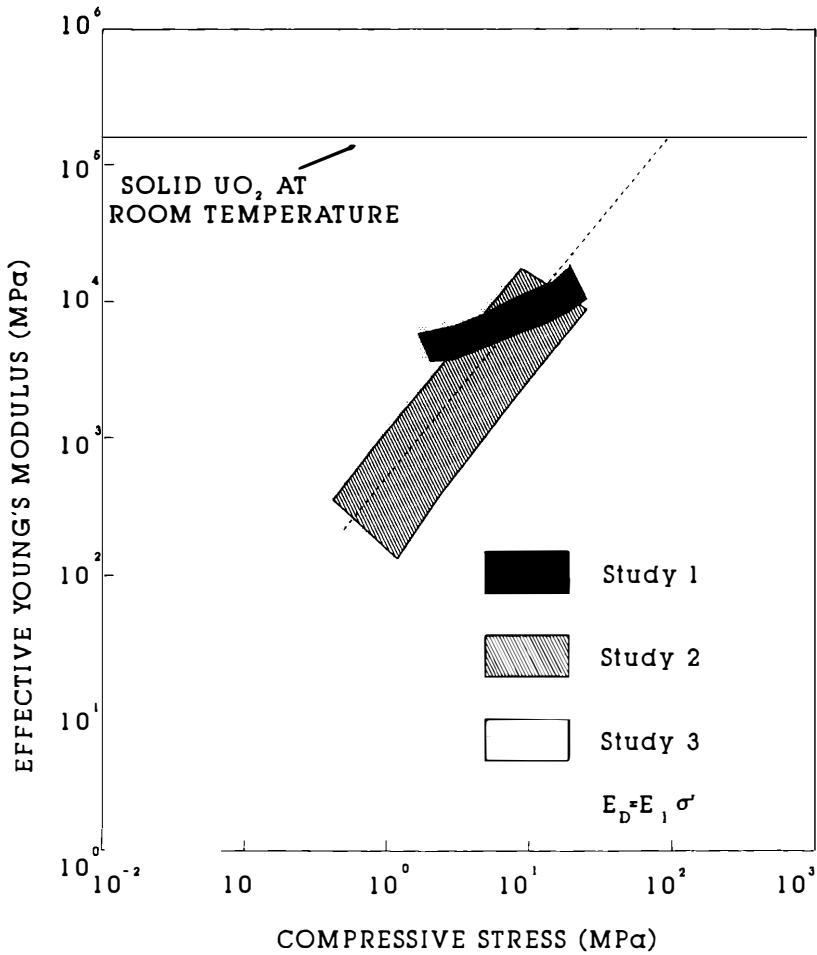


Fig. 2.22 The effective Young's modulus for cracked UO_2 fuel pellet columns [from Ref. 170].

plastic-elastic uncracked ring and an outer elastic but cracked region¹⁶⁸ (see Fig. 2.23). The contact pressure, P is determined by means of so-called "compliance" terms. The compliance represents the elastic strain produced by one unit of pressure. Hence, P_c , the contact pressure, is calculated by¹¹²

$$P_c = \frac{\Delta D_i}{D_F(W_F + W_c)} \quad (2.198)$$

where

ΔD_i = required elastic expansion of cladding to accommodate expanded pellet diameter

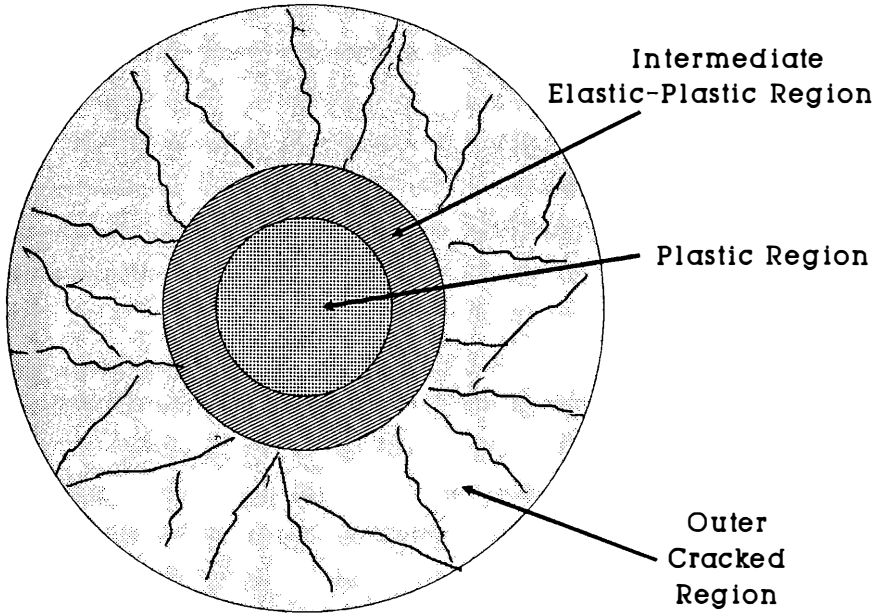


Fig. 2.23 Three-region pellet model.

D_F = hot restrained fuel diameter

$W_c W_F$ = compliance terms for fuel and cladding, respectively.

The compliance term for thin-wall cladding is given by¹¹²

$$W_c = \frac{D_i + D_o}{2E_c(D_o - D_c)} \quad (2.199)$$

where D_i , D_o are the inner and outer diameter, respectively, and E_c is Young's modulus for the cladding. The fuel compliance term is the sum of (1) the compliance of the intermediate cylinder for deflection of its outer radius and (2) the compliance for the outer cracked region (see Fig. 2.23). If the fraction of the outer pellet surface with an open crack width is small, then¹¹²

$$W_F = \left[\frac{1}{E_F} \ln \frac{D_F}{(D_F)_c} \right] + \frac{1}{E_i} \left[\frac{(D_F)_c^2 + (D_F)_p^2 (1 - 2P_i/P_o)}{(D_F)_c^2 - (D_F)_p^2} - \gamma_i \right] \quad (2.200)$$

where

D_F = hot outer diameter of the fuel pellet

$(D_F)_c$ = inner diameter of cracked pellet region (outer diameter of elastic region)

$(D_F)_p$ = outer diameter of plastic pellet region (inner diameter of elastic region)

E_F Young's modulus for fuel in outer cracked pellet region

E_c Young's modulus for intermediate ring of elastic fuel

γ_i Poisson's ratio for fuel in intermediate elastic ring

P_i = radial pressure at $(D_F)_F$

P_o = radial pressure at $(D_F)_c$.

The first term on the right-hand side of Eq. (2.200) represents the compliance of the cracked pellet region, while the second term represents the compliance of the intermediate ring.

Although the intermediate ring has enough plasticity to close the crack, elastic behavior is assumed in the cracked region. Therefore the pressure P_o at diameter D_{FC} is given by

$$P_o = \left(\frac{D_F}{D_{FC}} \right) P_c \quad (2.201)$$

The stress intensity, which in this case is $|\sigma_\theta - \sigma_z|$, is assumed to remain constant throughout the elastic-plastic intermediate ring. At the inner radius of this region (outer radius of fully plastic region), the stress intensity is equal to the yield stress. The location of $(D_F)_p$ and the value of P_i are determined by locating where the temperature produces a yield stress of the appropriate magnitude.

In the simplest approach to the determination of $(D_F)_c$, this diameter would be set at the location where finite plasticity begins ($T \approx 1200^\circ\text{C}$). However, a more accurate approach¹¹² evaluates the size of the region at a temperature below 1200°C where the contact pressure has forced the crack to close.

Calculations based on the elastic-plastic conditions yield lower contact pressures than the purely elastic case. The plastic core yields and, hence, what must be overcome is the stiffness of the intermediate ring less the compliance of the outer cracked annulus.

It should be recognized that the computational approaches for determining the pellet stresses described in this section are not the only paths taken. Other approaches are described by Sakagami *et al.*,¹⁶⁸ Nissen,¹⁷¹ Samei,¹⁷² and Olander,¹⁷³ among others.

2.5 FUEL PERFORMANCE

2.5.1 Typical Reactor Operating Conditions

Typical average linear power and average reactor power densities are listed in Table 2.VII for both conventional PWRs and CANDU reactors. Note that the newer 17×17 PWR assemblies and the 37-pin CANDU have somewhat

TABLE 2.VII
Typical Fuel Operating Conditions

	Conventional	PWR	CANDU	
Assembly design	15×15	17×17	28 pin	37 pin
Rod diameter (mm)	10.7	9.5	15.2	13.1
Average linear power (kW/m)	23.1	17.8	26.5	25.7
Average power density* (kW/l)	106	105	85	89

*Based on total fuel element and coolant volume.

lower linear power ratings than the earlier 15 × 15 and 28-pin designs. The linear power ratings are based on the total reactor power and do not allow for the small amount of power generated outside the fuel (about 2.6% of total power in a typical conventional PWR).

At the usual operating conditions of a conventional PWR, the maximum cladding surface temperature is about 347°C. Measurement⁵³ of fuel temperatures of typical helium-filled rods with 96% dense pellets at usual coolant conditions indicate centerline temperatures of 1200°C at 33 kW/m (~10 kW/ft) and 1760°C at 49 kW/m (~15 kW/ft). The measurements were taken after sufficient burnup for pellet cracking but before hard cladding to pellet contact was established.

With a 2.82 value of F_Q , as used in several conventional designs, the average linear power rating for the 17×17 design shown in Table 2.VII converts to a maximum linear power of 50.2 kW/m (15.3 kW/ft). The experimental observations reported above then indicate a maximum centerline fuel temperature of about 1800°C. The maximum fuel temperature at the first rise to power (before pellet cracking) would be expected to be about 200°C higher. With fuel temperatures that exceed 1800°C for only a brief period, one would not expect columnar grain growth but there would be a small equiaxed grain region near the center of the hot pellet. At the average linear power rating, no such grain growth would be observed.

If the temperature profile is taken as approximately parabolic, the volumetric average fuel temperature of the hot pellet is approximately 1055°C. The fractional fission gas release from the hot pellet would be less than 15% at moderate burnups (see Sec. 2.2.4.3). Average fractional fission gas releases for the entire rod, assuming the normal operating power is not exceeded, would be expected to be no more than 5%. However, conservative design calculations are usually based on assuming operation at about 110% of nominal power and these would result in somewhat greater calculated releases.

2.5.2 Fuel Behavior

The objective of all steady-state fuel performance analysis is to establish fuel element designs and performance specifications that lead to a low prob-

ability of fuel element failure. A fuel failure may be defined as a cladding breach releasing observable quantities of fission products into the coolant. Most plants are designed to operate with pin-hole size defects in up to 1% of the fuel elements. Fission product releases above this level, either due to an increase in number or size of the cladding defects, raise coolant activity levels above those considered acceptable. In addition to the pin-hole cladding or weld defects that could be present in the as-fabricated fuel, fuel cladding failures are generally ascribed to one of the following causes: (1) pellet-cladding interaction, (2) cladding corrosion, (3) brittle cladding failures, (4) overpressurization, (5) cladding collapse into pellet stack gaps, (6) cladding wear, (7) fuel rod bowing and (8) hydriding.

Fuel rod bowing will occur if the axial cladding growth due to irradiation-enhanced creep is sufficiently high so that the ends of the rod hit both the bottom and top nozzle. The rod then bows to accommodate further rod growth. This problem can be avoided by providing adequate clearance between the nozzles and fuel rods. A related problem is that of fuel assembly bow. In fuel assemblies supported by Zircaloy RCC guide tubes, differential growth of the guide tubes can cause the entire assembly to bow. Such differential growth is possible if the assembly is in an area with a high flux gradient (e.g., core periphery) and if the guide tubes show high irradiation growth rates. The growth of Zircaloy that has been annealed sufficiently to recrystallize is generally small enough so that this does not occur. However, this problem has been observed with stress-relief annealed tubing used for guide tubes.¹⁷⁴

Cladding wear failure can arise if the grid-springs holding the rod in place become loose enough for the rod to vibrate. Irradiation-enhanced creep of the springs holding the rod leads to a significant stress relaxation and hence a significant reduction in spring loads when Zircaloy springs are used. The initial spring load must be sufficient so that the reduced load is still adequate to retain the rod in position. Some assemblies designed for very high burnup use Inconel springs to avoid the problem.

Wear or vibration failures have also been seen in fuel rods adjacent to the core baffle in plants in which there is coolant downflow between the baffle and core barrel. Since the baffle plates are bolted together, water gets through the cracks and strikes the outer fuel pins. The jetting from this leakage can cause vibration of adjacent fuel rods. The wear failures caused by this vibration can be eliminated by locating extra rod support clips in the axial region where such jets may be encountered.¹⁷⁵ Alternatively, up-flow can be used in the baffle region. Since this leads to a much lower pressure differential, jetting is largely eliminated.

During the 1970s it was found that some low-density fuel pellets densified with exposure to such a degree that axial gaps in the fuel column occurred. Upon cladding creep-down, the cladding ovality in the axial gap

region became sufficient for the cladding to collapse. The cladding ductility is not sufficient to allow the collapse without cracking, thus producing a failure. Failures of this type are no longer a problem since all fuel manufacturers now produce somewhat more dense fuel with a stable porosity. The use of denser, stable fuel, to prevent gap formation, and pressurizing of the fuel rods, to decrease the cladding creep-down rate, have effectively eliminated this failure mode.

Overpressurization by fission gas release will not by itself lead to cladding breach under normal operating conditions. However, if the internal gas pressure were sufficiently high to force the cladding away from the fuel, fuel temperatures would substantially increase. This in turn would lead to high fission gas release fractions. Such releases lead to an increased concentration of volatile fission products, which can chemically attack the internal surface of the cladding. The increased presence of these products increases the likelihood of stress-corrosion cracking.

After fuel-densification failures were observed, fuel rods were pressurized during fabrication with helium at about 350 psi. This resulted in an internal pressure of about 900 psi at operating conditions. While this pressure decreased creep-down rates, it substantially increased the internal pressure at the end of life (EOL). Now that fuel densification has been greatly reduced, creep-down is seen as less of a problem. Fill pressures are often set at 250 psi or less. The reduction in EOL gas pressure thus produced allows higher fuel burnups to be achieved.

Although unirradiated Zircaloy can undergo significant elongation (strain) prior to failure, the irradiation embrittlement of Zircaloy, coupled with hydrogen pickup during corrosion, substantially reduces its ductility. A brittle failure will result if the failure strain is exceeded. Because of this, it has usually been assumed that the maximum strain-range seen at EOL must be limited to a small value (e.g., 1%) to prevent brittle failure. However, more recent experience with high burnup fuel indicates that this low a strain range limit may be unnecessarily conservative.

If the ductility of the cladding is sufficiently reduced during reactor exposure, the cladding will become brittle during cold shutdown. Fuel failures may then occur during the fuel handling required in refueling operations.

Cladding corrosion only becomes a problem for high-burnup fuels. Corrosion thins the cladding and therefore increases the applied stresses. In addition, the increased thickness of oxide on the surface of the cladding increases its temperature, further accelerating the corrosion rate. The hydrogen absorption of the Zircaloy is, at least to some degree, proportional to the degree of oxidation since some of the absorbed hydrogen comes from the oxidation process. This additional hydrogen pick-up further reduces cladding ductility. Proprietary zirconium alloys, which show corrosion

rates significantly below that of Zircaloy-4, particularly at long exposure times, have been developed in both the United States and Western Europe for use with high-burnup fuels.

An accelerated corrosion rate leading to cladding failures at modest burnup can occur in the presence of heavy buildup of deposits on the exterior cladding surface. This can occur with poor control of water chemistry but this problem appears primarily with BWR rather than PWR fuel. It has been established that, for PWR fuel, the external oxidation of the cladding should not exceed about 100 μm to avoid clad failure.¹⁷⁶

If the fuel pellet contains a hydrogenous contaminant, usually moisture, decomposition of the contaminant provides hydrogen, which can react with the internal surfaces of the cladding. It has been found that this reaction is often greatly enhanced at particular locations leading to massive local hydriding. Cladding failure generally follows. Hydriding failures were once a fairly common cause of fuel rod failure but now (1995) are relatively rare. Current manufacturing procedures appear to effectively remove any hydrogenous contaminants.

Pellet-cladding interaction, or PCI, failures are the most common cause of fuel rod failure and are of the greatest concern to fuel rod designers. PCI failures can be attributed to either (1) mechanical overload of cladding subject to tensile stress concentration or (2) stress-corrosion cracking. The last mechanism is recognized as the predominant failure mechanism.¹⁷⁷

Stress-corrosion cracking occurs when high stress levels are present and halogens are available at the inner surface. Cracking will occur only after the stress exceeds a threshold value and the halogen concentration is above a critical level. The halogen concentration is provided primarily by the iodine in the fuel fission products. The high stress levels are provided by pellet-cladding contact pressure. The threshold stress level for the onset of cracking decreases with irradiation exposure.¹⁷⁵ The degree of stress-corrosion cracking increases with increasing stress levels.

High stresses can be produced in fuel element cladding if the fuel has been operating at less than full power and then been brought rapidly to a higher power (power ramp). If the pellet-cladding gap was closed at the low power, the only way the cladding can respond to the rapid thermal expansion of the pellet is via elastic expansion. Such an expansion requires a high tensile stress and contact pressure. The maximum cladding stresses produced are considerably higher than would be calculated on the basis that the cladding tube remains cylindrical. A major reason for this is the fact that during irradiation the pellets tend to form an "hourglass" or "wheatsheaf" shape. That is, the diameter of the pellet at the axial centerline is smaller than at the pellet ends. As the cladding creeps down on the pellet, ridges are formed at the pellet junctions. At each ridge there is a significant stress concentration (Fig. 2.24). Additional regions of stress concentration occur where the cladding must bridge a crack in the fuel pellet.

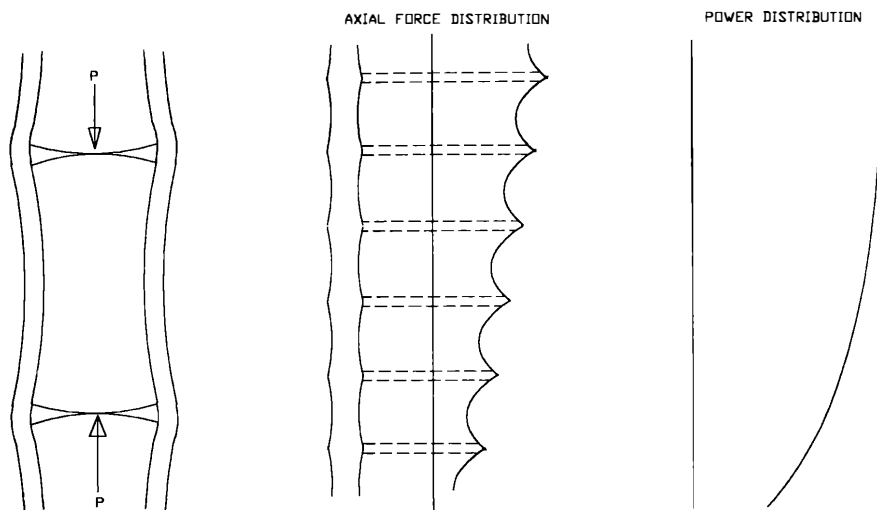


Fig. 2.24 Axial force distribution on cladding wall [from C. Kjaerheim and E. Rolstad, OECD Report, Halden Reactor Project, HPR-107, Halden, Norway (1969)].

When power ramping results in stress-corrosion cracking, there often appears to be a thermal feedback effect involved. The proposed sequence of events is¹⁷⁸: (1) fuel with considerable inventory of stored fission gas is ramped, (2) increased fuel temperature produces increased fission gas release, (3) fission gas decreases conductivity of the fill gas ($k_{FG} \approx 0.05k_{He}$) and decreases gap conductance, and (4) fission gas release is further increased. This sequence can lead to substantial increases in fuel temperature and hence greater thermal expansion and higher thermal stresses. This condition is particularly likely where fuel fragment misalignment impairs the improvement of gap conductance with contact pressure. Rods that are not prepressurized are most sensitive to thermal feedback effects. By prepressurizing the rod with several atmospheres of helium at room temperature, the thermal conductivity of the fuel gas is much less sensitive to the amount of fission gas released and thermal feedback effects are greatly reduced.

Improved plant operational strategies are generally successful in preventing PCI failures. In the so called "RSST" approach¹⁷⁹ all of the four "predictors" must exceed a critical value for a failure to occur. Minimum or critical values have to be exceeded for (1) the power Range (operate at a power above a burnup-dependent failure threshold) or (2) the power Step (size of power increase) or (3) the Speed of the power increase or (4) the Time at high power. If any one of the "predictors" is subcritical, failures should not occur. The development of these criteria were based on a very extensive test program carried out at the HFR reactor in Pelten, Holland.

Young analyzed over 700 irradiations and developed curves that simultaneously fit 80% of the failure and nonfailure data. He concluded that the best predictor of the failure boundary was the power increase. The allowable power change was found to be a function of burnup and initial power level. Figure 2.25 shows the PCI failure boundaries they developed.

2.5.3 Improved Fuel Rod Design

In addition to establishing revised plant operating procedures, fuel performance can be improved by a number of design modifications. Fuel assembly modifications leading to improved performance and higher burnups have been discussed previously in Sec. 2.1.2.3. It has also been previously observed that prepressurization of fuel rods with helium reduces thermal feedback effects and decreases the probability of PCI failure. In addition, the prepressurization reduces cladding creep-down, thus reducing the total strain range to which the cladding is subjected. For given EOL burnup, there is an optimum level for initial pressurization. Improved fuel behavior must be balanced against the limitation imposed by the EOL pressure of the helium itself.

PCI failures are also reduced by use of barrier coatings on the inner wall of the fuel element cladding. The first of these barriers was a colloidal dispersion of graphite as a cladding coating in CANDU reactors.¹⁸¹ Fuel failures due to power ramps were significant problems in the CANDU-type reactor where at-power refueling can impose rapid power changes on the fuel elements being moved. The thin, collapsed cladding used in CANDU reactors leads to a particularly high stress. At a given burnup level, there is a range of stresses above which failures are likely. The stress range within which failures begin decreases as burnup increases. The use of a graphite barrier coating substantially increases the threshold stress. It was originally thought that the graphite acted by decreasing friction between the fuel and cladding and thus decreasing mechanical stresses. It is now believed that the improved behavior with the graphite coating is due to the prevention of halogen fission products from reaching the Zircaloy and thus preventing stress-corrosion cracking.

PCI failures have also been a major problem in BWRs where control rod motion can subject fuel adjacent to the rod to a severe power ramp. The problem has been greatly ameliorated by the use of Zircaloy cladding lined with pure zirconium or zirconium alloyed with a small amount of iron.¹⁸² The inner layer is coextruded with the Zircaloy and metallurgically bonded to it. The total wall thickness (cladding plus liner) equals that of conventional cladding. The liner provides a soft ductile interface between the pellet and the cladding. Because of its high ductility, the liner itself has a very low susceptibility to stress-corrosion cracking. The liner also serves a

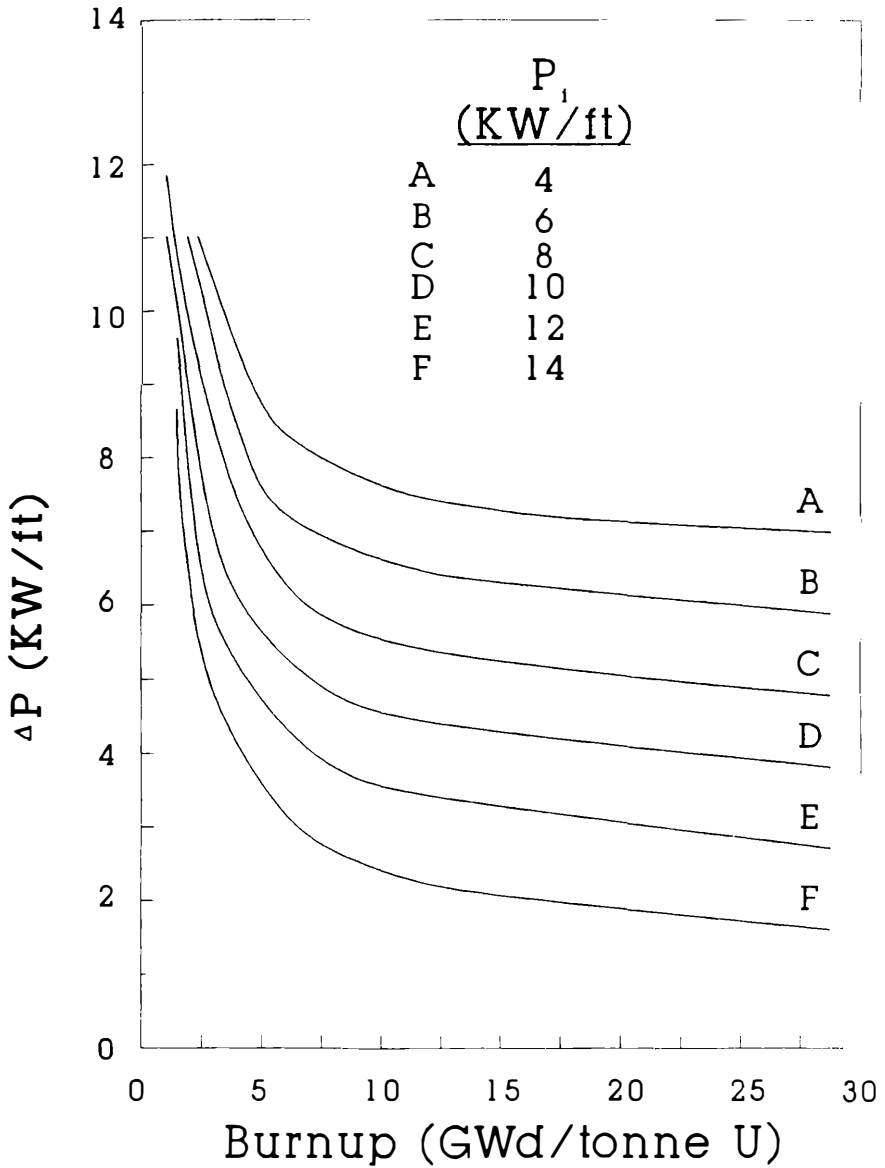


Fig. 2.25 Predicted change-in-power (ΔP) failure boundaries as a function of burnup [from Ref. 180].

barrier between the halogen fission products and Zircaloy base. While most PWR fuel manufacturers have not felt it necessary to go to zirconium-lined cladding, some manufacturers are proposing its use for fuel elements subject to extended burnup. Zirconium barriers, which range from 10 to 30% of the wall thickness, have been proposed. Analyses⁴⁸ indicate that the very ductile liner will carry no load and that the higher stresses in the Zircaloy lead to creep rates 15 to 70% above those of conventional cladding.

Since PCI failures are exacerbated by high halogen concentrations, improved fuel performance can be obtained by operation at lower peak powers. Some high-burnup designs call for operation at reduced linear heat rates. In addition to improved PCI performance, the lower fission gas releases mean low gas pressure at the end of fuel life.

Because pellet "hourglassing" is a major cause of stress concentration in PWR cladding, reduced hourglassing will further reduce PCI failure rates. It has been found that this can be accomplished by reducing the pellet length-to-diameter ratio.

The use of annular pellets is another way of reducing cladding stresses. The free volume at the pellet center allows some of the pellet swelling and thermal expansion to be taken internally and thus reduces cladding strain and stress. Further, the reduced temperature provided by such pellets reduces fission gas release and the central free volume reduces pressure buildup. Although extensive tests of fuel rods with annular pellets have shown their value,¹⁸³ no Western core has yet (1993) used this design. However, annular pellets have been used in the Russian VVER reactors.

Most designs for high-burnup fuel would use cladding with improved corrosion resistance (see Sec. 2.1.2.1). Such designs also tend to use the improved cladding material for the support grids and control-rod guide tubes.

2.5.4 Evaluation of Fuel Performance During Normal Operation

2.5.4.1 Computer Programs for Fuel Element Design

The complexity of the calculations required to treat fuel and cladding deformation and restructuring, as well as fission gas release, obviously requires that these calculations be performed via computer. As noted in Sec. 2.4.1, the fuel is divided into a series of axial segments and radial rings within each segment. The behavior of each of these fuel volumes is then modeled over a series of timesteps so that the behavior over life may be determined. The fuel power at each timestep is set so as to simulate the expected time variation in power. The relative power in the various axial segments is set on the basis of the expected axial power variation. To obtain peak stress levels it is important that the power ramps to which the fuel may be subjected be simulated at appropriate points during the exposure.

Timesteps need to be adjusted to allow such ramps to be adequately simulated.

A fuel-life simulation with an appropriate fuel element design code will determine at least the following: (1) maximum strain and strain range, (2) internal gas pressure at the end of life, and (3) maximum cladding stress. These computations are generally made assuming the cladding retains its cylindrical shape. The U.S. NRC Standard Review Plan¹⁸⁴ indicates that the uniform strain of the cladding should not exceed 1%. Some investigators replace the simple maximum strain range limit by a more sophisticated rule. Verbeek and Hoppe⁴⁷ note that, at a given constant temperature and stress, there is a maximum strain at which rupture will occur. They therefore assume that the strain produced at each stress level to which the rod is subjected consumes a portion of the total life of the cladding. If it is assumed that the applied stress variations over time is approximated by a series of timesteps at a constant stress, a total damage fraction, D' is defined as

$$D' = \sum \frac{|\Delta \epsilon_{pi}|}{(\epsilon_L)_i} \quad (2.202)$$

where $|\Delta \epsilon_{pi}|$ is the absolute value of plastic strain (less irradiation creep, which is taken as nondamaging) accumulated during the i 'th time period; $(\epsilon_L)_i = (\epsilon_u)_i (R_i)$, the ductile strain limit that applies during the i 'th time period; $(\epsilon_u)_i$ is the uniform ductility limit for the material condition (temperature, degree of corrosion, fast neutron fluence) prevailing in i 'th time period; and R_i is the factor dependent on strain rate and temperature. Failure is assumed to occur when D' equals unity.

Other investigators set the design limit as a fraction of the rupture strain. The circumferential strain at cladding rupture, ϵ_R , depends on the cladding temperature, degree of cold work, and neutron fluence. For the case of no circumferential temperature variation, Hargman *et al.*³⁴ have correlated ϵ_R by

$$\epsilon_R = (0.198 + 4.16 \times 10^{-4} T + 2.06 \times 10^{-7} T^2) F_R F_{cw} \quad \text{for } T < 1090 \text{ K} \quad (2.203)$$

where

T = cladding temperature, K

F_R = irradiation factor = $[1 + 2 \exp(-\phi t / 10^{23})] / 3$

F_{cw} = cold work function = $[\exp(-21 C_w) + 0.33] / 1.33$

C_w = fraction by which cross-sectional area is reduced by cold work

ϕt = neutron fluence, n/m²

The effect of cold work and neutron fluence on rupture strain are substantial. A sample with 20% cold work and a neutron fluence of approximately

10^{25} n/m² would show a rupture elongation of only about 10% of the value for unirradiated and unworked Zircaloy. Note that Eq. (2.203) does not include the effects of (1) hydrogen absorption, which further reduces ductility, and (2) stress annealing, which reduces the effect of cold work and increases ductility. To ensure an adequate margin of safety, the maximum design strain should be set substantially below the rupture strain (or $D' < 1$).

At the temperatures encountered in normal operation, the cladding will be fairly brittle at the end of life and cladding failure may be expected at the rupture strain ($\epsilon_L = \epsilon_p$). However, at the higher temperature of some transients, ductile behavior is encountered. With ductile behavior, failure would be encountered when the cladding strain is such that instability is encountered. Hagrman *et al.*³⁴ indicate that for $T < 1090$ K, this occurs when the strain reaches one fourth the rupture strain computed with $F_R = F_{cw} = 1.0$.

As previously observed, the usual internal gas pressure limit is that there be no cladding lift-off at full power at the end of life. Maximum cladding stresses (ignoring hourglassing) are set at levels such that PCI failures are rare. Roberts and Gelhaus¹⁸⁵ indicate that the threshold stress for PCI failure is approximately 190 MPa. A typical design stress is about 70% of this value.¹⁸⁶

One of the earliest computer programs to consider fuel element design in a sophisticated manner was the CYGRO code.^{166,187} Both fuel and cladding are divided into a series of concentric rings linked by continuity laws of force and displacement. The stresses, strains, and all properties are taken to be uniform throughout a given ring. During each time increment, strain rates and stress are taken as constraints in a given ring. Strain rates and stress are related by Prandtl-Reuss equations that are modified to include the effects of fuel growth, thermal expansion, plasticity, and creep. The size of a timestep is limited so changes in stress and strain are small.

Fuel swelling is computed via Nichols and Warner's BUBL model.⁹⁵ Porosity migration is treated by Nichols' method.²⁸ The total solid volume in each ring is held constant so that the radii of the boundaries follow the fuel. The heat generation rate in each ring is a constant fraction of the total heat generation throughout time history.

A comprehensive stress-strain analysis of the fuel is conducted. In the low-temperature region, allowance is made for the fact that if tensile stress exceeds a predetermined amount, the fuel cracks and tensile stress drops to near zero. Cracks are assumed to heal during compression. Frictional interaction between fuels and cladding is also considered. The finite element technique is used to solve stress-strain relations.

Although CYGRO conducts a very comprehensive analysis of the stress-strain relationship in the fuel, it considers only a single axial node. Fission gas pressure information as a function of time must come from another calculation.

The LIFE code¹⁸⁸ eliminated the need for obtaining fission gas release separately, since it is capable of considering a series of axial nodes at different power levels. This code was originally developed for fast reactor fuel, but versions satisfactory for use with LWR (SST¹⁸⁹ and LIFE-THERMAL) are available. The stress-strain analysis performed by the LIFE code is less elaborate than that done by CYGRO, since only three fuel regions (columnar, equiaxial, and unrestructured) are considered in the stress-strain analysis. The swelling and fission gas release analysis is more sophisticated since the GRASS code⁹⁶ is used for computation of these quantities and a number of axial nodes are considered.

Both CYGRO and LIFE are time consuming to run. In the first generation of easily usable design codes, approximate stress analyses are conducted and swelling and fission gas release are described by analytical expressions. The COMETHE (Ref. 91), FRAP-S (Ref. 190), and GAPCON (Ref. 191) codes are examples of this approach. In all of these codes, a series of axial segments is considered so that a reasonable estimate of overall fission gas release can be obtained and its effect on cladding and fuel behavior can be determined.

A second generation of design codes, which use improved correlations for fuel and cladding behavior but which also perform only a relatively simple stress analyses, are now (1992) available. These include COMETHE (Ref. 192), ESCORE (Ref. 48), TACO3 (Ref. 193) and other reactor vendor codes.

A number of the more recent fuel element design codes continue to use relatively simple approaches for fission gas release and fuel swelling but perform fairly rigorous stress-strain analyses. Some of the better known programs in this category are: BEAF (Ref. 169), SAMURA (Ref. 194), ENIGMA (Ref. 195), FEMAXI-III (Ref. 196), FRAPCON (Ref. 120), SLEUTH (Ref. 197), and TRANSURANUS (Ref. 198). The FREY code (Refs. 199 and 200), which uses a fully coupled two-dimensional finite element analysis, differs from the other programs in this category in that it can consider azimuthal variations.

In addition to the LIFE codes, several programs now incorporate fundamental models of fission gas release and fuel swelling and conduct a sophisticated stress analysis of fuel and cladding. FUTURE (Ref. 201) and WAFER-3 (Ref. 202) are examples of programs of this nature.

2.5.4.2 *PCI Fuel Failure Estimation*

The computer programs for fuel element design discussed in the previous sections allow the designer to determine whether his or her design exceeds any of the accepted thresholds. However, even though all design criteria are met, experience indicates that a small number of fuel failures will still occur.

Rolstad²⁰³ argued that stochastic elements play an important role in determining power range failures. He concluded that failures occurred when controlling parameters were at the most unfavorable ends of their tolerance bands, or when the most unfavorable fuel crack pattern coincided with the worst cladding defect. Therefore, he proposed a probabilistic approach. In Rolstad's original POSHO code,²⁰³ local power ramps resulting from planned fuel operation are simulated. These are then processed through a fuel failure probability routine. Failure probability estimates are based on fuel design parameters, burnup, and previous power ramp failure experience. The failure probabilities for a given power change are a function of local power level. The number of cladding cracks produced by the change is obtained by multiplying the number of pellet interfaces at a given power level by the average failure probability at that power level.

In the original POSHO program, the concept of "power shock" was used to represent cladding stresses resulting from rapid power increases. The "power shock" at any time is the difference between the real nodal power and the power corresponding to the zero gap and no contact pressure. In the newer POSHO-THERMAL,²⁰⁴ fuel temperatures and fission gas releases are computed. The concept of "temperature shock" (actual temperature less temperature for zero contact pressure) is now used as a stress index. This results in more accurate failure predictions since fuel temperatures are not always directly proportional to power (e.g., large fission gas releases decrease gap conductance).

The SPEAR program¹⁰¹ also allows the examination of structural patterns of fuel failure. Through application of a pattern matching program to a large base of fuel failure data, failure probabilities are assigned to regions in which various operating conditions (e.g., burnup, maximum fuel temperature, contact pressure, iodine concentration, ramp rate) or combinations of these conditions are within a given range. The SPEAR program then compares the time-varying operating conditions to these failure probability regions and assigns appropriate failure probabilities to the design core.

More mechanistic cladding failure models are also available. In one approach,¹⁰¹ the mean times to failure obtained from stress-corrosion cracking experiments are correlated as a function of maximum hoop stress, iodine concentration, and cladding texture angle. A damage parameter, d , is then defined as the ratio of the operating time under given conditions to the failure time. The total damage is the sum of these ratios. The fraction of failures for a given total d is obtained from a log-normal probability distribution derived from a fit of the laboratory cracking data. In using this correlation the cladding stress is based on an estimate of the maximum stress at the cladding ridges.

Yaung *et al.*¹⁸⁰ have developed a more elaborate stress-corrosion cracking model. They divide the fuel cladding into a number of radial elements and keep track of (1) the local stress-strain level (allowing for effects of ridges

and cracks) in each element and (2) the iodine environment in the first (innermost) unfailed radial element. Failure of a given radial element is determined by an experimentally established criterion.

2.6 TRANSIENT FUEL ROD BEHAVIOR

2.6.1 Fuel Rod Temperature Transients

2.6.1.1 Analytical Techniques

Under transient conditions when heat storage must be considered, the general conduction equation takes the form

$$c_p \rho \frac{\partial T}{\partial t} = \nabla(k \nabla T) + q''' \quad (2.204)$$

where t represents time, c_p specific heat, ρ density, and other symbols have their previous meanings.

We consider a fuel pellet of radius a surrounded by cladding with an outside radius b : If we ignore axial conduction and assume that k for the fuel is a constant, for the pellet region we write

$$k_1 \left(\frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} \right) + q''' = c_{p1} \rho_1 \frac{\partial T_1}{\partial t} \quad (0 < r \leq a) \quad (2.205a)$$

while for the cladding region

$$k_2 \left(\frac{\partial^2 T_2}{\partial r^2} + \frac{1}{r} \frac{\partial T_2}{\partial r} \right) = c_{p2} \rho_2 \frac{\partial T_2}{\partial t} \quad (a \leq r \leq b) \quad (2.205b)$$

We consider a step change in heat transfer coefficient at the outer cladding at $t=0$ and a subsequent reduction in heat generation rate. This corresponds to the situation arising during a transient when the fuel element is suddenly blanketed by steam [departure from nucleate boiling (DNB) occurs (Sec. 4.4.2)] and the film coefficient is drastically reduced. Heat generation rate is reduced on reactor scram.

A set of dimensionless groups is defined as follows:

$$B_1 = \frac{h_g b}{k_2} \quad B_2 = \frac{h b}{k_2} \quad (2.206)$$

where h_g is gap conductance and h is the film coefficient at rod surface

$$R_1 = \frac{r}{b} \quad R_p = \frac{a}{b} \quad \tau = \left(\frac{k_2}{c_{p2} \rho_2} \right) \frac{t}{a^2} \quad (2.207)$$

$$C_p(R_1) = \frac{c_p(r) \rho(r)}{c_{p2} \rho_2}, \quad K(R_1) = \frac{k_1(r)}{k_2}, \quad (2.208)$$

$$Q(\tau) = \frac{q'''(t)}{q'''(0)} \quad T^+ = \frac{k_2 T}{q'''(0) a^2} \quad (2.209)$$

Then Eqs. (2.205a) and (2.205b) can be written as

$$K \left(\frac{\partial^2 T_1^+}{\partial R_1^2} + \frac{1}{R_1} \frac{\partial T_1^+}{\partial R_1} \right) + Q = C_p \frac{\partial T_1^+}{\partial \tau} \quad (0 < R_1 \leq R_p) \quad (2.210a)$$

$$\frac{\partial^2 T_2^+}{\partial R_1^2} + \frac{1}{R_1} \left(\frac{\partial T_2^+}{\partial R_1} \right) = \frac{\partial T_2^+}{\partial \tau} \quad (R_p < R_1 \leq 1) \quad (2.210b)$$

The boundary conditions are

$$B_1(T_1^+ - T_2^+) = -K \frac{\partial T_1^+}{\partial R_1} \quad (\text{at } R_1 = R_p) \quad (2.211)$$

$$B_1(T_1^+ - T_2^+) = \frac{\partial T_2^+}{\partial R_1} \quad (\text{at } R_1 = R_p) \quad (2.212)$$

$$B_2(T_2^+ - T_\infty^+) = -\frac{\partial T_2^+}{\partial R_1} \quad (\text{at } R_1 = 1) \quad (2.213)$$

where T_∞^+ is the dimensionless bulk coolant temperature.

These equations can be solved by a Hankel transform²⁰⁵ or a finite integral transform.²⁰⁶ In both cases, the exact solutions are in terms of infinite series. By using a finite integral transform technique, Matsch²⁰⁶ obtained the following solution:

$$T^+(R_1, \tau) = \sum_n C_n(\tau) X_n(R_1) \quad (2.214)$$

where values X_n are obtained by considering an associated equation of the form

$$\nabla K(R_1) \nabla X(R_1) + (\lambda_n)^2 X(R_1) = 0 \quad (2.215)$$

to which the solutions are a set of eigenfunctions (X_n) true for discrete values of $(\lambda_n)^2$

A complex solution procedure is required to determine the eigenfunctions. Because of the complexity of this procedure, and because analytic techniques do not provide good accuracy when large temperature changes make the assumption of constant properties invalid, analytic techniques that attempt a complete solution of the problem are now rarely used.

2.6.1.2 Simplified Analytical Techniques for Temperature Transients

Analytical techniques, which are less tedious than described in Sec. 2.6.1.1, can be useful in some instances. They can provide a rapid means for obtaining approximate estimates under many conditions. In addition, for rel-

atively slow transients, the results obtained can be adequate and not require a more sophisticated procedure. The loss-of-flow accident (LOFA) with scram sometimes falls into this category. For a UO_2 fuel rod, maximum cladding temperature occurs several seconds after initiation of the accident and a lumped parameter procedure yields a reasonable estimate.

Lumped parameter techniques are among the earliest procedures used for examining temperature transients.²⁰⁷ In the lumped parameter procedure suggested by Tong,²⁰⁸ thermal resistances and the capacitances of the pellet and cladding are evaluated at their average condition in time and space. Each quantity is lumped at the middle of the physical geometry and axial conduction is neglected. Again, we consider the situation where there is a step reduction in the coolant heat transfer coefficient and a subsequent reduction in heat generation rate due to reactor scram.

Rate equations for heat transfer from the UO_2 fuel rod can be written as

$$q'_n = C_1 \frac{dT_1}{dt} + \frac{T_1 - T_2}{R_1} \quad (2.216)$$

and

$$\frac{T_1 - T_2}{R_1} = C_2 \frac{dT_2}{dt} + \frac{T_2 - T_c}{R_2} \quad (2.217)$$

where

q'_n = nuclear heating, Btu/s ft of fuel rod

C_1 = thermal capacitance of pellet, Btu/°F ft, $C_1 = \pi r_1^2 c_{p1} \rho_1$

C_2 = thermal capacitance of clad, Btu/°F ft, $C_2 = 2\pi r_2(\Delta r)c_{p2}\rho_2$ for thin cladding

R_1 = resistance of UO_2 and gap, s ft °F/Btu

= $[1/(8\pi k_1)] + [1/(2\pi r_1 h_g)]$, where k_1 is UO_2 thermal conductivity and h_g is gap conductance

R_2 = resistance of coolant film = $1/(2\pi r_2 h)$, s ft °F/Btu

T_1 = average pellet temperature, °F

T_2 = average cladding temperature, °F

T_c = bulk coolant temperature, °F.

After a pipe rupture, the system pressure and saturation temperature of the coolant drop with time. Hence, $T_c = T_c(t)$. Time t is counted from the instant of rupture. From the Laplace transformation of Eqs. (2.216) and (2.217), we get

$$T_1(s) =$$

$$\frac{\left(R_2 C_2 s + 1 + \frac{R_2}{R_1}\right) R_1 q'_n(s) + T_c(s) + \left(R_2 C_2 s + 1 + \frac{R_2}{R_1}\right) R_1 C_1 T_1(0) + R_2 C_2 T_2(0)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_2 C_1) s + 1} \quad (2.219)$$

By knowing coolant temperature and fuel rod power as a function of time, the histories of pellet and cladding temperatures can be computed from the inverse transformation of Eqs. (2.218) and (2.219).

In the analysis of loss-of-flow transient, two simplifications can be made: First, when the pumps lose power, system pressure does not change significantly and coolant temperature T_c remains approximately constant; second, maximum cladding temperature usually occurs within 10 s of the instant of loss of power to the pump. Hence, decay heat can be assumed constant during this short time period. Our boundary conditions now become

$$\begin{aligned} \text{for } t \leq 0: \quad q'_n &= q'_n(0) & R_2 &= R_{2,0} \\ \text{for } 0 < t \leq t_1: \quad q'_n &= q'_n(0) & R_2 &= R_{2, \text{ film boiling}} \\ \text{for } t \geq t_1: \quad q'_n &= \beta q'_n(0) & R_2 &= R_{2, \text{ film boiling}} \end{aligned} \quad (2.220)$$

By taking advantage of the constancy of T_c , we can solve Eq. (2.217) for T_1 and then differentiate with respect to t to obtain dT_1/dt . We then rewrite Eq. (2.216) as

$$q'_n = C_1 C_2 R_1 \frac{d^2 \theta}{dt^2} + \left(C_1 + C_2 + \frac{C_1 R_1}{R_2} \right) \frac{d\theta}{dt} + \frac{\theta}{R_2} \quad (2.221)$$

where $\theta = T_2 - T_c$.

When the Laplace transform is taken, we obtain

$$\begin{aligned} \frac{1}{s} [1 - \beta \exp(t_1 s)] &= C_1 C_2 R_1 \left[\bar{\theta} s^2 - \theta(0) s - \frac{d\theta(0)}{dt} \right] + \left(C_1 + C_2 + \frac{C_1 C_2}{R_2} \right) \\ &\quad \times [\bar{\theta} s - \theta(0)] + \frac{\bar{\theta}}{R_2} \end{aligned} \quad (2.222)$$

where $\bar{\theta}$ represents the Laplace transform of θ .

The value $\theta(0)$ is given by

$$\theta(0) = q'_n(0) R_{2,0} \quad (2.223)$$

and $d\theta(0)/dt$ from the relationship

$$T_1(0) = q'_n(0) (R_1 + R_{2,0}) \quad (2.224)$$

and Eq. (2.217). Substitution of these numerical values, followed by the inverse transformation, yields θ as a function of time.

Chen *et al.*²⁰⁹ extended the lumped parameter technique to consider phase changes (cladding and fuel melting). They considered the problem of loss of flow at constant power (failure to scram) and developed equations for estimating the times at which cladding and fuel melting begin. Steady-state conditions are computed first, assuming the coolant is at saturation. The average cladding temperature, T_{c_o} , at steady state is

$$T_{c_o} = T_{\text{sat}} + \frac{q''' r_F^2}{2} \left[\frac{1}{h_0 r_c} + \frac{1}{k_c} \left(\frac{1}{2} - \frac{r_F^2}{r_c^2 - r_F^2} \ln \frac{r_c}{r_F} \right) \right] \quad (2.225)$$

and the average steady-state fuel temperature, T_{F_o} , is given by

$$T_{F_o} = T_{\text{sat}} + \frac{q''' r_F^2}{2} \left(\frac{1}{4k_F} + \frac{1}{h_0 r_c} + \frac{1}{h_{\text{gap}} r_F} + \frac{1}{k_c} \ln \frac{r_c}{r_F} \right) \quad (2.226)$$

where

h_0 = steady-state (pre-DNB) coolant heat transfer coefficient

q''' = volumetric heat generation rate

r_F = outer radius of fuel region

r_c = outer radius of cladding

k_c, k_F = thermal conductivity of cladding and fuel, respectively

T_{sat} = saturation temperature.

They then assume that the coolant heat transfer coefficient goes to zero (vapor blanketing) and remains constant at that value. After defining the following symbols, they develop the working equations shown in Table 2.VIII,

where

t_d = time at which vapor blanketing (DNB) occurs

ρ_c, c_{pc}, H_c = density, specific heat, and latent heat of cladding, respectively

ρ_F, c_{pF}, H_F = density, specific heat, and latent heat of fuel, respectively

$h' = \left(\frac{r_F}{4k_F} + \frac{1}{h_{\text{gap}}} + \frac{r_c - r_F}{2k_c} \right)^{-1}$, average overall heat transfer coefficient

$C_F = r_F \rho_F c_{pF} / 2h'$, calculated at the steady-state maximum fuel temperature

$C'_F = r_F \rho_F c_{pF} / 2h'$, calculated at average temperature

$T_{\text{av}} = [T_{FP} + T_F(t_m)] / 2$

$C_c = [(r_c^2 - r_F^2) \rho_c c_{pc}] / 2R_F h'$, calculated at the cladding melting temperature, T_{cp}

$Q = r_F q''' / 2h'$.

TABLE 2. VIII
Working Equations for the Chen *et al.*²⁰⁹ Transient Conduction Model with Phase Change

Phases	Timing	Average Fuel and Cladding Temperatures
Transient phase before clad melting	$t_d < t \leq t_p$ (starting time of clad melting), where $t_p = \begin{cases} t_d + \left(\frac{T_{cp} - T_{co}}{T_{f0} - T_{co}} \right) C_c & \text{for } t_p - t_d \leq \frac{C_f C_c}{C_f + C_c} \\ t_d + \left[\frac{T_{cp} - T_{co}}{T_{f0} - T_{co}} - \left(\frac{C_f}{C_f + C_c} \right)^2 \right] (C_f + C_c) & \text{for } t_p - t_d > \frac{C_f C_c}{C_f + C_c} \end{cases}$	Case 1: $-t_d \leq \frac{C_f C_c}{C_f + C_c}$ $T_c = T_c, + \frac{Q}{C_c} (t - t_d)$ and $T_f =$ Case 2: for $t - t_d > \frac{C_f C_c}{C_f + C_c}$ $+ \frac{Q}{C_f + C_c} \left(\frac{C_f^2}{C_f + C_c} + t - t_d \right)$ $t_{f0} - \frac{C_f C_c}{(C_f + C_c)^2} (T_{f0} - T_{co}) + \frac{Q}{C_f + C_c} (t - t_d)$
Clad melting phase	(complete time of clad melting) Case 1: for $t_p - t_d \leq \frac{C_f C_c}{C_f + C_c}$ $m = \begin{cases} t_p + \frac{(r_c^2 - r_f^2) \rho_c H_c}{2 R_f h' (T_{f0} - T_{cp})} \\ t_p + \frac{(r_c^2 - r_f^2) \rho_c H_c}{2 R_f h' (T_{f0} - T_{co})} + C_f \frac{T_{cp} - T_{co}}{T_{f0} - T_{co}} \end{cases}$ $m - t_p \leq C_f$ $m - t_p > C_f$ Case 2: for $t_p - t_d > \frac{C_f C_c}{C_f + C_c}$ $m = \begin{cases} t_p + \frac{(r_c^2 - r_f^2) \rho_c H_c}{2 R_f h' (T_{f0} - T_{co})} \left(\frac{C_f + C_c}{C_c} \right) \\ t_p + \frac{(r_c^2 - r_f^2) \rho_c H_c}{2 R_f h' (T_{f0} - T_{co})} + \frac{C_f^2}{C_f + C_c} \end{cases}$	$T_c = T_{cp}$ (clad melting temperature) Case 1: for $t_p - t_d \leq \frac{C_f C_c}{C_f + C_c}$ $T_f = \begin{cases} T_{f0} + (T_{cp} - T_{co}) \frac{t - t_p}{C_f} & \text{for } t - t_p \leq C_f \\ T_{f0} + T_{cp} - & \text{for } t - t_p > C_f \end{cases}$ Case 2: for $t_p - t_d > \frac{C_f C_c}{C_f + C_c}$ $f = \begin{cases} T_{cp} + \frac{T_{f0} - T_{co}}{C_f + C_c} (C_c + t - t_p) & \text{for } t - t_p \leq C_f \\ T_{f0} + T_{cp} - & \text{for } t - t_p > C_f \end{cases}$

TABLE 2.VIII (continued)

Transient phase before fuel melting	$m < t \leq t_{pf} \text{ (starting time of fuel melting), where}$ $t_{pf} = \left\{ \begin{aligned} & t_m + \frac{[T_{fp} - T_f(t_m)]C_f'}{T_{f0} - T_{co} - T_f(t_m) + T_{cp}} \quad \text{for } t_{fp} - t_m \leq \frac{C_f' C_c}{C_f' + C_c} \\ & t_m + \frac{T_{fp} - T_f(t_m) + \frac{C_c}{C_f' + C_c} \left[T_f(t_m) - T_{cp} - \frac{C_c(T_{f0} - T_{co})}{C_f' + C_c} \right]}{(T_{f0} - T_{co})/(C_f' + C_c)} \quad \text{for } t_{fp} - t_m > \frac{C_f' C_c}{C_f' + C_c} \end{aligned} \right.$	$T_c = \left\{ \begin{aligned} & T_{cp} + \frac{1}{C_c} [T_f(t_m) - T_{cp}] (t - T_{cp} + \frac{T_{f0} - T_{co}}{C_f' + C_c} \left\{ \frac{C_f' [T_f(t_m) - T_{cp}]}{T_{f0} - T_{co}} + t - t_m - \frac{C_f' C_c}{C_f' + C_c} \right\}) \quad \text{for } t - t_m \leq \frac{C_f' C_c}{C_f' + C_c} \\ & T_f(t_m) - \frac{1}{C_f'} [T_f(t_m) - T_{cp} - T_{f0} + T_{co}] (t - t_m) \quad \text{for } t - t_m > \frac{C_f' C_c}{C_f' + C_c} \end{aligned} \right.$ $T_f = \left\{ \begin{aligned} & T_f(t_m) - \frac{T_{f0} - T_{co}}{C_f' + C_c} \left\{ \frac{C_c [T_f(t_m) - T_{cp}]}{T_{f0} - T_{co}} - t + t_m - \frac{C_c^2}{C_f' + C_c} \right\} \quad \text{for } t - t_m \leq \frac{C_f' C_c}{C_f' + C_c} \\ & T_f(t_m) - \frac{C_f' C_c}{C_f' + C_c} \quad \text{for } t - t_m > \frac{C_f' C_c}{C_f' + C_c} \end{aligned} \right.$
Fuel melting phase	$t_{pf} < t \leq t_{fm} \text{ (complete time of fuel melting)}$ $t_{fm} = \left\{ \begin{aligned} & t_{pf} + \frac{r_{fp} H_f}{2h' [T_{f0} - T_{co} - T_{fp} + T_c(t_{pf})]} \quad \text{for } t_{fm} \leq t_{pf} + C_c \\ & t_{pf} + \frac{r_{fp} H_f}{2h' (T_{f0} - T_{co})} + \frac{C_c [T_{fp} - T_c(t_{pf})]}{T_{f0} - T_c} \quad \text{for } t_{fm} > t_{pf} + C_c \end{aligned} \right.$	$T_c = T_{fp} - [T_{fp} - T_c(t_{pf})] \exp\left(-\frac{t - t_{pf}}{C_c}\right)$ $T_f = T_{fp} \text{ (fuel melting temperature)}$

2.6.1.3 Electrical Analog Techniques

The lumped parameter technique is basically equivalent to using an electrical analog. Conditions at the nodal points of the electrical network represent the average conditions of a particular region. To use such an analog, we note the correspondence between voltage and temperature, current and heat flow, and the product of electrical resistance and the capacity of the electrical system and thermal diffusivity. Thus, Fourier's law of $q = \Delta T/R$, is represented by Ohm's law for current flow $i = \Delta E/R$. The change in system thermal energy is given by

$$\frac{dQ}{dt} = c_p \rho V \frac{dT}{dt} \quad (2.227)$$

where

Q heat content, Btu

c_p specific heat

V = volume

ρ = density.

The change in electrical energy is represented by

$$\frac{dQ_e}{dt} = C \frac{dE}{dt} \quad (2.228)$$

where Q_e is the electric charge on the condenser and C is condenser capacitance.

The electrical analog technique was widely used for control and transient analyses and in nuclear power plant simulators. However, the rapid advance in digital computer capabilities has led to the virtual disappearance of analog computation.

2.6.1.4 Approximate Numerical Techniques

An analytical solution of the lumped parameter equations can only be obtained for relatively simple boundary conditions. During many accident situations, rod power and coolant heat transfer conditions vary in a complex manner. This necessitates a stepwise numerical integration of the governing equations [Eqs. (2.216) and (2.217)] written in finite difference form. Although we may continue to represent the cladding and fuel in terms of single average temperatures, it is also feasible to allow for a parabolic temperature profile in the fuel.

By assuming a parabolic temperature profile, improved accuracy can be obtained with little increase in computational effort. With the parabolic profile assumption, the pellet temperature profile may be written in terms of the fuel center temperature, T_{FC} , and the pellet surface temperature, T_{FS} . We then have

$$T_F = T_{FC} - (T_{FC} - T_{FS})(r/r_1)^2 \quad (2.229)$$

where T_F is the fuel temperature at radius r and other symbols have their previous meaning. In some approaches,²¹⁰ the cladding temperature is also assumed to have a parabolic profile. However, since the temperature drop across the cladding is small, it is more usual to assume a single cladding temperature, T_2 .

We obtain the changes in T_1 and T_2 over time period (Δt) by writing the fuel and cladding heat balances in finite difference form after adding a term accounting for any steam-cladding reaction. We then have

$$q'_n = \left(c_1 \frac{\Delta T_1}{\Delta t} \right) + \left(\frac{T_{FC} - T_{FS}}{R_F} \right) \quad (2.230)$$

$$q'_c + \left(\frac{T_{FS} - T_2}{R_g} \right) = \left(c_2 \frac{\Delta T_2}{\Delta t} \right) + \left(\frac{T_2 - T_c}{R_c} \right) \quad (2.231)$$

where

$R_c = (1/2\pi r_2 h) + (\Delta r/4\pi r_c k_2)$ = resistance of coolant and cladding

$R_F = 1/8\pi k_1$ = resistance of fuel (2.232)

$R_g = (1/2\pi r_1 h_g) + (\Delta r/4\pi r_c k_2)$ = resistance of gap and cladding

and

q'_n = rate of heat production in fuel (energy/ $L\Delta t$)

q'_c heat addition to cladding due to any zirconium-steam reaction (energy/ $L\Delta t$)

r_1 = outside radius of fuel pellet

r_2 = inside radius of cladding

k_1, k_2 = average thermal conductivity of fuel and cladding, respectively

c_1, c_2 = heat capacity of fuel and cladding, respectively (energy/ $sL\Delta T$)

T_1, T_2 = average temperature of fuel and cladding, respectively

ΔT_i = change in T_i over time period Δt

T_c = coolant temperature.

We may relate T_1 to T_{FS} and T_{FC} by making use of the parabolic profile and noting

$$T_1 = \int_0^{r_1} T_F(2\pi r) dr / (\pi r_1^2) = (T_{FC} + T_{FS})/2 \quad (2.233)$$

By equating the radial heat flow at the pellet boundary to the heat transfer across the gap we have

$$\frac{T_{FS} - T_2}{R_g} = -k_f \left. \frac{dT}{dr} \right|_{r=r_1} = \frac{2k_f(T_{FC} - T_{FS})}{r_1} \quad (2.234)$$

where k_f = thermal conductivity of fuel at T_{FS} . With the use of Eqs. (2.233) and (2.234), T_{FC} and T_{FS} can be eliminated in Eqs. (2.230) and (2.231). We then are left with two finite difference equations to be solved simultaneously for T_1 and T_2 at time $(t + \Delta t)$. A revised value of (q'_n) , (q'_c) , and material properties are then used to produce a new set of finite difference equations to be used for the next time period. Ghiassian *et al.*²¹⁰ compared parabolic-profile results to computation of fuel and cladding temperatures using a series of radial nodes. They concluded that the parabolic-profile approach gave sufficiently accurate predictions for use in a LOCA.

2.6.1.5 Numerical Methods for Computation of Transient Temperature Profiles

If a detailed examination of the fuel temperature profiles during a transient is desired, it can be readily accomplished by numerical techniques designed for digital computer use. In addition to accounting for the time variation in power and coolant conditions, these methods can allow for the changes in fuel and cladding properties with varying temperature profiles. Discrete time and space intervals are considered and the general conduction equation, Eq. (2.204), may be written in finite difference form. If axial conduction is ignored, for an annular region at a given elevation we write

$$V_j c_p [T_{j,i-1}] \times \left(\frac{T_{j,i} - T_{j,i-1}}{\Delta t_i} \right) = q'_j + \frac{T_{j-1,i-1} - T_{j,i-1}}{R_{j-}} - \frac{T_{j,i-1} - T_{j+1,i-1}}{R_{j+1}} \quad (2.235)$$

where

V_j = volume of region j , per unit of length

$c_p [T_{j,i}]$ = volumetric heat capacity of pellet at the temperature of region j at the end of timestep i

$T_{j,i}$ = temperature of region j at the end of timestep $i - 1$

Δt_i = time increment

q'_j = heat generation per unit of time and length in region j

$R_{j-1,j}$ = resistance between region $j - 1$ and j computed as

$$\frac{1}{R_{j-}} = \frac{k_p [T_{(j-1,j)i-1}] (A_{j-1,j})}{r_j - r_{j-}} \quad (2.236)$$

with

$k_p [T_{(j-1,j)i-1}]$ = conductivity of UO_2 at a temperature that is the average temperature of regions $j - 1$ and j at the end of timestep $i - 1$

$A_{j-1,j}$ = heat transfer area between regions $j-1$ and j
 r_{j-1} and r_j = average radii of these regions.

For the last region in contact with the cladding, we write

$$\frac{1}{R_{j,c}} = h_f \times A_{j,c} \quad (2.237)$$

where h_f is contact conductance.

The basic equation can be solved as

$$T_{j,i} = \frac{q'_j \Delta t_i}{V_j c_P [T_{j,i-1}]} + m_{j-1,j} T_{j-1,i-1} + m_{j,j+1} T_{j+1,i} \\ + (1 - m_{j-1,j} - m_{j,j+1}) T_{j,i-1} \quad (2.238)$$

where

$$m_{j,j+1} = \frac{\Delta t_i}{V_j c_P [T_{j,i-1}] R_{j,j+1}} \quad (2.239)$$

The foregoing formulation is called an "explicit" solution since temperatures at new time i can be calculated directly (explicitly) given the conditions at time $i-1$. Equations of this nature are subject to numerical instability if the timestep chosen is too large. Thus, Eq. (2.238) yields a negative (unstable) value for $T_{j,i}$ if

$$(m_{j-1,j} + m_{j,j+1} - 1) T_{j,i-1} \geq \frac{q'_j \Delta t_i}{V_j c_P [T_{j,i-1}]} \\ + m_{j-1,j} T_{j-1,i-1} + m_{j,j+1} T_{j+1,i} \quad (2.240)$$

Since the right side of Eq. (2.240) is always positive, we can avoid this condition by assuring ourselves that

$$m_{j-1,j} + m_{j,j+1} - 1 \leq 0 \quad (2.241)$$

Thus, the maximum value of Δt_i can be determined as

$$\Delta t_i \leq \frac{V_j c_P [T_{j,i-1}] (R_{j-1,j}) (R_{j,j+1})}{R_{j-1,j} + R_{j,j+1}} \quad (2.242)$$

The minimum of Δt_i values determined for the several segments is used.

There are two disadvantages to this procedure: First, we are computing the net heat flow into a given annular segment based on temperatures at the previous time. Second, restriction on the size of the time increment may require an excessive amount of computations for problems extending over long time periods. These difficulties can be obviated using the "implicit" form of the difference equation. Here we determine heat flows from temperature differences at the advanced position in time. Thus, Eq. (2.235) becomes

$$V_j c_p [T_{j,i}] \times \frac{(T_{j,i} - T_{j,i-1})}{\Delta t_i} = q'_j + \frac{(T_{j-1,i} - T_{j,i})}{R_{j-}} - \frac{(T_{j,i} - T_{j+1,i})}{R_{j,}} \quad (2.243)$$

Initially, we assume that the specific heat and R have values equal to those at the previous time. We then have a set of linear equations, which can be solved simultaneously for T_j . If properties change significantly with temperature, updated values for the current time are used in an iterative procedure.

While the implicit method removes tight restrictions on the value of Δt_i chosen, use of a large value of Δt_i increases the error due to discretization. This discretization error is of $O[\Delta t_i + (\Delta r)^2]$.

If temperatures change significantly over a time stop, it is more realistic to evaluate the temperature changes using average values for the temperature differences, properties, and rate of heat generation. This formulation, generally designated as the Crank-Nicholson approach, yields

$$V(\bar{c}_p)_j \times \left(\frac{T_{j,c} - T_{j,c-1}}{\Delta t_i} \right) = \tilde{q}'_j + \left[\frac{(T_{j-1,i}) + (T_{j-1,i-1} - T_{j,i-1})}{2\bar{R}_{j-}} \right] - \left[\frac{(T_{j,i} - T_{j+1,i}) + (T_{j,i-1} - T_{j+1,i})}{2\bar{R}_{j,j+1}} \right] \quad (2.244)$$

where

$$(\bar{c}_p)_j = \frac{c_p [T_{j,i-1}] + c_p [T_{j,i}]}{2}$$

$$\tilde{R}_{l,m} = \frac{\left(\frac{k_p}{2} [T_{(l,m)c-1}] + \frac{k_p}{2} [T_{(l,m)i}] \right) (\Delta t_{l,m})}{r_c - r_m}$$

$\tilde{q}'_j$ = average value of q_j over the timestep.

Since conditions at time i are unknown, the set of equations represented by (2.244) is solved initially using values from the previous time step for c_p and R . The equations are then resolved iteratively with updated property values.

The Crank-Nicholson approach is unconditionally stable and has a discretization error of $O[(\Delta t)^2 + (\Delta r)^2]$. It thus has less of an error due to the Δt than the original implicit procedure.

The foregoing finite difference procedures can also be used for calculating cladding temperatures by using cladding properties and setting q'_j to zero at interior nodes and equal to the heat generated by the zirconium-water reaction in the outermost region. For the node point adjacent to fuel,

R_j is defined in terms of gap conductance, while for the node point adjacent to the coolant, R_j is defined in terms of coolant heat transfer coefficient.

Finite difference procedures, similar to the types described in this section, are commonly incorporated in the computer codes used for analyzing core thermal and hydraulic transients (see Sha²¹¹ for overview of such analyses). Power generation versus time is usually input and the program computes fuel, cladding, and coolant temperatures plus heat input to the coolant.

Although the relative simplicity of the finite difference approach has led to its widespread use in nuclear fuel element analysis, other approaches such as the "finite element" method²¹² may be used. Comparisons of the finite element and finite difference methods²¹³ at conditions typical of fuel transients have been made. The results do not show that any clear improvement in accuracy or computing time is gained by use of the finite element technique.

Lassman²¹⁴ has proposed a solution technique based on the use of a quasi-steady-state approximate function:

$$T[r,t] = \frac{q_f''[t]}{4k} r^2 + C_1[t] \ln r + C_2[t] \quad (2.245)$$

where

$$q_f''[t] = q'''[t] - c_p \rho \frac{\partial T}{\partial t} = q'''[i] - c_p \left(\frac{(T[r,i] - T[r,i-1])}{\Delta t} \right)$$

The constants $C_1[t]$ and $C_2[t]$ are replaced in terms of the temperatures at the inner and outer boundaries of the given annular ring. For node j , this results in

$$T_j[r,t] = T_j^0 + \frac{q_f''}{4k_j} [(r_j^0)^2 - r^2] + \frac{\ln(r/r_j^0)}{\ln(r_j^0/r_j^1)} \times \{T_j^0 - T_j^1 + \frac{q_f''}{4k_j} [(r_j^0)^2 - (r_j^1)^2]\}, \quad (2.246)$$

where the superscripts 0 and 1 refer to the outer and inner radii of node j . The nodes are linked by setting the heat flux at the outer edge of node j equal to the heat flux at the inner edge of node $(j+1)$ and setting $T_j^0 = T_{j+1}^1$. The required heat fluxes are obtained by differentiating Eq. (2.246) with respect to r at each node. The resulting equations, representing the heat flux equalities at the node boundaries and the outer boundary, can be rearranged in the form

$$a_j T_{j-1,i}^0 + b_j T_{j,i}^0 + c_j T_{j+1,i}^0 = f(T_{j-1,i-1}^0, T_{j,i-1}^0, T_{j+1,i}^0) \quad (2.247)$$

when a_j , b_j , and c_j are constants for a given radius and set of properties. The simultaneous equations may be solved by Gaussian elimination or other matrix techniques. Alternatively, they may be written as

$$b_j T_{j,i}^0 = f(T_{j-1,i-1}^0, T_{j,i-1}^0, T_{j+1,i}^0) - a_j T_{j-1,i}^0 - c_j T_{j+1,i}^0 \quad (2.248)$$

Each equation can now be solved individually by an iterative process, using values derived from the previous iteration, for the current temperature on the right-hand side of Eq. (2.243). Lassman²¹⁴ concludes that, in view of the necessity for updating the fuel properties as the temperature changes, an iterative solution of the set of equations represented by Eq. (2.243) can be preferable to the repeated use of Gaussian elimination. The rapidity of the iteration scheme depends on the method used for assigning the temperatures on the right hand side of Eq. (2.243). Lassman finds that rapid convergence is obtained when the $(T_{j,i})_{m+1}$, the temperature values used at iteration $(m+1)$ iteration, be computed from:

$$(T_{j,i})_{m+1} = (T_{j,i})_m - \left\{ \frac{[(T_{j,i})_m - (T_{j,i})_{m-1}] [(T_{j,i})_{m^*} - (T_{j,i})_m]}{[(T_{j,i})_{m^*} - (T_{j,i})_m] - [(T_{j,i})_{m-1} - (T_{j,i})_{m-1}]} \right\} \quad (2.249)$$

where $(T_{j,i})_m$ is the value used on the right-hand side in the iteration m , and $(T_{j,i})_{m^*}$ is the value of $(T_{j,i})$ obtained as a solution (left-hand side) in iteration m . The iterative solution scheme indicated here can also be applied to the equations derived from the finite difference method.

All of the foregoing one-dimensional models make the tacit assumption that axial conduction is negligible. This has been found to be correct in nearly all cases. Two notable exceptions are when the cladding segment considered is (1) adjacent to the quench front in the core recovery phase of a loss of coolant accident or (2) is adjacent to the end of a partially inserted control rod.

2.6.2 Cladding Behavior During Transients

2.6.2.1 Cladding Behavior During Rapid Power Transients

The approaches and computer programs used to analyze behavior under normal operating conditions are readily extended to rapid transients. The modifications required to accomplish this generally consist of (1) use of a fission gas release model which incorporates transient effects (e.g., release due to grain boundary sweeping), (2) incorporation of an axial gas mixing model, which includes axial gas flow due to pressure gradients and diffusion, and (3) addition of a transient heat transfer model. Some programs also include a thermal-hydraulic analysis of the subchannel in which the hot fuel rod is located. Alternatively, the fuel rod analysis program may be linked to a separate thermal-hydraulic program, which then provides the necessary coolant boundary conditions. Computer programs designed for analysis of fuel element behavior during transients include COMETHE,²¹⁵ FEMAX,²¹⁶ FRAP-T,²¹⁷ and TRANSURANUS.¹⁹⁸

2.6.2.2 Zircaloy Behavior During a Hypothetical LOCA

While many early transient analyses were concerned with demonstrating that center fuel melting did not occur, more recent studies are primarily

concerned with cladding behavior. It is recognized that most analytical techniques are valid only as long as a coolable geometry is maintained. During a LOCA, cladding temperature can rise to sufficiently high levels so that the hottest rods swell and burst locally with a longitudinal split. Providing the cladding is not too brittle, the cladding remains in one piece and the pellets are restrained. The design limitations established are based on this requirement.

It was previously observed that Zircaloy reacts with steam at high temperatures to form zirconium oxide and hydrogen. The rate at which this reaction occurs was originally described by the well-known Baker-Just relation²¹⁸

$$W^2 = (33.3 \times 10^6 t) \exp(-45,000/RT) = K_p t \quad (2.250)$$

where

W = weight of zirconium oxidized, mg/cm²

T = temperature of unoxidized zirconium, K

t = time, s

R = universal gas constant, 1.987 cal/mol K

K_p = rate constant, (mg)²·cm⁻⁴·s⁻¹

Based on more recent data, Thomas and Chexl²¹⁹ recommend the following revised values for K_p

$$K_p = A \exp(B/T) \quad (2.251)$$

where

$A = 9.32 \times 10^3$	$B = -13,760$	$800 \leq T \leq 1089 \text{ K}$
$A = 9.815 \times 10^6$	$B = -21,340$	$1089 < T < 1353 \text{ K}$
$A = 3.58 \times 10^5$	$B = -16,860$	$1353 < T \leq 1850 \text{ K}$
$A = 33.6 \times 10^6$	$B = -22,900$	$T > 1850 \text{ K}$

The reaction rate may also be related to the distance, δ , that metal-metal oxide interface has moved from its original position by

$$\frac{d\delta}{dt} = K_p \times 10^3 / (2\rho_{\text{Zr}}^2 \delta) \quad (2.252)$$

where δ is the distance the interface has moved (mm) and ρ_{Zr} is the density of zirconium (kg/m³).

The rate constants of Eq. (2.251) describe the behavior when the reaction rate is limited by the rate of oxygen diffusion through the oxide layer. However, when hydrogen concentrations are high, the reaction rate can also be limited by the diffusion of steam across the gas boundary layer to the oxide surface. Under these latter conditions,

$$\frac{d\delta}{dt} = \frac{yM_{\text{zr}}K_g}{2\rho_{\text{zr}}} \quad (2.253)$$

where

K_g = mass transfer coefficient, mol/(m²·s·mole fraction)

M_{zr} = molecular weight of zirconium

y = mole fraction of steam in gaseous phase.

Because of the low steam flow rates in the steam-cooling phase of a LOCA, the mass transfer coefficient, K_g , is obtained using the analogy between heat and mass transfer in laminar flow:

$$K_g = \frac{D_f P'}{RT_g De} \text{Nu} \left(\frac{\text{Sc}}{\text{Pr}} \right)^{1/3} \quad (2.254)$$

where

De = mass transfer surface equivalent diameter, m

D_f = diffusivity of steam in steam-hydrogen mixture (m²/s)

P' = total pressure, atm

T = temperature of gas mixture, K

R = ideal gas constant, atm m³/mol K

Nu = Nusselt number $0.13 (\text{Gr Pr})^{1/3}$

Gr = Grashoff number $g\beta(T_w - T_g)De^3\rho_g^2/\mu_g^2$

Pr = Prandtl number

Sc = Schmidt number

T_g, T_w = temperature of wall and gas mixture, respectively, K

ρ_g = density of gas phase

μ_g = viscosity of gas phase

β = coefficient of volume expansion.

The reaction rate is determined by the minimum ($d\delta/dt$) of Eqs. (2.252) and (2.253).

The zirconium oxide film formed on the surface of the cladding acts as a thermal barrier. At the same time, zirconium metal is consumed, effectively reducing the wall thickness of the cladding. In addition, Zircaloy tends to dissolve its own oxide, producing a cladding with increased oxygen content and decreased ductility.

Earlier we noted that at temperatures above ~ 1040 K, alpha-phase zirconium is transferred to the beta phase. At temperatures where zirconium oxidation takes place (between 1400 and 1700 K) the outer surface is oxidized to ZrO_2 . This is adjacent to a region of alpha-phase zirconium stabilized with a high concentration of oxygen and then a region of mixed

alpha and beta phases inside this. The inner region is likely to be pure beta-phase material. The oxide and alpha-phase material are brittle; ductility of the cladding is primarily due to the beta phase.

The embrittlement of oxidized Zircaloy has been studied by Hobson and Rittenhouse²²⁰ and Hobson.²²¹ Hobson and Rittenhouse²²⁰ initially concluded that the ductility of Zircaloy depended on F_w , the fraction of original wall thickness remaining in the beta phase at operating temperature. However, Hobson²²¹ subsequently concluded that ductility depends not only on F_w , but also on the amount and distribution of oxygen in the beta phase. These latter quantities appear to be a function of the oxidizing temperature with increased oxygen dissolved at higher oxidizing temperatures. In view of these facts, the acceptance criteria for ECCS for water-cooled reactors²²² require that total oxidation of the cladding be limited to a maximum of 17% of the original cladding material and that the maximum cladding temperature during the transient be limited to 2200°F. Therefore, analyses of a hypothetical LOCA must demonstrate that neither of these limits are exceeded.

Metallurgical interaction between Zircaloy and stainless-steel or nickel-based alloys used as spring or spacer grid materials can occur at temperatures in excess of 900°C. The eutectic temperatures for the possible alloy systems²²³ are Zr-Fe, 934°C; Zr-Ni, 961°C; and Zr-Cr, 1300°C. Alloying is apparently inhibited to some degree by crud and oxide layers. Pickman²²³ notes that United Kingdom Atomic Energy Authority experiments show that Zircaloy-2 can braze to stainless-steel grid ferrules at temperatures in excess of 1200°C. The 2200°F (1204°C) temperature limit set by U.S. emergency core cooling acceptance criteria effectively serves to limit possible alloying.

Cladding failure can occur when internal gas pressure causes cladding ballooning and subsequent rupture at local spots. It is of interest to be able to calculate the degree to which such failures are possible under various hypothetical conditions. Several computer code models, such as SST (Ref. 189) and FRAP-T (Refs. 224 and 225) have been designed to perform such calculations. Initial conditions are determined by a steady-state model that simulates the power-time history of the fuel rod to the point at which the transient begins. The computer codes then attempt to estimate fuel and cladding behavior.

In the central region of the fuel rod, the temperature rise due to the substantial zirconium-steam reaction will lead to rapid expansion of the cladding. The general way in which this will be accomplished will be indicated using the relatively simple model in the earlier FRAP-T^{217,224} programs. It is assumed that gas flows from the gas plenum to the expanded region through the fuel-cladding gap. Laminar flow is assumed in the gap. Gas pressures are computed as a function of axial location and time. External and internal pressures are used to determine stresses in the cladding.

As cladding temperature rises, stresses can reach a level sufficient to cause ballooning at some location. The shape of the localized cladding bulge is taken as that of a rotated sine curve.

The equilibrium equation for a thin membrane element in the bulge area can be written as

$$(\sigma_z / r_a) + (\sigma_\theta / r_c) = P / L_c \quad (2.255)$$

where

P = differential pressure

σ_a, σ_θ = axial and circumferential stresses, respectively

axial and circumferential radii of curvature, respectively

L_c = cladding thickness.

Significant deformation is not obtained until axial and radial stresses in the cladding exceed the yield stress. This criterion may be expressed as

$$\sigma_y \frac{(1/r_a) + (1/r_c)}{P / L_c} = f \quad (2.256)$$

When $f < 1$ for a given axial node, the node will deform. Specification of these deformations is based on the following:

1. Nodal deformations are a function of the nodal instability at that node which is the most unstable ($f_m = \text{maximum } f_i$) and will deform the most.
2. The specified displacements must be small enough that adjacent stable nodes are not unrealistically affected.

In FRAP-T5,²²⁴ deformations are specified by the addition of a finite deformation to the nodal deformation calculated during the last timestep:

$$d_i = d_{i0} + dh_i \quad (2.257)$$

where d_i is the new radial coordinate of axial node i , d_{i0} is the old radial coordinate of axial node i , and dh_i is the specified incremental radial displacement of node i . Deformations are computed by

$$dh_i = dh_m \left[\frac{(1 - f_i)^2}{1 - f_m} \right] + 0.1 \quad (2.258)$$

where dh_m is the maximum displacement to be added to any node, f_i is the instability factor at node i , and f_m is the maximum instability factor. A value of dh_m equal to the cladding thickness was found to produce a rapid convergence. Although the deformation is not associated with a particular timestep, a limit on the maximum temperature change over a timestep assures that timesteps remain short during any rapid transient.

The effect of adding an incremented deformation to node i is to decrease the radius of curvature at i . Examination of Eq. (2.256) shows this decrease in curvature to increase the stability function f_i and, thus, the local stability at node i . An additional effect is that an increase in deformation at node i causes an increase in curvature at node $i+1$ and $i-1$. An increase in curvature decreases the stability function and, hence, local cladding stability. The effect, therefore, of locally deforming a weak spot is to strengthen the weak spot but propagate the weakness into the surrounding material, possibly causing additional new instability and further propagation. Careful examination of membrane instabilities on ballooning tubes indicates that initial deformation is quite localized and then proceeds to either rupture or an enlarged stable geometry.

In other approaches to rod ballooning problems, the rapid cladding strain rate in the plastic region is used to calculate the cladding deformation. In the FRETA program,²²⁷ the circumferential stresses are computed from thin-shell theory in which the radius of curvature replaces the cladding radius. The plastic strain $\dot{\epsilon}$, is then obtained from a relationship between stress and strain rate appropriate to the plastic region. In this region, the relationship between stress σ and strain rate $\dot{\epsilon}$ can be expressed as

$$\dot{\epsilon} = 10^{-3} \left(\frac{\sigma}{K \epsilon^n} \right)^{1/m} \quad (2.259)$$

where

σ = stress, Pa

ϵ = strain and $\dot{\epsilon}$ = strain rate, s^{-1}

K = strength coefficient, Pa, $K = \exp(8.755 + 8663/T)$ for $T > 900$ K

n = strain hardening coefficient, $n = 0.0279$ for $T > 850$ K

m = strain rate sensitivity coefficient, $m = -6.47 \times 10^{-2} + 2.203 \times 10^{-4} T$ for $900 < T < 1090$ K and $T > 1255$ K.

A strain rate correction is required for $1090 < T < 1255$ K.

In both the FRETA and early FRAP-T programs, the computed deformations provide a maximum circumferential strain, which is then compared to the allowable burst strain at the given temperature. Tests have shown that the burst or rupture strain depends on the rate at which the cladding is being heated. Powers and Meyer²²⁷ have examined an extensive set of Zircaloy rod burst strain obtained in an aqueous environment and have developed a tabular relationship between rupture temperature and burst strain for slow and rapid temperature ramps. A shortened version of their tabulation is shown in Table 2.IX. The table also shows the percent of the coolant flow area blocked by the rod burst. It may be observed that the rupture strain increases up to about 800°C, which is about where the transition from the α to β Zircaloy phase begins. The strain then drops sharply

TABLE 2.IX
Circumferential Strain at Rupture of Zircaloy Tubing in Hot Aqueous Environment*

Rupture Temperature (°C)	Slow Ramp Correlation (<25°C/s)		Fast Ramp Correlation (>25°C/s)	
	Burst Strain (%)	Flow Blockage (%)	Burst Strain (%)	Flow Blockage (%)
600	10	6.5	10	6.5
650	13	8.4	12	7.5
675	20	13.8	15	10.0
700	45	33.5	20	13.8
725	67	52.5	28	20.0
750	82	65.8	38	27.5
800	90	71.5	57	43.3
850	82	65.8	60	46.0
875	67	52.5	57	43.3
900	43	35.7	45	33.5
950	25	18.0	25	18.0
1000	33	24.1	35	25.7
1050	33	24.1	77	61.6
1075	25	18.0	80	64.5
1100	14	9.2	77	61.6
1125	11	7.0	39	28.5
1150	10	6.5	26	18.3
1200	10	6.5	36	26.2

*Table derived from data of Powers and Meyer (Ref. 227).

from 800 to 950°C. A rise in the burst strain then occurs. A final drop occurs at about 1075°C, which corresponds to the completion of the transition to the β phase.

In later versions of FRAP-T,^{217,228} burst strain has been abandoned as a failure criterion. In the BALON2 deformation subcode,²¹⁷ the true stress is used as the failure criterion. True stress, σ_{tr} , is defined as the force per unit area using the actual cross-sectional area at the time of measurement. This differs from engineering stress, σ_e , where the cross sectional area corresponds to the initial (undeformed) area. The two stresses are related by

$$\sigma_e = \sigma_{tr} / [\exp(\epsilon_{tr})] \quad (2.260)$$

where ϵ_{tr} is the true strain = $\Delta L / L_{act}$, ΔL is the change in length, and L_{act} is the length at instant of change. The deformation model is used to determine the true stress and strain. The true stress so determined is compared to the true ultimate strength, σ_u , which depends on the strain rate and cladding properties at the given temperature. According to Hagrman et al.,³⁴ this is given by

$$\sigma_u = K \left(\frac{\dot{\epsilon}}{10^{-3}} \right)^m \left(\frac{n}{1-m} \right)^n \quad (2.261)$$

where $\dot{\epsilon}$ is the true strain rate (s^{-1}), σ_u is the ultimate strength (Pa), and other quantities are as defined previously [see Eq. (2.259)].

Calculation of the deformation considers spatial variations by dividing the ballooning region into a network of axial and circumferential nodes. The model accounts for circumferential temperature gradients, fast or slow heating of the cladding and the effects of prior irradiation and cold work.

The FRAP-T model considers only a single rod and does not model the effect of neighboring rods. Some programs such as the FRETA²²⁶ and the CANSWEL subcode,²²⁹ consider the effect of adjacent rods in modifying the shape of the ballooning hot rod.

Significant swelling in the region near the spring can exceed the working range of the grid. This can jam the fuel rod against the grid preventing axial motion and inducing fuel rod bowing. The effects of ballooning can, however, be more significant. Interaction between adjacent ballooned rods can effectively block a channel. While such blockage does not have a deleterious effect at high flooding rates (rates in excess of one inch per minute), at lower rates the effect can be deleterious.²³⁰ Current (1994) regulations conservatively require that at these flooding rates (1) cooling calculations are based on the assumption that cladding temperatures are determined by steam cooling alone and (2) that the effects of flow blockage are considered. Hall and Duffy²³¹ suggest a procedure for calculating the effect of cladding ballooning on coolant flow. They estimate the flow in the blocked region by assuming that the flow must redistribute in such a manner that pressure drop across the blocked region must be equal to that of the unblocked region in a parallel channel.

2.6.2.3 *Stainless-Steel Behavior During a LOCA*

Stainless (austenitic) steel cladding will react to form hydrogen and metal oxides at high temperatures just as zirconium does. At temperatures above about 1000°C, the stainless-steel oxidation rate also follows a parabolic rate law when steam diffusion is not limiting.²³² If reaction rates are compared on the basis of the mass of oxygen reacted per unit area per unit time, the stainless steel rate is lower than that of zirconium until approximately 1400 K. However, the rate exceeds that of zirconium at higher temperatures. At about 1600 K, the reaction rate constant [see Eq. (2.250)] defined in terms of $\text{mg}^2 \text{O}_2 / (\text{cm}^4 \text{s})$ is nearly 5.5 times that of zirconium. At 1300 K, the rate constant is only about 35% of that of zirconium.²³²

Stainless steel melts and disintegrates ("froths") at about 2550°F. Because of this behavior, the U.S. NRC sets the maximum cladding temperature allowed during a LOCA at the same level as for Zircaloy (2200°F).

The heat release in the stainless-steel/steam reaction is about a factor of 20 lower than that for the zirconium/steam reaction.²³² This would significantly reduce the rate of cladding temperature rise during a LOCA. In addition the hydrogen release for unit mass of metal reacted is only two-

thirds that of zirconium. Since mechanical properties (strength and ductility) of stainless steel are superior to those of Zircaloy at temperatures below 2200°F, less cladding ballooning and deformation during a LOCA would be expected than with Zircaloy-clad fuel.

2.6.3 Fuel Melting

Although overpower transients due to a rapid reactivity insertion are not now believed to be a significant contributor to severe accident risk in PWRs, such accident scenarios have received considerable attention in the past. Based on experimental observations of actual Zircaloy-clad UO_2 fuel element behavior during rapid transients in test reactors such as TREAT,²³³ it has been concluded that fuel fragmentation occurs when a portion of the fuel becomes molten. The fragmentation is caused by the approximately 10% increase in volume (see Sec. 2.1.1.3) that occurs on UO_2 melting. The expansion, and possible contact of molten fuel with the cladding, results in catastrophic failure of the cladding.

The power burst due to a rapid reactivity insertion is generally completed before any significant amount of energy can be transferred from the fuel to the coolant. Hence, the parameter that primarily determines whether the fuel fails is the total energy input per unit mass of fuel. If this energy input is sufficient to cause fuel melting, failure will occur.

Experimental data from TREAT²³³ and similar tests indicate that catastrophic failure will occur at energy inputs of about 300 cal/gm fuel. This observation is the source of the U.S. regulatory requirement that the maximum fuel enthalpy during an anticipated transient without scram (ATWS) be below 280 cal/g (Ref. 234). This enthalpy limit should provide adequate protection against the loss of coolable geometry due to centerline fuel melting. High power levels can be encountered during an ATWS incident and rapid fuel melting is a possibility.

In the unlikely event of a LOCA in which the emergency core cooling system fails, fuel melting will also occur. Since the reactor is shut down shortly after the accident is initiated, fuel heat-up is primarily due to decay heat. Not only is there a significant time period before melting begins, but the fuel melting proceeds relatively slowly. The determination of the time at which centerline fuel melting produces fuel failure then requires a thermal analysis of the melt progression coupled with a mechanical analysis of cladding behavior. It should be recognized that loss of a coolable geometry due to failure of embrittled cladding may come prior to failure due to centerline fuel melting.

If the original fuel rod geometry can be assumed to be retained for a brief period, the extent of fuel melting can be estimated using the parabolic profile modification of the lumped parameter approach [Eqs. (2.230) and (2.231)]. Once the central fuel temperature, T_{FC} , reaches the melting temperature, T_m , we can assume that the central fuel region will remain at the

melting point while the remainder of the fuel melts. The average temperature of the remaining solid fuel, T_1 , is then simply

$$T_1 = (T_m + T_{FS}) / 2 \quad (2.262)$$

if a parabolic profile holds in the unmelted region. A heat balance, in finite difference form, on the fuel then becomes

$$q'_n = \left(\frac{m_t - m}{m_t} \right) c_1 \frac{\Delta T_1}{\Delta t} + \frac{[H_F(\Delta m)]}{\Delta t} + \left[\frac{(T_m - T_{FS})}{R'_F} \right] + \left(\frac{\Delta m}{m_t} \right) \frac{c_1(T_m - T_1)}{\Delta t} \quad (2.263)$$

where the revised thermal resistance of the solid fuel is given by

$$R'_F = [1 - (r_c / r_1)^2 - 2(r_c / r_1)^2 \ln(r_1 / r_c)] / (8\pi k_i)$$

and

m_t, m total mass and molten mass of fuel per unit length, respectively

Δm = increase in molten fuel mass per unit length during period (Δt)

H_F = heat of fusion (energy/mass)

r_c, r_1 = radius of molten fuel and outer pellet radius, respectively,

with other symbols having the meanings assigned in Sec. 2.6.1.4. Note that, as the melt region advances, both the heat of fusion and heat required to increase the melting mass temperature to T_m [last term of Eq. (2.263)] must be supplied. If we acknowledge that r_c and r_1 change but make the approximation that the effect of the change in density on melting is such that the ratio of the molten fuel radius to the outer solid radius is essentially unaffected by the density change, then

$$(r_c / r_1) = (m / m_t)^{1/2} \quad (2.264)$$

Equations (2.262) through (2.264), together with Eqs. (2.231) and (2.233) can now be solved simultaneously for T_1 , T_2 , T_{FS} , Δm , and (r_c / r_1) if initial conditions are known. The equations are linear if m is taken as having the value at the end of the previous timestep.

Programs computing fuel behavior during a LOCA do not generally consider the extent of fuel melting within the fuel rod. Under most circumstances, the cladding will fail prior to fuel melting. The fuel geometry will then be altered with pellets from failed fuel forming a rubble bed. Programs examining severe accidents do examine the progression of fuel melting in such beds. Under these conditions, the heat balance integral method of

Goodman^{235,236} or the moving boundary approach (see Sec. 6.8.5.5 and Ref. 234) can be used. The problem is greatly complicated by the possible relocation of the molten fuel.

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CHAPTER THREE

Hydrodynamics

The rate of heat removal from a reactor core and the dynamic forces on the core and internals depend strongly on the flow behavior of the system. In this chapter, the basic flow characteristics of friction, turbulence, void distribution, and depressurization are described. Specific applications such as hydrodynamic problems in the reactor vessel and flow instability in a core are then discussed.

The material presented here focuses on water systems and assumes an essentially intact reactor core. The larger class of hydrodynamic problems involved in severe accident analysis is not addressed here. Some discussion of severe accident issues is presented in Chapter 6.

3.1 BASIC FLOW CHARACTERISTICS

3.1.1 Transport Equations for Single-Phase Flow

Under normal reactor operating conditions, the flow along a pipe or channel is highly turbulent. We may then assume that the fluid velocity is essentially constant across the pipe cross section. Under these conditions, the conservation equations for adiabatic flow in the axial (z) direction may be written in differential form as

$$\rho \frac{\partial V}{\partial z} + \frac{\partial \rho}{\partial t} + V \frac{\partial \rho}{\partial z} = 0 \quad (3.1)$$

for mass conservation and

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial z} + \frac{g_c}{\rho} \frac{\partial P}{\partial z} + gF + g \frac{\partial y}{\partial z} = 0 \quad (3.2)$$

for conservation of momentum and

$$\begin{aligned} & \frac{\partial}{\partial t} \left(U + \frac{V^2}{2g_c J} \right) + \frac{\partial}{\partial z} \left(H + \frac{V^2}{2g_c J} \right) V \\ & + \frac{1}{J} \left[\frac{\partial P}{\partial t} \left(\frac{1}{\rho} \right) + \frac{V}{\rho} \left(\frac{\partial P}{\partial z} \right) + \frac{g}{g_c} V F + \frac{g}{g_c} \frac{dy}{dz} \right] \end{aligned} \quad (3.3)$$

for the conservation of energy. The symbols are defined as

F = frictional head loss per unit length, L/L

g = gravitational acceleration, L/t^2

g_c = gravitational conversion factor (1.0 in SI units)

H = enthalpy, energy/mass

J = mechanical equivalent of heat, mechanical energy/thermal energy

P = pressure, force/ L^2

U = internal energy, energy/mass

V = axial velocity, L/t

y = vertical elevation, L

z = axial distance, L

ρ = density, mass/ L^3

During steady state, the time derivatives vanish. For isothermal flow, H is constant and the energy equation becomes equivalent to the momentum equation. The resulting steady-state energy/momentum conservation equation may be rewritten as:

$$-\frac{\partial P}{\partial z} = \underbrace{\frac{g}{g_c} \rho F}_{\text{friction}} + \underbrace{\frac{g}{g_c} \rho \frac{dy}{dz}}_{\text{elevation}} + \underbrace{\frac{\rho}{g_c} V \frac{\partial V}{\partial z}}_{\text{acceleration}} \quad (3.4)$$

The total steady-state pressure drop is thus the sum of the friction, elevation and acceleration terms. If we integrate around a closed flow loop, we may write the total pressure drop, ΔP_{total} , as the sum of integrals of the frictional acceleration and elevation terms.

$$\Delta P_{\text{total}} = \Delta P_{\text{friction}} + \Delta P_{\text{acceleration}} + \Delta P_{\text{elev}} \quad (3.5)$$

However, in single-phase flow, the integrals of the acceleration and elevation changes around a loop are nearly zero (exactly zero under isothermal

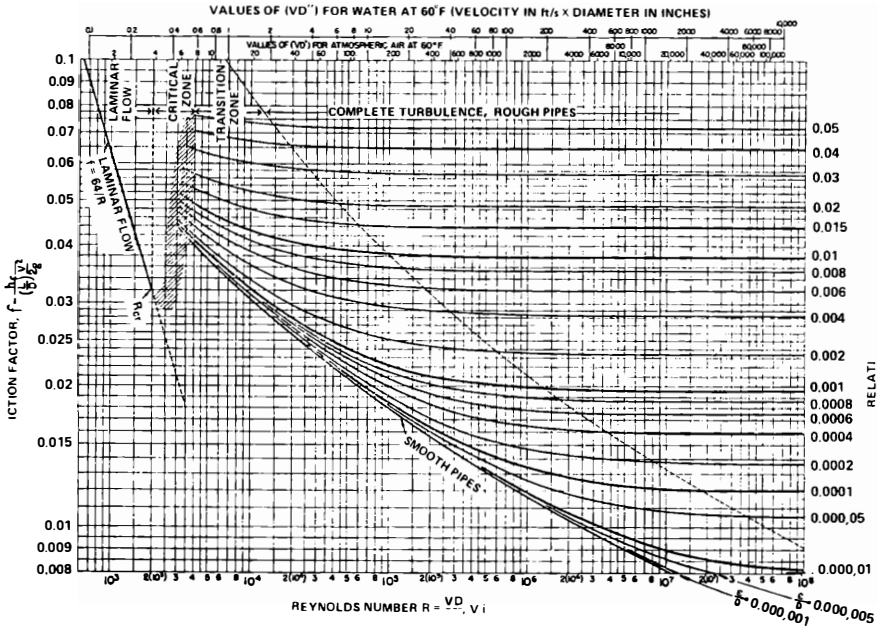


Fig. 3.1 Moody friction factor chart [from *Trans. ASME*, 671 (1944)].

conditions). Determination of the total pressure loss is then reduced to the determination of the frictional pressure loss. The pressure loss must, of course, be matched by the pressure rise across the pump. Hence, flow through the loop is fixed at the intersection of the head-flow curve of the pump and the flow-head loss curve of the loop.

3.1.2 Single-Phase Flow Friction

Single-phase flow pressure drop is generally calculated in accordance with

$$\Delta P_{\text{friction}} = \left(\frac{fL}{De} \right) \frac{\rho V^2}{2g_c} + \sum \left(K_i \frac{\rho V_i^2}{2g_c} \right) \quad (3.6)$$

where

f = skin friction factor due to wall shear in a straight flow channel. Empirical values for f can be obtained from the Moody curve¹ in Fig. 3.1.

K_i = form friction factor due to the i 'th change in the cross section or restriction in the flow channel. The value of K for sudden expansion K_e and for sudden contraction K_c are given by Kays and London.² Their values at high Reynolds numbers are reproduced in Fig. 3.2.

V = fluid velocity through the channel

V_i = fluid velocity in the smaller channel at area change i

De = equivalent diameter of the straight flow channel, $4 \times$ cross-sectional area/perimeter.

Analytical expressions have been developed to describe the friction characteristics of various flow regimes. For circular channels, the laminar friction factor can be calculated

$$f = 64/\text{Re} \quad (3.7)$$

where Re is the Reynolds number. The turbulent friction factor in a smooth channel can be calculated from the approximate relationship

$$f = C\text{Re}^{-} \quad (3.8)$$

where C equals 0.184 for isothermal flow in commercially available smooth tubes. The effect of surface roughness is indicated in Fig. 3.1 in terms of the ratio of the height of the surface projections, ϵ , to pipe diameter, D . For commercial steel pipe, ϵ can be taken as ~ 0.00015 ft.

3.1.2.1 Bends, Valves, Fittings, and Restrictions

Pressure losses across bends, valves, and fittings are expressed in either terms of a loss coefficient K_l , or a length of pipe, L_e , having an equivalent pressure drop. Thus, pressure loss ΔP_l across a given component can be calculated by

$$\Delta P_l = K_l \frac{\rho V^2}{2g_c} = \left(\frac{f L_e}{De} \right) \frac{\rho V^2}{2g_c} \quad (3.9)$$

The reader is referred to Refs. 3, 4, and 5 for tabulations of K_l and (L_e/D_e) for commonly encountered components.

Pressure loss across a long restriction in a flow path can be estimated by adding frictional loss to expansion and contraction losses estimate via the K_i values of Fig. 3.2. However, when the restriction is so short (e.g., a perforated plate) that the vena contracta occurs beyond the restriction, then the pressure drop should be obtained from

$$\Delta P_l = \frac{G^2}{2g_c \sigma^2 \rho C^2} (1 - 2\sigma C + \sigma^2 C^2) \quad (3.10)$$

where G = mass velocity upstream of restriction, C = vena contracta area ratio, and σ = ratio of restricted flow area to full flow area.

Values of C , the vena contracta area ratio, and the length to pipe diameter ratio, L/D , at which the vena contracta is located are indicated in Table 3.I.

Observe in Table 3.I that, for $\text{Re} < \infty$, the values of K_c and K_e given in Fig. 3.2 do not approach 1.0 as the area ratio approaches unity. Todreas

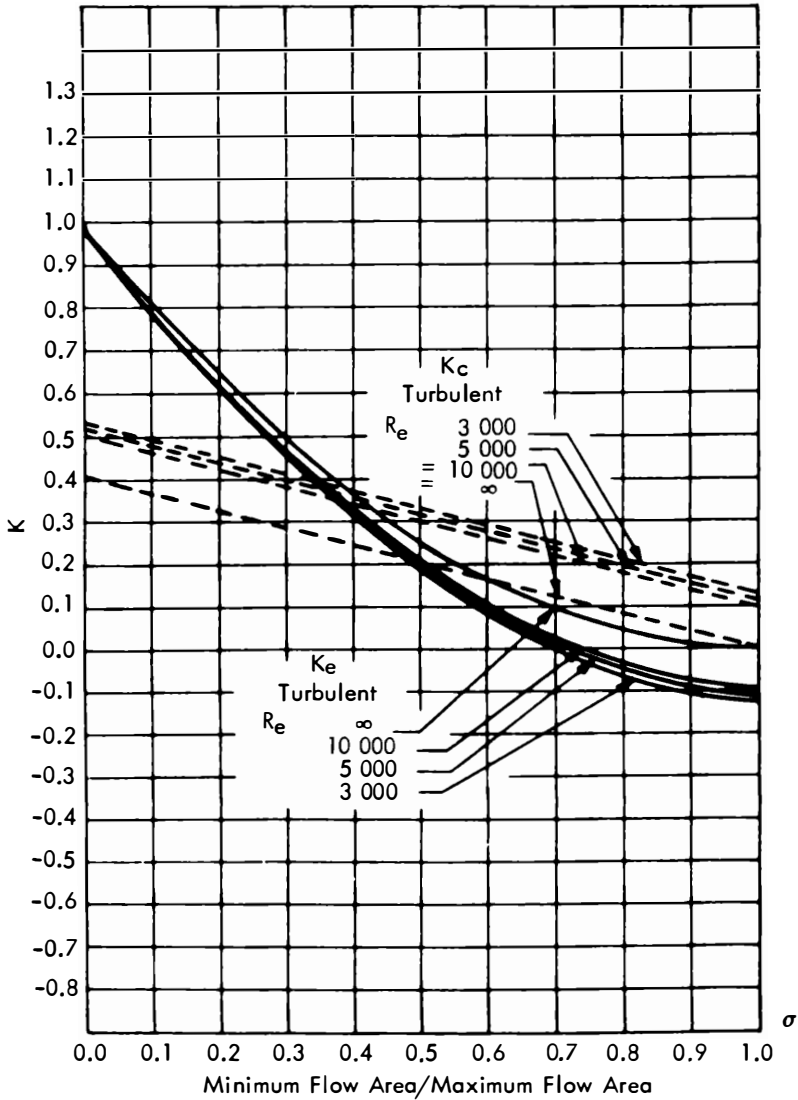


Fig. 3.2 Form friction factors for sudden expansions and contractions [from W.M. Kays and A. L. London, *Compact Heat Exchangers*, The National Press, Palo Alto, CA (1955)].

and Kazimi⁶ point out that this arises because the velocity distributions assumed in the computations are not identical on both sides of the area change. This is the case for entrance and exit area between a large flow area and a restricted flow area (e.g., inlet and exit of reactor fuel assemblies). However, for less drastic geometry changes, the flow distributions will be similar. For large values of σ , the curve for $Re = \infty$ is likely to be a closer approximation than the curve shown for the given Reynolds number.

TABLE 3.1
Vena Contracta Properties

$\sigma = \left(\frac{\text{Restriction Area}}{\text{Flow Area}} \right)$	C	Downstream Location (L/D)
0	0.617	0.65
0.3	0.643	0.55
0.6	0.712	0.3
0.8	0.813	0.25

The loss coefficients of Fig. 3.2 and Eq. (3.10) are for sharp or sudden area changes and the loss coefficients for more gradual area changes are significantly reduced (see Ref. 4). The loss coefficient for a sudden contraction is also reduced by simply rounding or chamfering the entrance. By rounding the entrance with a radius equal to 1/10 the diameter of the smaller pipe, the loss coefficient is reduced to 1/3 of that given in Fig. 3.2. A 45-deg chamfer at the edge of the entrance to the smaller line reduces the loss coefficient to 1/6 of that for a sharp entrance. Rounding or chamfering does not significantly affect enlargement loss coefficients.

3.1.2.2 Effect of Heating

In a channel where one wall is heated or cooled, the friction factor is changed due to the change of viscosity near the wall. For turbulent flow, Sieder and Tate⁷ suggested

$$f/f_{\text{ISO}} = \left(\frac{\mu_{\text{wall}}}{\mu_{\text{bulk}}} \right)^{0.14} \quad (3.11)$$

where f_{ISO} is the friction factor measured under isothermal conditions, and μ is fluid viscosity. For water flowing at 2000 psia, the effect has been correlated by Esselman *et al.*,⁸ LeTourneau *et al.*,⁹ and Mendler *et al.*¹⁰ for $\text{Re} > 3000$ as

$$f/f_{\text{ISO}} = 1 - 0.0019 \Delta T_f \quad (3.12)$$

where

$$\Delta T_f = q''/h$$

$$h = 0.023 (k_b/De)(\text{Re}_b)^{0.8}(\text{Pr}_b)^{0.4}$$

k_b = thermal conductivity of fluid at the bulk temperature

Pr_b , Re_b = Prandtl and Reynolds numbers, respectively, at bulk temperature.

A limiting value of 0.85 has been given for Eq. (3.12) by its authors.

Rohsenow and Clark¹¹ examined the Sieder and Tate⁷ relationship using water flowing at pressures of 1500 to 2000 psi. They found the exponent

for Eq. (3.11) was 0.60 for the frictional pressure loss alone, but 0.14 for the pressure loss due to both momentum and frictional losses. They noted that many of the data points gave values for f/f_{ISO} below 0.85. The relationship

$$\frac{f_{\text{friction}}}{f_{\text{ISO-friction}}} = \left(\frac{\mu_{\text{wall}}}{\mu_{\text{bulk}}} \right)^{0.6} \quad (3.13)$$

is recommended.

In laminar upflow, heating leads to an increase in the friction factor when the flow is sufficiently low to make buoyancy effects significant. As the power to flow ratio is increased, frictional losses are increased. For fully developed laminar flow, Bishop *et al.*¹² propose the empirical correlation

$$\left(\frac{f}{16} \right) \text{Re} = \left[1 + 2.97 \left(\frac{\text{Gr}/\text{Re}}{1000} \right) \right]^{0.655} = F' \quad (3.14a)$$

where

Gr = Grashof number = $\rho^2 \beta g D^4 q'' / (k \mu^2)$

q'' = heat flux, energy/area

k = thermal conductivity, energy/time length T

μ = viscosity of liquid, mass/length time

β = coefficient of volume expansion = $-(d\rho/dT) 1/\rho$

T = temperature

and other symbols have their previous meaning. For developing flow, Bishop *et al.* indicate that

$$\left(\frac{f}{16} \right) \text{Re} = 1 + \left(\frac{F' - 1}{1 + 0.36 Z^{-3}} \right) \quad (3.14b)$$

where $Z \leq (L/D) / 50$. Iannello *et al.*¹³ suggest a somewhat more complex approach that can also allow for downflow.

3.1.2.3 Core Pressure Drop

The reactor cores of almost all pressurized water reactors consist of an array of rod bundles. Hence, flow characteristics within the core are those of the bundles of which it is composed. Core pressure drop can be considered to be composed of three components:

1. pressure loss along the bundle of bare rods
2. pressure loss across the spacer grids or wires
3. pressure loss at core entrance and exit.

Entrance and exit losses can be evaluated as those due to sudden expansions and contractions, as previously described.

The pressure loss along bare rods is often predicted by assuming similarity to flow inside a tube and by using the equivalent diameter concept. In laminar flow, a somewhat better prediction can be obtained by using the so-called "equivalent annulus" approach. In this approach, the equivalent annulus flow area is assumed to be the fluid-dynamic equivalent of the section of annulus extending from the inner radius to the radius of maximum velocity, r_m . The ratio of r_m to the rod radius, is obtained from:

$$(r_m/r_o) = \left(\frac{2\sqrt{3}}{\pi} \right)^{1/2} (p/D) \quad (3.15a)$$

for triangular arrays and

$$(r_m/r_o) = \left(\frac{4}{\pi} \right)^{1/2} (p/D) \quad (3.15b)$$

for square arrays with (p/D) indicating the ratio of rod pitch to diameter. Todreas and Kazimi¹⁴ show that the product of the Reynolds number and friction factor, f , may then be obtained from

$$f \cdot \text{Re} = \frac{64(R^2 - 1)^3}{-3R^4 + 4R^4 \ln R - 1 + 4R^2} \quad (3.16)$$

where $R = r_m/r_o$ and the Reynolds number, Re , is based on the equivalent diameter of the flow passage.

In turbulent flow,* the assumption of similarity to flow in a tube using the equivalent diameter is also commonly applied. However, there is some evidence that the effect of the pitch-to-diameter ratio (p/D) is not fully described by the equivalent diameter approach. Deissler and Taylor¹⁵ conducted an early investigation of the validity of this approach for predicting rod bundle pressure drop. By an iterative procedure, they plotted the velocity profile in square and triangular lattices as shown in Figs. 3.3 and 3.4, respectively. These velocity profiles were calculated from a generalized velocity profile¹⁶ based on Deissler¹⁷ and Laufer's¹⁸ data. From these velocity profiles, friction factors and Reynolds numbers can be calculated by integrating the profiles to obtain bulk (or average) velocities, since both friction factors and Reynolds numbers are based on bulk velocities. Predicted friction factors for triangular and square arrays are shown in Figs. 3.5 and 3.6. Deissler and Taylor's¹⁵ predictions are in the vicinity of the line based on

*Studies by Haldar, Mohanty, and Gupta [*Heat & Mass Transfer 94—Proceedings of First ISHMT-ASME Heat and Mass Transfer Conf*, ISHMT, Bombay, India (1994)] indicate that there is a gradual transition from laminar to turbulent flow in rod bundles. They find the flow to be fully laminar at $\text{Re} \approx 800$ and fully turbulent at $\text{Re} \approx 10^4$. In the transition region, $f = C_1 / \text{Re} + C_2$ with C_1 and C_2 determined by the fully laminar and turbulent values at the endpoints of the transition curve.

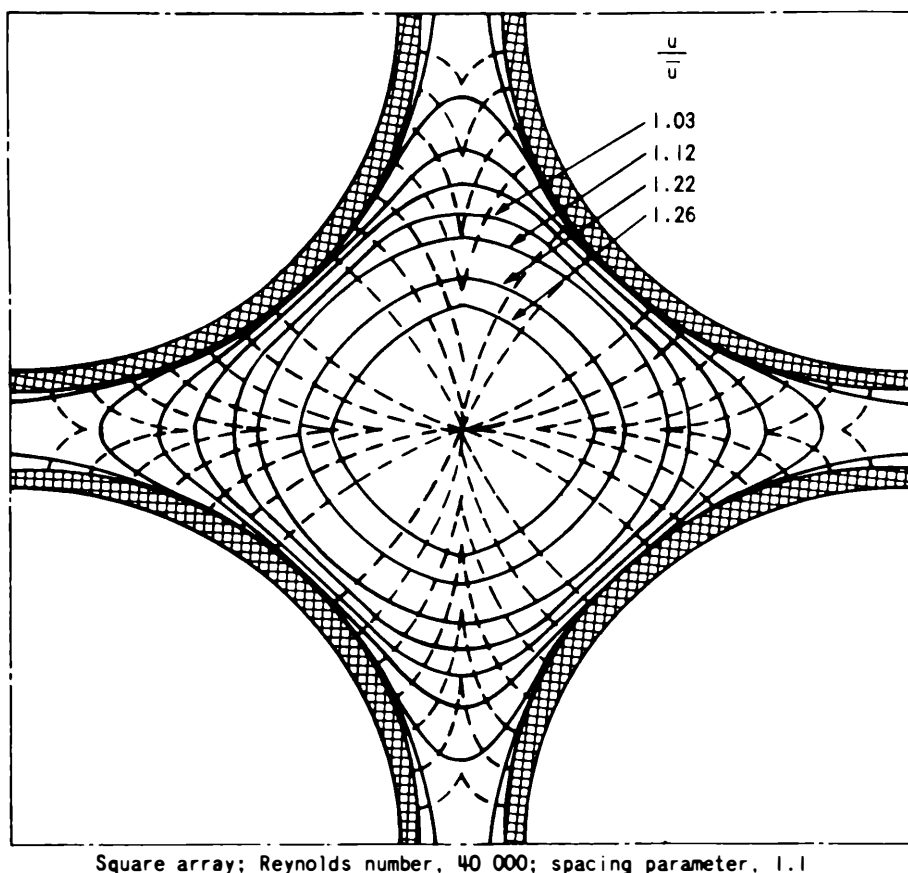


Fig. 3.3 Predicted velocity distribution between tubes (from Ref. 15).

circular tube data. However, there is an effect of p/D that would not be observed if the De concept strictly held. The experimental data shown have too much scatter to indicate any trend.

A number of other investigators have considered the effect of the p/D ratio: Miller *et al.*¹⁹ tested a 37-rod bundle of 3-ft heated length with 0.625-in.-o.d. rods in a triangular lattice of 1.46 p/D ratio and found the friction factor represented by

$$f = 0.296 \text{ Re}^{-1} \quad (3.17)$$

which is 60% higher than that obtained by fitting the Moody curve.

Le Tourneau *et al.*⁹ tested rod bundles of square lattice with p/D ratios of 1.12 and 1.20 and of triangular lattice with a p/D ratio of 1.12. These data

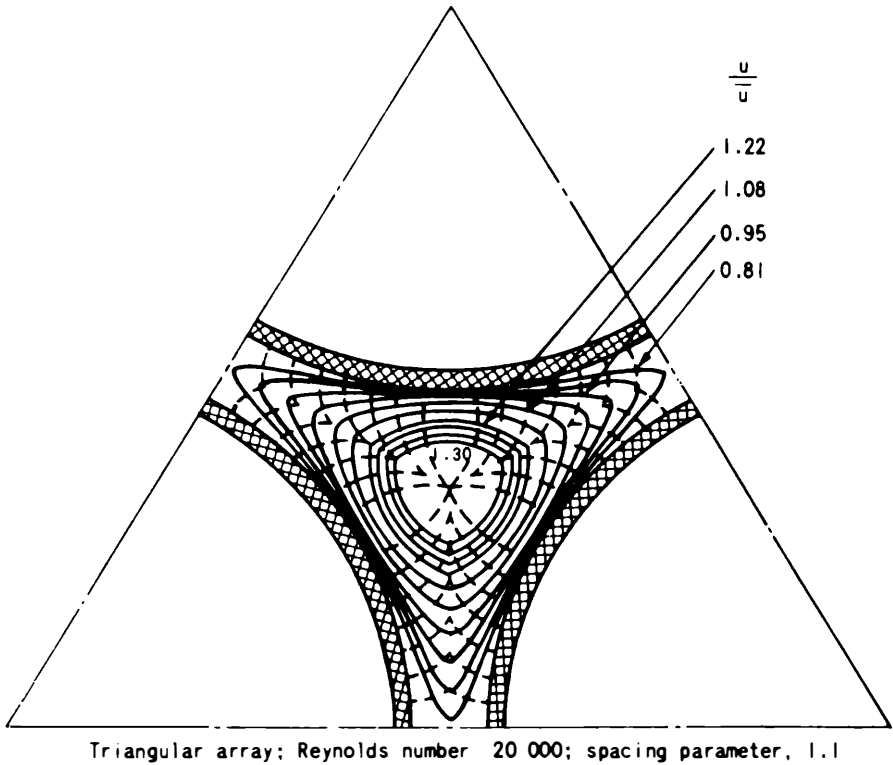


Fig. 3.4 Predicted velocity distribution between tubes (from Ref. 15).

fall within a band between the Moody curve (Fig. 3.1) for smooth tubes and a curve 10% below that in a Reynolds number range 3×10^3 to 10^5

Wantland²⁰ tested a 100-rod bundle of square lattice of $3/16$ -in. o.d. and 6-ft length with a p/D of 1.106 and 102-rod bundle of triangular lattice of $3/16$ -in. rod o.d. and 6-ft length with a p/D ratio of 1.190. For Prandtl numbers between 3 and 6, he obtained

1. For square lattices at $Re = 10^3 - 10^4$ and $Pr = 3 - 6$,

$$f = 1.76 Re^{-1} \quad (3.18)$$

which is $\sim 30\%$ higher than Eq. (3.8).

2. For triangular lattices at $Re = 2 \times 10^3 - 1 \times 10^4$,

$$f = 90.0 Re^{-1} + 0.0082 \quad (3.19)$$

which is $\sim 25\%$ higher than Eq. (3.8).

Dingee and Chastain²¹ tested 9-rod bundles in the water of 0.5-in. rod o.d., 24-in.-long square and triangular lattices with a p/D of 1.12, 1.20, and

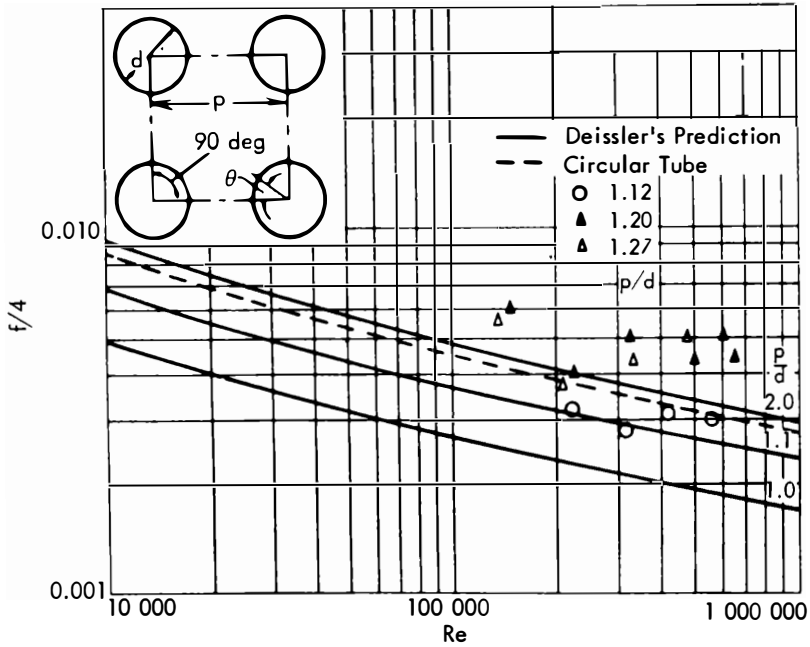


Fig. 3.5 Comparison of predicted and measured friction factors for square lattices (from Ref. 15).

1.27 Square lattice data are 5 to 20% higher than Deissler and Taylor's theoretical prediction.²² The triangular lattice data agree with the same prediction, but with a scatter of about $\pm 15\%$.

Subsequently, Trupp and Azad²³ measured velocity distributions, eddy diffusivities, and friction factors with air flow in triangular array bundles. These data indicated friction factors somewhat higher than Deissler and Taylor's predictions.¹⁵ For Reynolds numbers between 10^4 and 10^5 , their data at $p/D=1.2$ were $\sim 17\%$ higher than circular tube data. The data at $p/D=1.5$ were $\sim 27\%$ higher than circular tube data. Trupp and Azad²³ correlated their triangular array data by

$$f = C \text{Re}^{-n} \quad (3.20)$$

where

$$\left. \begin{aligned} C &= 0.287[(2\sqrt{3}/\pi)(p/D)^2 - 1.30] \\ n &= 0.368(p/D)^{-1.358} \end{aligned} \right\} \begin{aligned} 10^4 &\leq \text{Re} \leq 10^5 \\ 1.2 &\leq p/D \leq 1.5 \end{aligned}$$

The more recent estimates of p/D effects on rod bundle friction in turbulent flow have tended to rely on computations based on velocity and

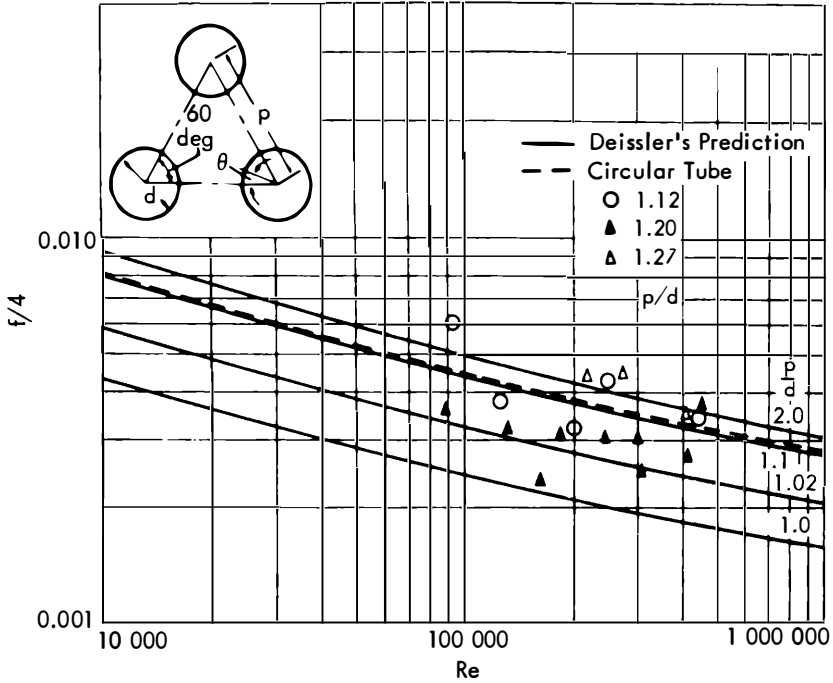


Fig. 3.6 Comparison of predicted and measured friction factors for triangular lattices (from Ref. 15).

eddy diffusivity profiles. Rehme²⁴ developed an approach for calculating rod-bundle friction factors based on a turbulent geometry factor that depended on $(f \cdot Re)$ for laminar flow in an equivalent annulus. He found that his predictions fit a variety of experimental data for rod bundles and non-circular channels. Cheng and Todreas²⁵ and Todreas and Kazimi¹⁴ have used Rehme's method to calculate rod-bundle friction factors as a function of p/D for interior channels and channels adjacent to a wall. They expressed their turbulent flow results as

$$f = C' Re^{-0.18} \quad (3.21)$$

where Re is based on the De for the channel under consideration. Values of C' are tabulated in Table 3.II for several geometries and p/D values. Computation of f via Eq. (3.21) and Table 3.II yields values which indicate the same general trend as shown in Figs. 3.5 and 3.6. The predictions of Eq. (3.21) are in reasonable agreement with Figs. 3.5 and 3.6 at low Reynolds numbers (e.g., $Re = 20 \times 10^3$) but they tend to lie above the values of the figures at high Reynolds numbers (e.g., $Re > 100 \times 10^3$).

TABLE 3.II
Values of Rod Bundle Friction Factor Coefficient as a Function of Geometry

Rod Arrangement	Channel Geometry	C'			
		$p/D = 1.05$	$p/D = 1.1$	$p/D = 1.3$	$p/D = 1.5$
Square array	Interior	0.120	0.140	0.152	0.154
	Edge	0.129	0.147	0.152	0.153
	Corner	0.138	0.147	0.150	0.150
Triangular array	Interior	0.142	0.147	0.154	0.156
	Edge	0.129	0.148	0.152	0.153
	Corner	0.152	0.144	0.151	0.151

Pressure losses across spacer grids or wires are from drag-type pressure losses. These are often calculated using pressure-loss coefficients for sudden contractions and expansions. Berringer and Bishop²⁶ tested the Yankee fuel rod bundle with both cylindrical ferrule spacers and egg-crate strap grids. The test section bundle was composed of 36 rods of 0.335-in. o.d. in a square lattice with a p/D of 1.27. The ferrule is 0.5 in. long with a 0.265-in. o.d. and a 20-mil wall thickness. The strap is 1 $\frac{1}{8}$ in. high and $\frac{1}{32}$ in. thick.

Comparison of the ferrule- and strap-pressure drops indicates that pressure drop across a level of straps was more than four times that across a full-ferruled section. Prediction of the pressure drop across the ferrule or strap using the loss coefficients for sudden contraction and expansion agrees with the data within 17%. Friction factors for the bare rod section are ~15% higher than the Moody curve in a pipe in the Reynolds number range of 10,000 to 30,000.

De Stordeur²⁷ measured the pressure drop characteristics of a variety of spacers and grids. He correlated his results in terms of a drag coefficient, C_s . Pressure drop ΔP_s across the grid or spacer was given by

$$\Delta P_s = \rho C_s V_s^2 s / 2 g_c A \quad (3.22)$$

where A = unrestricted flow area away from the grid or spacer, V_s = velocity in the spacer region, and s = projected frontal area of the spacer. The drag coefficient used was a function of the Reynolds number for a given spacer or grid type. At high Reynolds number ($Re \approx 10^5$) strap-type grids showed drag coefficients of ~1.65. The pressure drop across a crossed circular wire grid was ~10% lower.

On the basis of tests of several grids, Rehme²⁸ concluded that the effect of the ratio (s/A) is more pronounced than indicated by De Stordeur.²⁷ Rehme concludes that grid pressure drop data are better correlated by

$$\Delta P_s = C_V (\rho V_B^2 / 2 g_c) (s/A)^2 \quad (3.23)$$

where C_V = modified drag coefficient and V_B = average bundle fluid velocity. Drag coefficient C_V is a function of the average bundle Reynolds num-

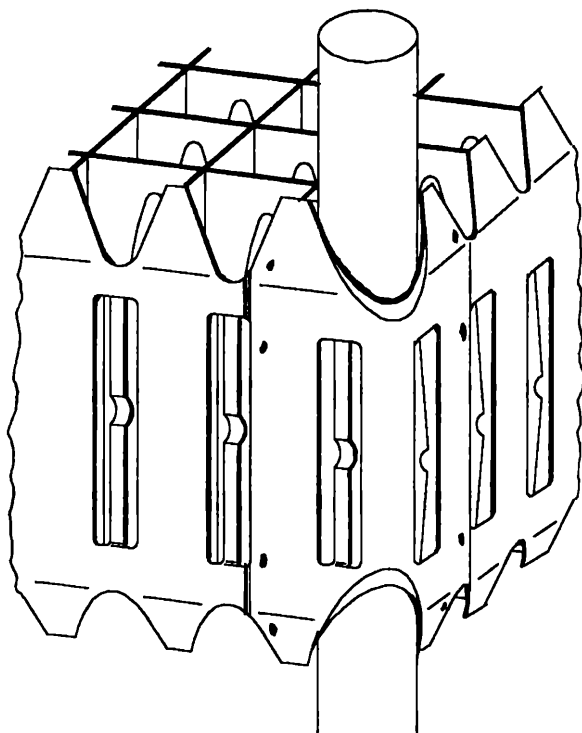


Fig. 3.7 Sketch of typical spacer grid (no mixing vanes) [from Kim *et al.*, *Nucl. Technol.*, **98**, 349 (1993)].

ber. Rehme's data indicate that for square arrays, $C_V = 9.5$ at $Re = 10^4$ and $C_V = 6.5$ at $Re = 10^5$.

Published information on the pressure drop across mixing vane grids is scarce. Yang *et al.*²⁹ report pressure drops across several mixing vane grids in terms of a K factor [see Eq. (3.6)]. Their values of K varied from approximately 0.4 to 0.8. Unfortunately, the exact geometries of the grids were not specified. A rough estimate of the pressure drop across such grids may be obtained by using Eq. (3.23) and incorporating the projected area of the mixing vane into the value of s . Figure 3.7 shows a sketch of a typical spacer grid for a square lattice.

Although mixing vane grids provide the benefits of substantial flow mixing, they also substantially increase the pressure drop and hence the upward hydraulic drag on the fuel assemblies. An appropriate design must ensure that the hydraulic forces do not lift the fuel assemblies. Where the assembly weight may be insufficient to provide the hold-down required, the mechanical arrangement at the upper core plate must provide the additional hold-down needed.

Rehme²⁸ also correlated a wide selection of data on the total pressure losses in wire-wrapped bundles. These pressure losses depend on the p/D ratio, helical spacer wire pitch H' and the number of rods. Rehme correlated rod-bundle pressure loss data in terms of a modified form of the friction factor, f' . Pressure loss is given by

$$\Delta P = f' \left(\frac{S_b}{S_t} \right) \frac{\rho V_{\text{eff}}^2 L}{2 g_c D e} \quad (3.24)$$

for $4 \times 10^3 \leq \text{Re} \leq 5 \times 10^5$ where

S_b / S_t = ratio of the wetted perimeter of rods and wires to the total wetted perimeter

V_{eff} = effective velocity accounting for swirl flow due to wire

$$\text{wraps} = (V_B) \{ (p/D)^{6.5} + [7.6(d_m / H') (p/D)^2]^{2.16} \}^{1/2}$$

d_m = mean diameter of wire wraps

f' = modified friction factor = $(64 / N'_{\text{Re}}) + 0.0816 / (N'_{\text{Re}})^{0.133}$

N'_{Re} = modified Reynolds number = $\rho V_{\text{eff}} D e / \mu$.

The total pressure drop across a core or reactor vessel is often expressed as

$$\begin{aligned} \Delta P_{\text{core}} &= K_{\text{core}} \frac{\rho V^2}{2 g_c} \\ \Delta P_{\text{vessel}} &= K_{\text{vessel}} \frac{\rho V^2}{2 g_c} \end{aligned} \quad (3.25)$$

where ρ is average density in the core and V is average flow velocity inside the core. For example, K values for the former Yankee Rowe reactor are

$$K_{\text{core}} = 15, \quad K_{\text{vessel}} = 34$$

and K values for the SELNI reactor are

$$K_{\text{core}} = 18, \quad K_{\text{vessel}} = 52$$

Lateral flow (normal to the fuel rod axis) occurs in a core where there are significant pressure differences between assemblies. In the lateral direction, the core appears as a large tube bank.

The pressure drop across bare tube bundles (rod axis perpendicular to flow) is generally computed by an equation of the form:

$$\Delta P = \frac{f' N G_{\text{max}}^2}{2 g_c \rho} \left(\frac{\mu_w}{\mu_b} \right)^{0.14} \quad (3.26)$$

where

G_{max} = mass flux based on minimum flow area

N = number of tube rows in flow direction

$f' =$ empirical friction factor

$\mu_b, \mu_w =$ viscosity at bulk and wall temperature, respectively.

The value of f' depends on the tube arrangement and the Reynolds number. Accurate values of f' were developed by Zukauskas³⁰ for a wide range of Reynolds numbers and are tabulated in heat exchanger design references (e.g., Ref. 31). However, approximate values of f' for the turbulent region may be obtained from³²

$$f' = \left\{ 0.25 + \frac{-0.118}{[(p-D)/D]^{1.08}} \right\} \text{Re}_{\max}^{-0.16} \quad (3.27)$$

for triangular arrays and

$$f' = \left\{ 0.044 + \frac{0.08(p/D)}{[(p-D)/D]^{0.43 + 1.13(D/p)}} \right\} \text{Re}_{\max}^{-0.15} \quad (3.28)$$

for square arrays. In the foregoing,

$$\text{Re}_{\max} = DG_{\max} / \mu_b$$

3.1.3 Simple Flow Transients

Fluid behavior during flow transients is governed by the basic conservation equations containing the time derivatives [Eqs. (3.1), (3.2) and (3.3)]. For incompressible single-phase flow under isothermal conditions, the density is constant and we need to consider only the momentum conservation equation. As a simple example, consider the loss-of-flow transient in a single closed horizontal loop of constant cross section. With the foregoing assumptions, Eq. (3.2) becomes

$$\frac{dV}{dt} = -\frac{g_c}{\rho} \frac{\partial P}{\partial z} - gF \quad (3.29)$$

Since $\partial P / \partial z$ is constant throughout the loop, we may equate it to the negative of the pressure rise across the pump, $(\Delta P)_p$, divided by the total length of the loop, L . We may also replace F in terms of the total frictional loss around the loop. We then have

$$\frac{dV}{dt} = \frac{g_c}{\rho} \frac{(\Delta P)_p}{L} - \frac{C_f}{2} \frac{V^2}{L} \quad (3.30)$$

where C_f is the total frictional loss coefficient for loop (assumed to be independent of flow rate). By making the very conservative assumption that the pump inertia is negligible and therefore the pressure change due to the pump is negligible during a loss-of-flow transient,

$$\frac{dV}{dt} = -\frac{C_f V^2}{2L} \quad (3.31)$$

Upon integration, we obtain

$$V = \frac{V_0}{(C_f / 2L) V_0 t + 1} \quad (3.32a)$$

where V_0 is the loop flow at time zero. This is sometimes rewritten in terms of the loop half time, t_l

$$V = \frac{V_0}{1 + t/t_l} \quad (3.32b)$$

where $t_l = 2L / C_f V_0$, which is the time for loop flow to decrease by factor of 2 in the absence of pump inertia.

If, at the opposite extreme, the pump inertia is very much larger than the loop inertia, then the flow during pump coastdown would be given in terms of the t_p , the pump half time³³

$$V = \frac{V_0}{1 + t/t_p} \quad (3.33)$$

where t_p is the time required for pump speed to decrease by a factor of 2 during coastdown.

The assumption of negligible fluid inertia means $t_p > t_l$ and hence a much longer coastdown would be observed. The coastdown flow under realistic conditions will lie between the values obtained from Eqs. (3.32) and (3.33) and depend on the ratio of (t_l / t_p) . A large pump inertia can be obtained by the addition of a flywheel. The increased inertia appreciably extends the coastdown period. An analysis of the loss-of-flow accident under realistic conditions is presented in Sec. 6.3.1.

Flow transients may also be caused by the sudden opening or closing of a valve or a rupture of a piping line. In such a case, pressure pulses are propagated through the system at sonic speeds.³⁴ This situation, commonly referred to as waterhammer, is discussed in Sec. 3.3.5.6.

3.1.4 Pump Behavior

PWR main coolant pumps are all centrifugal pumps. Such pumps have a drooping head flow characteristic as shown in Fig. 3.8. Note that the head versus flow curve is independent of the density of the fluid being pumped. As previously observed, the intersection of the pump performance curve with the flow rate-pressure loss curve of the reactor system (see Fig. 3.8) determines the operating point of the system. In a reactor with n symmetric loops, the system curve would be obtained considering the flow through

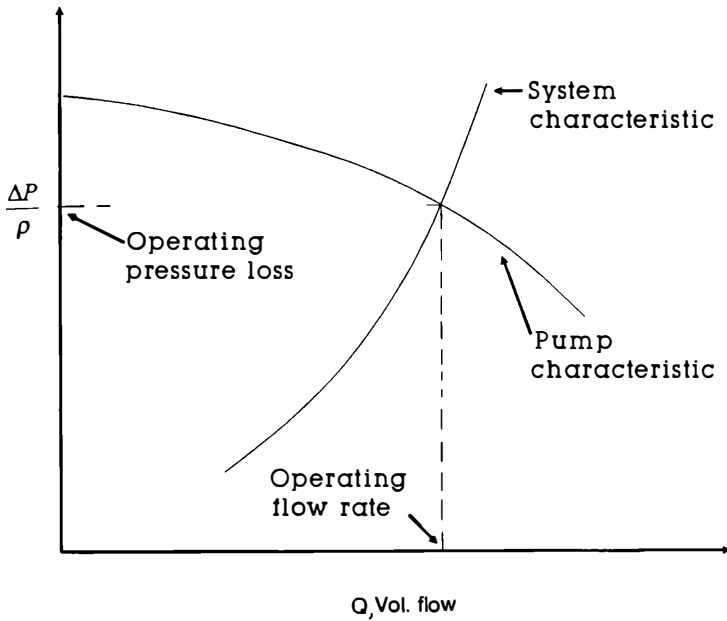


Fig. 3.8 Pump and system head loss curves.

one piping loop proceeding through an $(1/n)$ segment of the core and reactor vessel.

The total hydraulic pumping power, P'_h required is given by

$$P'_h = Q(\Delta P_s) = W/\rho (\Delta P_s) \quad (3.34)$$

where Q = total volumetric flow at operating point (vol/time), W = total mass flow rate (mass/time), and ΔP_s = system pressure drop (force/area). The actual power required is the hydraulic power divided by the pump efficiency. Since the power required increases as the density decreases, it is obvious why main coolant pumps are always located in the cold leg.

The pressure at the entrance to the impeller of a centrifugal pump is appreciably below that at the pump inlet. If this low pressure drops below the vapor pressure of the fluid at the temperature at which it is being pumped, vapor bubbles are produced and the pump is said to cavitate. Cavitation can lead to pump damage and must be prevented. Therefore, all pumps must operate with a net positive suction head (NPSH) as specified by the manufacturer. The NPSH corresponds to the (P/ρ) value at the suction nozzle less the lowest (P/ρ) that exists in the pump. The NPSH requirement is another reason for placing main coolant pumps in the cold leg.

The head versus flow curve is valid only for the pump speed at which it was determined. Thus, it could not be used during a flow coastdown. Further, during a loss-of-coolant accident (LOCA), the flow through the pump may reverse and, under some conditions, the shaft speed may also reverse. It is therefore necessary to have a complete set of pump characteristics.

The behavior of a centrifugal pump may be compactly represented by the so-called "homologous curves." For any given centrifugal pump, the effect of pump speed may be eliminated by plotting

$$(\Delta P/\rho)/(n/n_r)^2 \quad \text{versus} \quad Q/(n/n_r)$$

where n , n_r = actual and rated speed, respectively. When the head is expressed in terms of h' (the ratio of the actual head produced by the pump to the rated head), and flow is expressed in terms of Q' (the ratio of flow to rated flow), the resulting curves of $[Q'/(n/n_r)]$ versus $[h'/(n/n_r)^2]$ are the homologous curves.

In addition to the region of normal operation, where both Q' and (n/n_r) are positive, three other conditions may be distinguished. Therefore, four quadrants are defined as:

Quadrant	Pump Behavior	$Q' = Q/Q_r$	(n/n_r)
I	Normal pumping	>0	>0
II	Turbine pumping	>0	<0
III	Normal turbine	<0	<0
IV	Energy dissipation	<0	>0

Figure 3.9(a) illustrates the homologous curves obtained for a typical centrifugal pump. Note that one curve is obtained for quadrants I and IV and a second curve is obtained for quadrants II and III. During a large LOCA due to a hot-leg break, the pump could pass from quadrant I through quadrant IV with reverse flow and positive rotation. It would subsequently pass through quadrant III where both flow and speed are reversed. Quadrant II is rarely encountered. Quadrant IV can be avoided by adding a nonreversing ratchet on the pump shaft.

In addition to the head versus flow relationship, it is also necessary to know the pump torque, T' as a function of flow. Homologous curves, which hold for all pump speeds, are obtained by defining β as the pump torque/rated pump torque and plotting $\beta/(n/n_r)^2$ against $Q'/(n/n_r)^2$. As may be seen in Fig. 3.9(b), two separate curves are again obtained. Wylie and Streeter³⁴ show that it is possible to replace the four homologous curves by two curves by expressing them as

$$\frac{(\Delta P/\rho)}{(n/n_r)^2 + (Q')^2} = \phi_1 \{ \pi + \tan^{-1} [Q'/(n/n_r)] \} \quad (3.35)$$

and

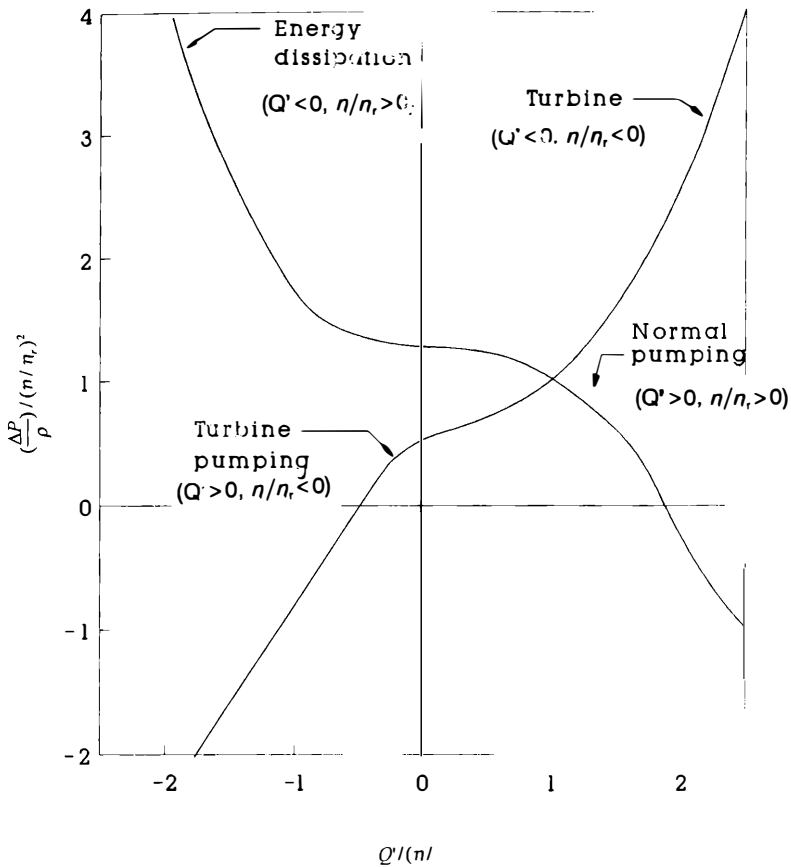


Fig. 3.9(a) Homologous curves for pump head for typical centrifugal pump.

$$\frac{\beta}{(n/n_r)^2 + (Q')^2} = \phi_2 \{ \pi + \tan^{-1} [Q' / (n/n_r)] \} \quad (3.36)$$

where ϕ_1 and ϕ_2 indicate a functional relationship. Use of Eqs. (3.35) and (3.36) simplify the computer representation of the homologous curves.

3.2 REACTOR COMPONENT HYDRODYNAMICS

3.2.1 Velocity Profiles and Turbulence in Fuel Rod Bundles

3.2.1.1 Velocity Profiles and Wall Shear Stress

Velocity profiles in rod bundle channels have been measured by several investigators. The data of Renksizbulut and Hadaller³⁵ for the central chan-

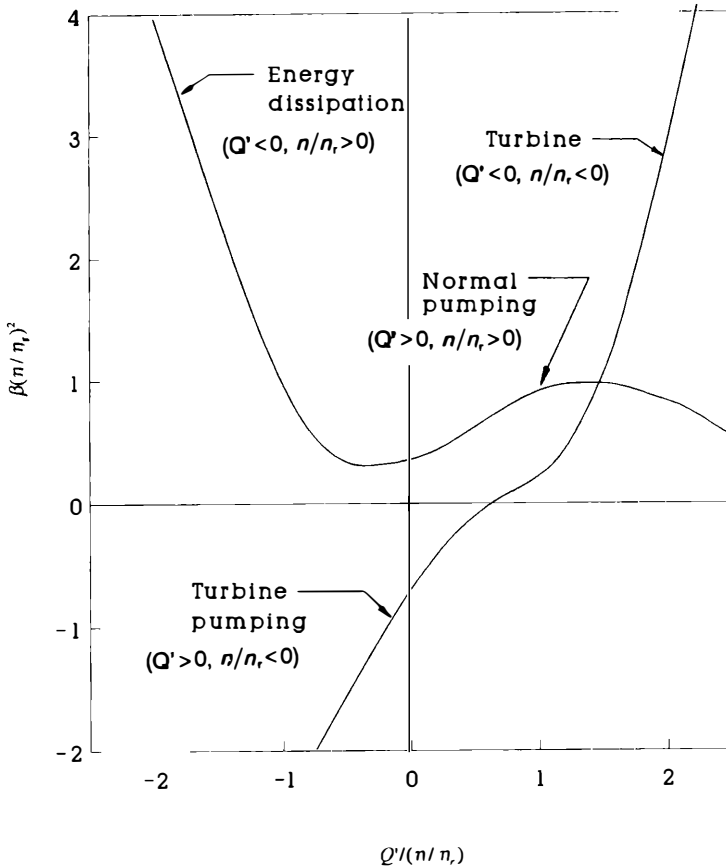


Fig. 3.9(b) Homologous curves for pump torque for typical centrifugal pump.

nel of a six-rod bundle with a square array show velocity contours very similar in shape to the predicted contours shown in Fig. 3.3. Similarly, the measurements of Ouma and Tavoulares³⁶ for central channels in a rod bundle on a triangular spacing show velocity contours very similar to the predicted velocity contours of Fig. 3.4. The velocity profiles depend to some degree on the (p/D) ratio and the profiles are, of course, distorted if the channel includes a portion of a boundary wall.

The local wall shear stress varies with azimuthal position. In a square array, the wall shear stress is a minimum at the position of minimum rod gap (azimuthal angle $= \theta = 0^\circ$) and a maximum where the distance between rods is a maximum ($\theta = 45^\circ$).^{35,37} The ratio of local to average wall shear stress appears to be independent of Reynolds number in both turbulent and laminar flow. The azimuthal variation appears to be enhanced as the

(p/D) ratio is increased. However, the azimuthal variations are generally no more than $\pm 20\%$ in turbulent flow.

Similar azimuthal wall shear variations are seen in triangular lattices. The shear stress is at a minimum at the smallest gap and a maximum in the open flow region.³⁶

Measurements of the variation of the average wall velocity across the minimum gap region in a square rod array are reported by Hooper,³⁷ Rehme,³⁸ Renksizbulut and Hadaller,³⁵ and Kjellstrom.³⁹ All of these investigators found that, when expressed in terms of universal variables, the velocity variation was very close to that seen in a round tube. For (p/D) values between 1.15 and 1.217 the data for the turbulent region were close to

$$u^+ = 2.7 \ln y^+ + 4.0 \quad (3.37)$$

where

$$u^+ = \bar{u} / u^*$$

$$y^+ = y u^* / \nu \quad u^* = (\tau_w / \rho)^{1/2}$$

$$\bar{u} = \text{mean value of axial velocity}$$

$$\tau_w = \text{wall shear stress at minimum gap}$$

$$\nu = \text{kinematic viscosity} = (\mu / \rho).$$

Detailed flow measurements within rod bundles have been essentially restricted to single-phase flow (1993). The effect of the presence of vapor on the detailed velocity and shear stress distribution remains to be determined.

3.2.1.2 Turbulent Flow Structure

In turbulent flow, the velocity varies chaotically with time in magnitude and direction. The instantaneous values at a point are usually expressed as the sum of a mean value and a fluctuating value. That is

$$u = \bar{u} + u' \quad v = \bar{v} + v' \quad w = \bar{w} + w' \quad (3.38)$$

where u, v, w are the instantaneous values of velocity in the $z, y,$ and x directions, respectively, and u', v', w' are the components of fluctuating velocity. For irrotational one-dimensional flow in the z direction, $\bar{v}$ and $\bar{w}$ are zero.

The magnitude of the turbulent velocity fluctuations is generally characterized in terms of the root mean square (rms) of the fluctuating velocities. The turbulent intensity i_b is defined as

$$i_b = \left\{ \frac{1}{3} [(\bar{u}')^2 + (\bar{v}')^2 + (\bar{w}')^2] \right\}^{1/2} / \bar{u} \quad (3.39a)$$

Alternatively, we may define a turbulent intensity in a particular direction. In that case, the value is often normalized using the frictional velocity, u^* . For example,

$$i_x = \sqrt{(\overline{u'}_x)^2} / u^* \quad (3.39b)$$

The fluctuating velocities give rise to momentum and energy transfer between adjacent positions in addition to the transfer by molecular diffusion. This is quantified by defining an eddy diffusivity, e . For the asymmetric flow in a rod bundle, the value of e depends on the direction. Thus, for the direction normal to the wall we define

$$(e_m)_y = - \frac{\overline{u' v'}}{\partial u / \partial y} \quad (3.40)$$

and parallel to the wall

$$(e_m)_x = - \frac{\overline{u' w'}}{\partial u / \partial x} \quad (3.41)$$

For irrotational one-dimensional flow, the effective shear stresses are then given by

$$\tau_{zy} = \left[\mu + \rho(e_m)_y \right] \left(\frac{\partial \bar{u}}{\partial y} \right) \quad (3.42a)$$

$$\tau_{zx} = \left[\mu + \rho(e_m)_x \right] \left(\frac{\partial \bar{u}}{\partial x} \right) \quad (3.42b)$$

and the rate of energy transfer within the fluid is

$$c_p \rho \left[\alpha + (e_q)_y \right] \frac{\partial T}{\partial y} = q''_y \quad (3.43a)$$

$$c_p \rho \left[\alpha + (e_q)_x \right] \frac{\partial T}{\partial x} = q''_x \quad (3.43b)$$

where

$(e_q)_z, (e_q)_x$ = eddy diffusivity of heat in the y and x directions, respectively

α = thermal diffusivity $k / \rho c_p$

q''_x, q''_y = heat fluxes in the x and y directions, respectively

c_p = specific heat at constant pressure.

It is commonly assumed that the eddy diffusivity of heat, e_q , equals the eddy diffusivity of momentum as determined from experimental velocity fluctuation measurements together with Eqs. (3.40) or (3.41). However, a more exact approach sets

$$e_q = e_m / (\text{Pr})_t \quad (3.44)$$

where Pr_t = turbulent Prandtl number. Zeggel and Monir⁴⁰ suggest that, for rod bundles, $(\text{Pr})_t$ may be correlated by

$$(\text{Pr})_t = 0.98 + 2.87 / (\text{Pr} \text{ Re}^{1/2}) \quad (3.45)$$

The turbulent intensity in a round tube may be related to the eddy diffusivity via Prandtl's mixing length hypothesis.* The mixing length theory states that for one-dimensional flow

$$e_m = l^2 \left| \frac{du_z}{dy} \right| \quad (3.46)$$

$$|\bar{u}'_z| = l \left| \frac{du_z}{dy} \right| \quad (3.47)$$

where l is the Prandtl mixing length. By assuming isotropic turbulence and using Eq. (3.39), we then have

$$e_m = i_b l (\bar{u}_z) \quad (3.48)$$

In the near-wall region, l is directly proportional to y , the distance from the wall. The value of l then increases more slowly until it reaches about 14% of the radius at the tube center.

Considerable effort has been devoted to the determination of the distribution of eddy diffusivities and turbulent intensities in rod bundles. This has been of particular concern to designers of fast reactors who would like to be able to describe the complete temperature distribution in their high flux, tightly packed, wire-wrapped bundles. This approach has generally been termed the *distributed parameter approach*.

The distributed parameter approach has also been of some interest to the designers of tightly packed, triangular lattice, water-cooled reactors. Based on extensive measurements, Rehme⁴¹ has concluded that the wall-parallel momentum transport across gaps of tightly or irregularly packed rod bundles exceeds the wall perpendicular transport by up to an order of magnitude. In more open lattices, the wall-parallel and wall-perpendicular diffusivities are more nearly of the same magnitude.

Based on the extensive experimental data of Rehme,⁴² Zeggel and Monir⁴⁰ have developed an empirical correlation for the radial distribution of the wall-parallel eddy diffusivity $e_\theta(r, \theta)$ in a tightly packed bare-rod bundle with a triangular lattice. In dimensionless form, this is

*The eddy diffusivity may also be obtained via the more sophisticated "K- ϵ model." Although not generally used in reactor core modeling, the approach has been used in conjunction with "porous media" modeling and it is discussed in Sec. 4.2.2.2.

$$e_{\theta}^{+}(r, \theta) = \overline{e_{\theta}^{+}(\theta)} \left[1.0 + A_R (B_R^2 - B_R + 1/3) - A_R \left(B_R - \frac{y/R}{Y_{m(\theta)}} \right)^2 \right] \quad (3.49)$$

where

$$e_{\theta}^{+}(r, \theta) = e_{\theta}(r, \theta) / [u^{*}(\theta) Y_m(\theta) R]$$

$$Y_{m(\theta)} = [r_m(\theta) - R] / R$$

R = rod radius

$r_m(\theta)$ = radial coordinate of line of maximum velocity

y = distance from rod surface

$$\ln[\overline{e_{\theta}^{+}(\theta)}] = 0.118 \exp[-13.8 Y_m(\theta) + (De/R)^{0.236} + 3.52 + a^1]$$

$$a^1 = 0.215(De/R)^{3.4} + 5.1(C^1/De)^{0.149} - 6.94$$

C^1 = azimuthal distance between planes of symmetry in neighboring channels.

For the central region of the bundle

$$A_R = 2.0, \quad B_R = 0.6$$

If the bundle is surrounded by a duct, then in the region near the wall (wall to line of maximum velocity),

$$A_R = 2.0, \quad B_R = 0.75$$

By use of Eq. (3.49) together with a wall perpendicular velocity profile, the axial momentum balance equation can be solved. Note that Eq. (3.49) holds strictly only for a p/D ratio of 1.10. However, it should be a reasonable approximation of the behavior in the tight-lattice, triangular pitch cores with p/D ratios close to 1.1.

In an actual reactor core, behavior is complicated by the presence of spacer and mixing vane grids or wire wraps. The vapor produced by the boiling in the hottest channels of the core also makes the behavior substantially more complicated than in a bare rod bundle with single-phase flow. Because of these complications, most current (1993) analyses of PWR behavior are based on the *lumped parameter* or *subchannel* approach. Here, the properties of a single subchannel (region between four adjacent rods in a square array, region between three adjacent rods in a triangular array) are assumed to be uniform. With this approach, the information needed is that required to determine the momentum and mass transfer between subchannels. Therefore, the turbulent mixing of greatest importance is the wall parallel mixing at the minimum gap between rods.

Rehme's⁴² data indicate that the turbulent intensity in the minimum gap region is very much higher than in the central region of the channel. Axial and azimuthal turbulent interstices have a similar order of magnitude, but the azimuthal turbulent intensities are about 30% higher than the axial val-

ues. The turbulent intensities across the central 80% of the gap appear to be fairly constant.

Perhaps the most important conclusion regarding the turbulent intensities in the rod gap region is that they are above those for round tubes. The difference is greatest when comparing the central region of the gap and the central region of a round tube where they can differ by a factor of 2. Hooper³⁷ and Hooper and Rehme⁴³ also show that the axial and azimuthal turbulent intensities in the gap region increase significantly as the gap size is decreased. Further, compared to triangular rod bundles with similar p/D ratios, the square bundles show higher levels of turbulence in the gap region.

Since eddy diffusivity varies directly with turbulent intensity [see Eq. (3.48)], the rate of turbulent momentum and energy exchange per unit area will increase as the gap size is decreased. In addition, for bare rod bundles, the exchange rate per unit area will be higher in a square array than in a triangular array with the same p/D ratio.

3.2.1.3 Effect of Mixing Vanes and Two-Phase Flow

The square array rod bundles of vessel-type PWRs often contain mixing vanes placed on the upper edge of the grid spacers.* The vanes are produced by bending tabs of metal which extend above the grid. The entire tab may be bent in one direction or the tab may be split and the halves bent in opposite directions. A typical mixing vane arrangement is shown in Fig. 3.10. Note that the vane angle, α' , represents the angle of the blade relative to the flow direction.

Although most of the information on mixing vane performance is proprietary, some data have appeared in the open literature. The laser-Doppler velocimeter measurements of Shen *et al.*⁴⁴ provide a picture of the action of the split vanes illustrated in Fig. 3.10. The lateral velocity profile at two distances downstream of the vane is shown in Fig. 3.11. At a short distance downstream of the vanes, flow enters from two of the adjacent channels and exits to the remaining two. In addition, a vortex is created in the center of the subchannel. Similar behavior is seen further downstream but the direction of the flows from the adjacent channels have reversed and the magnitude of the flows is substantially reduced. The cross-flows produced by the vanes lead to substantial mixing of the coolant in the bundle. This mixing leads to a considerably more uniform enthalpy rise across the bundle.

*Some later designs achieve improved interchannel mixing without mixing vanes. In these designs, each grid strap is composed of two thin segments. The segments are attached to each other in a way that produces small channels between the segments in addition to the main channels within the spacer grid. The flow through these small channels, which are tilted at an angle to the axis of the fuel rod, causes the desired mixing improvement. Performance data on these grids remains (1995) proprietary.

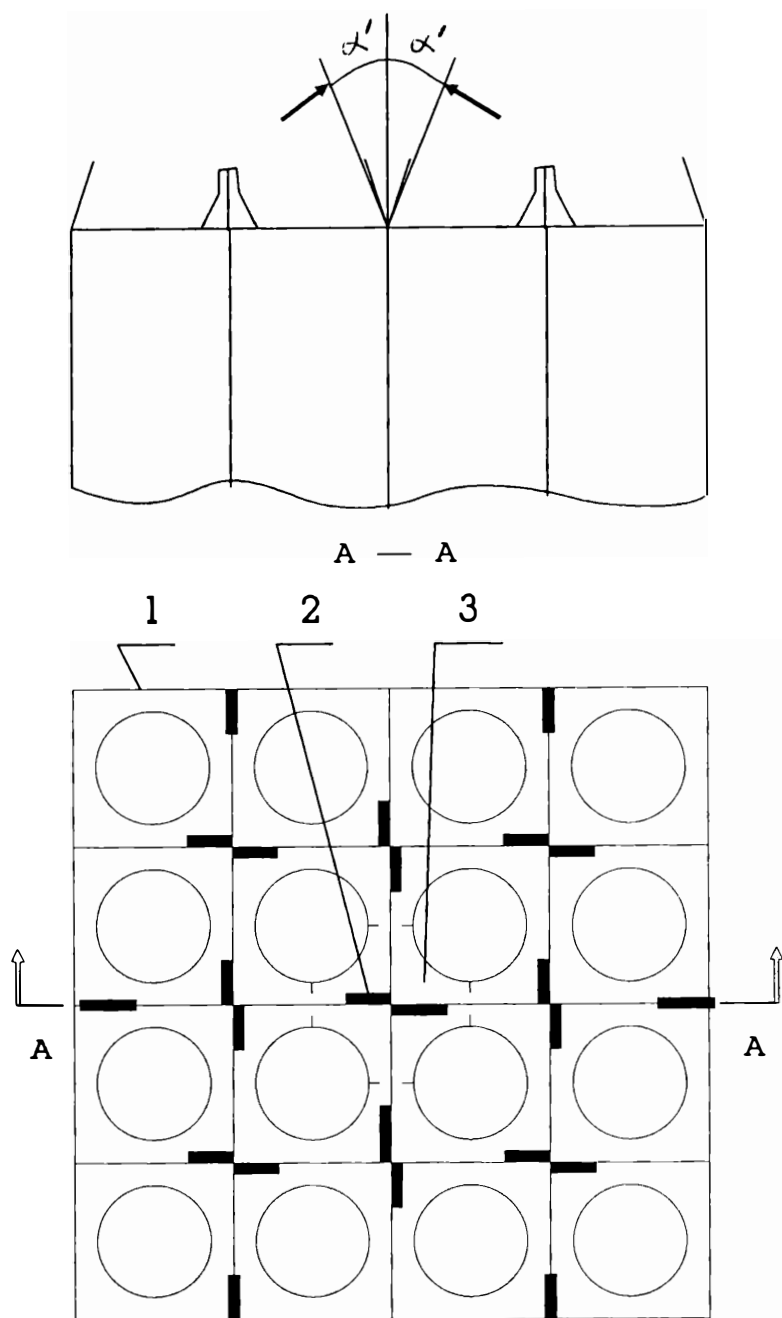


Fig. 3.10 Arrangement of mixing blades on the grid spacer. 1, grid spacer; 2, ripped-open blades; 3, central subchannel [from Ref. 44].

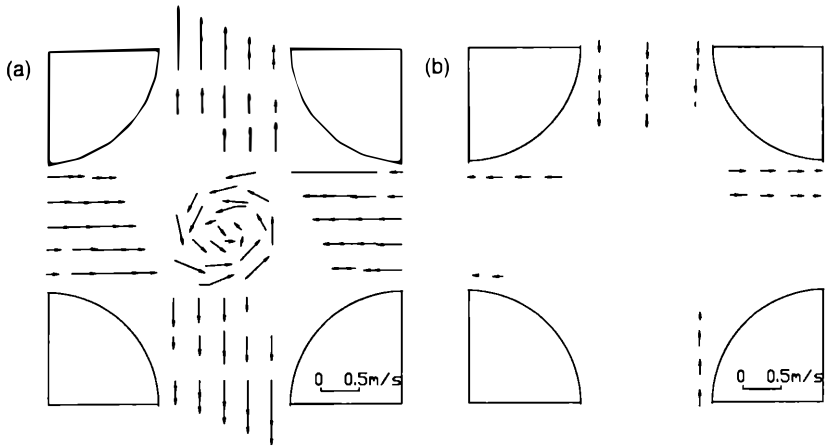


Fig. 3.11 Lateral velocity profile for $\alpha = 25^\circ$: (a) $L/De = 6$, (b) $L/De = 25$ [from Ref. 44].

Yang *et al.*⁴⁵ measured total mixing downstream of several grids. The results of this study also showed a high rate of mixing immediately downstream of the grid. The mixing rate decayed rapidly and then remained nearly constant at L/D ratios above 10. Yang *et al.*⁴⁵ observed that the degree of mixing depended significantly on the grid design. The grid having a pressure loss coefficient substantially above the others tested showed the highest mixing rates. However, there was no obvious relationship between loss coefficient and mixing rates among the other grids. However, Shen *et al.*⁴⁴ found that the degree of mixing depended on the vane angle (α' in Fig. 3.11). Mixing and pressure drops increased as α' increased, but, at large values of α' , the mixing rates became nonuniform across the subchannel. After taking this nonuniformity into account, they recommend $\alpha' = 25$ deg as optimum.

In addition to providing gross interchannel mixing, mixing vane grids also somewhat increase the turbulent intensity immediately downstream of the grids.⁴¹ The axial turbulent intensity was about 60% higher than bare-rod values immediately downstream of the grid. However, after an L/D of about 10, the turbulent intensity differed little from that seen in a bare bundle of the same p/D . This result indicates that the major effect of a vane is the gross mixing it produces.

The turbulent intensity is also affected by the presence of vapor bubbles. The measurement of the detailed distribution of eddy diffusivity and turbulent intensities in rod bundles under two-phase flow conditions is quite difficult. The effects need to be inferred from limited data on pipe flow and on overall mixing coefficients. Michiyoshi and Serizawa⁴⁶ examined two-phase turbulence during bubbly flow in round tubes using dual-sensor hot

film anemometry. They found the turbulence to be highly promoted by the presence of bubbles. Their study, which was limited to the low void region, showed that eddy diffusivities in the central region of the tube could be more than double the single-phase values. Although detailed measurements of the eddy diffusivity distribution in rod bundles are not (1993) available in the presence of vapor, data on average rates of turbulent mixing with two-phase flow⁴⁷⁻⁵⁰ confirm the significant effect of the presence of vapor.

In view of the difficulties in determining local values of eddy diffusivities when mixing vanes and two-phase flow are present, most computations are based on the use of empirical correlations of gap average values.

If $\bar{\epsilon}$, an average value of the transverse (wall-parallel) eddy diffusivity in the gap region is known, the value of w' the turbulent transverse fluctuating flow rate between adjacent subchannels per unit length of rod (mass/ hL), may be obtained from

$$w' = \rho \bar{\epsilon} (l/De) \quad (3.50)$$

where

ρ = fluid density, mass/volume

$\bar{\epsilon}$ = average eddy diffusivity, area/time

l = Prandtl mixing length, L

De = equivalent diameter of channel, L .

Since the ratio (l/De) does not change very much for reactor channels, it is common to combine this ratio with $\bar{\epsilon}$ and define an average mixing coefficient, ϵ , such that

$$w' = \rho \epsilon \quad (3.51)$$

where ϵ and $\bar{\epsilon}$ leave the same units. For a PWR assembly without mixing vanes, ~ 0.06 (lb/s ft) is a typical value for w' . The various empirical correlations that have been used for w' and ϵ , and which combine the effects of turbulent mixing and gross mixing due to mixing vanes or wire wraps, are described in Sec. 4.2.1.3.

3.2.2 Flow in Plenums

Flow streams or vortices in the lower plenum can affect flow distribution at the core inlet. The downward jet formed by the flow coming down between the thermal shield and vessel wall causes a low-pressure region at the edge of the core inlet, which will reduce flow in the outer assemblies. Pressure at the center of a vortex with a horizontal plane of rotation is usually lower than the surrounding pressure; thus, the flow rate to any fuel assembly located above a vortex is reduced.

The most effective way to even off the inlet flow is to increase flow resistance at the bottom of the core. According to Prandtl's equation for velocity distribution improvement,⁵¹

$$\frac{\Delta V_{\text{after screen}}}{A_{\text{avg}}} = \frac{1}{K+1} \frac{\Delta V_{\text{before screen}}}{V_{\text{avg}}} \quad (3.52)$$

where

$$K = \frac{\Delta P_{\text{screen}}}{\rho V_{\text{avg}}^2 / 2g_c} \quad (3.53)$$

A similar relationship can be applied to the core inlet. For example, consider the situation where the Euler number at the core inlet is

$$\text{Eu} = \frac{\Delta P_{\text{core inlet}}}{\rho V_{\text{jet}}^2 / 2g_c} \leq 10 \quad (3.54)$$

Under the most adverse conditions, K is then equal to 10. To obtain the pressure drop at the inlet of the hot channel, we add the stagnation pressure of the jet to the average inlet pressure drop. This leads to $(\Delta P \text{ of maximum flow channel} / \Delta P \text{ of average channel}) = 11/10$. Since ΔP is proportional to V^2 , the ratio of the maximum inlet flow rate to the average inlet flow rate is $(11/10)^{1/2}$

Several analytical procedures based on potential flow theory have been used for predicting the velocity distribution in the lower plenum. Yeh and Tong⁵² used conformal mapping to solve a two-dimensional approximation of the flow field. Yeh⁵³ obtained a solution for the potential field in the three-dimensional case. He divided the field into several regions (downcomer, lower plenum, and cylindrical region above bottom of downcomer) and solved the differential equations for each region by the separation of variables technique. Figure 3.12 shows the results of a typical calculation. This figure shows the benefit of a downcomer skirt that extends an appreciable distance below the core. While the flow is clearly not evenly distributed at the edge of the downcomer, distribution becomes more uniform as one moves upward.

Although potential flow theory can be considered useful in giving a qualitative view of behavior and an approximate estimate of velocities, analytical methods must simplify the geometry and ignore the many structural members in the lower plenum. Of course, any vorticity introduced cannot be considered. In view of the limitations of previously available analytical techniques, most reactor designers determined the velocity distribution in the lower plenum from experimental studies using scale models.

Hetsroni⁵⁴ describes hydraulically testing a model of a PWR vessel and core. Of the four dimensionless groups involved, geometry, relative roughness, Reynolds number, and Euler number, $[\Delta P(2g_c)/\rho V^2]$, he concludes

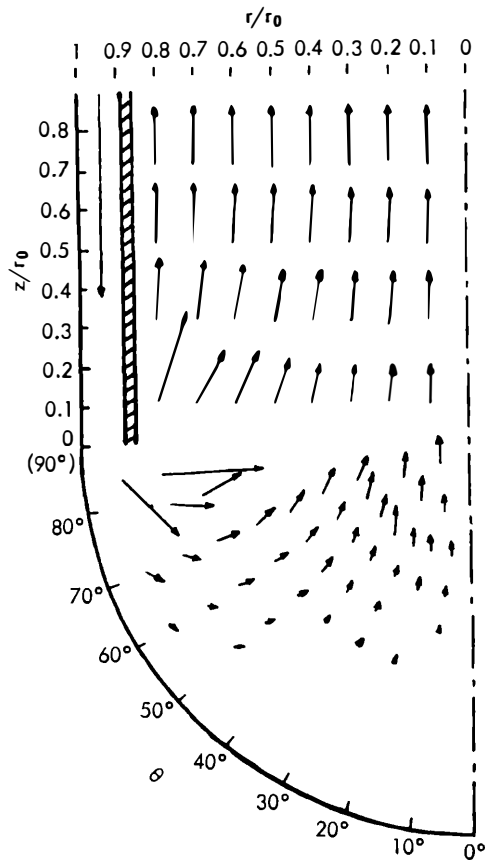


Fig. 3.12 Typical velocity distribution in lower plenum. The arrows indicate direction and relative magnitude of fluid velocities [from Ref. 53, courtesy *Nucl. Eng. Des.*].

that geometry and Euler number are most important. His tests used a $\frac{1}{7}$ scale model that preserved geometry. Care was taken to keep the Euler number similar in the prototype and model. The $\frac{1}{7}$ scale factor was also adjusted for relative roughness in small flow passages, but it was ignored in the large passages. The test results obtained a significantly improved flow distribution when a straight downcomer skirt extended below the core. A tapered skirt did not lead to satisfactory flow distribution. Flow distribution was also found to depend on the geometry of the lower plenum.

Application of the experimentally determined velocity distribution to an actual core design was somewhat of a problem. Under operating conditions where high power regions generate appreciable quantities of steam, the inlet flow redistributes to maintain a nearly constant pressure at the core

outlet. Khan⁵⁵ suggested that the measured velocity distribution at the core inlet be used to determine an inlet pressure distribution. Further, he suggested that the inlet pressure distribution be assumed to hold under operating conditions. An alternative procedure would be to use inlet flow distribution to determine a set of flow resistances between the downcomer and various core assemblies. These flow resistances then might be assumed to be the same under operating conditions.

The degree to which the coolant entering the reactor vessel through the several loops is mixed in the lower plenum of the vessel is of concern under asymmetric flow conditions. Such conditions can develop between reactor cold-leg loops due to instrumentation and control uncertainties, pump flow uncertainties, or variations in the secondary side flow in a steam generator. If such asymmetries lead to asymmetry in the temperature of the coolant entering the core, core power distribution and burnup could be altered. More severe asymmetries are possible during accidents where such things as pump failure, a stuck-open relief valve on a steam generator, or steam-line break can significantly alter cold-leg temperature.

Mixing tests, conducted on a large PWR having an external system with two steam generators and four reactor pumps, showed very incomplete mixing during normal (four-pump) operation.⁵⁶ In the lower plenum, only about 15% of the flow was mixed completely and another 25% was partially mixed. Greater mixing was attained with asymmetric operation but the mixing was still far from complete. Little mixing was seen in the core but significant mixing occurred in the upper plenum. Under normal operation, mixing in the upper plenum accounted for 60 to 75% of the total mixing. It would appear that the common accident analysis assumption of a uniform core inlet temperature may not always be justified. If an allowance for minimal mixing in the lower plenum is made in the asymmetric case, an allowance for the mixing in the upper plenum should be included in evaluating the temperature of the coolant entering the various loops.

Upper plenum behavior can significantly affect a number of accident sequences. In reactor accidents that involve substantially reduced flows through the reactor vessel, the possibility of thermal stratification in the upper plenum should not be overlooked. During the reflooding phase of a LOCA, the control-rod guide tubes in the upper plenum can cause significant deentrainment from the droplet laden steam leaving the core (see Sec. 3.5.2). If a severe accident involving core damage were to occur, the upper plenum would play an important role in determining molten fuel transport and sweepout as well as debris collection.

The advances of computational fluid mechanics have now considerably changed the situation for designers of future reactors. Large computer programs, based on the so-called "porous media" concept, are now available for treating large scale flow distribution problems. Such programs use finite difference approximations of the three-dimensional conservation equations

and include a model for turbulent energy dissipation. Boundaries are generally described by local isotropic porosity factors and fluid-solid source terms. The effect of structural members in the plenum can thus be included. In addition, it is possible to model both a symmetric section of the plenum and the reactor core. It is feasible to model the reactor core on an assembly-by-assembly basis with the appropriate levels of heat addition. After adjusting the model so that agreement with experiment is obtained for isothermal conditions, the model may be used with appropriate heat addition to establish core inlet pressures or flows at design conditions. Porous media computations should also be able to determine the degree of mixing in the upper and lower plenums. The porous media approach is discussed in detail in Sec. 4.2.2 and in Refs. 79 through 81 of Chapter 4.

3.2.3 Flow-Induced Vibration

It is generally agreed that flow-induced vibration of nuclear reactor components is due to one of four mechanisms, namely: (1) fluid elastic instability, (2) periodic vortex shedding, (3) turbulence-induced excitation, and (4) acoustic resonance. The relative importance of the various vibration mechanisms depends on the particular component and flow situation.

Fluid-elastic instabilities occur when there is a coupling between fluid-induced dynamic forces and structural motion. Such instabilities occur when the velocity is high enough that the energy imparted by the fluid force exceeds the energy dissipated by damping.

Periodic vortex shedding occurs downstream of a structure placed in a cross-flow. If the shedding frequency coincides with a natural frequency of the structure, damaging resonance may occur.

Turbulence-induced excitation produces random pressure fluctuations, which cause vibration. Turbulence excitation is most important in axial flow.

Acoustic noise is present in all systems. It is a vibration problem only if the frequency of the noise is close to one of the natural frequencies of the structure.

3.2.3.1 *Hydraulic Vibration of Core Structure*

When a high-speed flow passes through a gradually narrowing passage, the pressure head of the stream is converted into a velocity head, which creates a suction force on the wall. If the wall is movable or flexible, the passage is automatically reduced to zero and flow is stopped. As soon as flow becomes stagnant, the stream pressure increases to its maximum value, pushing the wall back, thus providing a wide passage again. These suction and pushing forces can act periodically and, thus, vibrate the core structure. This fluid-elastic instability mechanism appears to govern the vibration of parallel fuel plates.

Parallel fuel plate vibration has been analyzed by Miller,⁵⁷ Scavuzzo,⁵⁸ and Remick.⁵⁹ Miller applied a method of "neutral equilibrium" where the pressure and plate restoring forces were balanced. He was then able to determine a critical velocity, V , above which significant vibration will occur. He expresses this vibration-inducing velocity for fuel plate assemblies with long edges attached to side plates as

$$V_c = [15g_c E (\Delta L)^3 d / \rho b^4 (1 - \gamma_1^2)]^{1/2} \quad (3.55)$$

where

ΔL = plate thickness

b = plate width

d = flow channel depth

E = Young's modulus of elasticity

γ_1 = Poisson's ratio.

Miller's results are frequently applied, and Wambsganns,⁶⁰ who extended the work to include second-order terms, concluded that critical flow velocities could be from 0.63 to 0.85 times those derived by Miller.

Miller and Kennison⁶¹ analyzed the flow-induced vibration of a blade suspended in a flow channel, which widens at a point downstream of the leading edge—a condition similar to that observed by a control-rod blade. They concluded that there are three possible motions: (1) decaying vibrations, (2) divergent vibrations, and (3) static deflection against the channel wall. The motion observed is dependent on insertion length and flow rate. For short insertion lengths, there is a critical velocity above which divergent oscillations are obtained. They also conclude that a critical insertion length exists above which no oscillations can be sustained, but where deflection against the channel wall is possible. Wambsganns⁶² concludes that the analysis explains experimental observations qualitatively, not quantitatively.

Acoustically induced vibrations can be significant for reactor internals (core barrel and thermal shield). Acoustic pulsations generated by coolant pumps can cause large responses in shell structures. This is most serious when several coolant pumps are present. The minute differences in impeller rotation speeds means that at some point in time the acoustic pressure generated by these pumps will reinforce each other. This reinforcement is known as "multipump beating" and has been observed in operating reactor systems.⁶³ The frequencies at which acoustic oscillations may be expected correspond to the pump shaft rotation speed, the vane-passing frequency or their higher harmonics. Serious damage can occur if the natural frequency of the structure coincides with any of the foregoing.

If the natural frequency of the structure is substantially above the frequency of the expected oscillation, vibration problems will not be significant. The response of complicated structures to an arbitrary dynamic

excitation is difficult to predict. Bohm⁶⁴ developed an analytical model that approximates the internals as an interconnected beam-type structure and uses a transfer-matrix approach for determining natural frequencies. Bowers and Horvay⁶⁵ adopted a similar approach for estimating vibration frequencies of the core barrel and thermal shield. They noted that the narrow gap between the shell and vessel was responsible for a substantial increase in the virtual mass of the shell. They expressed their results in terms of ν , a nondimensional frequency parameter. The actual frequency of vibration, ω , is given by

$$\omega = (\nu/L)^2(EI/m)^{1/2} \quad (3.56)$$

where

L = length of the shell

$m = m_s + m_h$

m_s, m_h = mass per unit length of shell and entrained water, respectively

E = Young's modulus

I = moment of inertia of shell

M = attached mass (tubesheet, part of fuel, etc.).

They found ν to be a function of the weight ratio (mL/M), support stiffness, and bottom restraint. Typical values for ν were in the range of 1.5 to 2.2 for the fundamental mode; 3.4 to 4.2, 6.3 to 7.2, and 9.4 to 10.2 for the second, third, and fourth harmonics, respectively.

Actual operating experience indicates that the natural frequencies of the core barrel and thermal shields may not be significantly above the frequency of the exciting pulses. Therefore, in addition to making certain that the natural frequency of the structure does not closely correspond to one of the acoustic frequencies expected from pump noise, it is necessary to show that the actual response of the system is not excessive. This computation usually involves⁶³ (1) determination of variation in the spatial acoustic pressure distribution as a function of time (see Ref. 66), (2) an analysis of the transient dynamic response of the structure to determine that structural response is not beyond the limit during pump startup or coastdown, and (3) a steady-state vibration analysis to assure that structural response is not excessive during normal operation.

Procedures for analyzing the responses of core barrel and shield structures to specific pressure loadings have been developed by several investigators. Penzes⁶⁷ shows how a known pulsating pressure at the inlet to downcomer region can be used to determine the pulsating pressure distribution on the cylindrical core barrel or thermal shield. Penzes introduces the concept of a time-dependent body force (time-dependent additional thickness) in the governing differential equation. With this conceptual sub-

stitution for actual loading, the time-dependent mixed boundary value problem can be represented by a vibration problem with homogeneous boundary conditions. Bowers and Horvay⁶⁸ applied Penzes' approach to predicting internals response, but obtained vibration amplitudes considerably smaller than those predicted by Penzes.

The most sophisticated approach devised is that of Au-Yang.⁶⁹ He observes that flow-induced vibration is a stochastic process. Since stress and strain within a structure can be obtained from displacement distribution, Au-Yang develops expressions for predicting rms displacement and displacement power spectral density as a function of location. The prediction requires a knowledge of the natural frequency and mode shapes of the structure, virtual mass, pressuring loading function, and damping ratios. The loading function and damping ratios must be developed on the basis of experimental information (Penzes' analysis could be used to determine loading function if pressure fluctuations are known). In-fluid frequencies and modal shapes can be predicted by currently available finite element computer programs providing the virtual mass is supplied to the program. The virtual masses, in-fluid natural frequencies, mode shapes, and experimentally-based loading functions and damping ratios are used to compute displacement power spectral density in terms of Powell's acceptances.⁷⁰

The complexity of most reactor coolant systems makes it difficult to predict the frequency and magnitude of hydraulic forcing functions; therefore, scale models of system components are often tested under simulated operating conditions.*

3.2.3.2 Hydraulic Vibration of Fuel Rods

A cylinder exposed to axial flow will exhibit fluid-elastic instability if subjected to a sufficiently high flow. The critical velocity, V_c , above which fluid-elastic instability will occur with a cylinder supported at both ends in an unrestricted medium⁶³ is given by

$$V_c = (\pi/L) \left(\frac{EI}{\rho A} \right) \quad (3.57)$$

where L = unsupported rod length, ρ = fluid density, A = cross-sectional area of rod, and other symbols have their previous meaning. This critical velocity is normally considerably above the values encountered in reactor conditions.

*Au-Yang and Jordan [*Nucl. Eng. Des.*, **58**, 113 (1980)] obtained forcing function data for the barrel and shield structure of a 1/6 scale model of a commercial PWR. After expressing their observations in terms of dimensionless parameters, they found that the nondimensional results also represented the observed behavior in a geometrically similar full-size reactor. Forcing function data for a 1/9 scale model of an advanced reactor with larger diameter barrel and shield were obtained by Au-Yang *et al.* [*Nucl. Eng. Des.*, **157**, 93 (1995)].

At reactor conditions, the hydraulic vibration of fuel rods with their axes parallel to the flow is due to turbulence excitation. This problem has been studied by Burgreen *et al.*,⁷¹ Quinn,⁷² Sogreah,⁷³ Pavlica and Marshall,⁷⁴ Paidoussis,⁷⁵ and Morris.⁷⁶ The earliest correlation is that due to Burgreen, who found that single rods vibrate at their natural frequency (reduced slightly by viscous damping in water) and that amplitudes of the vibrations are given by

$$\left(\frac{\delta}{D}\right)^{1.3} = (0.83 \times 10^{-10}) C \left(\frac{\rho_1 V^2}{\mu \omega_n}\right) \left(\frac{\rho_1 V^2 L^4}{EI}\right)^{0.5} \quad (3.58)$$

where

δ = maximum rod deflection

D = rod diameter

C = 5 for pin-ended rod, 1 for rigidly held rod, and 2.08 for one end rigidly held, the other pin ended

ω_n = the natural frequency of vibration (Hz) = $\sqrt{EI/(M'L^4)}$

ρ_1 = water density

μ = water viscosity

E = Young's modulus of elasticity

I = moment of inertia

V = axial velocity

M' = rod mass per unit length

L = rod length.

Paidoussis^{75,77} extended the work of Burgreen *et al.*⁷¹ His later correlation for deflection amplitudes is

$$\frac{\delta}{D} = \alpha_1^{-4} \left[\frac{u^{1.8} \left(\frac{L}{\alpha}\right)^{1.8} \text{Re}^{0.25}}{1 + u^2} \right] \left(\frac{De}{D}\right)^{0.4} \left(\frac{B^{2/3}}{1 + 4B}\right) (5 \times 10^{-4} \times K) \quad (3.59)$$

where

α_1 = first-mode beam eigenvalue

$B = M + m/M$, M = rod mass, m = fluid mass

$u = (M/EI)^{1/2} (VL)$

V = axial velocity

De = hydraulic diameter of the flow channel

K = constant that is 1 for quiet flow and 5 for realistic disturbance levels.

This correlation shows much less dependence on fluid viscosity than Paidoussis' original proposal. It brings it into close agreement with the data of Quinn⁷² and Pavlica and Marshall,⁷⁴ which show almost no fluid temperature effect.

Paidoussis⁷⁷ also examined the vibration of 19-element wire-wrapped bundles and found vibration amplitudes differed substantially from those of Eq. (3.58) and that they did not increase significantly with flow. Kinsel⁷⁸ studied the vibration characteristics of 217-pin spiral wire-wrapped fuel assemblies in water. He observed very low water pin displacements, generally 5 mm or less. The 0.23-in.-diameter pins showed distinct resonances at low flows, but broad-band Gaussian distributions at high flows.

Pavlica and Marshall⁷⁴ examined the vibration of rod bundles with spring clip spacers. In correlating the data, they used the stiffness (EI) of the bundle rather than that of the individual rods. Since intermediate rod-to-rod connectors are not rigid, they used experimental values of (EI) rather than an estimate of single-element parameters. With this change, they found their room temperature data agreed with the original Paidoussis correlation. The discrepancy between their 150°F data and correlation appears to be remedied by the reduced viscosity effect in the revised correlation.

Gorman^{79,80} investigated the effect of void fraction on single-pin vibration in both an annulus and a seven-rod bundle. Pin dimensions and bundle arrangement were similar to those found in CANDU designs. Gorman found that rms pin displacement increased substantially under two-phase conditions. The maximum rms displacement occurred at simulated qualities in the range of 12 to 20%. At a constant mass flow, the maximum rms displacement under two-phase conditions in a rod bundle could be up to eight times that observed in single-phase flow.

Paidoussis⁸¹ recommends that the following conditions be adhered to in the design of reactor fuel assemblies: (1) The axial velocity be below the critical velocity for fluid-elastic instability (generally not a problem). (2) The frequency of flow perturbations be at least 25% above or below ($2\omega_n$) where ω_n is the natural frequency of the fuel rod. (3) The amplitude of the turbulence-induced vibrations be such that adjacent fuel rods will not touch.

In actual practice, conservatism requires the maximum vibration amplitude to be considerably smaller than would be allowed by Paidoussis' third criterion. Vibration amplitudes of 0.2 to 0.25 mm are common design criteria.⁸² To obtain such low vibration amplitudes, a considerable force must be maintained by the spacer-grid springs. Further, if fretting wear at the support is to be prevented, the sinusoidally varying reactions at the support should be less than the spring preload. Preumont⁸² suggests that these forces may be computed by using a finite element model of the fuel rod with distributed masses and spring forces at each of the support points. He assumes that damping is negligible and that the rod vibration is in its fundamental mode with the maximum deflection at the design value. He then

determines the forces at the support locations required to bring the rod displacements at these locations to zero. For a typical fuel rod in a 14×14 PWR assembly, he computes that spacer-grid forces in excess of 1 N are required at the end supports (1.5 N at top) but considerably lower forces are needed in the center region of the rod.

Note that, due to creep, grid springs relax under neutron irradiation. After a fast fluence of 2.5×10^{21} n/cm², the residual spring force on an Inconel spring is only about 1/7 of the original force.⁸² Thus, the spring preload at the beginning of life needs to be substantially higher than that calculated in order to provide the needed preload at the end of life. However, the highest forces are needed at the assembly top and bottom where fluxes are lower. At a fast fluence of $\sim 0.85 \times 10^{21}$ n/cm², the residual force on an Inconel spring has only dropped to 1/3 of its original value.

3.2.3.3 *Vibration of Heat Exchanger Tubing and In-Core Instrumentation Thimbles*

Vibration of heat exchanger tubing can be a significant cause of tube failure. Such tube failures may be due to either fatigue or impact-fretting wear.

Vibration problems are most severe in those regions of the heat exchanger where the tubes are subject to appreciable cross-flow. This occurs in (1) the inlet region of the heat exchanger where there is a flow of liquid from the entrance ports across the lower portion of the tube bundle, and (2) in the U-bend region of a U-tube heat exchanger where the two phase mixture flows across the bundle.

Vibration in cross-flow may be due to either fluid-elastic instability or periodic vortex shedding. With an isolated cylinder, periodic vortex shedding is a well-understood phenomenon. The shedding frequency is represented by the Strouhal number, St ,

$$St = f_v L / V \quad (3.60)$$

where f_v = vortex shedding frequency, L = characteristic dimension (D for cylinder), and V = stream velocity. When the Reynolds number is in the range of 3.6×10^6 to 1×10^7 , the vortex is in the transcritical range. The drag remains constant and the Strouhal number remains at ~ 1.2 as the velocity is changed.⁸³ Behavior in tube bundles is not as well understood. However, the problem appears to be of lesser importance. While difficulties can arise if the shedding frequency corresponds to the natural frequency of the vibrating tube and resonance occurs, fluid-elastic instability appears to be the main problem.

Just as in axial flow, fluid-elastic instability in cross-flow is avoided if the cross-flow velocity is kept below a critical value. Pettigrew *et al.*⁸⁴ recommend that

$$V_r / \omega_n D < K [2\pi \zeta_T m' / (\rho D^2)]^n \quad (3.61)$$

where

D = rod diameter

$V_r = V_\infty p / (p - D)$

V_∞ = superficial fluid velocity

p = tube pitch

m' = mass per unit length (includes hydrodynamic mass)

ζ_T = total damping ratio

ω_n = natural frequency of vibrating tube (Hz).

For values of $(2\pi\zeta m / \rho D^2)$ below 2, Pettigrew *et al.*⁸⁴ recommend that $K = 4.0$ and $n = 0.5$. For values above 2, they suggest that taking $K = 4.9$ and $n = 0.08$ will provide a lower bound for the critical velocity.

The total damping ratio is the sum of the viscous damping, ζ_v , and the support damping ζ_s . Pettigrew *et al.*⁸⁴ recommend

$$\zeta_s = \left(\frac{N-1}{N} \right) \left(\frac{2200}{\omega_n} \right) \left(\frac{\rho D^2}{m'} \right) \left(\frac{\Delta L}{L} \right)^{0.6} \quad (3.62)$$

where N = number of spans, ΔL = support thickness, L = characteristic span length (average of three longest spans), and other symbols are as previously defined. The viscous damping in a single-phase liquid may be obtained from the theory of Chen *et al.*⁸⁵ but a simplified version due to Rogers *et al.*⁸⁶ is valid for the usual range of heat exchanger parameters

$$\zeta_v = \frac{100\pi}{\sqrt{8}} \left(\frac{\rho D^2}{m} \right) \left(\frac{\nu}{\pi \omega_n D^2} \right)^{1/2} \left\{ \frac{1 + (D/D_c)^2}{[1 + (D/D_c)^2]^2} \right\} \quad (3.63)$$

where ν = kinematic viscosity and D_c = effective diameter of surrounding tubes.

Viscous damping in a two-phase fluid is considerably more complicated than the single-phase damping and no simple equation is available for its calculation. The relative damping of a two-phase air-water mixture has been measured for a p/D ratio of 1.47⁸⁴. At a void fraction of 0.4, the viscous damping has been increased by a factor of about 3.5. The maximum damping was reached at a void fraction of about 0.6. Damping then declines but there was considerable scatter in the measurements. Two-phase effects should be considered in flow situations such as in the U-bend region of the steam generator. However, use of Eq. (3.63) would give a conservative value for the critical velocity.

To determine whether the critical velocity defined by Eq. (3.61) is actually reached in a particular heat exchanger, an analysis of the flow field within the tube bundle is required. This can be accomplished by means of computer programs using the porous media approach (see Sec. 4.2.2). The THIRST program⁸⁷ is an example of such a program developed specifically for steam generator design.

Vibration of in-core instrumentation thimbles must also be considered. In-core instrumentation is brought to the core via tubes or thimbles that penetrate the lower head of the reactor vessel. These instrumentation thimbles are subjected to the cross-flow produced as the reactor coolant flows from the thermal shield area to the fuel assembly inlets. Fluid-elastic instability must be considered in essentially the same manner as for the heat exchanger tubing. Determination of the flow field in the lower plenum of the reactor vessel via a fluid mechanics computation (e.g., porous media approach) is the first step in this analysis.

3.2.4 Hydraulic Design of Control Rods

To obtain a simplified method of analysis for estimating the drop time of a control rod, the basic assumption is made that all the hydraulic forces are proportional to the square of the rod velocity. The equation-of-motion of the control rod is then

$$\frac{M+m}{g_c} \frac{dV}{dt} = \Sigma F \quad (3.64)$$

where M is the mass of control rod and m is the virtual mass of the body due to fluid acceleration; i.e.,

$$m = M\rho_l / \rho_M \quad (3.65)$$

The equation-of-motion can be simplified to

$$\frac{dV}{dt} = C_2 - C_3 V^2 \quad (3.66)$$

Solution of the above equation is

$$t = \frac{1}{2(C_2 C_3)^{1/2}} \ln \frac{1 + [1 - \exp(-2C_3 x)]^{1/2}}{1 - [1 - \exp(-2C_3 x)]^{1/2}} \quad (3.67)$$

where x represents distance traveled.

Asymptotically, as $x \rightarrow \infty$ (or $x \geq 2/C_3$),

$$t = \frac{C_3 x + \ln 2}{(C_2 C_3)^{1/2}} \quad (3.68)$$

At the end of the rod's travel, it enters a dashpot with a small clearance between it and the rod. This clearance produces a high frictional force that rapidly decelerates the rod. The resisting force in the dashpot is linearly proportional to the distance of insertion, Z . The equation-of-motion when the rod enters the dashpot is

$$\frac{dV}{dt} = C_2 - (C_3 + C_4 Z) V^2 \quad (3.69)$$

The solution to this equation is

$$V^2 = V_0^2 \exp(-2C_3Z - C_4Z^2) + 2 \exp(-2C_3Z - C_4Z^2) \times C_2 \int_0^Z \exp(2C_3Z + C_4Z^2) dz \quad (3.70)$$

where V_0 is the entering velocity. This can be rearranged to

$$V^2 = V_0^2 \exp(C_3^2/C_4) \exp(-u^2) + (2C_2/\sqrt{C_4}) \exp(-u^2) \int_{C_3/\sqrt{C_4}}^u \exp(u^2) du \quad (3.71)$$

where

$$u = C_3/\sqrt{C_4} + \sqrt{C_4}Z$$

Hence, rod velocity in the dashpot can be evaluated at each Z .

An additional concern in the hydraulic design of rod cluster control rods is the vibration of the rodlets when the control rod is in the fully withdrawn or nearly fully withdrawn position. Such vibration leads to wear and eventual penetration of the tube containing the poison material.

The upper portion of the control assembly is contained in a solid guide tube and is thus protected from fluid buffeting. However, in the region just above the fuel assembly, a solid guide tube cannot be provided because provisions must be made for the exiting coolant to leave. In this region, where the coolant is flowing out of the assembly and across the core to an exit port, separate partial shrouds are provided for each rodlet. The shrouds are usually hollow tubes with a portion of the back side cut away to allow access for the spider. The open area means that the rodlets feel some effect of the cross-flow in this region. Since the tubes are not fully exposed to the cross-flow stream, the equations for the previous section are not directly applicable. Because of the complexity of the geometry, it would appear that a determination of whether a proposed design is satisfactory requires experimental verification.⁸⁸

3.3 TWO-PHASE FLOW

Although there is no net void at the outlet of a PWR under normal operating conditions, there is substantial subcooled boiling within the cores of modern plants. Bulk boiling is allowed in the hottest channels; further, during most accident situations, boiling occurs throughout most of the core. Neither steady-state nor transient behavior can be understood without a knowledge of two-phase flow fundamentals. Since the details of two-phase flow behavior are available in a number of references (see Refs. 89, 90, and 91), only a summary is presented here.

As noted previously, the discussion of this chapter is confined to steam-water behavior and does not deal with two-phase flow situations that might be encountered in severe accidents.

3.3.1 Two-Phase Flow Structure (Flow Patterns)

When liquid and vapor flow together in a channel, there must be an interface between the phases. In most situations, the flow takes on a particular structure or configuration. The various configurations seen are generally designated as *flow patterns*.

The typical flow patterns encountered in cocurrent flow through a horizontal tube are illustrated in Fig. 3.13(a). At the lowest gas and liquid flow rates, the liquid flows along the bottom of the tube and the gas along the top, which is designated stratified flow. At higher gas flow rates, waves are found on the gas-liquid interface; this is called *wavy flow*.

At higher liquid flow rates than those that occur during stratified flow, we encounter *bubble flow* if the gas flow rate is very low. Here, small bubbles of gas tend to flow in the upper portion of the tube. When the gas flow rate is increased, large bullet-shaped bubbles, called *plugs*, are formed. In plug flow, these plugs move through the liquid along the upper portion of the tube. An increase in gas flow leads to *slug flow*, where frothy slugs of liquid move rapidly across the upper region of the tube. Between the slugs there is a wavy layer of liquid at the bottom of the tube.

At high gas velocities *annular flow* appears. Here the liquid flows around the outside of the tube while the gas flows in a central core. Further increases in gas velocities lead to entrainment of some of the liquid as droplets carried along in the central gas core.

At very high mass flow rates, we encounter dispersed flow. At low qualities this is seen as a froth of tiny bubbles essentially uniformly distributed in the liquid. At high qualities it is seen as a mist of fine droplets suspended in the vapor.

A significant change in flow patterns occurs when a line is inclined slightly to the horizontal. The stratified flow pattern disappears and is replaced by intermittent flow. Weisman and Kang⁹² suggest that stratified flow will disappear when

$$\sin\theta > D/L \quad (3.72)$$

where θ is the angle of inclination and L is the length of line being examined.

Wavy flow is still seen in inclined lines, but the wavy flow region is restricted. As the inclination angle increases, the gas flow rate at which wavy flow begins also increases. At a sufficiently sharp angle of inclination, wavy flow disappears altogether. However, the bubble flow area expands as inclination is increased.

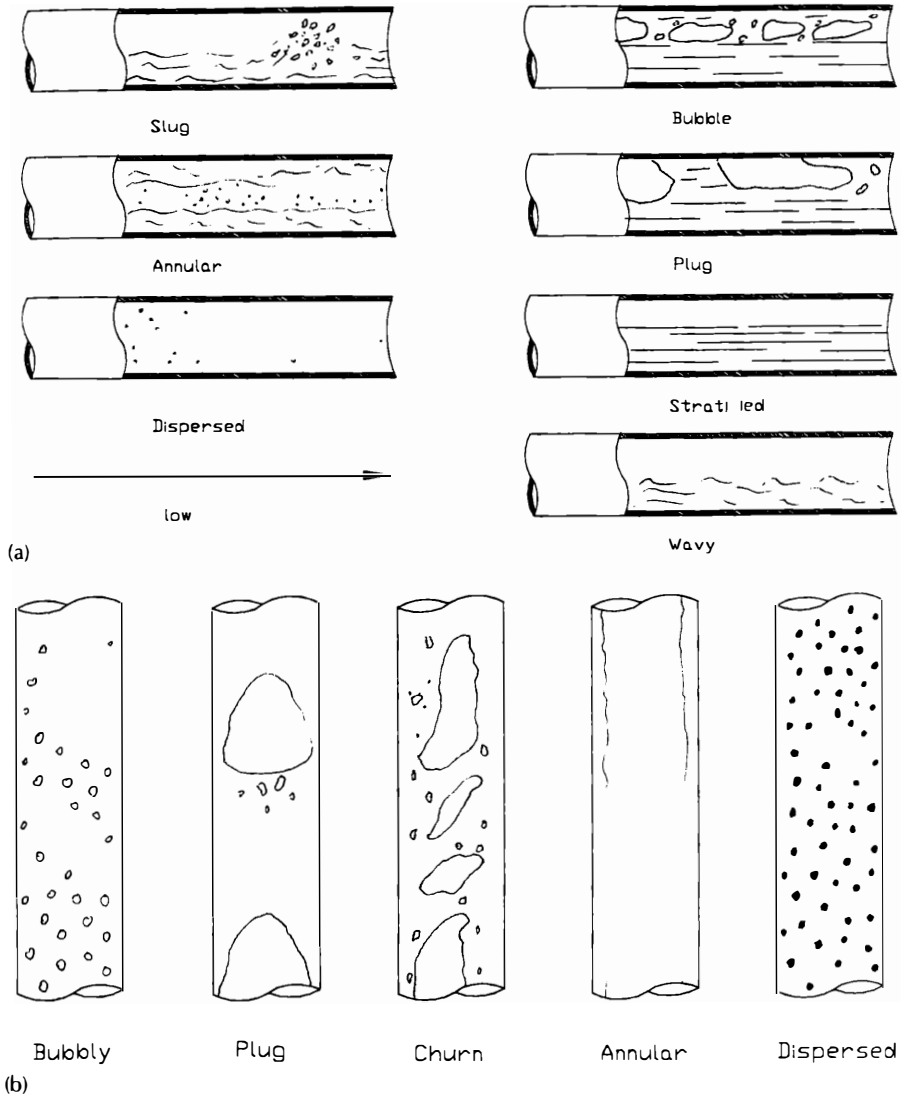


Fig. 3.13 Flow patterns in (a) horizontal flow and (b) vertical flow [from T. Crawford, Ph.D. Thesis, Drexel University, Philadelphia, PA (1983)].

In vertical lines, and lines at very sharp angles of inclination, slug flow disappears and the only flow patterns seen are bubbly, plug, churn, and annular (Fig. 3.13). Churn flow is a chaotic mixture of large packets of gas and liquid, which seem to have a churning motion.

Since flow patterns can affect pressure drop and clearly will affect inter-phase mass and momentum transfer, it is desirable to be able to predict the

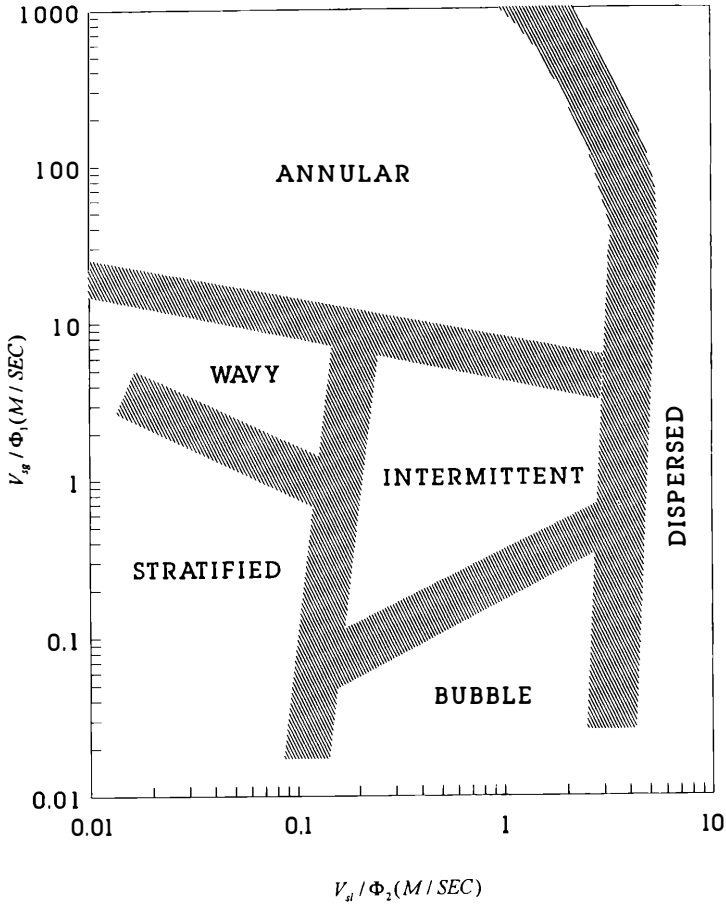


Fig. 3.14(a) Generalized flow pattern map for horizontal flow.

flow pattern that will exist at a particular set of conditions. Early attempts at correlating the transitions from one flow pattern to another⁹³ attempted to use the same dimensionless groupings for all transitions. Subsequent analyses^{94,95} have indicated that this is inappropriate and that separate mechanisms govern each transition. Despite this complexity, simplified flow maps in terms of superficial gas and liquid velocities have been devised.^{92,95,96} The steady-state maps applicable to horizontal and vertical flow are shown in Figs. 3.14(a) and (b). The fact that different mechanisms govern the individual transitions is accounted for by plotting the maps in terms of (V_{SG} / ϕ_1) versus (V_{SL} / ϕ_2) where V_{SG} , V_{SL} are the superficial gas and liquid velocities, respectively (velocity obtained assuming the specified phase is flowing alone), and ϕ_1 , ϕ_2 are the property area pipe diameter

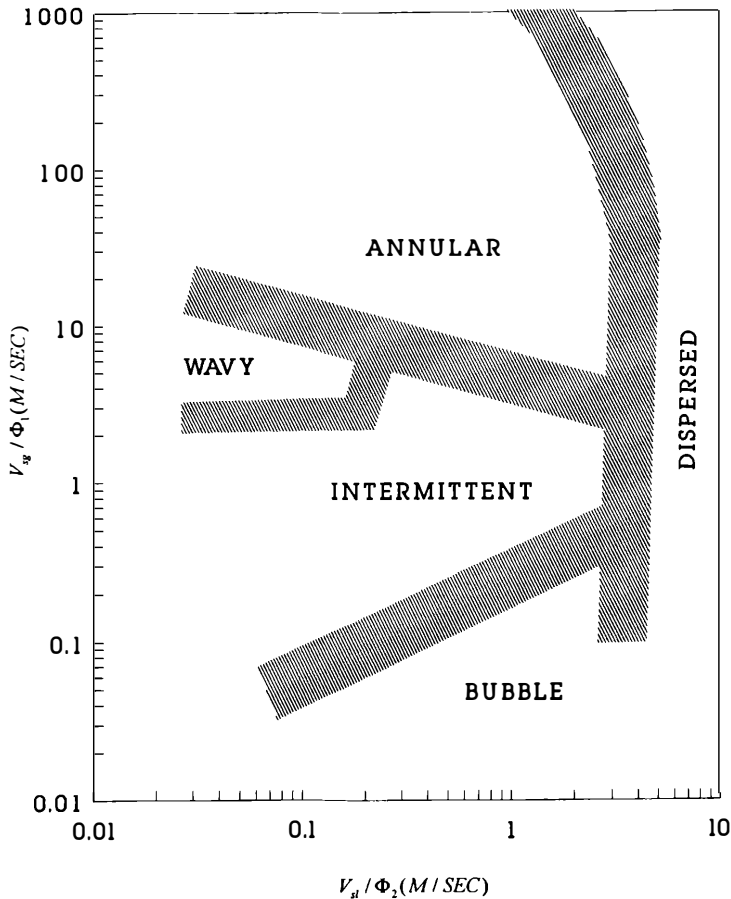


Fig. 3.14(b) Generalized flow pattern map for vertical flow.

correction factors obtained from Table 3.III. To simplify the maps, the slug, plug and churn flow patterns have all been combined and considered to be "intermittent" flow.

Crawford *et al.*⁹⁶ found that the flow maps of Fig. 3.14 could be extended to inclined and downward flow by appropriately modifying Φ_1 and Φ_2 for these cases. These modifications are shown in Table 3.III. Note that since Fig. 3.14(b) is used for both upward and downward flow, the unstable region, seen at very low liquid velocities in downward flow, is not shown. This region occurs when the buoyancy-induced upward velocity of the vapor is equal to or exceeds the downward velocity of the liquid.

Crawford and Weisman⁹⁷ have compared the flow pattern map of Fig. 3.14(b) for vertical flow in simple tubes under adiabatic conditions to data

TABLE 3.III
Property and Pipe Diameter Corrections to Overall Flow Map*

Transition	Flow Orientation	ϕ_1	ϕ_2
To dispersed flow	All	$\left(\frac{\rho_{GS}}{\rho_G}\right)\left(\frac{\rho_L}{\rho_{LS}}\right)^{0.67}\left(\frac{D}{D_s}\right)^{0.21}\left(\frac{\mu_{LS}}{\mu}\right)^{0.09}\left(\frac{\sigma}{\sigma_s}\right)^{0.24}$	$\left(\frac{\rho_L}{\rho_{LS}}\right)^{0.33}\left(\frac{D}{D_s}\right)^{0.21}\left(\frac{\mu_{LS}}{\mu_L}\right)^{0.09}\left(\frac{\sigma}{\sigma_s}\right)^{0.24}$
To annular flow	All	$\left(\frac{\rho_{GS}}{\rho_G}\right)^{0.23}\left(\frac{\Delta\rho}{\Delta\rho_s}\right)^{0.11}\left(\frac{\sigma}{\sigma_s}\right)^{0.11}\left(\frac{D}{D_s}\right)^{0.415}$	1.0
Intermittent-separated	a) Horizontal and slightly inclined	1.0	$\left(\frac{D}{D_s}\right)^{0.45}$
	b) Sharply inclined	separated flow disappears $\left[1 + 4(\sin 1.95 \theta) \exp\left(\frac{12}{3 + \theta }\right)\right]$	
	c) Downward		
Bubble-intermittent	a) Horizontal and upward	$\left(\frac{D}{D_s}\right)^{0.8} - 0.65 \cos\theta$	$\frac{D}{D_s}$
	b) Downward	$\left(\frac{D_s}{D}\right)^{0.19} [0.65 - 0.3 (\cos\theta)^{0.3}] \left[0.6 \left(\frac{V_{SL}}{V_{SG}}\right)^{0.73}\right]$	1.0

*S denotes standard conditions. $D_s = 1.0$ in. = 2.54 cm, $\rho_{GS} = 0.0013$ kg/l, $\rho_{LS} = 1.0$ kg/l; $\mu_{LS} = 1$ cP; $\sigma_s = .07$ N/m (70 dynes/cm) $\Delta\rho = \rho_L - \rho_G$
Source: Based on T. J. Crawford, Ph.D. Thesis, Drexel University, Philadelphia, PA (1983).

obtained with adiabatic flow (with heating). They concluded that although considerable scatter was observed, most of the data were in reasonable agreement with the map. However, they recommended that the intermittent-bubble correction factor be modified at high values of (V_{SL}/ϕ_2) . They suggested that if $(V_{SL}/\phi_2) \geq 0.15$, then the correction factors in Table 3.III for the transition be changed to

$$\phi_1 = (D_S/D)^{0.1} \quad \text{and} \quad \phi_2 = 1.0 \quad (3.73)$$

Crawford and Weisman⁹⁷ also compared the flow map of Fig. 3.14 to high-pressure water-steam data taken in rod bundles under adiabatic conditions. The map was used replacing D by De . They found appreciable scatter but that the available data at pressures of 69 bar and above were in rough general agreement with the proposed map.

It is not clear that steady-state flow pattern maps based on superficial velocities are applicable under transient conditions. It has been suggested by both McFadden *et al.*⁹⁸ and Mishima and Ishii⁹⁹ that, for transient conditions, a map in terms of the total mass flux, G , and the volume fraction of vapor, α , would be more appropriate. Crawford *et al.*¹⁰⁰ tested this supposition in a series of flow transients. They translated Fig. 3.14(b) into G - α coordinates for his system conditions and compared his flow pattern observations to the resulting map. They found generally reasonable agreement but that there was a tendency for the existing flow pattern to persist slightly past the steady-state boundary.

The RELAP-5 accident analysis program¹⁰¹ follows the G - α approach and uses the flow pattern map shown in Fig. 3.15. The map has the same general features as the map for vertical flow shown in Fig. 3.14(b) but it does not include a wavy flow region. It also distinguishes between dispersed bubble and dispersed droplet flow. Since the same flow pattern map is used for all pressure, it does not allow for the effect of fluid properties on the transition locations.

The TRAC accident analysis program¹⁰² does not use a central flow pattern calculation but it does distinguish between flow patterns in the calculation of interfacial transfer rates. The criteria used are:

Bubbly/churn flow for $\alpha < \alpha_{\text{tran}}$

Annular flow for $\alpha_{\text{tran}} < \alpha < \alpha_{\text{tran}} + 0.25$

Dispersed droplet flow for $\alpha > \alpha_{\text{tran}} + 0.25$

The parameter α_{tran} is a function of the density ratio, (ρ_l/ρ_g) . It may be seen that TRAC does not consider mass velocity effects.

Under normal PWR operating conditions, the mass velocity is generally above 2×10^6 lb/hr ft². At this velocity, and the water properties found at PWR temperatures, dispersed flow is to be expected in the hot channels.

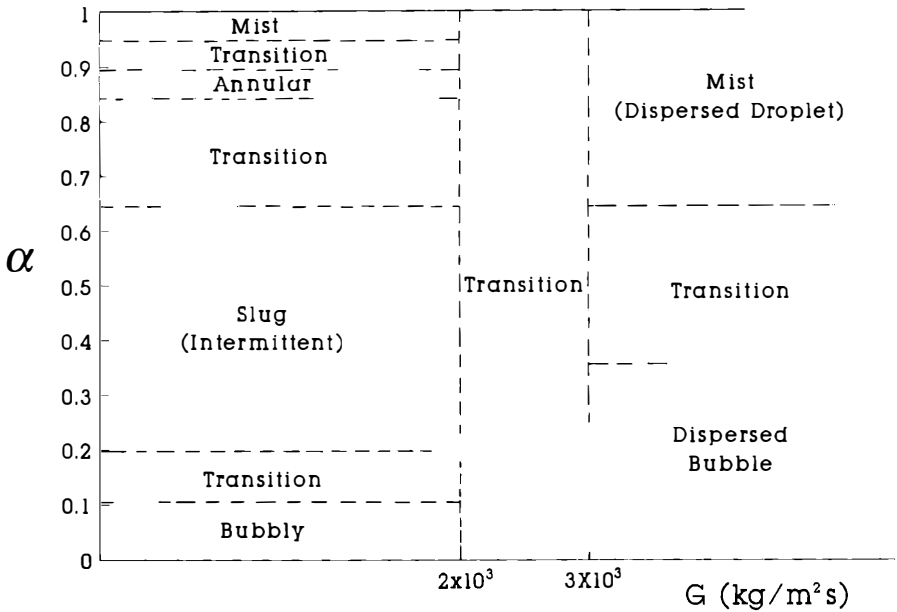


Fig. 3.15 Flow pattern map used by RELAP-5 [from Ref. 101].

At somewhat lower velocities, bubble and intermittent (churn) flow would be encountered. Because of the relatively low vapor volume fractions at normal or near-normal operating conditions, annular flow would not be seen. However, annular flow could be encountered during the reflood phase following a LOCA.

In addition to annular upflow in the core, countercurrent annular flow may be encountered in the downcomer during a LOCA.

3.3.2 Void Fraction

Because of the relatively low density of steam compared to liquid water, the steam vapor phase is called void. Void fraction, α , is defined as the ratio of local vapor volume to the total local flow volume, that is,

$$\alpha = \frac{A_g}{A_g + A_l} \quad (3.74)$$

where A_l is the cross-sectional area occupied by the liquid and A_g is the cross-sectional area occupied by the vapor. The average density, $\bar{\rho}$, of a two-phase mixture then becomes

$$\bar{\rho} = \alpha \rho_g + (1 - \alpha) \rho_l, \quad (3.75)$$

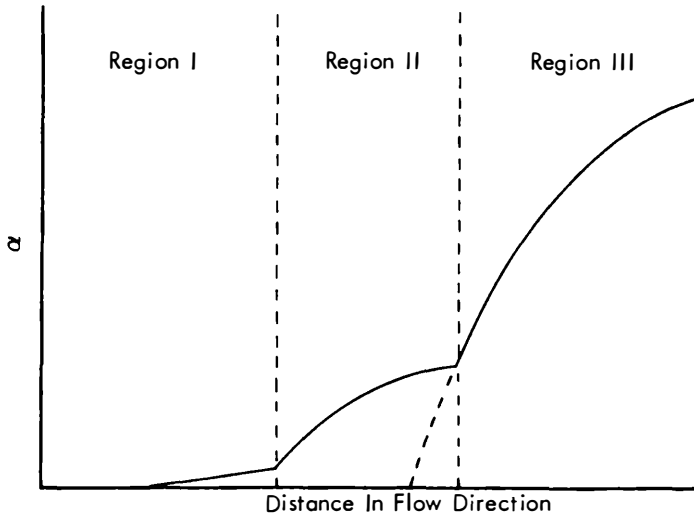


Fig. 3.16 Void fraction in boiling region (from Ref. 104).

where ρ_g and ρ_l are the vapor and liquid densities, respectively. Slip ratio S is defined as the ratio of vapor velocity to liquid velocity

$$S = \frac{V_g}{V_l} = \frac{\chi}{1 - \chi} \frac{1 - \alpha}{\alpha} \frac{\rho_l}{\rho_g} \quad (3.76)$$

where χ = flow quality = mass flow rate of vapor/mass flow rate of mixture.

Void fraction in a heated channel can be described with the aid of Fig. 3.16. Initially, the entire fluid is highly subcooled and no void is present. When the liquid adjacent to the wall becomes superheated, bubbles form. In region I, where the bulk of the liquid is still highly subcooled, the voids are a thin layer attached to the wall. In region II, where subcooling is low, void fraction increases and bubbles are detached from the wall and recondense very slowly. In region III, the bulk liquid has reached saturation and bulk boiling occurs.

Maurer¹⁰³ correlated void fraction in region I in terms of δ , the effective thickness of the vapor film on the wall. Thus,

$$\alpha = \frac{4\delta}{De} \quad (3.77)$$

and δ is the maximum of 0.00033 ft or

$$\delta = \frac{q_b'' k_l Pr_l}{1.07 h^2 (T_{sat} - T_{bulk})} \quad (3.78)$$

where

q_b'' = heat transfer rate due to boiling = $q''_{\text{total}} - h(T_{\text{sat}} - T_b)$

h = forced convection heat transfer coefficient

k_l = thermal conductivity of the liquid

Pr_l = Prandtl number of the liquid.

The value of α computed from Eqs. (3.77) and (3.78) is always very low and therefore it is often ignored.

Early work by Bowring¹⁰⁴ showed that the detached region begins at a critical fluid subcooling that depends on flow velocity and heat flux. Correlations for the detachment point criterion have been presented by Bowring¹⁰⁴ and several other investigators.^{105–108} Of these, the criteria of Levy¹⁰⁷ and Saha and Zuber¹⁰⁸ appear to be the most successful.

The Levy¹⁰⁷ model is based on the idea that at the bubble tip, which is located at dimensionless distances, Y_B^+ from the wall, the liquid temperature should be as warm or warmer than the saturation temperature. The departure criterion is then established by assuming that the liquid temperature follows Martinelli's universal temperature profile and exceeds the saturation temperature at the departure point. The value of Y_B^+ is given by

$$Y_B^+ = 0.01 \frac{(\sigma g_c D_h \rho_l)^{1/2}}{\mu_l} \quad (3.79)$$

and the departure criteria are:

$$\text{if } 0 \leq Y_B^+ \leq 5.0, \quad H_f - H_{ld} = c_p \frac{q''}{h} - \frac{q''}{G(f/8)^{1/2}} \text{Pr } Y_B^+ \quad (3.80a)$$

$$\begin{aligned} \text{if } 5.0 \leq Y_B^+ \leq 30.0, \quad H_f - H_{ld} = c_p \frac{q''}{h} - \frac{5.0q''}{G(f/8)^{1/2}} \\ \{ \text{Pr} + \ln[1 + \text{Pr}(Y_B^+ / 5.0 - 1.0)] \} \end{aligned} \quad (3.80b)$$

$$\begin{aligned} \text{if } Y_B^+ \geq 30.0, \quad H_f - H_{ld} = c_p \frac{q''}{h} - \frac{5.0q''}{G(f/8)^{1/2}} \\ \{ [\text{Pr} + \ln(1.0 + 5.0\text{Pr}) + 0.5 \ln(Y_B^+ / 30.0)] \} \end{aligned} \quad (3.80c)$$

where

c_p = specific heat of liquid

G = mass flux, mass/area time

h = single-phase forced convection heat transfer coefficient

H_f = enthalpy of saturated liquid

H_{ld} = enthalpy of liquid at point of bubble departure

f = friction factor

D_h = hydraulic diameter

q'' = total heat flux, energy/ L^2t

Pr = liquid phase Prandtl number

σ = surface tension

μ_l = liquid phase viscosity.

A somewhat simpler departure criterion, which appears to work about as well, has been presented by Saha and Zuber.¹⁰⁸ They concluded that at low values of the Peclet number, bubble departure was thermally controlled, while at high Pe values it was hydrodynamically controlled. Their analysis yields the following criteria:

$$\text{for } Pe \leq 70,000, \quad T_{\text{sat}} - T_L = 0.0022 \left(\frac{q'' D}{k_l} \right) \quad (3.81a)$$

$$\text{for } Pe > 70,000, \quad T_{\text{sat}} - T_L = 153.8 \frac{q''}{c_p G} \quad (3.81b)$$

where

T_{sat} = saturation temperature

T_L = bulk liquid temperature

Pe = Peclet number = $Re Pr$

k_l = thermal conductivity of liquid.

At fluid enthalpies above that at the bubble departure point, a significant void fraction may exist even though the liquid enthalpy is below that of saturated liquid. Lahey and Moody¹⁰⁹ present a mechanistic model for computing the quality in this region. The model is based on estimating the vapor produced by the fraction of the applied heat flux going to boiling and subtracting the condensation of vapor by the subcooled liquid core. Their expression has the form

$$x^1(z) = \frac{1}{GAH_{fg}} \left[\int_{z_d}^z \left(\frac{p_h q_b''}{1 + \epsilon^1} \right) dz - \int_{z_d}^z p_h q''_{\text{cond}} dz \right] \quad (3.82)$$

where

A = flow area

$x^1(z)$ = actual quality at location z ($z > z_d$)

z_d = location at which bubble departure occurs

p_h = heated perimeter

q_b'' = heat flux due to boiling and bubble agitation = $q'' \left(\frac{H_l - H_{ld}}{H_f - H_{ld}} \right)$

H_l = enthalpy of liquid at location $z = H(z) - x^1(z)H_{fg}$

$H_{(z)}$ = total fluid enthalpy at location z

q'' = total heat flux

q''_{cond} condensing heat flux $= h'_0 \frac{H_{fg}}{v_{fg}} \frac{A}{p_h}$

H_{fg}, v_{fg} = enthalpy and specific volume change on evaporation, respectively

h'_0 = condensation related coefficient $= 150 (h \text{ } ^\circ\text{F})^{-1}$

$\epsilon^1 = \rho_l(H_f - H_l) / [\rho_g H_{fg}]$.

Note that once $H_l = H_f$, the liquid enthalpy remains at H_f and all of the heat flux goes toward boiling.

In view of the cumbersome nature of the foregoing relationship, simplified approaches are often used in design. A very simple approach is the so-called "profile-fit" method where the actual vapor mass fraction $x^1(z)$ is related to the "thermodynamic quality," $x(z)$. The relationship used is

$$x^1(z) = x(z) - x_{ld} \exp \left[\frac{x(z)}{x_{ld}} - 1 \right] \quad (3.83)$$

where

$$x(z) = H(z) / H_{fg}$$

$$x_{ld} = (H_{ld} - H_f) / H_{fg}$$

Note that x_{ld} will be negative.

Empirical correlations have also been used. Thom *et al.*¹¹⁰ correlated void fractions in subcooled boiling for the 750- to 1000-psi range. Their correlation, which is in terms of apparent density $\bar{\rho}$, appears somewhat simpler. Tong¹¹¹ extended their correlation to 2000 psi; the modified correlation is

$$\bar{\rho} = \rho_l - \frac{\gamma' X' (\rho_l - \rho_g)}{1 + X' (\gamma' - 1)} \quad (3.84)$$

where

ρ_l, ρ_g = liquid and vapor densities, respectively

$$X' = \left(H_{in} + \frac{4Lq''}{D_c G} - \beta H_f \right) / (H_v - \beta H_f)$$

$$\gamma' = \rho_l / (\rho_v S) = \exp \{ 4.216 [(y - 8.353)^2 / (8.353)^2 - 1]^{1/2} \}$$

$$y = \ln(P / 3206)$$

P = pressure in psia

β = ratio of enthalpy at inception of detached region to the saturation enthalpy $= 1 - 0.15 q'' / G$

H_{in} inlet enthalpy to channel, Btu/lb

H_v, H_f = saturation enthalpies of vapor and liquid, respectively, Btu/lb

G = mass velocity, lb/(h ft²)

S = slip ratio

L = heated length, ft.

The above equation was developed from uniform heat flux data; although it has been applied to channels with nonuniform heat flux and mixing by replacing $(4L_q/DeG)$ by the enthalpy rise to the point in question, experimental verification is lacking.

When bulk boiling begins, quality χ is determined from fluid enthalpy H by

$$H = \chi H_v + (1 - \chi) H_f \quad (3.85)$$

Void fraction is obtained from the quality. However, void fraction and slip ratio during bulk boiling vary with the flow pattern. At the pressures and flows encountered in a PWR, the flow pattern existing at the onset of boiling will be small vapor bubbles dispersed through a liquid core (dispersed or bubbly flow). At higher void fractions, small bubbles may coalesce into large elongated bubbles, and churn flow may be seen. If enthalpy is increased beyond the range normally observed in a PWR, the vapor forms a continuous core with an annulus of liquid adjacent to the wall (annular flow).

The simplest treatment of dispersed or bubbly flow is Bankoff's.¹¹² By assuming a power law distribution for velocity and void fraction, the following equation relating x and α was derived:

$$1/\chi = 1 - \rho_l/\rho_g(1 - K'/\alpha) \quad (3.86)$$

with the flow factor K defined by

$$K' = \alpha + (1 - \alpha)/S \quad (3.87)$$

Bankoff¹¹² proposed that constant K' can be obtained from

$$K' = 0.71 + 0.0001 P \quad (3.88)$$

where P is in psia. Revised values for K' were proposed by Kholodovski¹¹³ and Hughmark.¹¹⁴ By correlating available data, Hughmark concluded that K' was a function of parameter Z , where

$$Z = \left[\frac{D_c G}{\mu_l(1 - \alpha) + \mu_g \alpha} \right]^{1/6} \left(\frac{G^2}{\rho_{hg}^2 D_c} \right)^{1/8} \left[\frac{(1 - \chi)\rho_g + \chi\rho_l}{(1 - \chi)\rho_g} \right]^{1/4} \quad (3.89)$$

where ρ_h = mixture density obtained by assuming no slip and μ_l, μ_g = liquid and vapor viscosities, respectively. All other symbols have their previous meaning. The relationship between K' and Z proposed by Hughmark is

TABLE 3.IV
Relationship Between Flow Factor K and Parameter Z

Z	K	Z	K	Z	K	Z	K	Z	K
1.3	0.185	3.0	0.49	6.0	0.72	15	0.808	70	0.93
1.5	0.225	4.0	0.605	8.0	0.767	20	0.83	130	0.98
2.0	0.325	5.0	0.675	10	0.78	40	0.88		

given in Table 3.IV Use of Hughmark's relationship leads to slip ratios that approach 1.0 as mass velocity increases. This is in accord with experimental observations. The Hughmark correlation tends to give poor results for alphas (α) greater than ~ 0.8 . At high α , it is probably preferable to obtain the relationship between α and χ by simply extrapolating the Hughmark χ versus α curve developed for $\alpha < 0.75$ so that the curve goes through $\alpha = 1$ at $\chi = 1.0$.

More recently, Zuber and Findlay¹¹⁵ proposed a more sophisticated model which can now be applied to all flow patterns, although it is most useful in the bubble, dispersed and intermittent regions. They define local superficial velocities $u = (Q_v + Q_l)/A$, $u_v = (Q_v/A)$, and $u_l = (Q_l/A)$ based on total flow area A and volumetric flows, Q . Velocity and void distributions were assumed to follow

$$\frac{u'}{u_c} = 1 - \left(\frac{r}{R} \right)^n \quad (3.90a)$$

$$\frac{\alpha - \alpha_w}{\alpha_c - \alpha_w} = 1 - \left(\frac{r}{R} \right)^n \quad (3.90b)$$

where r is radial distance, R is tube radius, and subscripts c and w refer to centerline and wall conditions, respectively. For adiabatic flow, α_w is taken as zero.

If locally the vapor were to have the same velocity as the liquid, there still would be a difference between the average vapor and liquid velocities since the vapor is not uniformly distributed over the cross section. Under the condition of equal local vapor and liquid velocities, we may define a distribution factor C_o such that:

$$C_o = \beta / \bar{\alpha}' = \frac{n+2}{n+1} \quad (3.91)$$

where $\beta = Q_v / (Q_v + Q_l)$, with Q_v , Q_l as the volumetric flow rates of vapor and liquid, respectively, and $\bar{\alpha}'$ as the mean value of α assuming locally equal vapor and liquid velocities. Note also that with equal local velocities, C_o equals $1/K'$ in the Bankoff¹¹² and Hughmark¹¹⁴ void models [Eq. (3.86)].

In actuality, the local liquid and vapor velocities are not identical and the true average vapor velocity, V_v , is given by

$$V_g = C_o u + \bar{u}_{gl} \quad (3.92)$$

where $V_g = u_v / \alpha$ and $\bar{u}_{gl}$ = weighted mean drift velocity of vapor. It is also usual to define a quantity, j_{gl} , called the *drift flux*:

$$j_{gl} = \bar{u}_{gl} \bar{\alpha} \quad (3.93)$$

Drift flux j_{gl} represents the volumetric rate at which vapor flows through (forward in upflow, backward in downflow) a unit area of a plane normal to the channel axis and moving with the flow at a velocity u . This approach to void estimation is commonly referred to as the *drift flux model*.

The value of u_{gl} , the drift velocity, is closely related to the terminal rise velocity of a bubble through the liquid and it is often assumed to be identical with this rise velocity. The value of u_{gl} is positive in vertical upflow, negative in downflow, and often taken as zero in horizontal flow. Once the value of C_o and u_{gl} are specified, the average void fraction, α , may be computed from

$$\alpha = \frac{\beta}{C_o + u_{gl}/u} = \frac{x/\rho_g}{C_o[x/\rho_g + (1-x)\rho_l] + u_{gl}/G} \quad (3.94)$$

At high pressure (typical PWR conditions), u_{gl} is small and the exact value used will probably not affect the results significantly.¹¹⁶

The slip ratio, S , may be determined if C_o and u_{gl} are known by

$$S = \frac{[(C_o G / \rho_l) + u_{gl}] \rho_l (1 - \alpha)}{(1 - \alpha C_o)[G - \alpha \rho_g u_{gl} / (1 - \alpha C_o)]} \quad (3.95)$$

Early attempts at defining C_o generally were based on determining the appropriate values of n in Eq. (3.90) to be used with the various flow patterns. It can be shown that $C_o < 1$ when the void concentration at the wall is greater than in the central region and $C_o > 1$ for the reverse conditions. For homogeneous flow, $C_o = 1$ and $u_{gl} = 0$. Most recent applications have tended to regard C_o simply as an empirical quantity to be fit to the available data. Chexal and Lellouche¹¹⁷ have developed a very comprehensive set of expressions for C_o and u_{gl} , which fit a wide variety of data. However, the model is complex and cumbersome to use. A much simpler model, which has been shown to perform well,¹¹⁸ is that of Dix¹¹⁹ as modified by Lahey and Moody.¹⁰⁹ In this model

$$C_o = \beta \{1 + [(1/\beta) - 1]^b\} \quad (3.96)$$

where

$$b = (\rho_g / \rho_l)^{0.1}$$

$$u_{gl} = 2.9 \left[\frac{(\rho_l - \rho_g) \sigma g g_c}{\rho_l^2} \right]^{1/4} \sin \theta \quad (3.97)$$

σ = surface tension

θ = angle of inclination (positive if upward).

Chexal *et al.*¹¹⁹ indicate that the model generally performed well except in large pipes.

Kawanishi *et al.*¹²⁰ present a more complex set of relationships for C_o and u_{gl} , which are, however, less complex than the Chexal-Lellouche equations.¹¹⁷ The proposed relationships are provided in Table 3.V Kawanishi *et al.*¹²⁰ compare their correlation to a wide variety of upflow and counterflow data and obtain better agreement with the data than with the Chexal-Lellouche approach.

The drift-flux model is now (1994) the most widely used approach for high-velocity forced convection systems. However, it appears to provide

TABLE 3.V
Drift Flux Parameters of Kawanishi *et al.*¹²⁰

Average Velocity of Mixture u (m/s)	Pressure P (MPa)	Tube Diameter D (m)	Drift Velocity u_{gl}
$u \leq 0$			$\sqrt{2} \left\{ \frac{\sigma g (\rho_l - \rho_g)}{\rho_l^2} \right\}^{1/4}$
$0 < u < 0.24$	$P \leq 18$	$D \leq 0.61$	Interpolate
$u \geq 0.24$	$P \leq 1.5$	$D \leq 0.05$	$0.35 \left\{ \frac{g D (\rho_l - \rho_g)}{\rho_l} \right\}^{1/2}$
		$0.05 \leq D$	$0.52 \left\{ \frac{g D (\rho_l - \rho_g)}{\rho_l} \right\}^{1/2}$
	$1.5 < P \leq 18$	$D \leq 0.02$	Same as $P \leq 1.5$ MPa
		$0.02 < D < 0.24$	Interpolate
		$0.24 \leq D \leq 0.46$	$0.048 \sqrt{\frac{\rho_l}{\rho_g}} \left\{ \frac{g D (\rho_l - \rho_g)}{\rho_l} \right\}^{1/2}$
Average Velocity of Mixture u (m/s)	Distribution Parameter C_o		
$u \leq -3.5$	$1.2 - 0.2 \sqrt{\rho_g / \rho_l}$		
$-3.5 \leq u \leq -2.5$	$0.9 + 0.1 \sqrt{\rho_g / \rho_l}$		
	$-0.3(1 - \sqrt{\rho_g / \rho_l})(2.5 + u)$		
$-2.5 \leq u < 0$	$0.9 + 0.1 \sqrt{\rho_g / \rho_l}$		
$0 \leq u$	$1.2 - 0.2 \sqrt{\rho_g / \rho_l}$		

less satisfactory results at low velocities and natural circulation conditions. At the low velocities present during the reflood phase of a LOCA, Yeh and Hochreiter¹²¹ recommend

$$\alpha = 0.925(\rho_g/\rho_l)^{0.239} \left(\frac{u_v}{u_{cr}} \right)^b \left(\frac{u_v}{u_v + u_l} \right)^{0.6} \quad (3.98)$$

where

$$u_{cr} = 1.53g^{1/2}(g_c \sigma / g \rho_l)^{1/4}$$

$$b = 0.67 \text{ if } u_v/u_{cr} < 1$$

$$b = 0.47 \text{ if } u_v/u_{cr} \geq 1$$

u_v, u_l = superficial velocities of vapor and liquid, respectively.

For natural circulation systems operating between 10 and 100 bar, Mochizuki and Ishi¹²² recommend that the void fraction be obtained from the slip ratio, S , where

$$S = K^1 + (1 - K^1) \left[\frac{(\rho_l/\rho_g) + K^1(1/x - 1)}{1 + K^1(1/x - 1)} \right]^{0.5} \quad (3.99)$$

and $K^1 = 0.95 \tanh(5x) + 0.05$.

In the annular flow region, the effect of mass flux appears to be relatively small. The void fraction relationship of Martinelli and Nelson,¹²³ which is presented as a plot of α versus quality, for various pressures, has therefore often been used. Thom¹²⁴ modified these curves on the basis of improved data and proposed a correlation of the form

$$\alpha = \frac{\gamma_1 x}{1 + x(\gamma_1 - 1)} \quad (3.100)$$

where x is flow quality and γ_1 is an empirical constant equal to $P/(\rho_l S)$. Thom proposed the values of γ_1 shown in Table 3.VI.

All of the relationships between x and α indicated in this section apply strictly to steady-state conditions only. During rapid transients, the relationship between x and α may not be identical to that seen in the steady state. This question is considered in Sec. 3.3.4.3.

TABLE 3.VI
Parameter Values for the Thom Void Fraction Correlation

Pressure (psia)	Values of γ					
	14.7	250	600	1250	2100	3000
γ	246	40.0	20.0	9.80	4.95	2.15
						3206
						1.6

3.3.3 Two-Phase Pressure Drop

3.3.3.1 Pressure Drop in Pipes and Rod Bundles

In the highly subcooled boiling region where only attached wall voidage is present, it has sometimes been assumed that single-phase pressure drop calculations are adequate. However, this is not the case at high heat fluxes typical of reactor cores. The measured pressure drop is significantly above the single-phase value.

Correlations of subcooled boiling pressure drop were proposed by Owens and Schrock,¹²⁵ Le Tourneau and Troy,¹²⁶ and Tarasova et al.¹²⁷ The correlation of Tarasova et al.,¹²⁷ which includes both friction and acceleration losses, is particularly convenient. Tarasova gives the ratio of boiling pressure drop, $(dP/dz)_b$, to isothermal (nonboiling) pressure drop, $(dP/dz)_0$, as

$$(dP/dz)_b / (dP/dz)_0 = 1 + [20N / (1.315 - N)] \times [q'' v_g / (H_{fg} V_{in})]^{0.7} (v_g / v_l)^{0.08}, \quad (3.101)$$

where

$$N = (H - H_0) / (H_{sat} - H_0)$$

$$q'' = \text{wall heat flux, Btu/h ft}^2$$

$$v_g, v_l = \text{specific volume of gas and liquid, respectively, ft}^3/\text{lb}$$

$$H = \text{liquid enthalpy, Btu/lb}$$

$$H_{sat}, H_0 = \text{liquid enthalpy at saturation and inception of boiling, respectively, Btu/lb}$$

$$V_{in} = \text{inlet velocity, ft/h}$$

$$H_{fg} = \text{enthalpy change on evaporation, Btu/lb.}$$

The foregoing correlation was developed for data with heat fluxes between 0.18 to 0.55×10^6 Btu/h ft², pressures between 142 and 2840 psi, and G between 1.0 and 2.2×10^6 lb/h ft².

In the detached and bulk boiling regions, the increase in fluid volume significantly increases the velocity of the stream and, therefore, its momentum; consequently, all three pressure drop components indicated by Eq. (3.5)—frictional loss, momentum change, and elevation pressure drop—need to be considered separately under these conditions. At the low void fractions ($\alpha < 0.30$) and high mass flows usually encountered in a water-cooled reactor, two-phase pressure drop can be evaluated from the homogeneous model proposed by Owens.¹²⁸ The elevation pressure drop is given by

$$\Delta P_{\text{elev}} = \int_0^L \rho \, dL \approx \bar{\rho} \Delta L \quad (3.102)$$

For a local boiling flow, $\bar{\rho}$ can be obtained from Eq. (3.94). The friction pressure drop is obtained from

$$\Delta P_{\text{friction}} = \frac{f \Delta L}{De} \frac{\bar{v} G^2}{2g_c} \quad (3.103)$$

and the acceleration pressure drop term is obtained from

$$\Delta P_{\text{acc}} = \frac{G^2}{g_c} \frac{d\bar{v}}{dL} \Delta L \quad (3.104)$$

where G is the mass velocity (lb/h ft^2) and $\bar{v}$ is the average specific volume of the mixture at the location considered.

Evaluation of friction factor f in Eq. (3.103) requires estimating a Reynolds number. Owens¹²⁸ simply used the liquid viscosity here but other investigators have proposed using a two-phase mixture viscosity. In some cases, the formulation suggested by McAdams *et al.*¹²⁹ has been used:

$$\bar{\mu} = \chi \mu_g + (1 - \chi) \mu_l \quad (3.105)$$

where μ_g, μ_l = viscosities of gas and liquid, respectively, and $\bar{\mu}$ = viscosity of two-phase mixture. However, Choe¹³⁰ obtained improved agreement with available data by assuming that the viscosity of a bubbly mixture is actually higher than that of a pure liquid. Choe¹³⁰ suggests that fluid viscosity can be obtained from the following:

$$\bar{\mu} = \mu_l \exp[2.5/(1 - 39\alpha/64)] \quad \alpha \leq 0.5$$

or

$$\bar{\mu} = \mu_g + (\mu_l - \mu_g) \left(\frac{1}{\alpha} - 1 \right)^3 \quad \alpha > 0.5 \quad (3.106)$$

where

$$\mu_c = \bar{\mu} \quad \text{at} \quad \alpha = 0.5$$

The pressure drop obtained during high void-fraction flow again depends on the flow pattern; however, a general empirical correlation for calculating the two-phase frictional pressure drop was developed by Baroczy.¹³¹ His work is considered an extension of that of Martinelli and Nelson¹²³ and Lockhart and Martinelli¹³² who defined the two-phase pressure drop in terms of a single-phase pressure drop. Baroczy writes

$$\phi_{f0}^2 = \frac{(\Delta P / \Delta L)_{\text{TP}}}{(\Delta P / \Delta L)_l} \quad (3.107)$$

where

- ϕ_{f0}^2 = two-phase frictional pressure drop multiplier
- $(\Delta P / \Delta L)_{\text{TP}}$ = two-phase pressure drop per unit length
- $(\Delta P / \Delta L)_l$ = single-phase pressure drop obtained at the same mass velocity when the fluid is entirely liquid.

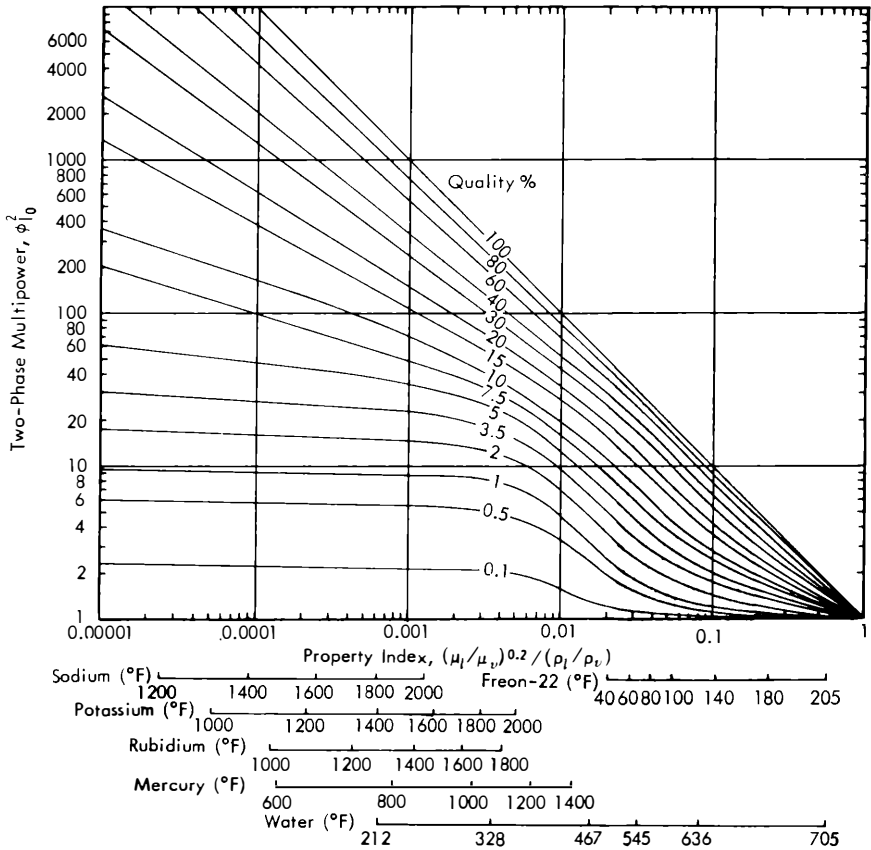


Fig. 3.17 Two-phase friction pressure drop correlation for $G = 1 \times 10^6$ lb/(h ft²) [from Ref. 131].

By defining a property index $[(\mu_l/\mu_v)^{0.2} (\rho_l/\rho_v)]$, Baroczy¹³¹ obtained a correlation for ϕ_{l0}^2 that was independent of pressure. He also observed that his correlation could be used with the gas-phase pressure drop $(\Delta P/\Delta L)_g$ by noting

$$\frac{1}{\Gamma^2} = \frac{(\Delta P/\Delta L)_l}{(\Delta P/\Delta L)_g} = \frac{(\mu_l/\mu_v)^{0.2}}{(\rho_l/\rho_v)} \quad (3.108)$$

and $(\Delta P/\Delta L)_g$ = single phase pressure drop obtained at the same mass velocity, G , when fluid is entirely vapor. His correlation is given in two sets of curves:

1. a plot of the two-phase multiplier ϕ_{l0}^2 as a function of property index $(\mu_l/\mu_v)^{0.2}/(\rho_l/\rho_v)$, as shown in Fig. 3.17

2. plots of a two-phase multiplier ratio as a function of property index, quality, and mass velocity as shown in Fig. 3.18.

The latter ratio multiplies ϕ_{l0}^2 whenever G deviates from 1×10^6 lb/h ft²

The somewhat whimsical way in which the mass velocity correction factors of Fig. 3.18 vary makes interpolation between the plotted mass velocities and extrapolation outside the velocity range difficult. Chisholm and Sutherland¹³³ found that they could represent the Baroczy curves in a compact and orderly form. They wrote

$$\phi_{l0}^2 = 1 + CX + \frac{1}{X^2} \quad (3.109)$$

where $X = \text{Martinelli parameter} = (\rho_g / \rho_l)^{1/2} [(1-x)/x]^{0.9} (\mu_l / \mu_g)^{0.1}$ for fully turbulent liquid and gas flow. The parameter C depends on the mass velocity and the Baroczy property index as shown in Fig. 3.19. Note that Fig. 3.19 is given in terms of Γ , while Fig. 3.17 is in terms of $1/\Gamma^2$

An alternative approach, which shows good accuracy when $(\mu_l / \mu_g) > 1000$ and $G > 100$ kg/m² s, is provided by Chisholm's correlation¹³⁴:

$$\phi_{l0}^2 = 1 + (\Gamma^2 - 1)[Bx^{2-n/2}(1-x)^{2-n/2} + x^{2-n}] \quad (3.110)$$

where B is determined from Table 3.VII(a) and $n = 0.2$ for axial flow.

The values of ϕ_{l0}^2 obtained from the Baroczy curves or Eq. (3.109) or (3.110) are strictly applicable to axial flow inside tubes or axial flow in rod bundles and annuli. For cross-flow in rod bundles, Eq. (3.110) may be used¹³⁵ with the revised constants shown in Table 3.VII(b).

In bulk boiling, the effect of wall heat flux on friction factors has often been ignored. However, Tarasova *et al.*¹²⁷ recommend that the adiabatic multipliers be corrected by

$$\phi_{\text{heat}}^2 / \phi_{\text{adiabatic}}^2 = 1 + 0.99(q''/G)^{0.7} \quad (3.111)$$

where q'' is given in Btu/h ft² and G is expressed in lb/h ft² (for $710 < P < 2840$ psia, $0.37 \times 10^6 < G < 1.9 \times 10^6$, and $0.032 \times 10^6 < q'' < 0.53 \times 10^6$).

The frictional pressure drop during boiling should be calculated in a stepwise manner for length steps for which the changes in quality and fluid properties are only moderate.

At high void fractions, use of the homogeneous model for computation of the acceleration pressure drop is not desirable since the effect of slip can be significant. It is convenient to write the acceleration pressure drop as

$$\Delta P_{\text{acc}} = r \frac{G^2}{g_c \rho_l} \quad (3.112)$$

where r is a dimensionless acceleration multiplier. When the inlet fluid contains no voids, r is given by

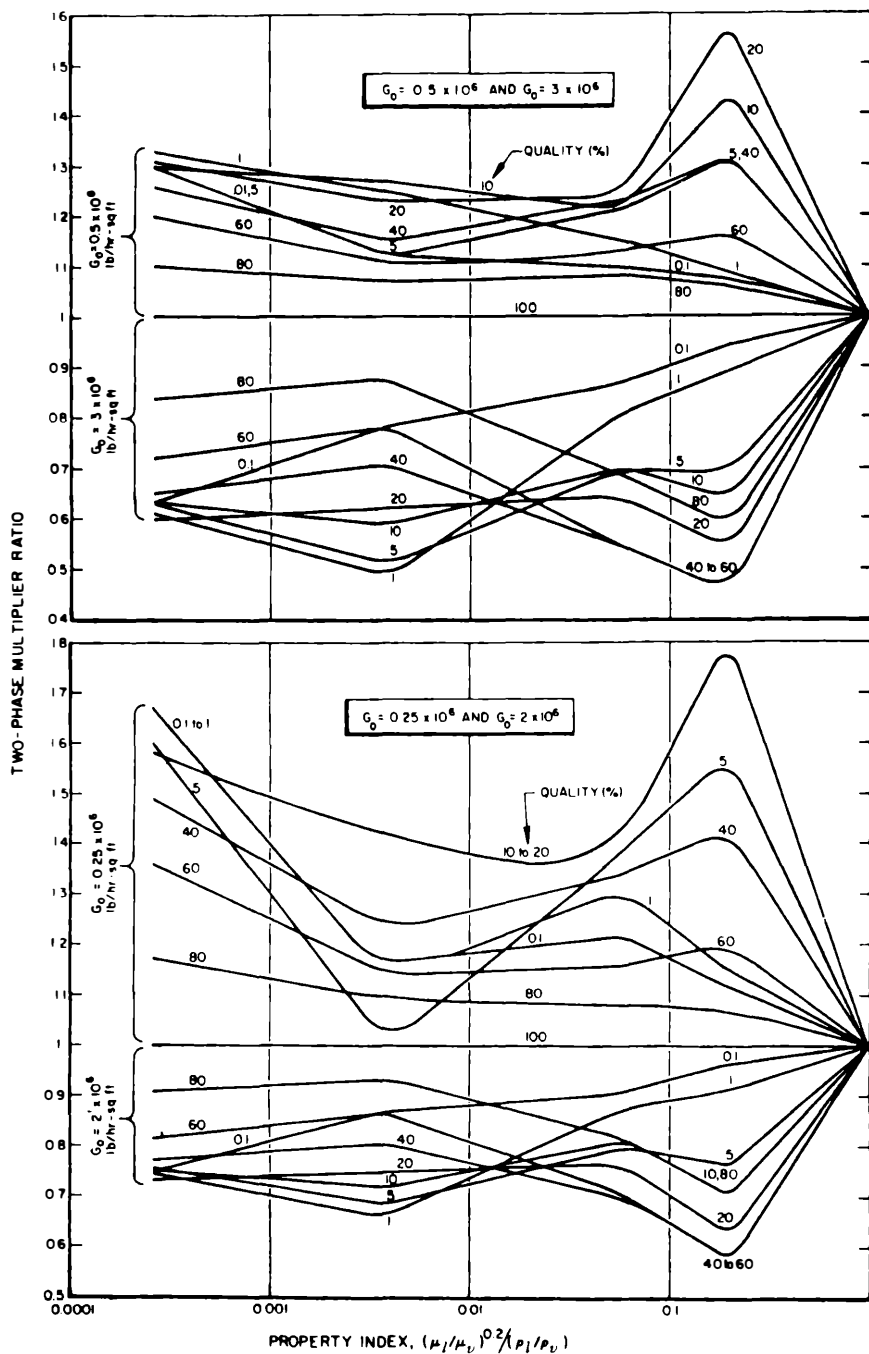


Fig. 3.18 Mass velocity correction versus property index [from Ref. 131].

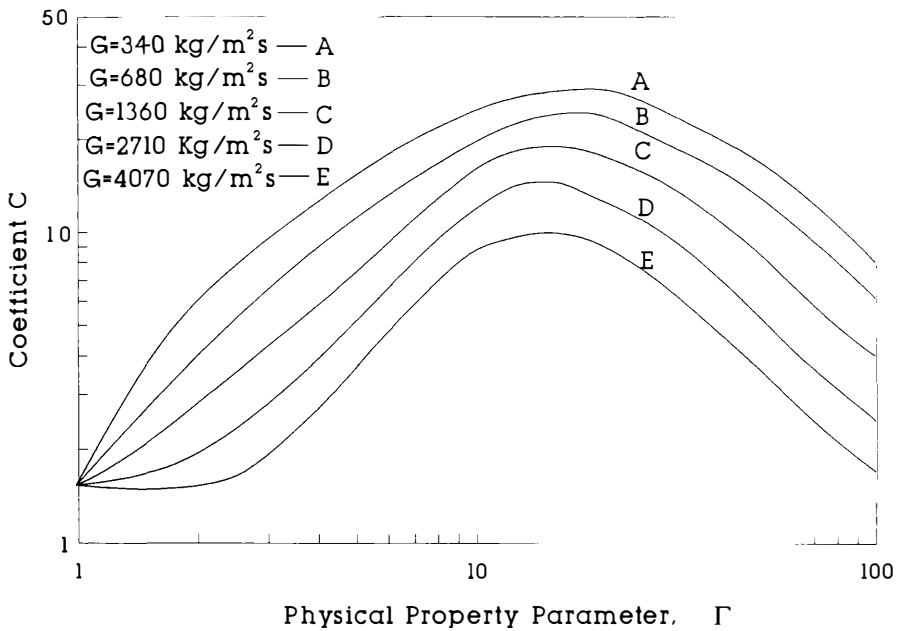


Fig. 3.19 Coefficient C of Eq. (3.109) as a function of mass velocity, G , and property index [from Ref. 133].

TABLE 3.VII
Constants for Chisholm's Two-Phase Multiplier Equation

(a) Axial Flow	Γ	$G \text{ (kg/m}^2\text{s)}$	B
	≤ 9.5	< 500	4.8
		$500 < G < 1900$	$2400/G$
		≥ 1900	$55/G^{0.5}$
	$9.5 < \Gamma < 28$	≤ 600	$520/(\Gamma G^{0.5})$
		≥ 600	$21/\Gamma$
	≥ 28		$15000/(\Gamma^2 G^{0.5})$
(b) Cross-Flow	Flow Pattern	B	n
	Vertical, bubble, dispersed annular	1	0.37
	Horizontal (side to side), bubble, dispersed annular	0.75	0.46
	Horizontal (side to side), separated	0.25	0.46

$$r = \frac{(1 - \chi_e)^2}{(1 - \alpha_e)} + \frac{\chi_e^2}{\alpha_e} \left(\frac{\rho_l}{\rho_g} \right)_{\text{sat}} - 1 \quad (3.113)$$

where χ_e = flow quality at channel exit and α_e = void fraction at channel exit. By proper evaluation of α , the effect of slip is included. The value of r is independent of the manner of heat addition (sinusoidal, uniform, etc.) along the channel.

3.3.3.2 Pressure Drop Across Area Changes, Restrictions, and Fittings

Pressure differences across area changes can be computed by one-dimensional momentum balances. For an abrupt expansion, Lottes¹³⁶ reports that the total pressure difference can be obtained from

$$\Delta P_e = \frac{G^2 \sigma^2}{\rho_l g_c} \left(\left[\frac{\rho_l}{\rho_g} \chi^2 \left(\frac{1}{\alpha_1 \sigma} - \frac{1}{\alpha_2} \right) \right] + \left\{ (1 - \chi)^2 \left[\frac{1}{\sigma(1 - \alpha_1)} - \frac{1}{(1 - \alpha_2)} \right] \right\} \right) \quad (3.114)$$

where

α_1, α_2 = void fractions upstream and downstream, respectively, of expansion

σ = area ratio (small area/large area)

G = total mass flow rate, mass/time area

ρ_l, ρ_g = liquid and gas densities, respectively

χ = mixture quality.

Janssen¹³⁷ gives the pressure loss across a contraction as

$$\begin{aligned} \Delta P_c = & \frac{G^2}{2g_c \rho_l} \left(\frac{1}{\sigma^2} \left\{ \frac{\rho_l}{\rho_g} \chi^2 \bar{\alpha}_1 \left(\frac{1}{C^2 \alpha_3^2} - \frac{1}{\alpha_2^2} \right) \right. \right. \\ & + (1 - \chi)^2 (1 - \bar{\alpha}_1) \left[\frac{1}{C^2 (1 - \alpha_3)^2} - \frac{1}{(1 - \alpha_2)^2} \right] \Big\} \\ & - \frac{2}{\sigma^2} \left\{ \frac{\rho_l}{\rho_g} \chi^2 \left(\frac{1}{C \alpha_3} - \frac{1}{\alpha_2} \right) + (1 - \chi)^2 \left[\frac{1}{C(1 - \alpha_3)} - \frac{1}{(1 - \alpha_2)} \right] \right\} \\ & \left. + \frac{\rho_l}{\rho_g} \chi^2 \bar{\alpha}_2 \left(\frac{1}{\sigma^2 \alpha_2^2} - \frac{1}{\alpha_1^2} \right) + (1 - \chi)^2 (1 - \bar{\alpha}_2) \left[\frac{1}{\sigma^2 (1 - \alpha_2)^2} - \frac{1}{(1 - \alpha_1)^2} \right] \right) \end{aligned} \quad (3.115)$$

where

C = vena contracta area ratio

α_1, α_2 void fractions upstream and downstream, respectively, of contraction

$$\bar{\alpha}_1 = (\alpha_3 + \alpha_2)/2$$

$$\bar{\alpha}_2 = (\alpha_1 + \alpha_2)/2$$

α_3 = void fraction at vena contracta.

On the basis of an extensive analysis of available data, Weisman et al.¹³⁸ concluded that for abrupt expansions, α_1 and α_2 should be evaluated by assuming slip flow. They recommend Hughmark's¹¹⁴ relationship for obtaining α from χ . For abrupt contractions, they again recommended that α_1 and α_2 be obtained by assuming slip flow and Hughmark's correlation.¹¹⁴ However, substantial mixing occurs at the vena contracta and

$$\alpha_3 = \alpha_{\text{slip}} + (\alpha_{\text{homog}} - \alpha_{\text{slip}})K' \quad (3.116)$$

where $\alpha_{\text{slip}} = \alpha$, which would be computed for vena contracta size pipe in the absence of contraction:

$$\begin{aligned} K' &= 1 & \text{if } \alpha_2 \leq 0.5 \\ K' &= 1.5 - \alpha_2 & \text{if } \alpha_2 > 0.5 \end{aligned}$$

It was also found that assuming homogeneous flow everywhere provided nearly as good a correlation of the data as Eq. (3.115) did. Then, total pressure drop across a contraction can be approximated by

$$\Delta P_c = \frac{G^2}{2\sigma^2 g_c} \left[\left(\frac{1}{C} - 1 \right)^2 + (1 - \sigma)^2 \right] \left[\frac{\chi}{\rho_g} - \frac{(1 - \chi)}{\rho_l} \right] \quad (3.117)$$

Although this equation does not represent what is actually occurring, it is a useful tool.

When contraction and expansion are well separated, the total pressure loss is

$$\Delta P_t = \Delta P_c - \Delta P_e \quad (3.118)$$

where ΔP_c and ΔP_e are obtained from Eqs. (3.114) and (3.115) and α_3 is obtained from Eq. (3.116). However, for restrictions with a length-to-diameter (L/D) ratio of <2 , only partial mixing at the vena contracta is obtained. For these conditions, α_3 can be estimated from Eq. (3.116) with mixing factor K' calculated from

$$K' = C'(L/D)^b \quad (3.119)$$

where $C' = 1.48 - 1.57\alpha$, and $b = 1.5\alpha - 0.34$. If the foregoing leads to a value of K' above 1 or below zero, K' is set equal to 1 or zero.

When the vena contracta is outside (beyond) the restriction (see Table 3.I for vena contracta location), then a revised momentum balance must be used and we obtain

$$\Delta P_t = \frac{G_t^2 v_l}{2g_c \sigma^2} \frac{1}{C^2} \left\{ \frac{\rho_l}{\rho_g} \chi^2 \alpha_3 \left(\frac{1}{\alpha_3^2} - \frac{\sigma^2 C^2}{\alpha_2^2} \right) + (1 - \chi)^2 (1 - \alpha_3) \left[\frac{1}{(1 - \alpha_3)^2} - \frac{\sigma^2 C^2}{(1 - \alpha_2)^2} \right] \right. \\ \left. - 2\sigma C \left[\frac{\rho_l}{\rho_g} \chi^2 \left(\frac{1}{\alpha_3} - \frac{\sigma C}{\alpha_2} \right) + (1 - \chi)^2 \left(\frac{1}{1 - \alpha_3} - \frac{\sigma C}{1 - \alpha_2} \right) \right] \right\} \quad (3.120)$$

where

α_1, α_2 = void fractions upstream and downstream, respectively, of restriction

α_3 = void fraction at vena contracta

C = vena contracta area ratio

σ = ratio of restricted flow area to full flow area.

For short restrictions ($L/D \leq 0.5$), there appears to be little mixing at the vena contracta and α can be estimated, assuming slip flow; e.g., from Hughmark's correlation.¹¹⁴

At L/D ratios above 0.5, some mixing at the vena contracta is obtained and K' the mixing factor at the vena contracta, can be estimated from Eq. (3.119) with $C' = -1.8\alpha + 1.05$ and $b = 3.1$ ($\alpha - 0.36$). Again, if this leads to a value for K' above 1 or below zero, K' is set at 1 or zero.

Chisholm has correlated the available data on two-phase pressure drop across bends.¹³⁹ His later correlation¹³⁹ is based on the use of Eq. (3.110) with a redefinition of Γ^2 and B . If we assume that the single phase loss coefficient K_l [see Eq. (3.9)] is the same for both liquid at gas flow, then for 90-deg bends

$$\Gamma^2 = \rho_l / \rho_g \quad (3.121a)$$

and

$$B_{90} = 1 + \frac{2.2}{K_l(2 + R/D)} \quad (3.121b)$$

where R = bend radius and D = pipe outer diameter.

For bends other than 90 deg, Chisholm recommends

$$B_\theta = 1 + (B_{90} - 1) \frac{K_{l90}}{K_{l\theta}} \quad (3.122)$$

where $K_{l90}, K_{l\theta}$ are the single-phase loss coefficients for bends with angles of 90 deg and θ , respectively.

Based on limited data, Chisholm and Sutherland¹³⁹ suggested that the two-phase pressure drop across tees be modeled using

$$(\Delta P_{TP})/(\Delta P_l) = 1 + C(\Delta P_g/\Delta P_l)^{1/2} + (\Delta P_g/\Delta P_l) \quad (3.123)$$

where

$$C = [1 - (C_2 - 1)(v_{fg}/v_g)^{1/2}][(v_g/v_l)^{1/2} + (v_l/v_g)^{1/2}]$$

and $C_2 = 1.75$. More recent data have shown this to be a very considerable simplification of two-phase behavior in tee junctions. These experimental data show that appreciable phase separation occurs at tees¹⁴⁰ and that models that treat each phase separately are required for good accuracy.¹⁴¹ Such computations are discussed in Sec. 3.5.6.

Correlations of two-phase pressure drops across valves have been proposed by Beattie¹⁴² and Simpson *et al.*¹⁴³ for the intermittent and separated flow regions. Fairhurst¹⁴⁴ proposed a model in which different multipliers were used depending on whether or not the flow quality was above a critical value. The tests of Kiederle and Mayinger¹⁴⁵ indicate that both the Beattie and Simpson *et al.* correlations provide reasonable results for the separated and intermittent flows. The Simpson *et al.*¹⁴³ correlation, given by

$$\phi_{f0}^2 = \left\{ 1 + x \left[\left(\frac{\rho_l}{\rho_g} \right)^{1/6} - 1 \right] \right\} \left\{ 1 + x \left[\left(\frac{\rho_l}{\rho_g} \right)^{5/6} - 1 \right] \right\} \quad (3.124)$$

shows no liquid viscosity effect while that of Beattie¹⁴² does. Since Kiederle and Mayinger's data also show no viscosity effect, Eq. (3.124) is recommended. For the dispersed flow (homogeneous flow) region, use of the single-phase loss coefficient together with the homogeneous mixture density is recommended.

Very little data are available in the open literature on the effect of two-phase flow on the pressure drop characteristics of core spacer grids or wires. Since at reactor design conditions, the flow is generally in the dispersed region, a common approach is to use single-phase equations together with the two-phase homogeneous mixture density and velocity.

3.3.4 Basic Two-Phase Flow Modeling

The proper description of the steady-state thermal-hydraulic behavior of a nuclear reactor core requires an analysis based on a valid mathematical model of the two-phase fluid encountered in the hotter channels of the core. In addition, the appropriate description of reactor system behavior during accident situations requires a mathematical model for transient two-phase flow. In this section, the major approaches to two-phase flow modeling will be described and the basic equations used will be indicated. The discussion here will be confined to one-dimensional flow since most systems analyses assume that the flow can be taken as one dimensional. In Chapter 4, the

revisions needed for the cases where multidimensional flow must be considered will be indicated.

3.3.4.1 Single-Fluid Models

The simplest approach to two-phase modeling is to assume that a two-phase mixture can be treated as a single fluid. A further simplification is obtained if we assume that a homogeneous mixture exists with vapor and liquid traveling at the same velocity. An analysis based on this approach would use the time-averaged governing equations, which are also averaged over the flow cross-sectional area. In their simplest form, the Eulerian versions of the differential equations for conservation of mass, momentum, and energy are identical to the single-phase conservation equations [Eqs. (3.1) to (3.3)] except that V , H , and ρ now represent mixture average velocity, enthalphy, and density and the frictional head loss, F , is based on the two-phase frictional loss.

To obtain the values V , H , and ρ at a given time, appropriate constitutive equations are required. When the homogeneous model is used, it is usual to assume that thermodynamic equilibrium exists and then properties are based on the appropriate thermodynamic tabulations. The model based on these assumptions is called the homogeneous-equilibrium (HEM) model.

Computer programs that need thermodynamic properties find the use of tables time consuming. Simple equations that can represent these properties are greatly preferred. Garland *et al.*¹⁴⁶ provide a highly accurate set of simple equations for this purpose.

Lahey¹⁴⁷ has written the homogeneous flow equations in slightly different form. Lahey's model accounts for the effect of gravitation and can be used in a duct with a varying cross-sectional area.

$$\text{Conservation of mass: } \frac{\partial}{\partial Z}(\rho VA) + A \frac{\partial \rho}{\partial t} = 0 \quad (3.125)$$

$$\begin{aligned} \text{Conservation of momentum: } \frac{1}{g_c} \left[\frac{\partial G}{\partial t} + \left(\frac{1}{A} \frac{\partial (G^2 A / \rho)}{\partial Z} \right) \right] = \\ - \frac{\partial P}{\partial Z} - \frac{g}{g_c} \rho \sin \theta - \frac{\tau_w p_f}{A} \end{aligned} \quad (3.126)$$

where the identity, $G = \rho V$, has been used and A = cross-sectional flow area, area; p_f = frictional wetted perimeter, ft, (length); and τ_w = wall shear stress, force/area.

$$\text{Conservation of energy: } \rho \left(\frac{\partial E}{\partial t} + V \frac{\partial E}{\partial Z} \right) = q'' \left(\frac{p'}{A} \right) + q''' + \frac{1}{J} \frac{\partial P}{\partial t}, \quad (3.127)$$

where

$$E = \left(H + \frac{V^2}{2g_c J} + \frac{g}{g_c} \frac{z \sin \theta}{J} \right)$$

and

q'' = heat flux, energy/area time

q''' volumetric heat generation rate, energy/vol time

p' = the heated perimeter, length

J mechanical equivalent of thermal energy (1.0 in SI units).

Single-fluid models can also be obtained without making the assumption of equal phase velocities. Tentner and Weisman¹⁴⁸ derived a simplified, thermal equilibrium, single-fluid model in terms of the slip ratio. When we use the slip ratio, S , to define the phasic velocities, we can write

$$SV_l = V_g \quad (3.128)$$

With this notation, and taking the cross-sectional area as constant, the equations for a horizontal system become:

$$\text{Mass conservation: } \frac{\partial}{\partial t}[\alpha \rho_g + (1 - \alpha) \rho_l] + \frac{\partial}{\partial Z}[\alpha \rho_g SV_l + (1 - \alpha) \rho_l V_l] = 0 \quad (3.129)$$

$$\begin{aligned} \text{Momentum conservation: } \frac{\partial}{\partial t}[\alpha \rho_g SV_l + (1 - \alpha) \rho_l V_l] + \frac{\partial}{\partial Z}[\alpha \rho_g S^2 V_l^2 \\ + (1 - \alpha) \rho_l V_l^2] + \frac{\partial p}{\partial z} = -\frac{\tau'_w}{A} \end{aligned} \quad (3.130)$$

$$\begin{aligned} \text{Energy conservation: } \frac{\partial}{\partial t}[\alpha \rho_g E_g + (1 - \alpha) \rho_l E_l] + \frac{\partial}{\partial Z}[\alpha \rho_g V_l SE_g \\ + (1 - \alpha) \rho_l V_l E_l] - \frac{\partial P}{\partial t} = \frac{q'_w}{A} \end{aligned} \quad (3.131)$$

The notation used in the above equations is:

$$E_g = H_g + S^2 V^2 / (2g_c J)$$

$$E_l = H_l + V^2 / (2g_c J)$$

q'_w = rate of heat transfer per unit length from wall to fluid, energy/length time

V_l, V_g = liquid and gas velocities, respectively

τ'_w = shear force per unit length exerted by the wall, force/length.

The foregoing model assumes that $(\partial S / \partial z)$ and $(\partial S / \partial t)$ are negligible and that enthalpy and internal energy differ by a negligible amount. The slip

model would normally be coupled with the assumption of thermodynamic equilibrium. Such a model is designated as an unequal velocity, equal temperature (UVET) model.

3.3.4.2 Two-Fluid Models

Two-fluid models are attempts to provide a reasonably complete and accurate description of the flow of a mixture of different phases of one or more components without going to the usually impractical extreme of tracking the interface separating the volumes occupied by the different phases. Such models can allow for the significant differences between the velocities and temperatures of the gas and liquid (departure from homogeneous flow and departure from thermodynamic equilibrium) that may be present during transient boiling flows.

Models of this nature are based on the use of separate conservation equations for the vapor and liquid phases. Thus the three conservation equation model resulting from the single-fluid approximation is replaced by a six-equation model.

The simplest of the available two-fluid models is that of Wendroff.¹⁴⁹ By defining the term α_1 and α_2 as

$$\alpha_1 = \alpha \quad \alpha_2 = 1 - \alpha$$

the conservation equations for each phase can then be written in identical form. If SI units are used, we have

$$\frac{\partial \alpha_i \rho_i}{\partial t} + \frac{\partial \alpha_i \rho_i V_i}{\partial z} = \Gamma'_i \quad i = 1, 2 \quad (3.132)$$

$$\frac{\partial (\alpha_i \rho_i V_i)}{\partial t} + \frac{\partial (\alpha_i \rho_i V_i^2)}{\partial z} + \alpha_i \frac{\partial P}{\partial z} = M_{zi} \quad i = 1, 2 \quad (3.133)$$

$$\frac{\partial (\alpha_i \rho_i E_i)}{\partial t} + \frac{\partial (\alpha_i \rho_i E_i V_i)}{\partial z} + P \left(\frac{\partial \alpha_i}{\partial t} + \frac{\partial (\alpha_i V_i)}{\partial z} \right) = E_i \quad i = 1, 2 \quad (3.134)$$

where

E_i = total internal energy density $[U_i + (V_i^2/2g)]$

Γ'_i = mass transfer rate to the i 'th phase, mass/time vol (*note:*

$\Gamma'_g = -\Gamma'_l$)

M_{zi} = rate of axial momentum transfer to the i 'th phase, momentum/time vol

E_i = rate of energy transfer to i 'th phase, energy/vol time

U_i = internal energy, energy/mass.

If these equations are closed by specifying equations of state, say, $\rho_i = \rho_i(P_i, H_i)$, and interfacial transfer rates, then there are six partial differ-

ential equations and six state variables: V , α , V_1 , V_2 , H_1 , H_2 . These six equations provide one of the simplest two-fluid models and can be considered to apply to a variety of flow situations provided one can safely neglect viscosity, surface tension, and virtual mass. Nonequilibrium effects are introduced by appropriately defining Γ_i , M_{zi} and E_i .

Wendroff's basic model is perhaps the simplest computationally feasible full nonequilibrium model. The model is rudimentary in that the means of interphase transfer are unspecified. Further, the basic model is "ill posed" (differential equations are not hyperbolic). Introduction of additional effects are required to obtain a well-posed system. Despite the fact that this basic model is ill posed, it is possible to obtain numerically stable computational schemes for it when a coarse mesh is used. The meaning of ill-posedness will be explored further in Sec. 3.3.4.5.

More complex two-fluid models, in which interphase transfer mechanisms are partially specified within the conservation equations, are available.¹⁵⁰ Unfortunately, most of these are also ill posed. Despite the fact that such an equation set may be ill posed, it has been possible to use such sets as a basis for numerical computation of two-phase system behavior. Several major transient analysis computer programs (e.g., RELAP5/Mod2¹⁵¹) are based on an ill-posed model.

Various schemes have been used to obtain a hyperbolic set of differential equations. The model used in the CATHARE code¹⁵² illustrates one such approach. A somewhat simpler approach is indicated by Lahey.¹⁵³ He incorporates Reynolds stresses, surface tension, and virtual mass effects to achieve a well-posed set. The mass conservation equations are identical to those indicated by Eq. (3.132). He writes the momentum conservation equations as

$$\begin{aligned} \frac{\partial}{\partial t}(\alpha_i \rho_i V_i) + \frac{\partial}{\partial z}(\alpha_i \rho_i V_i^2) - \alpha_i \frac{\partial P_i}{\partial z} + (P'_i - P_i) \frac{\partial \alpha_i}{\partial z} + \Gamma'_i V_i \\ + \frac{\partial}{\partial z}(\alpha_i \tau_{Ri}) + \alpha_i \rho_i g \cos \theta + M_{zi} \quad i = 1, 2 \end{aligned} \quad (3.135)$$

and the energy conservation equation as

$$\begin{aligned} \frac{\partial}{\partial t}(\alpha_i \rho_i H_i) + \frac{\partial}{\partial z}(\alpha_i \rho_i V_i H_i) = \alpha_i \frac{D_i P_i}{Dt} - V_i M_{zi} + q''_i a_i + \Gamma'_i H_{fg} \\ - V_i \frac{\partial}{\partial z}(\alpha_i \tau_{Ri}) + (P'_i - P_i) \frac{D_i \alpha_i}{Dt} \quad i = 1, 2 \end{aligned} \quad (3.136)$$

where

$D(\)/Dt$ = substantial derivation of () with respect to phase i

a_i = interfacial area per unit volume, area/vol.

q_i'' = interfacial heat transfer rate, energy/area time

P_i' = interfacial pressure of phase i , force/area

P_i = pressure of phase i , force/area

τ_{Ri} = Reynolds stresses in phase i .

The rate of axial momentum transfer, M_{zi} , is the sum of the effects of wall drag, interfacial drag, and virtual mass. For the liquid phase we would have

$$M_{zi} = (\tau_w p' / A) + \frac{1}{8} \rho_l C_D |V_g - V_l| (V_g - V_l) a_i - \alpha \rho_l C_{vm} \left(\frac{D_g V_g}{Dt} - \frac{D_l V_l}{Dt} \right) \quad (3.137)$$

where

τ_w = wall shear stress, force/area

p' = perimeter

A = flow area

C_D = drag coefficient

C_{vm} = virtual mass coefficient.

A similar equation is written for the vapor phase but negative signs precede the virtual mass and drag terms.

In the foregoing formulation, the phase pressures are not assumed to be equal but are related by

$$P_g - P_l = (2\sigma/R_b) - (\rho_l/4)(V_g - V_l)^2 \quad (3.138a)$$

where R_b = average bubble radius. The interfacial liquid pressure, P_l' , is defined by

$$P_l' - P_l = -(\rho_l/4)(V_g - V_l)^2 \quad (3.138b)$$

but P_g' is taken as equal to P_g .

The Reynolds stresses in the gas phase are considered negligible but the bubble-induced Reynolds stresses in the liquid phase are computed from

$$\tau_{Rl} = \frac{-1}{5} \alpha_l \rho_l (V_g - V_l)^2 \quad (3.139)$$

The equations used for phase pressure difference and Reynolds stresses in the Lahey model strictly apply only to bubble or dispersed flows. These should be replaced if the model is applied to separated or annular flow. The exact form of these terms and the virtual mass term is still a subject of debate (1995).

With an equation of state giving the density of a given phase as a function of the enthalpy of that phase and the pressure, the six conservation equations can be solved for H_g , H_l , V_g , V_l , α , and P providing interfacial

transfer rates are determined. Constitutive equations giving interfacial friction and heat transfer are therefore required for closure. A two-fluid model will provide greater accuracy than a single-fluid model only if reasonably accurate interfacial transfer rates can be established.

Note that a two-fluid model does not assume, as required in a single-fluid model, that the transient slip ratio be the same as that seen in steady state. If interfacial friction is properly modeled, the model will determine the appropriate slip ratio during the transient.

3.3.4.3 Four Equation Models

The ill-posedness problem and relatively long computing times associated with two-fluid models have led some investigators to the use of a simpler model that uses only four conservation equations. In the most widely used approach, differential equations for mixture mass, energy, and momentum conservation are coupled with an equation describing the time-dependent (dynamic) behavior of the phase velocity difference.⁹⁸

In the so-called "dynamic slip model" used in the RETRAN program,⁹⁸ the equation describing the time-dependent velocity difference (dynamic slip equation) is obtained by subtracting appropriate formulations of the liquid and vapor equations to obtain a single equation. This equation is then solved for the derivative of the slip velocity. We then have

$$\begin{aligned} \frac{\partial(V_{gl})}{\partial t} \left[1 + \frac{(M_a)\rho_c}{\alpha_2\alpha_1\rho_l\rho_g} \right] = & - \left(\frac{1}{\rho_l} - \frac{1}{\rho_g} \right) \frac{\partial P}{\partial z} + \frac{\rho A_{gl} B_{gl} V_{gl}}{\alpha_1\alpha_2\rho_l\rho_g} + V_g \frac{\partial V_g}{\partial z} \\ & - V_l \frac{\partial V_l}{\partial z} + \frac{A_{wg} B_{wg} V_g}{\alpha_1\rho_g} - \frac{A_{wl} B_{wl} V_l}{\alpha_2\rho_l} - \Gamma' \left[\left(\frac{\bar{V}_g - V_g}{\alpha_1\rho_g} \right) + \left(\frac{\bar{V}_l - V_l}{\alpha_2\rho_l} \right) \right] \quad (3.140) \end{aligned}$$

where

$$V_{gl} = V_g - V_l$$

$$V_l = u_l / (1 - \alpha)$$

$$V_g = u_v / \alpha$$

$$\bar{V}_g = \langle \alpha V_g \rangle / \langle \alpha \rangle$$

$$\bar{V}_l = \langle (1 - \alpha) V_l \rangle / \langle 1 - \alpha \rangle$$

$$\langle \phi \rangle = \int_A \phi dA / A$$

u_v = superficial velocity of liquid and vapor, respectively (note:
In RETRAN, $\bar{V}_g = V_g$, $\bar{V}_l = V_l$)

A_{gl} = liquid-gas interfacial area per unit volume

A_{wg} , A_{wl} = wall-gas and wall-liquid contact areas per unit volume

B_{gl} = interfacial friction coefficient = $1/8\rho_c V_{gl} C_{gl}$

B_{wi} = friction coefficient between wall and phase $a = 1/8\rho_i V_i f_i$

M_a = added (virtual) mass coefficient = $b(1 - \alpha)\alpha\rho$

$b = (0.5 + \alpha)(1 - \alpha)$ for $\alpha < 0.5$, $b = (1.5/\alpha) - 1$ for $\alpha \geq 0.5$

f_i = Moody friction factor between phase i and wall

C_{gl} = interfacial drag coefficient

Γ' = rate of vapor generation per unit volume, mass/vol time

ρ_c = density of continuous phase, mass/vol.

Equation (3.140) coupled with mixture mass, momentum, and energy conservation, forms a well-posed set.

The dynamic slip model is usually coupled with the assumption of thermodynamic equilibrium. However, this is not a requirement of four-equation models. Lack of thermodynamic equilibrium may be modeled by using the conservation equations for mixture mass, vapor mass, mixture energy, and mixture momentum. Alternatively, a liquid mass balance may be used in place of the mixture mass balance. By providing a separate vapor mass balance, one can incorporate nonequilibrium vaporization or condensation rates. Models of this nature have been described by Ishi¹⁵⁴ and Kroe-ger¹⁵⁵ among others.

The two-mass conservation equations coupled with the energy and momentum conservation equations allow the determination of x , H_g , H_l , and the velocity of one of the phases, V_g or V_l . The velocity of the second phase is determined via a constitutive equation specifying the velocity difference between phases. This specification has usually been accomplished by means of the drift flux approach (see Sec. 3.3.2). Because of this, the entire model is generally referred to as a *drift flux model*.

Drift flux models have the disadvantage that steady-state velocity differences are assumed to hold during rapid transients. They have the desirable features of well posedness and fairly rapid computation times. Nevertheless, there is relatively little current (1994) interest in such models and most models of unequal temperature conditions use a two-fluid model.

3.3.4.4 Constitutive Equations for Nonequilibrium Models

Closure for either the two-fluid or four-equation models requires the use of constitutive equations to define interfacial transfer rates. The form of these equations depends both on the formulation of the conservation equation in which it is used and on the physical model assumed for the interfacial transport process.

Most modelers are agreed that interfacial momentum transfer may be obtained by exploiting the relationship between void fraction and interfacial shear that exists in the steady state. Crawford *et al.*¹⁰⁰ show that if the void fraction α and the two-phase pressure drop, $\Delta P/\Delta z$, are known for a given quality, then $A_{gl}C_{gl}$, the product of the interfacial drag and interfacial area per unit volume in the steady state, may be obtained from solution of the

dynamic-slip equation [Eq. (3.140)] with vapor generation and time-dependent terms set to zero:

$$A_{gl}C_{gl} = \left\{ \frac{-f}{2D\rho_1^2} \left[\frac{\phi_{l0}^2 \Delta p}{\rho_g} + \frac{(1-x)^2}{(1-\alpha)^2} \right] + \frac{\bar{\rho}g\Delta p}{G^2\rho_g\rho_l} \right\} \frac{8[\alpha(1-\alpha)\rho_g\rho_l]^3}{[(1-x)\alpha\rho_g - x(1-\alpha)\rho_l]^2 \bar{\rho}\rho_1^2} \quad (3.141)$$

where

$A_{gl}C_{gl}$ = product of interfacial drag coefficient and interfacial area per unit volume

$\bar{\rho}$ = average density of mixture, $\Delta p = \rho_l - \rho_g$

f = Moody friction factor.

It is assumed that the steady-state value applies to transient situations and hence the steady-state values of $A_{gl}C_{gl}$ may be used to obtain B in Eq. (3.140). In obtaining Eq. (3.141), we assumed that (1) $\partial V_g / \partial z \approx \partial V_l / \partial z \approx 0$ for the adiabatic area and (2) the vapor is not in contact with the pipe wall. Hence, this equation should not be used for separated flow. Equation (3.141) allows interfacial drag to be computed from well-known pressure drop and void correlations.

Other investigators take a somewhat different point of view and conclude that for bubble and intermittent flows the interfacial drag can be determined directly from drift flux parameters. Bestion¹⁵² writes the drift velocity, u_{gl} , as

$$u_{gl} = C_m g [(\rho_l - \rho_g) De / \rho_l]^{1/2} \quad (3.142)$$

where C_m = constant.

For vertical flow, where buoyancy dominates, he expresses the interfacial friction force per unit volume, F_i , as

$$F_i = \alpha(1-\alpha)(\rho_l - \rho_g)g \quad (3.143)$$

By means of Eq. (3.85) together with Eqs. (3.142) and (3.143) we obtain, by elimination of $(\rho_l - \rho_g)$,

$$F_i = \alpha(1-\alpha) \frac{1}{C_m^2} \frac{\rho_l}{De} (1 - C_0\alpha)^2 \left[V_g - \frac{C_0(1-\alpha)}{1 - C_0\alpha} V_l \right]^2 \quad (3.144)$$

Since Eq. (3.144) is limited to only a few flow patterns in vertical flow, Eq. (3.141) has a more general applicability.

The early formulations of interfacial mass and heat transfer tended to be of the relaxation type in which the rate of interphase mass transfer depended on the difference between actual and equilibrium values and some characteristic relaxation time. Kroeger¹⁵⁵ thus described the volumetric rate of vapor generation, Γ' , during a pressure transient as

$$\Gamma' = \rho_m \left(\frac{\partial c}{\partial P} \right)_s \psi \left(\frac{dP}{dt} \right) \quad (3.145)$$

where

c = vapor concentration {static quality = $\alpha / [(\rho_l / \rho_g) - (\alpha \rho_l / \Delta \rho)]$ }

$(\partial c / \partial P)_s$ = change in c with respect to pressure based on maintenance of constant entropy

$\psi = 0$ if $c_E < c_A$ or $1 + (c_E - c) / \tau$ if $c_E > c_A$

τ = relaxation time

c_E = static quality at thermodynamic equilibrium

c_A = threshold value of c_E up to which no vapor generation is assumed

ρ_m = mixture density, mass/vol.

More recent models tend to relate the mass transfer rate to interfacial heat transfer rates. This approach is used in the TRAC code.¹⁰² TRAC assumes that the steam-water interface is always at the saturation temperature and that the rate of interphase exchange of mass is defined by the energy exchange needed to maintain the saturation temperature. The volumetric rate of energy exchange at the interface, q_i''' (energy/time vol), is then given by

$$q_i''' = (ha)_{il}(T_l - T_{\text{sat}}) + (ha)_{ig}(T_g - T_{\text{sat}}) \quad (3.146)$$

$$\Gamma' = \Gamma'_{\text{wall}} + q_i''' / H_{fg} \quad (3.147)$$

where $(ha)_{ig}$, $(ha)_{il}$ = product of interfacial heat transfer coefficient and interfacial area for gas and liquid, respectively, and T_g , T_l , T_{sat} = gas, liquid, and saturation temperatures respectively. TRAC provides separate correlations for interfacial areas and heat transfer coefficients for each flow pattern. Tables 6.IV and 6.V summarize correlations for calculating interfacial areas and interfacial heat and momentum transfer, which were developed for TRAC evaluation of LOCAs.

It should be recognized that modeling of interfacial transfer rates remains (1995) an area of ongoing investigation and there is no general agreement on the most appropriate models.

3.3.4.5 Ill-Posed Equation Sets

The conservation equations for any of the previously described two-phase flow models can be written in the following general form:

$$\mathbf{A} \frac{\partial \mathbf{Y}}{\partial z} + \mathbf{B} \frac{\partial \mathbf{Y}}{\partial t} - \mathbf{F} = 0 \quad (3.148)$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{F}$ are known functions of z , t , and $\mathbf{Y}$. It may be shown¹⁵⁶ that the partial differential equations are total differential equations along the directions, dz/dt , obtained by solution of the determinantal equation

$$|\mathbf{A} - \mathbf{B} \, dz/dt| = 0 \quad (3.149)$$

The directions dz/dt so obtained are called the *characteristic directions*. When all of the characteristic directions are real, the problem is said to be well posed. When one or more of the characteristic directions are imaginary, the problem is said to be ill posed. The latter designation is used since the presence of an imaginary slope in problems describing propagation phenomena is usually considered as indicative of a lack of physical reality.

Some of the widely used two-fluid models are found to have complex characteristics. At the present time (1995), opinions still differ as to the correct interpretation of such complex characteristics and the proper course of action. Some workers have intimated that complex characteristics are necessary to the description of physical instabilities. However, Ramshaw and Trapp¹⁵⁷ have shown that systems with real characteristics can exhibit physical instabilities and that complex characteristics are not necessary to describe such behavior. They show that although stability and real characteristics are not related in general, they become essentially equivalent at high frequencies. One might instead adopt the related position that equations having complex characteristics may correctly describe physical instabilities. The analysis of Ramshaw and Trapp proves that although this might be the case at sufficiently long wavelength, the results become increasingly erroneous as the wavelength is decreased and eventually, at very short wavelengths, equations having complex characteristics will predict unphysical instabilities, or, in other words, the problem is improperly posed.

Krishnamurthy *et al.*¹⁵⁸ have extensively examined the stability of the RELAP5/MOD3 computer code, which is based on an ill-posed, one-dimensional, two-fluid model. They found that model exhibited numerically convergent (stable) behavior within the discrete spatial bounds consistent with current (1992) usage. They concluded that a well-posed equation set is not necessary and that it is sufficient to require numerical stability only for the shortest waves of interest, i.e., $\lambda = 2 \, (\Delta z)$ where Δz is the shortest spatial increment to be used.

3.3.5 Sonic Velocity and Critical Flow

A sudden change in fluid velocity causes a large pressure change that is seen as a pressure wave, across which there is discontinuity in pressure and velocity. The pressure wave, which can be either a compression wave (high pressure behind a wave front) or a rarefaction wave (low pressure behind a wave front), propagates at sonic velocity relative to the flow. It

can be set up by suddenly closing a valve in a flow line, by rupturing a high-pressure piping system (e.g., a LOCA), or by a steam burst in a reactor power excursion. The steam burst imparts kinetic energy, which is converted into a pressure pulse when the flow is interrupted. Pressure waves of this general nature are often called steam hammer or water hammer.

Pressure and velocity changes across an acoustic wave in one-dimensional flow can be derived from the momentum equation

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} = -\frac{1}{\rho} \frac{\partial P}{\partial x} \quad (3.150)$$

where V is fluid velocity, P is pressure, and x is distance. Since velocity change with respect to distance is relatively small in a straight pipe, the $\partial V / \partial x$ term is negligible, and Eq. (3.150) becomes

$$\frac{\partial V}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x} \quad (3.151)$$

The pressure wave travels at the velocity of sound a_1 ; therefore, it will travel distance ($a_1 t$) in time t . Let

$$\theta = x \pm a_1 t \quad (3.152)$$

where the negative sign is required for a wave moving in a direction opposite to V . By substituting Eq. (3.152) into Eq. (3.151), we obtain

$$\pm a_1 \frac{dV}{d\theta} = -\frac{1}{\rho} \frac{dP}{d\theta} \quad (3.153)$$

By integrating, this yields

$$\frac{P_2 - P_1}{V_2 - V_1} = \pm \rho a_1 \quad (3.154)$$

Equation (3.154) shows that in subcooled water with a density of 62 lb/ft³ and a sonic velocity of 5000 ft/s, a 3 ft/s change in fluid velocity across the wave front causes a pressure pulse of 200 psi.

When a one-dimensional acoustic wave travels in a fluid of equal and opposite velocity, the wave becomes stationary with respect to the earth. We then have critical flow at this point (fluid at sonic velocity), and downstream pressure signals can no longer be transmitted to the upstream fluid. Thus, the critical flow rate is limited by the upstream and critical point conditions, but not by downstream conditions. In the case of a rupture of a PWR piping system, the critical flow rate at the break determines the severity of the accident.

In single-phase flow, sonic velocity a_1 and critical mass flow rate are directly and simply related:

$$a_1^2 = g_c v^2 / -(dv/dP)_s, \quad (3.155)$$

where v is specific volume and subscript s indicates the derivative is evaluated at constant entropy. Critical mass flow rate G_c is given by

$$G_c^2 = a_1^2 \rho^2 = g_c / -(dv/dP)_s \quad (3.156)$$

Two-phase critical flow is, however, more complex: sonic velocity in a two-phase mixture varies with the amount of void present; velocity is usually measured at the wave front, where no phase change can take place instantaneously. On the other hand, critical flow rate is determined by the sonic velocity behind the wave front, where phase change does take place. This change depends on wave shape and bubble delay time which, in turn, depends on saturation pressure of the fluid. Therefore, it is necessary to consider sonic velocity and critical flows of two-phase mixtures separately.

3.3.5.1 Sonic Velocity

There are two limits for the response of a mixture of liquid and vapor to a pressure pulse: (1) mass transfer between the phases maintaining thermodynamic equilibrium, and (2) no mass transfer (frozen state) with liquid and vapor being independently isentropic. If no mass transfer is assumed, the computed velocity is often called the "frozen sonic velocity." If thermodynamic equilibrium is assumed, the computed velocity is substantially below frozen sonic velocity and the velocity actually observed at low and moderate pressures.

If vapor and liquid can be considered a homogeneous mixture, sonic velocity can be quite simply derived from Eq. (3.155) by using the mean density $\bar{\rho}$, where

$$\bar{\rho} = \alpha \rho_g + (1 - \alpha) \rho_l \quad (3.157)$$

where α is void fraction and ρ_g , ρ_l are vapor and liquid densities, respectively. The frozen two-phase sonic velocity, a_{TP} , can be written as:¹⁵⁹

$$a_{TP}^2 = \left[\frac{\alpha \bar{\rho}}{\rho_v a_v^2} + \frac{(1 - \alpha) \bar{\rho}}{\rho_l a_l^2} \right]^{-1} \quad (3.158)$$

where a_v , a_l are sonic velocity in vapor and liquid, respectively. At low pressures where $a_l \gg a_v$, Eq. (3.158) becomes

$$\frac{a_{TP}^2}{a_v^2} = \left[\alpha^2 + \frac{\alpha(1 - \alpha)\rho_l}{\rho_v} \right]^{-1} \quad (3.159)$$

Dvornichenko¹⁶⁰ derived a slightly different expression for the frozen sonic velocity

$$\frac{a_{TP}}{a_v} = \left[\sqrt{\chi} + \frac{(1 - \chi)\rho_g}{\sqrt{\chi}\rho_l} \right], \quad (3.160)$$

where χ is quality. Grolmes and Fauske¹⁵⁹ found that Eq. (3.158) fitted their low void fraction data very well. Therefore, it can be used to predict velocity of the sonic wave front in a low-void bubbly flow in the absence of mass transfer. At high void fractions, the assumption of a homogeneous mixture is no longer valid, and increasing deviations from Eq. (3.158) are observed.

In evaluating a_{TP} by either Eq. (3.159) or Eq. (3.160), it is not obvious what path should be used to evaluate a_v^2 . If adiabatic behavior is assumed, then $a_v^2 = g_c P \gamma / \rho_v$, where γ is the ratio of specific heat at constant pressure to that at constant volume. If isothermal conditions are assumed, $a_v^2 = g_c P / \rho_v$.

Henry *et al.*¹⁶¹ studied the situation at higher void fractions where slip must be considered. They used $\bar{\rho} = 1 / [\chi v_v + (1 - \chi) v_l]$ in Eq. (3.155) and expressed a^2 in terms of liquid density ρ_l , gas density ρ_g , and velocity slip ratio S . Equation (3.155) becomes

$$a_{TP}^2 = [(1 - \chi)\rho_g + \chi\rho_l]^2 \left/ \chi\rho_l^2 \left(\frac{\partial \rho_g}{\partial P} \right)_{\text{sat}} + (1 - \chi)\rho_g^2 \left(\frac{\partial \rho_l}{\partial P} \right)_{\text{sat}} \right. \\ \left. - (\rho_l - \rho_v)\rho_l\rho_g \left(\frac{\partial \chi}{\partial P} \right)_s + \chi(1 - \chi)(\rho_l - \rho_g)\rho_l\rho_g \left(\frac{\partial S}{\partial P} \right)_s \right] \quad (3.161)$$

where subscript s indicates the derivative is evaluated at constant entropy. When it is assumed that no phase change occurs, Eq. (3.155) becomes

$$a_{TP}^2 = \frac{[(1 - \chi)\rho_v + \chi\rho_l]^2}{\left[\chi\rho_l^2 \left(\frac{\partial \rho_g}{\partial P} \right) + (1 - \chi)\rho_g^2 \left(\frac{\partial \rho_l}{\partial P} \right) + \chi(1 - \chi)(\rho_l - \rho_g)\rho_l\rho_g \left(\frac{\partial S}{\partial P} \right)_s \right]} \quad (3.162)$$

Equation (3.162) can be simplified for frozen annular flow (or stratified flow) as follows: For a stationary wave, Henry *et al.*¹⁶¹ suggested

$$\frac{\partial S}{\partial P} = \frac{1}{a_{TP}} \left(\frac{\partial V_v}{\partial P} - \frac{\partial V_l}{\partial P} \right) \quad (3.163)$$

For the vapor phase,

$$\frac{\partial P}{\partial z} + \rho_g a_{TP} \frac{\partial V_v}{\partial z} = 0$$

And, for the liquid phase,

$$\frac{\partial P}{\partial z} + \rho_l a_{TP} \frac{\partial V_l}{\partial z} = 0 \quad (3.164)$$

Therefore,

$$\frac{\partial S}{\partial P} = \frac{-1}{a_{TP}^2} \left(\frac{1}{\rho_g} - \frac{1}{\rho_l} \right)$$

By substituting Eq. (3.164) into Eq. (3.162),

$$a_{TP}^2 = \frac{\{[(1-\chi)\rho_g + \chi\rho_l]^2 + \chi(1-\chi)(\rho_l - \rho_g)^2\}}{\frac{\chi\rho_l^2}{a_v^2} + \frac{(1-\chi)\rho_g^2}{a_l^2}} \quad (3.165)$$

For compressible flow at low pressures ($a_l \gg a_v$), sonic velocity can be approximated as

$$\left(\frac{a_{TP}}{a_v} \right)^2 = 1 + \frac{1-\chi}{\chi} \left(\frac{\rho_g}{\rho_l} \right)^2 \quad (3.166)$$

or, in case of no slip,

$$\left(\frac{a_{TP}}{a_v} \right)^2 = 1 + \frac{1-\alpha}{\alpha} \left(\frac{\rho_g}{\rho_l} \right)^2 \quad (3.167)$$

In high-quality or high void fraction flow, both Eqs. (3.166) and (3.167) indicate that $a_{TP} \rightarrow a_v$.

Henry *et al.*¹⁶¹ compared their air-water mixture data with Eqs. (3.158) and (3.155). For the bubbly-flow region, they found sonic velocity agrees approximately with the frozen model, Eq. (3.158); for stratified flow (non-flowing system), Eq. (3.165) was applicable.

The stratified model of Eq. (3.165), which essentially gives the sonic velocity of vapor, is applicable for high-void steam-water (annular) flow. This appears to be verified by steam-water data obtained by England *et al.*,¹⁶² Diech *et al.*,¹⁶³ and DeJong and Firey.¹⁶⁴

Henry¹⁶⁵ subsequently observed that the velocity of a wave front at low pressures and very low void fractions tends to approach that given by Eq. (3.158) (frozen model) with a_v^2 evaluated at isothermal conditions. For void fractions above those where Eq. (3.158) strictly applies ($\alpha > 0.1$), and below those of the annular region ($\alpha \leq 0.6$), Henry¹⁶⁵ correlated the available data by

$$a_{TP}/a_H = 1.035 - 1.671\alpha \quad (3.168)$$

where a_{TP} = actual propagation velocity in two-phase mixture and a_H = propagation velocity determined from Eq. (3.158) with a_v^2 evaluated under isothermal conditions.

Equation (3.168) may simply be reflecting the difference in behavior between large and small bubbles. Hijikata *et al.*'s¹⁶⁶ study of shock waves in air-water mixture containing large and small bubbles showed that the two

bubble sizes behaved differently. They concluded that the small bubbles had the same temperature as the liquid and moved with the liquid. However, larger bubbles were isentropically compressed and moved at a velocity different from the liquid behind the shock. As the void fraction is increased in a flowing system, the proportion of large bubbles is increased, thus leading to deviations from the homogeneous isothermal model.

Despite the fact that the flow is probably not homogeneous under most conditions, Beattie¹⁶⁷ finds that the two-phase sonic velocity may be approximated by an expression involving only flow quality and fluid specific heats. For those cases where $\rho_l \gg \rho_g$ and $V \ll a_{TP}$, Beattie proposed

$$a_{TP} = a_v (\rho_g / \rho_l)^{1/2} (\gamma^1 / \gamma)^{1/2}$$

where

$$\gamma = (c_p / c_v)_g \quad (3.169)$$

$$\gamma^1 = \frac{x(c_p)_g + (1-x)(c_p)_l}{x(c_v)_g + (1-x)(c_v)_l}$$

and subscripts g and l indicate gas and liquid phases, respectively. Beattie was able to fit low-pressure air-water and steam-water data in the intermediate void fraction range by assuming that qualities could be computed from measured void fractions using a drift flux model.

The question of whether there is always negligible mass transfer at the wave front is not completely settled. On the basis of their data, Hamilton and Shrock¹⁶⁸ suggest that when the vapor bubble is very small, mass transfer may not be negligible. Such a situation is likely in high-velocity dispersed flow where very small bubble sizes are present. Mass transfer rates may be enhanced at high temperatures where some investigators¹⁶⁹ indicate that the time required to nucleate new bubbles becomes very short. Hence, additional nucleation during the brief period of wave passage, which would enhance mass transfer and reduce the front velocity, might be possible. There is no agreement on this point. Direct measurements at high fluid velocity, high temperatures, and low or moderate values of α do not appear to be available. Measurements of sonic velocity in high-pressure (69-bar) steam-water flow in the dispersed annular region give acoustic velocities close to that of single-phase gas flows. This is in accord with the previously cited observations at low pressures.

The effect of phase change slows down the pressure wave behind the front and makes it flatter than obtained in a fully frozen system (e.g., air-water). Grolmes and Fauske¹⁵⁹ also observed this effect while studying the characteristics of both compression and rarefaction waves. Behind the wave front, the effects of heat and mass transfer become important and appear to prevent compression waves from steepening into shock waves, signifi-

cantly lengthening the shape of the rarefaction waves. This effect has led Henry¹⁶⁵ to distinguish between the propagation velocity of a pressure pulse and the propagation velocity of a sound wave. Henry states that the frozen model holds strictly for a pressure pulse only. Propagation of a sound wave involves response to a continuous wave. At low frequencies, there is time for appreciable mass and heat transfer and, hence, lower propagation velocities can be expected. High frequency waves can be expected to have propagation velocities that approach those of a pressure pulse.

The detailed analysis of Ardron and Duffy¹⁷⁰ is in accord with this point of view. They show that the speed of sound becomes a function of frequency when thermal nonequilibrium effects are considered in a one-component system. Ruggles *et al.*¹⁷¹ experimental data for the air-water system show that sonic velocity also varies with frequency in a two-component system. However, in the nonvaporizing air-water system, the observed reduction in sonic velocity with decreasing frequency was considerably smaller than would be seen in a vaporizing system.

3.3.5.2 Critical Flow in Long Pipes

From the previous discussion, it is clear that there is an approach to thermodynamic equilibrium behind the wave front. Since critical flow is determined by conditions behind the front, some phase change must be considered. In a long pipe line, where there is adequate time for bubble nucleation and growth, thermodynamic equilibrium can be assumed.

The homogeneous equilibrium model (HEM) is the simplest analytical model that can be postulated. The model assumes that

1. both phases move at the same velocity
2. the fluid is in thermal equilibrium
3. flow is isentropic and steady.

By applying these assumptions to continuity and energy conservation equations, mass flow rate G is found by

$$G = [(2g_c J)(H_0 - H)/v^2]^{1/2} \quad (3.170)$$

where

$$H_0 = \text{upstream reservoir enthalpy} = H + (G^2 v^2)/(2g_c J)$$

$$H = \text{local fluid enthalpy} = H_f - \chi H_{fg}$$

$$v = \text{local fluid specific volume} = (1 - \chi)v_l + \chi v_g$$

$$J = \text{thermal energy to mechanical energy conversion factor.}$$

For fixed stagnation (upstream reservoir) conditions, the critical mass flow rate is obtained by assuming the fluid entropy remains constant and finding the downstream pressure for which G exhibits a maximum. The results of these calculations for steam-water systems are presented in Fig. 3.20.

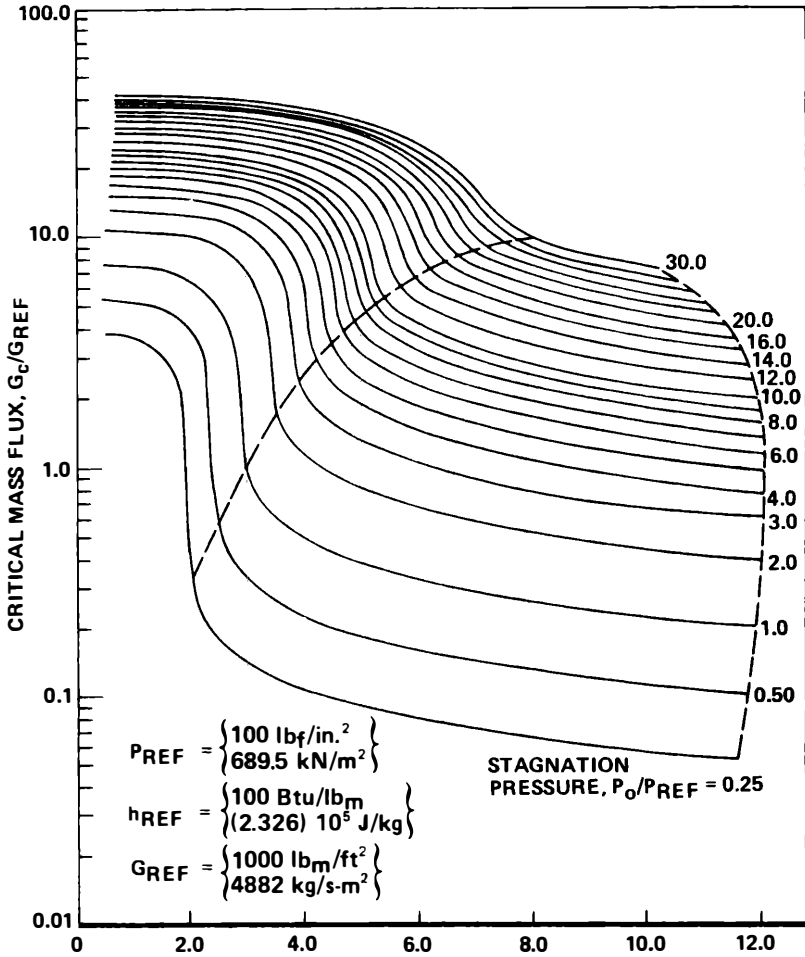


Fig. 3.20 Critical mass flux and homogeneous equilibrium, steam-water mixture [reproduced from NEDO-21052, General Electric Co. (Sep. 1975)].

Moody¹⁷² considered the available data for blowdown in long pipes. From an examination of nonequilibrium behavior during depressurization, Moody concludes that if pipe length is over five inches, equilibrium states can be expected. He found that available data on the blowdown of reservoirs through pipes of lengths >5 in. were predicted by Fig. 3.20 when calculations were based on reservoir conditions. This is in agreement with Edwards¹⁶⁹ observations.

Moody¹⁷² points out that although homogeneous theory using reservoir conditions provides a good prediction of critical flow rates, it provides a

poor prediction of pressure at the exit of the blowdown pipe. He concludes that mass fluxes are limited by homogeneous choking near the pipe entrance, but that a transition to slip flow occurs before reaching the exit. A second choked condition near the exit is produced and, if critical flow rates are to be evaluated on the basis of local conditions near the pipe exit, a slip flow model must be used.

Critical flow models for use with exit conditions, and based on high slip ratios at the exit, were developed by Moody,^{172,173} Fauske,¹⁷⁴ Zivi,¹⁷⁵ Cruver and Moulton,¹⁷⁶ and others. A major difficulty with these models is that the slip ratios used are very much higher than those observed in high velocity flow under nonchoked conditions. In addition, it has been shown^{148,177,178} that the unrealistically high slip ratios assumed by these models lead to imaginary characteristics. Hence, they render the system of conservation equations ill-posed for much of the domain of interest. Tentner and Weisman¹⁴⁸ showed that, when realistic slip ratios were used together with the single-fluid conservation equations of Eq. (3.129) through (3.131), real characteristics were always obtained. They found that the computed critical flow rates were very close to those of the HEM model.

It is now generally agreed that thermal nonequilibrium effects account for the apparently anomalous behavior at the break. Experimental data show a very sharp pressure gradient at the exit.¹⁷⁹ Such a gradient would be expected to produce nonequilibrium with void fractions and quality which are lower than predicted by equilibrium theory. Thus, if discharge predictions are to be made on the basis of break conditions, a nonequilibrium theory should be used.

Nonequilibrium effects also need to be considered in some instances where reservoir conditions are used to compute the discharge flow rate. If the liquid in the upstream reservoir is sufficiently subcooled so that very low qualities are observed at the break, discharge flow rates are found to be significantly higher than predicted by the HEM.

Henry¹⁸⁰ devised a simple model for pipes of intermediate lengths where thermodynamic equilibrium is not attained. He described the nonequilibrium behavior in terms of the parameter N , which relates the actual change in quality with pressure, dx/dP , to dx_e/dP , the rate of change in quality occurring under equilibrium conditions:

$$dx/dP = N(dx_e/dP) \quad (3.171)$$

On the basis of experimental data, Henry concluded that the actual exit quality, x_{LT} , at the break of a long tube was given by

$$x_{LT} = Nx_e \quad (3.172)$$

where x_e = thermodynamic equilibrium quality at exit and

$$\begin{aligned} N &= 20x_e & 0 < x_e < 0.05 \\ N &= 1 & 0.05 \leq x_e \end{aligned}$$

For tubes having a length to diameter ratio (L/D) of less than 12, he assumed vapor generation could be neglected. For longer tubes, he assumed that the actual exit quality x_{ex} exponentially approached the long tube value. That is

$$x_{ex} = x_{LT} \{1 - \exp[-B(L/D - A)]\} \quad (3.173)$$

where $A = 12$ for tubes with sharp entrance and 0 for a smooth inlet and $B = 0.0523$. With the assumption of isothermal behavior, the critical mass flow is calculated, assuming slip is unity, as

$$G_c^2 = \frac{1}{\left[\frac{xv_v}{P} - (v_v - v_l) N \frac{dx_e}{dP} \right]_{ex}} \quad (3.174)$$

where v_v and v_l are the specific volumes of vapor and liquid and the subscript ex indicate that all quantities are evaluated at the exit conditions. To do so, the exit pressure P_{ex} is required. This is obtained from

$$\eta = \frac{P_{ex}}{P_0} = 1 - \frac{G_c^2}{P_0} \frac{v_{l0}}{2K_c^2} + x_{ex}(v_{g,ex} - v_{l,0}) \quad (3.175)$$

where K_c is the contraction coefficient at the pipe exit. The subscript 0 indicates reservoir conditions. By iterative application of Eqs. (3.173), (3.174), and (3.175), G_c may be evaluated.

Nonequilibrium effects are also encountered in the discharge from long pipes when annular flow at very high void fractions exists at the exit. At the high velocities seen in choked flow, most of the liquid carried by the fluid is in the form of entrained droplets. The critical velocity is close to the critical velocity of the vapor flowing alone. Since the droplets move at a velocity close to the sonic velocity of the vapor, the mass flux discharge is significantly higher than HEM predictions. Grolmes *et al.*¹⁸¹ have proposed a simple correlation that merges the high-quality flashing flow data with the equilibrium model at low qualities. The correlation is stated to apply over the entire quality range,

$$G_c = \left\{ 1 + \left[\frac{x_0 b}{0.6(1 - \ln P^*/b)} \right]^2 \frac{R}{m_w c_p} \right\}^{1/2} \left(\frac{H_{fg}}{v_{fg} \sqrt{c_p T_0}} \right) \quad (3.176)$$

where

G_c = critical mass flux, $\text{kg/m}^2 \text{ s}$

x_0 = reservoir quality

b = 13 for water, 10.6 for most other substances

R = ideal gas constant, 8314 J/kg mol K

T = temperature, K

- c_p = specific heat of liquid, J/kg K
 m_w = molecular weight of vapor
 $P^* = \exp[b(1 - 1/T^*)]$
 T^* = T /(temperature at normal boiling point at atm pressure).
 H_{fg} = enthalpy change on vaporization, J/kg
 v_{fg} = change in specific volume at vaporization, m^3/kg .

Note that Eq. (3.176) is written in a manner which allows it to be applied to a variety of fluids.

3.3.5.3 Critical Flow in Short Pipes, Nozzles, and Orifices

When a reservoir discharges through a short passage a high degree of thermal nonequilibrium is encountered. If the length of the flow passage is negligibly small (e.g., discharge across an orifice), vapor formation has little influence and the discharge mass flux, G , may be calculated by the orifice equation¹⁸¹

$$G = C_d \sqrt{2\rho_m (P_o - P_a) g_c} \quad (3.177)$$

where

- ρ_m = density of fluid mixture upstream of orifice
 C_d = orifice discharge coefficient (0.61 at high Reynolds numbers of discharge flow)
 P_o, P_a upstream and ambient pressures, respectively
 g_c gravitational conversion constant (1.0 in SI units).

At very high pressure differentials, true choking can be encountered. The flow is then limited to a mass flux equal to the product of the mixture density and frozen sonic velocity over the area of the vena contracta. Both mass flux and sonic velocity are evaluated at the pressure upstream of the orifice.

Vaporization can influence the discharge of a reservoir through a short pipe or nozzle. A commonly encountered condition is one in which there is subcooled fluid just upstream of the break. Under these conditions, two types of choking mechanisms were seen by Zaloudek.¹⁸² Zaloudek observed that an upstream choke can form at a vena contracta and a downstream choke can form as the back pressure is built up at the exit edge by flashing. He found that when choking occurred at the vena contracta, critical mass velocity was given by Eq. (3.177) with P_a replaced by the saturation pressure and C_d is an empirically determined constant found to range from ~0.61 to 0.64. When downstream choking occurred, the critical flow rate agreed with Burnell's¹⁸³ correlation.

Burnell¹⁸³ previously recognized the existence of a metastable state in the flow of flashing water through nozzles and hypothesized that water

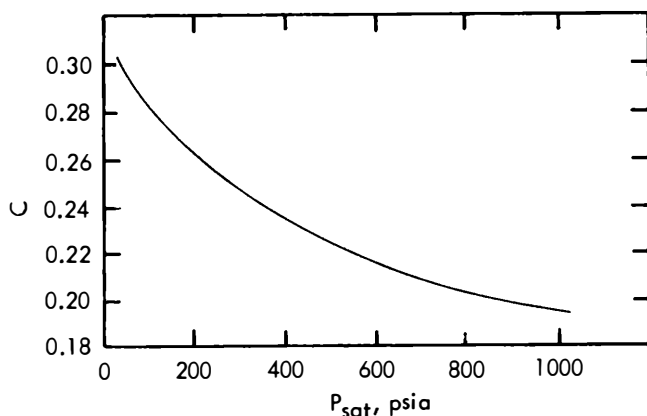


Fig. 3.21 Pressure coefficient for Burnell critical flow equation [from Ref. 183].

surface tension retarded the formation of vapor bubbles, thus causing the water to be superheated. He developed a semi-empirical method for predicting the flow of flashing water through short nozzles and correlated the discharge mass flow rate as

$$G_{crit} = \{2g_c \rho_l [P_{upstream} - (1 - C)P_{sat}]\}^{1/2} \quad (3.178)$$

where C was related to bubble delay time, which is a function of surface tension. The value of C that is recommended for design use is given in Fig. 3.21. The value of C determines the pressure undershot at the exit due to superheating of the liquid.

Sozzi and Sutherland¹⁸⁴ also observed nonequilibrium effects in blow-down through short nozzles. They saw a strong dependence on nozzle length with the shorter nozzles giving higher flow rates. They found that fluid passing through a very short length will not have sufficient time to completely nucleate before leaving the tube. For nozzle lengths of about five inches, agreement between observations and homogeneous predictions was obtained. They also observed that critical mass flux decreased with increasing throat diameter.

Abuaf *et al.*¹⁸⁵ provide a more fundamental analysis of flashing of sub-cooled liquid across a nozzle. They replaced the term $(1 - c)P_{sat}$ in Eq. (3.178) and wrote

$$G_c = [2g_c \rho_l (P_{upstream} - P_{sat} + \Delta P_F)]^{1/2} \quad (3.179)$$

where ΔP_F = pressure difference corresponding to the maximum superheat which can be sustained without flashing. The value of ΔP_F is based on a

modification of Alamgir and Lienhard's¹⁸⁶ static depressurization superheat, which allows for turbulence effects and change in pressure with position. It is given by

$$\Delta P_F = \max \left\{ 0, \Delta P_0 - 13.5(i_b)^2 V_l^2 \rho_l \right\} \quad (3.180)$$

where

$$\Delta P_0 = 0.253 \frac{\sigma^{3/2}}{\sqrt{k T_c}} T_r^{13.73} \frac{\sqrt{1 + 14(\Sigma)^{0.8}}}{1 - (\rho_g / \rho_l)}$$

$$\Sigma = \frac{\partial P}{\partial t} - V_l \frac{\partial P}{\partial z} = \frac{G_c^2}{\rho_l^2} \frac{d(\ln A)}{dz} + \Sigma'$$

and

A = nozzle flow area

Σ' = reservoir depressurization rate $= dP_0 / dt$

i_b = turbulent intensity

k = Boltzmann's constant

ΔP_0 = pressure undershoot in absence of turbulence effects

T_c = critical temperature

$T_r = T_{\text{sat}} / T_c$

σ = surface tension.

While this approach correlates available data and provides a more mechanistic view of nozzle behavior, the Burnell equation [Eq. (3.178)] appears to be adequate for design purposes.

Henry and Fauske¹⁸⁷ developed a model for critical flow in nozzles and short tubes which allows for nonequilibrium effects and considers a two-phase mixture upstream of the break. They proceed in a manner similar to that used by Fauske,¹⁷⁴ but do not assume thermodynamic equilibrium between phases. They use the empirical parameter to relate actual dx/dP to that (dx_e/dP) under equilibrium conditions in accordance with Eq. (3.171). The experimental parameter N is given by

$$N = x_e / 0.14 \quad \text{for } x_e \leq 0.14$$

$$N = 1.0 \quad \text{for } x_e > 0.14$$

They concluded that the critical mass flow rate was given by

$$G_c^2 = \left[\frac{\chi_0 v_v}{n P} - (v_v - v_{l0}) \left\{ \frac{(1 - \chi_0) N}{s_{ve} - s_{le}} \frac{dx_e}{dP} - \frac{\chi_0 c_p [(1/n) - (1/\gamma)]}{P(s_{v0} - s_{l0})} \right\} \right] \quad (3.181)$$

where

χ_0 = quality at stagnation conditions

n = polytropic exponent for vapor compression

γ = isentropic exponent for vapor compression = c_p/c_v

c_p, c_v = specific heats of vapor at constant pressure and volume, respectively

s_{ve}, s_{le} = entropy under equilibrium conditions of vapor and liquid, respectively

s_{t0}, s_{l0} = entropy at stagnation conditions of vapor and liquid, respectively.

Equation (3.181) is coupled with the momentum equation describing the pressure history to obtain a solution in terms of stagnation conditions

$$(1 - \chi_0)v_{l0}(P_0 - P_t) + \frac{\chi_0\gamma}{\gamma - 1}(P_0v_{v0} - P_tv_{vt}) = \frac{[(1 - \chi_0)v_l + \chi_0v_{vt}]^2}{2}G_c^2 \quad (3.182)$$

where the subscript t indicates throat conditions. By substituting this equation into Eq. (3.181) and by rearranging, we get the ratio of throat pressure, P_t , to stagnation pressure, P_0 .

$$\eta = P_t/P_0 = \left[\frac{\frac{1 - \alpha_0}{\alpha_0}(1 - \eta) + \frac{\gamma}{\gamma - 1}}{\frac{1}{2\beta\alpha_t^2} - \frac{\gamma}{\gamma - 1}} \right]^{\gamma/(\gamma - 1)} \quad (3.183)$$

where

$$\beta = [(1/n) + (1 - v_{l0}/v_{vt})] \left[\frac{(1 - \chi_0)NP_t}{\chi_0(s_{ve} - s_{le})_t} \frac{ds_{le}}{dP} \right] - \frac{c_p[(1/n) - (1/\gamma)]}{(s_{v0} - s_{l0})}$$

$$\alpha_0 = \frac{\chi_0v_{v0}}{(1 - \chi_0)v_{l0} + \chi_0v_{v0}}$$

$$\alpha_t = \frac{\chi_0v_{vt}}{(1 - \chi)v_{l0} + \chi_0v_{vt}}$$

$$v_{vt} = v_{v0}(\eta^{-1/\gamma}),$$

and subscript t indicates throat conditions. Once η is obtained by solving the transcendental equation, G_c is obtained from Eq. (3.182).

Note that the correlation given for N holds only for a dispersed mixture of vapor and liquid. In the discharge of a saturated or subcooled liquid through an orifice or short tube, a separated flow pattern is often observed.

TABLE 3.VIII
Applicability of Simple Models for Calculation of Critical Flow Rate

Apparatus	Upstream Conditions	Applicable Calculation Procedure
Long pipe	Two phase ($\alpha < \alpha_n$) to moderately subcooled liquid	HEM model (Fig. 3.20)
Long and intermediate length pipe	Two phase ($\alpha > \alpha_n$)	Grolmes <i>et al.</i> [Eq. (3.176)]
Long pipe	Highly subcooled liquid	Henry [Eq. (3.174)]
Intermediate length pipe	Subcooled liquid to two-phase with $\alpha < \alpha_n$	Henry [Eq. (3.174)]
Nozzle	Saturated or subcooled liquid	Burnell [Eq. (3.178)]
Short pipe	Saturated or subcooled liquid	Burnell [Eq. (3.178)] or orifice equation if back pressure builds up
Nozzle or short pipe	Two-phase mixture ($\alpha < \alpha_n$)	Henry and Fauske [Eq. (3.181)]
Orifice	Saturated or subcooled liquid	orifice equation [Eq. (3.177)]
	Two-phase mixture	minimum of value from Eq. (3.177) or mass flux based on frozen sonic velocity

Note: α_n = void fraction at which annular flow begins.

The proposed correlation for N is not applicable under these conditions and Burnell's equation should be used for G_c prediction.

The solution of the transcendental equation for η is time consuming. Because of this, when this approach is used as part of an accident analysis computer program, it is often represented by a set of polynomial equations that have been fit to the results over the parameter range of interest.¹⁸⁸

The applicable regions of the various simple critical flow models are summarized in Table 3.VIII.

3.3.5.4 Critical Flow Across Safety and Pressure Relief Valves

Safety and pressure relief valves are designed for the release of vapor. Section III of the American Society of Mechanical Engineers (ASME) Boiler and Pressure Vessel Code considers such valves to behave like an orifice having a flow area, A_0 , equal to the circumference of the valve seat, πD , times, L , the valve lift (distance valve seat moves on opening). At the high-pressure difference expected in nuclear vessel operation, the flow is assumed to be choked. The design equations recommended by the ASME may be written as¹⁸⁹

$$W = (0.9) A_0 G_c C_v, \quad (3.184)$$

where

$$A_0 = (\pi D)L$$

W = mass flow rate, mass/time

C_v = valve coefficient correcting for nonidealities

G_c = theoretical critical mass flux through nozzle of throat area A_0 .

The value of G_c is based on the sonic velocity of steam and is determined by

$$G_c = \frac{P_0}{(ZRT_0)^{1/2}} \left[\gamma \left(\frac{2}{\gamma+1} \right)^n \right]^{1/2} \quad (3.185)$$

where

P_0 = upstream pressure

R = ideal gas constant

Z = compressibility factor

$\gamma = c_p/c_v = 1.13$ for steam

$n = (\gamma+1)/(\gamma-1)$

T_0 = upstream temperature.

Sallet¹⁸⁹ observed that the C_v values used in Eq. (3.184) are based on the assumption that the back pressure is atmospheric pressure. This is not the case in a LOCA. He notes that when the back pressure increases significantly the value of C_v is reduced. His data show increasing the back pressure to 50% of the discharge pressure decreases C_v by about 13%.

When a liquid or low-void fraction two-phase mixture is discharged through a safety or relief valve, the sonic velocity is higher than for all vapor flow and one would expect that the nonchoked orifice discharge equation would apply [Eq. (3.177)]. In Sallet's¹⁸⁹ study of the blowdown of refrigerant liquid, the discharge mass velocity was predicted by the orifice equation, with flow area of A_0 and back pressure at that of the downstream tank, when P_0 was significantly above the set pressure. However, the discharge rate fell significantly below these predictions as P_0 approached the set pressure.

Orifice-like behavior when discharging liquid was also observed by Chen *et al.*¹⁹⁰ They examined the behavior of a spring-loaded safety valve with a short throat. Their tests examined the discharge of refrigerant 113 with both liquid and two-phase mixtures upstream of the valve. At the relatively low pressure differentials (up to 4.5 bar) of their tests, the valve acted as an orifice with a variable area. Their results could be correlated by

$$W/(g_c \rho_m \Delta P)^{1/2} = \frac{K_1}{P_{\text{set}}} \left(P_0 - P_{\text{set}} + \frac{\rho_m V_m^2}{g_c} \right) + K_2, \quad (3.186)$$

where

W = discharge mass flow rate, mass/time

V_m = mixture velocity based on upstream conditions

ρ_m = mixture density based on upstream conditions

K_1 = constant for given valve indicating change in flow area produced by additional force on valve seat

K_2 = constant for given valve indicating flow area provided by initial valve lift

P_{set} = set or relief pressure

P_0 = system pressure.

The value of the right side of Eq. (3.186) reaches a maximum value when the valve is fully open. It then acts as an orifice of constant area. If much higher pressure differentials had been used, a choked flow condition would be expected when the mixture velocity reached the frozen sonic velocity. The Chen *et al.*¹⁹⁰ study explains the reduced flows observed by Sallet at low pressures. As the upstream pressure approached the set pressure, the valve acted as an orifice with a variable area.

The results of CISE tests¹⁹¹ of safety valves discharging (60- to 120-bar) water and water-steam mixtures were entirely different. The valves, which had relatively long throats ($80 \text{ mm} < L < 116 \text{ mm}$) showed choking in the vicinity of the valve exit for all test conditions. With the flow taken as that of the valve throat, critical flow rates were closely matched when HEM results were multiplied by a coefficient slightly below unity. This result agrees with the recommended procedure for sizing safety relief valves in the chemical process industry.¹⁹² It would seem that tests of the specific valve geometry used are needed to establish definitively whether the valve behaves as an orifice with an area determined by the valve lift or if the valve will exhibit choking at the exit of the throat.

3.3.5.5 Comprehensive Critical Flow Models

All of the previously described critical flow models make particular assumptions regarding the flow behavior and ignore some of the conservation equations. Many of the more recent attempts to determine critical flow rates have been based on models that consider all of the conservation equations and thermodynamic nonequilibrium. The models generally allow for delay in the initial vaporization of subcooled liquid and a finite rate of bubble growth or heat transfer between phases once bubble nucleation has occurred. The resulting equations are then numerically integrated in a step-wise fashion along the flow passage. When critical flow actually occurs, a pressure discontinuity exists across the standing wave at the flow-passage exit. Most of the models consider that critical flow has occurred when the assumed mass flux leads to an axial pressure gradient at the exit which is

equal to or above some arbitrary high value. Models of this nature have been developed by Ardron,¹⁹³ Dobran,¹⁹⁴ Elias and Chambré,¹⁹⁵ Dagen *et al.*,¹⁹⁶ and others.

Dagen *et al.*¹⁹⁶ use five conservation equations that include mass and momentum equations for each phase and a combined energy equation. A sixth equation is added describing the growth of vapor bubbles along the flow path and a seventh equation relates time to the axial position along the pipe. The seven variables described by these equations are liquid temperature, gas velocity, liquid velocity, bubble radius, flow quality, pressure, and time.

The Dagen *et al.*¹⁹⁶ model is self-consistent and it may be used all along the discharge line. It also was able to predict the results of the large pipe experiments at Marviken.¹⁹⁷ The ability to predict large-pipe data is significant since such data represent behavior in a large loss of coolant accident. Many models do not provide very satisfactory predictions of these data.

Instead of using the exit pressure gradient as a criterion for critical flow, other comprehensive models have used the characteristic slopes. Recall (see Sec. 3.3.4.5) that the solution of the determinantal equation, Eq. (3.149), for the slopes (dz/dt) provides the characteristic directions. These characteristic directions represent propagation velocities. Since at the critical condition information from downstream cannot be propagated upstream, we require that the lowest value of dz/dt be zero when critical flow is attained

$$\min[(dz/dt)_i] = 0 \quad \text{for } i = 1, 2, \dots, n \quad (3.187)$$

where n is the number of equations defining the **A** and **B** matrices. In most cases, analytic solutions for (dz/dt) cannot be found and numerical solutions are required. Note that unless the nonequilibrium terms in the defining equations are associated with a derivative term, they do not affect the values of the characteristic directions.

One of the simpler models based on the method of characteristics is that due to Kroeger.¹⁵⁵ He used a five-equation drift flux model in which the vapor phase was assumed saturated and local pressure was used. Kroeger was able to solve the resultant characteristic equations and found for the case of zero drift velocity

$$(dz/dt)_{1,4} = V_m \pm a_{\text{HF}} \left(\frac{1}{1 - Z' \beta a_{\text{HF}}^2} \right) \quad (3.188)$$

$$(dz/dt)_{2,3} = V, \quad (3.189)$$

where V_m is the mixture velocity. Hence, at critical conditions:

$$V_m = a_{\text{HF}} \left(\frac{1}{1 - Z' \beta a_{\text{HF}}^2} \right) \quad (3.190)$$

where a_{HF} is the homogeneous frozen sound speed given by

$$a_{\text{HF}} = v \left\{ \frac{1}{c_{pl}} \left(\frac{\partial v_l}{\partial T_l} \right)_p \left[x \left(\frac{dH_g}{dP} \right)_{\text{sat}} - v \right] - x \left(\frac{dv_g}{dP} \right)_{\text{sat}} - (1-x) \left(\frac{\partial v_l}{\partial P} \right)_T \right\}^{1/2} \quad (3.191)$$

where v = mixture specific volume and Z' is given by

$$Z' = \frac{1}{v} \left[(v_g - v_l) + \frac{H_g - H_l}{c_{pl}} \left(\frac{\partial v_l}{\partial P} \right)_p \right] \quad (3.192)$$

The factor β accounts for nonequilibrium mass transfer:

$$\beta = \frac{dx_E}{dP} \left(1 + \frac{x_e - x}{\tau'} \right) \quad (3.193)$$

where x_e = equilibrium thermodynamic quality corresponding to the local mixture enthalpy, τ' = relaxation time (Amos and Schrock¹⁹⁹ recommend $\tau' = 1.1 \times 10^{-3}$ s), and

$$\frac{dx_c}{dP} = \frac{(1-x) \left(\frac{dH_f}{dP} \right)_{\text{sat}} + x \left(\frac{dH_g}{dP} \right)_{\text{sat}}}{H_{fg}} \quad (3.194)$$

Note that the conversion of thermal energy into kinetic energy has been neglected in Eq. (3.194) and the static mixture enthalpy is taken as constant. It should also be noted that the characteristic directions can be determined without setting the drift velocity to zero. However, a numerical solution is then required. The assumption of zero slip at the choke can be justified on the basis of the previously cited studies showing that the slip ratio must be close to 1.0 in order to obtain real characteristics.

Use of the characteristic directions for determination of critical flow rates requires that the critical flow calculations be coupled to a model that can compute the local conditions at the choke location. Shrock and coworkers^{198,199} have shown that, when used with their fluid modeling, Eq. (3.190) can satisfactorily predict the subcooled blowdown behavior seen in the large-pipe experiments at Marviken.¹⁹⁷ Schrock *et al.* based their computations on the use of mixture conservation equations written in terms of the mixture velocity, V , liquid temperature, T_l , pressure, P , and quality, x . The conservation equations employed were

$$\begin{aligned} \text{Mass:} \quad & -\frac{1}{v} \left[(1-x) \left(\frac{\partial v_l}{\partial P} \right)_{T_l} + x \left(\frac{dv_g}{dP} \right)_{\text{sat}} \right] \frac{dP}{dz} + \frac{1}{V} \frac{dV}{dz} \\ & + \frac{v_g - v_l}{v} \frac{dx}{dz} + (1-x) \left(\frac{\partial v_l}{\partial T_l} \right)_p \frac{dT_l}{dz} = 0 \end{aligned} \quad (3.195)$$

$$\text{Momentum: } \frac{dP}{dz} + \frac{V}{v} \frac{dV}{dz} = - \left(\frac{dP}{dz} \right)_f + \frac{g}{v} \quad (3.196)$$

$$\text{Energy: } \left[(1-x) \left(\frac{\partial H_l}{\partial P} \right)_{T_l} + x \left(\frac{dH_g}{dP} \right)_{\text{sat}} \right] \frac{dP}{dz} + V \frac{dV}{dz} + (H_g - H_l) \frac{dx}{dz} = 0 \quad (3.197)$$

where $(dP/dz)_f$ is the frictional pressure drop, v , v_g , v_l are specific volumes of mixture, gas, and liquid, respectively, and other symbols have their previous meaning. For blowdown conditions, the vapor is taken as at saturation and vapor properties evaluated from an appropriate equation of state. Properties of the superheated liquid were estimated by extrapolating subcooled properties to the superheated region. Since only mixture velocities are considered, this implies that slip is taken as negligible throughout the blowdown line.

The liquid superheat was correlated by

$$\frac{dP}{dz} - \left(\frac{dP}{dT} \right)_{\text{sat}} \frac{dT}{dz} = \Delta P_b \exp(-t_r/\tau') \quad (3.198a)$$

where

$$t_r = \int_{z_f}^z \frac{dz}{V(z)}$$

is the residence time of two phase mixture in duct, ΔP_b is the pressure undershoot at inception of flashing, and τ' is the relaxation time as used in Eq. (3.193).

Lee and Schrock¹⁹⁸ provide a correlation for ΔP_b that leads to better results than obtained with earlier estimates of this quantity. For saturated liquids, they obtain improved results by using the empirical expression

$$\Delta P_b = 0.013 \sigma' P^{1.38} \quad (3.198b)$$

where σ' (surface tension) and P are evaluated at inlet conditions and all quantities are in SI units.

The ability of the approach of Schrock and coworkers to predict subcooled blowdown data confirms the importance of thermal nonequilibrium and the minor influence of slip on this behavior.

Determination of critical flow conditions via characteristic directions determined by a full two-fluid model has been accomplished by Lahey,²⁰⁰ Sami and Duong,²⁰¹ and Micaelli *et al.*²⁰² Lahey's model²⁰⁰ [Eqs. (3.132), (3.135), and (3.136)] does not contain any heat or mass transfer terms associated with the spacial or time derivations. The characteristic directions obtained from his model therefore do not reflect the effect of interphase mass transfer.

Sami and Duong²⁰¹ introduce derivative terms into the mass transfer rate. However, the formulation of these terms is such that they only appear

in the characteristic equations as a multiplier for the difference between the gas and liquid velocities. Since this difference approaches zero at the high velocities characteristic of critical flow, nonequilibrium mass transfer will have little direct effect on the results. However, the use of these nonequilibrium models in the line leading to the break will produce qualities below the equilibrium quality at the choke point. Therefore, use of such two-fluid models will significantly affect the predicted break flow even though the characteristics do not themselves reflect nonequilibrium effects.

Integration of any of the comprehensive models into one of the computer programs used for PWR accident analysis is difficult. The accurate determination of break conditions requires the use of much shorter length steps in the break line than those commonly used. In the explicit and semi-implicit solution schemes of most programs, reduction of the length step anywhere reduces the allowable time step and hence increases solution time. The CATHARE program²⁰² avoids this difficulty. This code uses a fully implicit solution scheme that allows the use of a very fine mesh near the break without a reduction in the time step. CATHARE determines the critical flow rate from the characteristic equations of a six equation two-fluid model. This requires the solution of a sixth-degree polynomial equation.

While many systems analysis programs avoid lengthy solution times by using very simple critical flow models, others (e.g., RETRAN⁹⁸) use approximations of more complex analytical models. This is done by developing an algebraic equation that fits the solutions obtained from the complex model of interest over the range of conditions expected in a LOCA.

3.3.5.6 Decay of Pressure Waves

When a pressure wave reaches a rigid dead end, it is reflected as a wave of equal magnitude and sign; that is, a rarefaction wave is reflected as a rarefaction wave and a compression wave as a compression wave. When a wave reaches an open end or large reservoir, it is reflected as a wave of equal magnitude but of opposite sign. When a wave encounters a change in area, a portion of the wave is transmitted in the original direction of travel and a portion is reflected back. With the assumption of one-dimensional streamline flow, it can be shown²⁰³ that the amplitude of the transmitted wave is given by

$$\frac{\Delta P_{\text{transmitted}}}{\Delta P_{\text{incident}}} = \frac{2A_{\text{incident}}}{A_{\text{incident}} + A_{\text{transmission}}} \quad (3.199)$$

and the ratio of reflected ΔP to the incident ΔP is

$$\frac{\Delta P_{\text{reflected}}}{\Delta P_{\text{incident}}} = \frac{A_{\text{incident}} - A_{\text{reflection}}}{A_{\text{incident}} + A_{\text{reflection}}} \quad (3.200)$$

This behavior is illustrated in Fig. 3.22.

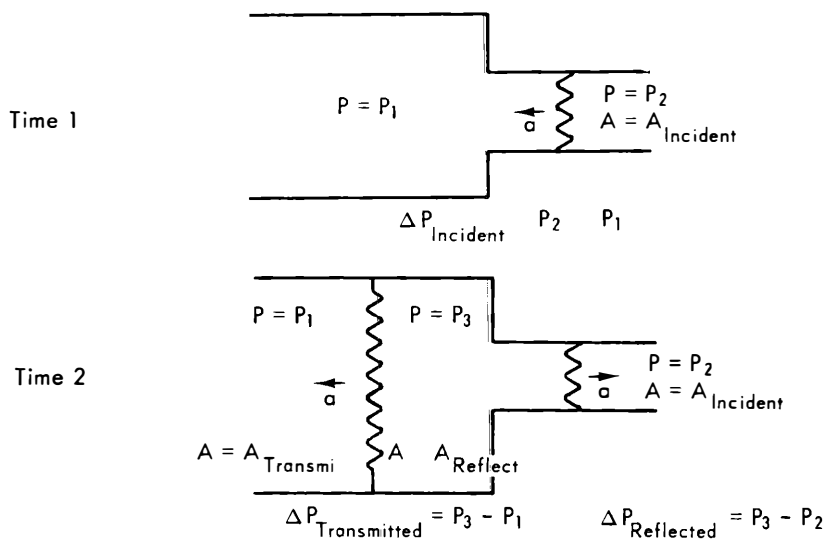


Fig. 3.22 Transmission and reflection of acoustic wave at area change.

In a frictionless system, the combination of forward and reflected waves is undamped and sets up a standing wave. In any real system, friction at the walls gradually reduces the amplitude of the waves. In a straight pipe, Lieberman and Brown²⁰³ give decay with time t as

$$\Delta P = (\Delta P_0) \exp(-fVt/D) \quad (3.201)$$

where

D = diameter of the pipe

f = Fanning friction factor

V = fluid velocity

$\Delta P_0 = \Delta P$ at zero time.

For a pipe diameter of 1 ft, a Fanning friction factor of 0.005, and water velocity of 20 ft/s, time constant D/fu is 10 s; thus, the frictional effect is relatively small.

When a pressure wave is reflected by a body that can significantly deflect, the amplitude of the reflected wave is appreciably reduced. The reader is referred to Streeter and Wylie²⁰⁴ for discussion of this case. In a reactor system, the piping and major components deflect very little and can usually be considered as rigid; however, deflection of any thin wall internal structure should be considered.

The many area changes in a reactor loop lead to the appreciable attenuation of an acoustic wave as it proceeds through the system. However,

the reflected waves set up at the area changes and the subsequent transmission and reflection of these reflected waves require an analysis that can follow this complex behavior in its entirety. The transmitted and reflected waves can act to reinforce each other in a particular region and to produce a high pressure across a particular component.

The arithmetic method²⁰⁴ is conceptually the simplest procedure for establishing system behavior. The method neglects friction and generally assumes a horizontal system. The procedure basically requires that each wave be followed through the system. Pressure change across each wave is determined by using Eq. (3.154). When reflected and transmitted waves are created at area changes, their subsequent behavior must be followed. The pressure-time history at a given location is determined by summing the pressure changes that occur as the various waves pass through the location in question.

Tediously tracking individual waves makes the arithmetic method unsuitable for all but the simplest of systems. The method of characteristics²⁰⁴ is most commonly used for solution of waterhammer problems. For single-phase flow, the continuity equation is written as

$$(\partial H / \partial t) + (a_1^2 / g)(\partial V / \partial x) + V(\partial H / \partial x) = 0 \quad (3.202)$$

the momentum equation is written as

$$g(\partial H / \partial x) + V(\partial V / \partial x) + (\partial V / \partial t) + (f V / 2D)|V| = 0 \quad (3.203)$$

where

H = fluid head

D = pipe diameter

x = axial distance

a_1 = sonic velocity

V = fluid velocity.

These partial differential equations are then converted into total differential equations that are subsequently expressed in finite difference form. The system is then divided into a series of control volumes and a set of finite difference equations is written for each control volume. Details of the numerical solution procedure are described in Sec. 4.1.2.3.

The simplest system modeling that can be used in this analysis is one that considers the reactor system to be one dimensional and, thus, to consist of a series of linear segments. While this is a reasonable way to simulate piping segments, it is not a good model of multidimensional regions within the reactor vessel. Streeter and Wylie²⁰⁵ pointed out that a more realistic modeling of such regions can be obtained by using an equivalent interconnected piping network instead of a simple linear arrangement. Fabric²⁰⁶ suggested further that when such network modeling is used, sonic velocities

should be adjusted for the fact that piping network paths are longer than those actually traveled by the sonic waves. He proposed that the sonic velocities be multiplied by $\sqrt{2}$ and $\sqrt{3}$, respectively, for each two- and three-dimensional region. Comparison of the calculated and experimental results shows improved agreement when the equivalent piping network with modified sonic velocities is used.²⁰⁷

The pressures produced by wave propagation within a PWR primary coolant system are of greatest concern in the brief period while the system is still subcooled after a hypothetical LOCA. Once boiling begins, sonic velocity is greatly reduced and the pressure waves are damped out. The WHAM code²⁰⁸ has been widely used for analyzing pressure wave behavior in subcooled reactor systems.

3.3.6 Pump Behavior with Two-Phase Flow

Two-phase fluid within a centrifugal pump leads to substantial degradation of performance over a wide range of void fractions. Two different approaches have been used to treat this problem. In one approach, analytic models are used to predict two-phase pump performance^{209,210} from single-phase performance. Model constants are adjusted to conform as well as possible to experimental data. Due to the complexity of the problem, this approach has been only moderately successful.

The second approach uses experimental data more directly via homologous curves for degraded operation. Extensive tests of centrifugal pumps similar to those used in PWRs have been carried out in the semiscale system at Idaho National Engineering Laboratory.²¹¹ These tests indicate that for α between about 0 and 20% there is little degradation in performance. The performance then falls sharply and remains nearly constant until $\alpha \approx 80\%$. Performance in this region is said to be fully degraded. Performance then rises sharply as α increases above 80% and then performance remains nearly undegraded. Based on this information, the RELAP4 and TRAC codes¹⁰² contain equations that fit semiscale pump homologous curves for head and torque in the fully degraded region. Performance is considered undegraded until voids reach about 7%. Over a narrow region, the pump head and torque are brought to the fully degraded level via a linear interpolation between the degraded and undegraded performance curves. The performance then remains fully degraded until voids exceed approximately 80% where an interpolation scheme brings the performance to undegraded levels.

3.4 FLOW INSTABILITY

The term *flow instability* refers to flow oscillations of constant or variable amplitude that are analogous to vibrations in a mechanical system. Mass flow rate, pressure drop, and voids can be considered equivalent to the

mass, exciting force, and spring of the mechanical system. In this connection, the relationship between flow rate and pressure drop plays an important role. Flow oscillations can be aggravated when there is thermohydrodynamic coupling between heat transfer, void, flow pattern, and flow rate; however, oscillations can occur even when the heat source is held constant. Both hydrodynamic and thermohydrodynamic instabilities are discussed, but the more elaborate nuclear thermohydrodynamic instabilities of boiling channels in water-cooled reactors are beyond the scope of this book.

Flow oscillations are undesirable in boiling, condensing, and other two-phase flow devices for several reasons:

1. Sustained flow oscillations can cause undesirable forced mechanical vibration of components.
2. Flow oscillations can cause system control problems of particular importance in liquid-cooled nuclear reactors where the coolant (such as water) also acts as a moderator.
3. Flow oscillations affect local heat transfer characteristics and boiling crisis.

Thus, the critical heat flux (CHF) was found by Ruddick²¹² to be reduced 40% when the flow was oscillating. This adverse effect was also found by Lowdermilk *et al.*²¹³ Mayinger *et al.*²¹⁴ found a similar reduction of CHF in an oscillating water flow at 2000 psia.

3.4.1 Nature of Various Flow Instabilities

3.4.1.1 Flow Pattern Instability

This occurs when flow conditions are close to the point of transition between churn flow and annular flow. A temporary increase in bubble population in churn flow (arising from a temporary reduction in flow rate) can change the flow pattern to annular flow with its characteristically lower pressure drop. When the driving pressure drop over the channel remains constant, flow rate attains a greater value. However, as flow rate increases, the vapor generated (even for unchanged heat transfer characteristics) can become insufficient to maintain annular flow. The flow pattern then reverts to that of churn flow and the cycle is repeated. The low-pressure drop characteristics of annular flow have been experimentally demonstrated by Wallis.²¹⁵ The cyclic behavior is partly due to the delay incurred in acceleration and deceleration of the flow.

3.4.1.2 Ledinegg Instability

Consider the operation of a boiler tube to which there is a constant heat input. At low flow, the boiler exit quality and velocity are high since the fluid is entirely, or almost entirely, vapor. This high vapor velocity accounts

for much of the pressure drop. When the flow is increased, slightly more vapor is generated and the pressure drop increases. However, as flow increases further, exit quality begins to decrease and velocity at the tube exit decreases. The pressure drop decreases and continues to decrease until the tube contents are all water. The pressure drop then increases as the water velocity increases. Experimental observations of this phenomenon are shown in Fig. 3.23 (Ref. 216).

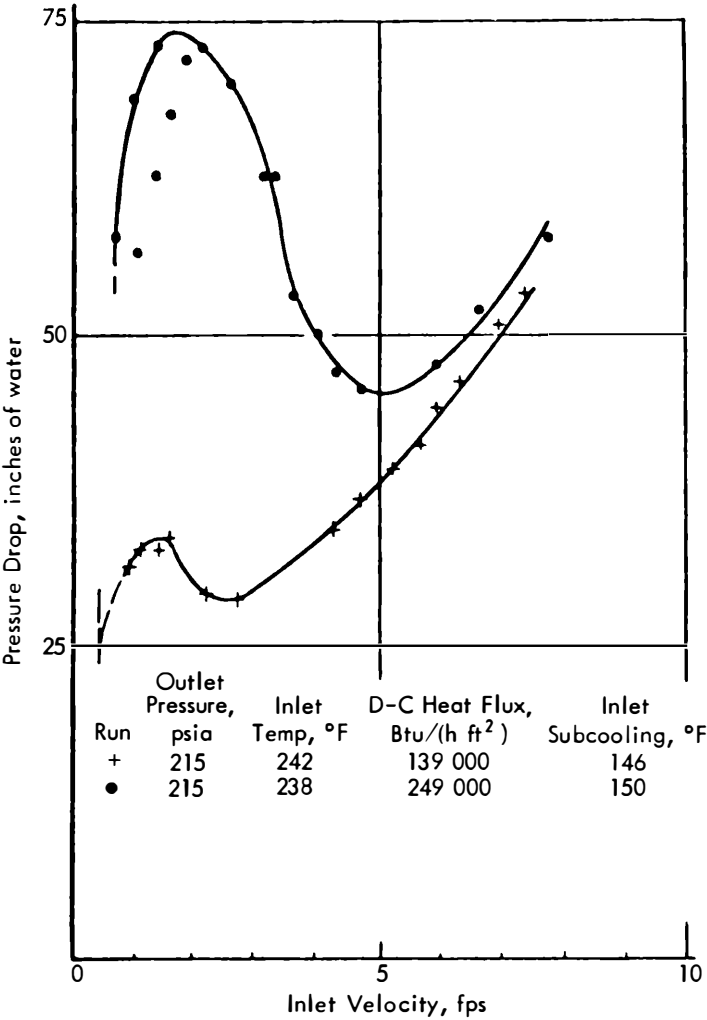


Fig. 3.23 Static pressure drop through a 0.174-in.-i.d. stainless-steel heated single tube (from Ref. 216).

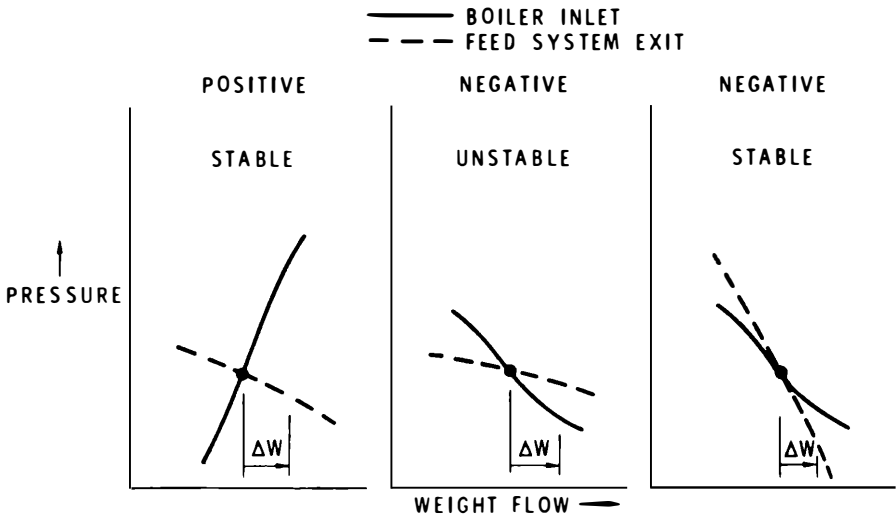


Fig. 3.24 Boiler resistance (from Ref. 217).

The effect of this phenomenon on stability can be understood by reference to Fig. 3.24, where we consider the operation of a boiling channel with a constant exit pressure. A portion of the pressure versus flow curve, which occurs at very low flow rates, is shown at the left. The corresponding curve for the boiler feed system is also shown. The system will operate where the two curves intersect. If flow to the boiler increases by a small amount, ΔW , boiler pressure becomes greater than supply pressure. This decelerates the flow and the system returns to the original operating point. On the other hand, if $\partial(\Delta P)/\partial G$ is negative, as shown in the central portion of Fig. 3.23, then an increase in flow decreases the inlet pressure so that the supply flow increases and continues to increase. This flow excursion can be predicted by the Ledinegg criterion,²¹⁸ which states that a necessary condition for the excursion is $\partial(\Delta P)/\partial G < 0$, where the steady-state characteristic of channel $G(\Delta P)$ is not single valued with identical inlet enthalpy and heat addition.

The flow can be stabilized in this region by providing a pump-supply curve whose slope is even more negative than that of the boiler (right side of Fig. 3.24). If boiler flow increases, the feed system cannot supply the high pressure needed and flow returns to the operating point. The system is then statically stable even though $\partial(\Delta P)/\partial G$ is negative. Therefore, the criterion for Ledinegg or static stability can be written as

$$\frac{\partial(\Delta P)_{\text{feed}}}{\partial G} - \frac{\partial(\Delta P)_{\text{loop}}}{\partial G} < 0 \quad (3.204)$$

where $(\Delta P)_{\text{feed}}$ is the driving head supplied by the feed pump and $(\Delta P)_{\text{loop}}$ is the flow friction head demanded by the loop.

A system that is statically unstable can exhibit flow oscillations, rather than a one-time excursion, if there is sufficient interaction and delayed feedback between the flow rate and mass accumulation (i.e., compressibility) in the heated section. Stenning and Veziroglu²¹⁹ call this type of oscillation *pressure drop oscillation*. It can generally be avoided by increasing the pressure drop at the channel inlet or by using a pump with a large head reduction at high flow rates. The techniques used for the analysis of dynamic instability will also determine whether pressure drop instability exists.

3.4.1.3 Dynamic Instability

A temporary reduction in the inlet flow to a boiling channel increases the rate of evaporation, thereby raising the average void fraction. This disturbance affects elevation, acceleration, and frictional pressure drop, as well as heat transfer behavior. For certain conditions of channel geometry, thermal properties of the heated wall, flow rate, inlet enthalpy, and heat flux, resonance with sustained oscillation can occur. Stenning and Veziroglu²¹⁹ found that the frequency of a density wave oscillation is higher than that of a pressure drop oscillation (Fig. 3.25). Neal and Zivi²²⁰ recognized that this oscillation is caused by flow-void feedback when a 180-deg phase shift exists between a flow disturbance and its void volume response. This can occur even when the boiler is operating in a region where $\partial(\Delta P)/\partial G$ is positive. We consider a single, closed loop containing pump, boiler tube with constant heat input, and condenser and assume that some sinusoidal flow oscillation at a specified frequency is introduced. If pressure drop across the boiler is examined, pressure oscillations are observed which are not necessarily in-phase with the flow oscillation; they can lag the flow oscillations.

We assume that this process is repeated for a number of different imposed frequencies. At each frequency, the lag (phase angle) between the velocity and pressure oscillations is determined as well as the ratio of amplitude of flow oscillation A_F to the amplitude of the pressure oscillation, A_P . A polar coordinate plot, usually called a Bode diagram, of a typical set of results is shown in Fig. 3.26 (Ref. 217). At steady state (zero frequency), pressure and flow are in phase, and the boiler has a completely positive resistance. Where the curve crosses the horizontal axis again, pressure and velocity are out-of-phase by 180 deg, and the boiler has a completely negative resistance.

To determine whether instability will occur in our system, we must perform a similar experiment with our feed system and expect to get results

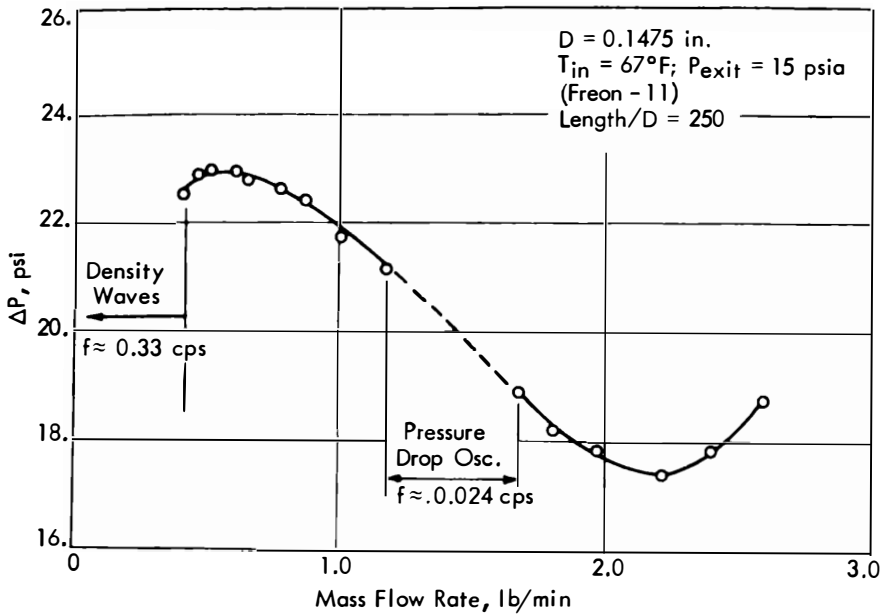


Fig. 3.25 Density and pressure wave oscillation regions [from *Proc. 1965 Heat Transfer and Fluid Mechanics Institute*, Stanford University Press, Stanford, CA (1965)].

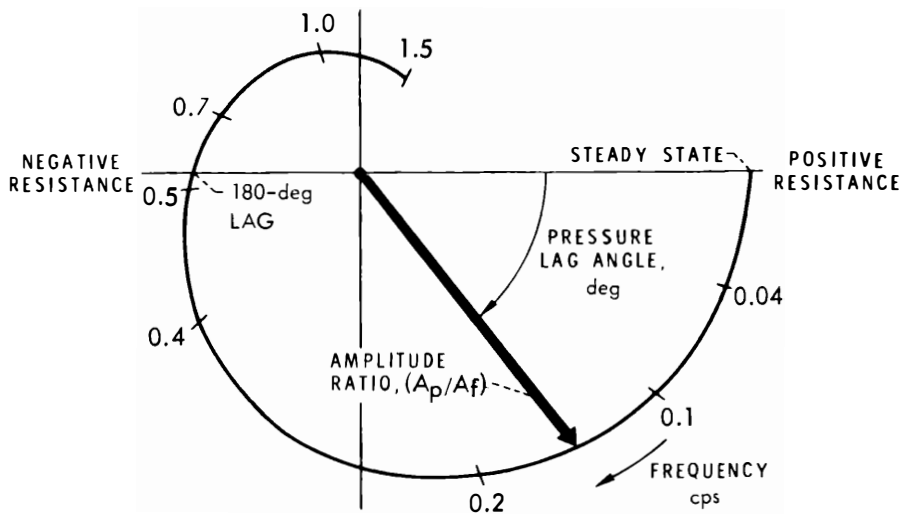


Fig. 3.26 Boiler-inlet impedance in polar coordinate form [from Ref. 217].

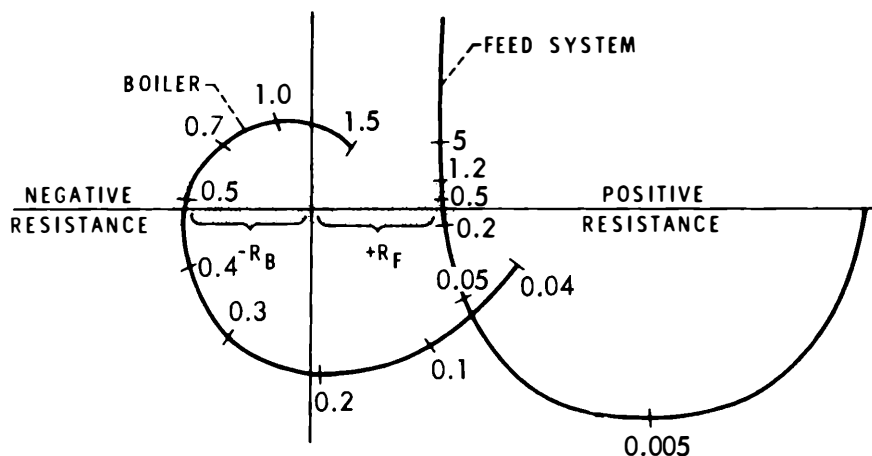


Fig. 3.27 Boiler-inlet and feedwater impedance [from Ref. 217].

such as those in Fig. 3.27 (Ref. 217). The feed system has a positive resistance at all frequencies. For the frequency range where the boiler has a negative resistance, $-R_B$, the feed system has a positive resistance, R_F . As long as $R_F > R_B$, the system is stable. If this is not the case, dynamic instability results.

3.4.1.4 Parallel Channel Instability

Parallel channel instability can arise when a number of channels are connected at common headers. Under these conditions, total flow can remain constant, but oscillations in the flow to some channels can occur. This problem is of particular concern in a pressure tube reactor where parallel channels are connected only at the inlet and exit. The problem can also arise in once-through steam generators. Moxon²²¹ notes that parallel channel instability is seen in such generators when a pressure drop perturbation arising from the evaporator and superheater is roughly equal to and 180-deg out of phase with a perturbation originating at the inlet.

3.4.1.5 Thermal Oscillation

This oscillation occurs in a film boiling region during rapid heating of a cryogenic system. It is a thermal response of the vapor film to flow disturbances. Disturbances that can be reinforced by a mechanism such as acoustic resonance become dominant in the system; these oscillations are often called acoustic instabilities. The velocity of sound is greatly reduced by the

presence of small quantities of void; this can lead to “pipe organ” resonances in short pipe lengths.

Observed frequencies have been related to the harmonics of a pipe that is open at both ends, and to Helmholtz resonance. This phenomenon has been reported by Edeskutz and Thursten.²²² The inception of oscillations can be predicted by

$$q''/GH_{fg} = 0.005(v_l/v_v) \quad (3.205)$$

where H_{fg} is enthalpy change on vaporization and the range of (v_v/v_l) is 9 to 25.

Cornelius and Parker²²³ detected acoustic instability (5 to 30 cps) of refrigerant Freon-114 in a natural circulation loop at a bulk temperature below critical temperature, but at critical pressure. Walker and Harden²²⁴ suggested that the region of pressure and flow fluctuations associated with a natural circulation loop operating at supercritical pressures occurs near maximum in a plot of density enthalpy product versus fluid temperature. Bergles *et al.*²²⁵ examined thermal oscillation in boiling water flow at 1000 psia. They found oscillations characterized by a frequency > 35 cps and most pronounced in the subcooled region. They also found pressure-drop amplitudes very large compared to steady-state values. The oscillations are considered to be acoustic in nature and induced by the collapse of subcooled voids.

3.4.1.6 Chugging

The term *chugging* is usually reserved for the cyclical phenomenon characterized by the periodic expulsion of coolant from a flow channel. The effects range from simple transitory variations of the inlet and outlet flow rates to a violent ejection of large amounts of coolant, usually through both ends of the channel. The chugging cycle consists of incubation, nucleation, expulsion, and reentry of the fluid. In the double-ended-expulsion case, when the two liquid slugs entering the channel collide, the vapor phase probably collapses. Although chugging was used by Fleck²²⁶ to describe gross instabilities in the SPERT-1 reactor system in 1959, little evidence of similar phenomena has been observed since that time.

Direct condensation chugging has been observed in BWRs where steam is directly vented into a water pool so as to limit the pressure rise in the containment vessel. At low steam flow rates (or at higher flow rates with low pool temperatures), water is forced out of the downcomer (injection pipe) and a steam bubble grows at the end of the downcomer. The bubble then condenses and water surges up the downcomer in waterhammer fashion. Since vapor suppression pools have not been used in PWR design, this phenomenon is not a problem for the PWR designer.

3.4.2 Effects of Various Parameters on Flow Instability

The effects of various flow-instability parameters, observed in studies previously mentioned, are as follows:

1. High heat input aggravates flow instability because of the resulting high rate of vaporization.
2. Increased inlet subcooling within a critical value reduces stability. Beyond that value, a further increase in subcooling can reverse the trend and stabilize flow.
3. Increased resistance at the exit strongly decreases stability since it increases exit pressure drop when a large void is generated in the channel.
4. Increased resistance at the inlet strongly increases stability. When flow is slowed by a disturbance, a large pressure head is retained at the inlet and is available for forcing the fluid through the channel, thereby stabilizing the flow.
5. Increased system pressure increases stability in proportion to the increase in reduced pressure, primarily because the difference in specific volumes of vapor and liquid then becomes smaller.
6. Increased static head in a long vertical channel increases the stability of upward boiling flow and decreases the stability of downward boiling flow. This occurs because the buoyancy of the vapor aids upward flow and opposes downward flow. With the latter, there is a tendency for the vapor to agglomerate and remain in the channel.

Acoustic or thermal oscillations are primarily associated with startup and transient situations²²⁰ and are usually not the major concern of the designer. Flow-pattern instabilities are imperfectly understood. Experimental observations are recommended. At present, the designer directs his major consideration to parallel channel instability and to Ledinegg and dynamic instability. Analysis for the last two problems can be carried on simultaneously since Ledinegg instability can be considered a special dynamic case occurring at zero frequency. These problems are fairly well understood and a theoretical analysis can be made.

3.4.3 Dynamic Instability Analysis

3.4.3.1 Basic Thermohydrodynamic Equations

Tong⁸⁹ reviewed this area extensively; we follow his discussion. First, we write the conservation equations for one-dimensional flow, with slip, in a vertical channel. We slightly modify equations (3.125), (3.126), and (3.127) for the mass balance to obtain

$$\frac{\partial}{\partial t}[\rho_l(1-\alpha) + \rho_v\alpha] + \frac{\partial}{\partial z}[\rho_l(1-\alpha)V_l + \rho_v\alpha V_v] = 0 \quad (3.206)$$

where z indicates axial distance and V_l and V_v are the liquid and vapor velocities, respectively. For the energy balance, we have

$$\frac{\partial}{\partial t}[\rho_l(1-\alpha)H_l + \rho_v\alpha H_v] + \frac{\partial}{\partial z}[\rho_l(1-\alpha)V_l H_l + \rho_v\alpha V_v H_v] = q''' \quad (3.207)$$

where energy dissipation is neglected, q''' is the heat input per unit volume of fluid and H_l and H_v are the liquid and vapor enthalpies, respectively. The foregoing assumes operation at a nearly constant pressure so that $dH = dE$. The momentum balance can be written as

$$\begin{aligned} & \frac{1}{g_c} \frac{\partial}{\partial t}[\rho_l(1-\alpha)V_l + \rho_v\alpha V_v] + \frac{1}{g_c} \frac{\partial}{\partial z}[\rho_l(1-\alpha)V_l^2 + \rho_v\alpha V_v^2] + \frac{\partial P}{\partial z} + F \\ & \pm [\rho_l(1-\alpha) + \rho_v\alpha] \frac{g}{g_c} = 0 \end{aligned} \quad (3.208)$$

A positive sign for the final term indicates upward flow; a negative sign indicates downward flow; F is the frictional force per unit volume of fluid.

These conservation equations assume that mixture behavior is adequately defined by overall conservation equations obtained from summing the conservation equations for the individual phases. Furthermore, each individual phase is assumed to occupy a given fraction of the channel cross section and to have a velocity that is constant across the cross section occupied.*

If the system remains at a nearly constant pressure, the equation-of-state can be written as

$$\rho_l \approx \rho_l(H_v) \quad (3.209a)$$

$$\rho_v \approx \rho_v(H_v) \quad (3.209b)$$

Note that in subcooled boiling, the mixture density should be calculated on the basis of the amount of local boiling void present (see Sec. 3.3.3.2).

The preceding equations can be simplified by introducing the quality defined as

$$\chi = \frac{w_v}{w_v + w_l} = \frac{\alpha \rho_v V_v}{\alpha \rho_v V_v + (1-\alpha) \rho_l V_l} \quad (3.210)$$

*Two-phase flow conservation equations have been written in a variety of forms. For a more comprehensive view of this subject, the reader is referred to Refs. 227, 228, and Sec. 3.3.4.

If local mass velocity is defined as

$$G = \alpha \rho_v V_v + (1 - \alpha) \rho_l V_l \quad (3.211)$$

the continuity balance becomes

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial G}{\partial z} = 0 \quad (3.212)$$

The volume-weighted mean density, $\bar{\rho}$, is given by Eq. (3.75) and the energy balance is given by

$$\bar{\rho} \frac{\partial H}{\partial t} - H_{fg} \frac{\partial \psi}{\partial t} + G \frac{\partial H}{\partial z} = q''' \quad (3.213)$$

The quantity ψ defined by

$$\psi = \rho_l \chi (1 - \alpha) - \rho_v \alpha (1 - \chi) \quad (3.214)$$

is a slip correction for the energy balance and is zero for nonslip flow. Mixing cup enthalpy H is defined by

$$H = H_l + \chi (H_v - H_l) \quad (3.215)$$

The momentum balance is

$$\frac{1}{g_c} \left[\frac{\partial G}{\partial t} + \frac{\partial}{\partial z} (v' G^2) \right] + \frac{\partial p}{\partial z} + F \pm \frac{\bar{\rho} g}{g_c} = 0 \quad (3.216)$$

where v' the effective specific volume for spatial acceleration, is

$$v' = \frac{1}{G^2} [\rho_l V_l^2 (1 - \alpha) + \rho_v V_v^2 \alpha] = \frac{\chi^2}{\alpha \rho_v} + \frac{(1 - \chi)^2}{(1 - \alpha) \rho_l} \quad (3.217)$$

This becomes $v' = 1/\bar{\rho}$ for nonslip flow, and the equation-of-state can be written as

$$\bar{\rho} = \bar{\rho}(H) \quad (3.218)$$

If channel size is small (e.g., between fuel rods in a PWR core) and vapor quality is sufficiently low so that bubbly flow exists, slip is negligible at high pressures. This is particularly true during a flow oscillation, since void distribution changes as flow changes from accelerating to decelerating.

3.4.3.2 Linearization and Laplace Transformation (Frequency-Domain Analysis)

When we consider a steam generator consisting of a large number of boiler tubes, each generating approximately the same amount of steam, it is usually adequate to consider the steam generator as a single channel. If we

utilize the technique of small perturbations, the variables in Eqs. (3.206), (3.207), and (3.208) can be described by

$$\begin{aligned} G(z,t) &= G(z,0) + \Delta G(z,t) \\ H(z,t) &= H(z,0) + \Delta H(z,t) \\ \bar{\rho}(z,t) &= \bar{\rho}(z,0) + \Delta \bar{\rho}(z,t) \\ q'''(z,t) &= q'''(z,0) + \Delta q'''(z,t) \end{aligned} \quad (3.219)$$

where $\Delta G(z,t)/G(z,0) \ll 1$, etc. If these variables are substituted into the conservation equations and if the second-order terms are ignored, the equations can be linearized to yield

$$\frac{\partial \Delta G}{\partial z} = -\frac{\partial \Delta \rho}{\partial t} \quad (3.220)$$

$$\Delta q''' = G \frac{\partial \Delta H}{\partial z} + \Delta G \frac{\partial H}{\Delta z} + \rho \frac{\partial \Delta H}{\partial t} \quad (3.221)$$

$$\frac{1}{g_c} \left[\frac{\partial \Delta G}{\partial t} + \frac{\partial}{\partial z} \left(\frac{2G\Delta G}{\rho} - \frac{G^2}{\rho^2} \Delta \rho \right) \right] + \frac{\partial \Delta P}{\partial z} + \Delta \rho \frac{g}{g_c} + \frac{fG}{g_c D_e \rho} \Delta G - \frac{fG^2}{2g_c D_e \rho^2} \Delta \rho = 0 \quad (3.222)$$

$$\Delta \rho = \frac{\partial \rho}{\partial H} \Delta H + \frac{\partial \rho}{\partial G} \Delta G + \frac{\partial \rho}{\partial q'''} \Delta q''' \quad (3.223)$$

Assume that we wish to examine the system stability with respect to inlet mass flux perturbations. In the usual approach, the linearized continuity, energy, and density equations [Eq. (3.220), (3.221), and (3.223)] are integrated to obtain the time-dependent distribution of density, mass flux, and enthalpy along the channel. The integration assumes that properties are independent of pressure and hence the continuity, energy, and density expressions are decoupled from the momentum equation. The result of the integration is then used in the momentum equation to calculate the pressure drop perturbation due to an inlet mass flux perturbation. The Laplace transformation of the resultant equation is then obtained.

Recall that the Laplace transform, $\mathcal{L}$, of a function of time, $f(t)$, is given by

$$\mathcal{L} f(t) = \int_{-\infty}^{\infty} f(t) e^{-st} dt \quad (3.224)$$

where s is a complex number ($\sigma + j\omega$). The real part, σ , gives the amplitude coefficient whereas the imaginary part, ω , represents the angular frequency of oscillation. The Laplace transform technique converts a partial differential equation into an algebraic equation in terms of s . The result of the

transformation yields a characteristic equation $H(s)$, which represents the dynamic response of the system in the frequency domain. That is,

$$\delta(P) = \delta(G)/H(s) \quad (3.225)$$

where $H(s)$ = open loop transfer function expressed as an algebraic equation in s , $\delta(G) = \Delta G e^{st}$, and $\delta(P) = \Delta P_{\text{exit}} e^{st}$. The stability of the channel is determined by setting $H(s)$ equal to zero and finding the roots of the equation. The system is stable only if all the roots have negative real parts.

For the stability analysis of an entire system, rather than just a boiling channel, the behavior of all the system components must be modeled. If the system can be modeled as a sequential set of components in a single loop, the system transfer function, $C(s)$, is then determined from

$$C(s) = \sum_i H_i(s) \quad (3.226)$$

where $H_i(s)$ is the transfer function for component i . Equation (3.226) requires that all components be approximated by linear differential equations with constant coefficients. Expressions for the transfer functions of a variety of components are available in the literature.²²⁹ In determining system stability, the same criterion is applied to $C(s)$ as previously applied to $H(s)$.

Equation (3.226) may also be used to avoid the difficulties of analytic integration along a boiling channel. The channel, or the entire system, may be divided into i subsections in which all variables and parameters are treated as unchanged and having their average steady-state value over the node. The equation set for each node is perturbed, linearized, and Laplace transformed to obtain the transfer function for each node. The transfer function for the channel or system is then the sum of the transfer functions for the individual nodes.

Linearized models using a frequency-domain analysis, such as exemplified by the STABLE 3 program²³⁰ have been shown²³¹ to be capable of providing reasonable predictions of existing data.^{219,231-235} A more recent example of the frequency domain approach is the DYNAM/US²³⁶ program.*

It is possible to apply the frequency-domain approach to systems containing a number of parallel channels. Lahey²³⁷ indicates how a nodal approach with an appropriately defined transfer function may be used. The DENSITY-PARA program²³⁸ provides an example of the application of this approach.

*Those readers interested in frequency-domain stability analyses, which include reactivity feedback effects, should see Chapter 7 of *The Thermal-Hydraulics of a Boiling Water Nuclear Reactor*, 2nd. ed, by R. T. Lahey, Jr., and F. J. Moody, American Nuclear Society, LaGrange Park, IL (1993).

3.4.3.3 Nonlinear Techniques

The chief advantage of linearized approaches is that perturbation of the governing equations around the steady-state operating conditions and conversion of the resulting linearized model to the frequency domain yields analytical solutions for the system transfer functions. These functions can then be examined using well-established criteria to obtain rigorous conditions for system stability. The approach is useful in determining (1) whether a particular operating point is stable and (2) the conditions needed for the onset of instability. However, the approach does not determine the range of stability for conditionally stable systems (system is stable for infinitesimal perturbation but unstable with a finite perturbation of sufficient magnitude) or the responses of an unstable system. In addition, the linear response assumed may not be a very good approximation of actual system behavior.

Nonlinear approaches to dynamic instability have been used by a number of investigators (e.g., Refs. 239 through 242). The basic approach first divides the system examined into a number of control volumes. The conservation equations are then written for each control volume in finite difference form. Steady-state conditions are assumed and the conservation equations are numerically integrated along the channel or channels. One of the system parameters (i.e., pressure, inlet flow, heat flux) is subjected to a finite-sized perturbation. The time variation of system behavior is then determined by numerically integrating the conservation equations along the channel for a series of time steps. If the perturbation produces oscillations in flow rate that damp out with time, the system is stable; if the oscillations persist, the system is unstable. Rosa and Podowski²⁴¹ point out that care must be taken to assure that an adequate number of axial nodes is provided. Failure to do this can lead to misleading results.

3.4.3.4 Simplified Criterion for Dynamic Instability

It is desirable to be able to obtain a preliminary estimate of the dynamic stability boundary under many circumstances. If the boiling channel can be considered to be placed between two plenums across which a constant pressure drop is maintained (equivalent to a feed pump with a flat characteristic in the range of interest), Ishii's²⁴³ stability criterion can be used. At high inlet subcooling, he concluded that the system is stable when

$$N_{\text{pch}} \leq N_{\text{sub}} + \frac{2[K_i + (f_m/2D_h^*) + K_e]}{1 + \frac{1}{2}[(f_m/2D_h^*) + 2K_e]} \quad (3.227)$$

where

$$\begin{aligned} N_{\text{pch}} &= \text{equilibrium phase change number} = \Omega_{\text{eq}} L / V_i \\ \Omega_{\text{eq}} &= \text{equilibrium frequency of phase change, rad/s} \\ &= \Gamma_g \Delta \rho / (\rho_g \rho_l) \end{aligned}$$

Γ_g = mass rate of vapor generation per unit vol, mass/vol s

N_{sub} = subcooling number $= (\Delta\rho/\rho_g)(\Delta H_{\text{sub}}/\Delta H_{fg})$

ΔH_{sub} = inlet subcooling $= H_{\text{sat}} - H_{\text{in}}$

ΔH_{fg} = enthalpy change on evaporation

$H_{\text{in}}, H_{\text{sat}}$ = enthalpy of liquid at inlet and saturation, respectively

ρ_g, ρ_l = gas and liquid densities, respectively

$\Delta\rho = \rho_l - \rho_g$

f_m two-phase mixture friction factor

D_h hydraulic diameter, $D_h^* = D_h/L$

L = length of heated channel

K_i, K_e = inlet and exit orifice coefficient, $\Delta P = KG^2/\rho g_c$

V_i fluid velocity at channel inlet.

The stability criterion of the previous equation holds only if $N_{\text{sub}} > (N_{\text{sub}})_{\text{cr}}$ and $(N_{\text{sub}})_{\text{cr}}$ is obtained from²⁴⁴

$$(N_{\text{sub}})_{\text{cr}} = 0.0022 (\text{Pe})(A_c/p_h L)(N_{\text{pch}})_0 \quad (3.228)$$

$$(N_{\text{sub}})_{\text{cr}} = 154(A_c/p_h L)(N_{\text{pch}})_0 \quad \text{if } \text{Pe} \geq 70,000 \quad (3.229)$$

where

Pe = Peclet number for liquid at inlet conditions

A_c = cross-sectional area of boiling channel

p_h = heated perimeter of boiling channel

$(N_{\text{pch}})_0$ = value of N_{pch} determined from linearized stability calculations for zero subcooling at the inlet.

Under most conditions, $(N_{\text{pch}})_0$ occurs at channel exit quality x_e , which is close to 1.0. Therefore, an approximate value for $(N_{\text{pch}})_0$ can be obtained from this fact and the relationship

$$(N_{\text{pch}}) = x_e (\Delta\rho/\rho_g) \quad (3.230)$$

At inlet subcooling numbers below $(N_{\text{sub}})_{\text{cr}}$, the system is expected to be stable if

$$(N_{\text{pch}}) \leq \left[\frac{(N_{\text{pch}})_{\text{cr}} - (N_{\text{pch}})_0}{(N_{\text{sub}})_{\text{cr}}} \right] N_{\text{sub}} + (N_{\text{pch}})_0 \quad (3.231)$$

where $(N_{\text{pch}})_{\text{cr}}$ = the value of N_{pch} obtained from Eq. (3.227) when $N_{\text{sub}} = (N_{\text{sub}})_{\text{cr}}$. The Freon loop stability data of Saha *et al.*²⁴⁴ appear to be in good agreement with this simplified approach.

3.4.4 Parallel Channel Instability

Momentum integration is particularly useful in treating the problem of parallel channel instability. Nonlinear transient momentum equations can be

conveniently solved for several channels by integrating the momentum equation along each channel. Fluid properties are obtained from the energy equation and the equation-of-state; integrated equations are then solved simultaneously. These techniques have been incorporated into the HYDNA computer code,²⁴⁵ which predicts flow oscillation in parallel channels. To describe the HYDNA computational method, we again follow Tong.⁸⁹

It is useful to separate inlet mass velocity and the instantaneous axial mass flow rate distribution. If $\Phi(z,t)$ is the axial mass flow rate distribution in the channel and $G(0,t)$ is the inlet mass flow rate, which can vary with time, we have

$$G(z,t) = G(0,t)\Phi(z,t) \quad (3.232)$$

Substituting this, where $G(z,t)$ is mass velocity as a function of z and t , into the momentum balance over the channel length gives

$$\frac{dG(0,t)}{dt} = - \frac{\Delta P + \int_0^L F dz + \int_0^L \pm \rho dz + \frac{1}{g_c} \Delta(v'G^2) + \frac{1}{g_c} G(0,t) \int_0^L \frac{\partial \Phi}{\partial t} dz}{\frac{1}{g_c} \int_0^L \Phi dz} \quad (3.233)$$

where ΔP is the pressure difference between the common plenums and $v' = 1/\rho$ for homogeneous flow. Factor $\int_0^L \Phi dz$, which accounts for the effect of axial variation of the mass velocity, comes close to unity for a slow transient if G remains nearly constant throughout the passage. However, $\partial \Phi / \partial t$ vanishes if the shape of $G(z)$ remains constant. All terms of Eq. (3.237), except $dG(0,t)/dt$ and ΔP , can be easily determined. Let C and B be given as

$$C = - \frac{1}{\frac{1}{g_c} \int_0^L \Phi dz} \quad (3.234)$$

and

$$B = - \frac{\int_0^L F dz + \int_0^L \pm \rho dz + \frac{1}{g_c} \Delta(v'G^2) + \frac{1}{g_c} G(0,t) \int_0^L \frac{\partial \Phi}{\partial t} dz}{\frac{1}{g_c} \int_0^L \Phi dz} \quad (3.235)$$

Substitution into the momentum balance yields

$$\frac{dG(0,t)}{dt} = C \Delta P + B \quad (3.236)$$

Equation (3.236) can be written for each of n parallel channels

$$\begin{aligned}\frac{dG_1(0,t)}{dt} &= C_1 \Delta P + B_1 \\ \frac{dG_2(0,t)}{dt} &= C_2 \Delta P + B_2 \\ \frac{dG_n(0,t)}{dt} &= C_n \Delta P + B_n\end{aligned}\quad (3.237)$$

With a given total inlet flow rate $W(t)$,

$$A_1 G_1(0,t) + A_2 G_2(0,t) + \dots + A_n G_n(0,t) = W(t) \quad (3.238)$$

where $A_1, A_2, \dots, A_n$ are the respective flow areas of channels 1, 2, $\dots, n$. Differentiation of Eq. (3.238) gives

$$A_1 \frac{dG_1(0,t)}{dt} + A_2 \frac{dG_2(0,t)}{dt} + \dots + A_n \frac{dG_n(0,t)}{dt} = \frac{dW}{dt}$$

or

$$A_1 (C_1 \Delta P + B_1) + \dots + A_n (C_n \Delta P + B_n) = \frac{dW}{dt} \quad (3.239)$$

The latter equation can be used to evaluate the pressure drop that is common to all channels; i.e.,

$$\Delta P = \frac{(dW/dt) - \sum B_n A_n}{\sum C_n A_n} \quad (3.240)$$

Equation (3.239) can be used to predict the pressure drop as a function of time by iterating on it along with n simultaneous equations of Eq. (3.237). The steps in the calculation for a small time increment are given by Tong⁸⁹ with a method that can predict the thermal and hydraulic behavior of the system in response to a specified variable inlet flow rate or a given variable power input to the channel.

The general nonlinear approach can also be used for parallel channels that are interconnected. Of course, the conservation equations for each channel modeled must be appropriately altered to allow for interchange between the channels if an interconnection is assumed. The PARALLEL²⁴⁶ and LOOP²²¹ codes are examples of such alternatives. The PARALLEL program uses the nonlinear technique described in Sec. 3.4.3.3 but provision for parallel boiling channels is included.

3.4.5 Flow Pattern Instability

An instability that appears to fall into this classification may be seen in a LOCA. During the later stages of blowdown, refill of the lower plenum, and reflood of the core, the subcooled emergency core coolant (ECC) may interact with steam flowing in the cold legs to produce flow and pressure oscillations or rapid transients. Oscillations have been observed in scale model tests (1/30 to 1/3 PWR scale) of the ECC injection region. It appears that this is primarily a flow pattern transition instability due to the change from dispersed liquid to a liquid slug near the injection point. The heat transfer rate to droplets is high with the result that the pressure is reduced. A plug of injected liquid then moves upstream and covers the injection port. The condensation rate is reduced and the pressure increases so that the steam from the steam generator flows into the injection area.

Capiello and Liles²⁴⁷ were able to predict the oscillation in ECC flow observed in a 1/14-scale experiment. By using a version of TRAC that contained a stratified-slug criterion in the code's generalized flow map, oscillatory flow behavior was indicated. The predicted oscillation frequency was, however, somewhat lower than observed.

3.4.6 PWR Instability Concerns

Under the conditions that may be expected during anticipated transients and normal operation of vessel-type PWRs, no instabilities are observed in the primary system. There is relatively little boiling in the core and hence little void reactivity feedback. Further, all the core channels are interconnected. It has been shown, both experimentally and analytically, that a set of interconnected channels will have a much larger area of stable operation than a set of isolated parallel channels.²⁴⁸

The situation in PWR cores is in marked contrast to that seen in BWRs. In a BWR, the coolant channels are isolated and the large void fraction in the core leads to a substantial void-power feedback. BWRs therefore operate closer to the dynamic instability threshold. Under natural circulation conditions, flow and neutron flux instability have actually been observed in some operating reactors.²⁴⁹

Instability in pressure tube PWRs is more of a concern than in vessel-type PWRs. Such systems must be carefully analyzed to determine whether parallel channel instability presents a problem. The situation is of most concern in some newer CANDU designs where boiling in the core at high power is utilized giving an average outlet quality of up to 4%.²⁵⁰ This leads to considerably greater void fraction in the core than in the usual vessel-type PWR. Void reactivity feedback can no longer be neglected.

To improve the stability of the new CANDU designs, some system modifications have been made. As usual, the heat transfer system is arranged in two essentially independent circuits; one on each side of the vertical axis

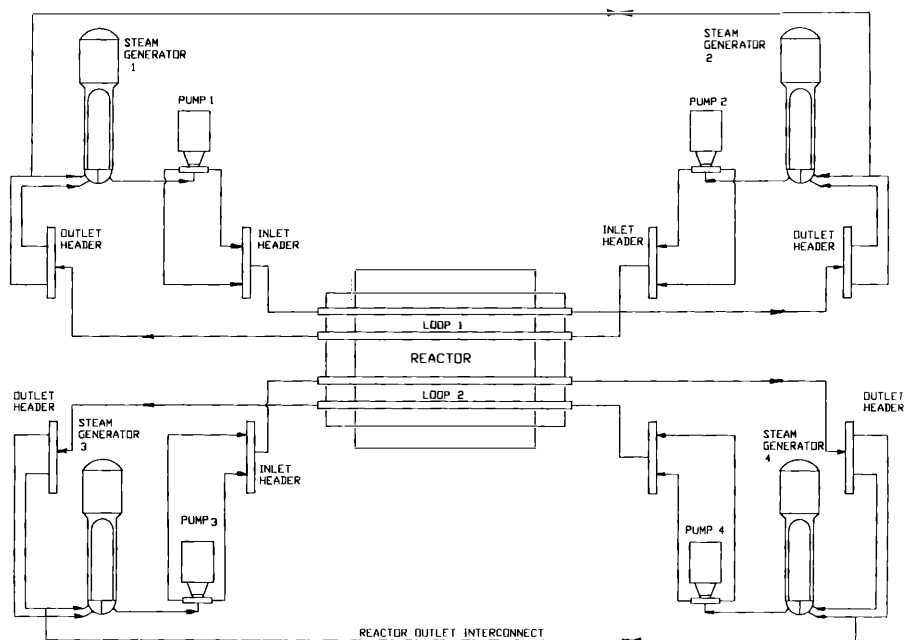


Fig. 3.28 Use of reactor outlet interconnect to enhance CANDU stability [from Garland and Pang, *Nucl. Technol.*, 75, 241 (1986)].

of the core. Each circuit contains two inlet headers, two steam generators, and two outlet headers (see Fig. 3.28). The new systems are modified by an interconnection line, which connects the outlet headers of each circuit. The system stability analysis shows the system to be stable during normal operation with the interconnection line in place but that parallel channel instabilities would occur if it were removed. If a flow perturbation occurs, the interconnection reduces the effect of the perturbation by allowing fluid to move from the remote header to the perturbed header. The ability of the fluid to move from the perturbed header is enhanced if the interconnect contains vapor. To increase the fraction of steam in the interconnect line, the line is elevated and drainage is provided.

Flow stability is also a concern in the Savannah River Site (SRS) production reactors. These vertical pressure tube reactors are more susceptible to flow instability than power reactors because of the low pressure (2 bar) at which SRS reactors operate. While SRS reactors are stable at normal operating conditions, local flow excursions are possible under accident conditions.²⁵¹ During a postulated accident in which reactivity is inserted in one flow channel, the increased power would produce substantial boiling and void production. The sharply increased channel pressure drop leads to

a situation where the Ledinegg stability criterion is not met. The channel flow is then sharply reduced, leading to inadequate cooling of the fuel. The channel power level at which the flow excursion occurs is lower than the power level at which conventional correlations predict the occurrence of the critical heat flux (see Sec. 4.42 for critical heat flux discussion). The margin to the onset of flow instability is therefore the limiting factor in setting the reactor operating power.

In addition to analyzing the primary system, a stability analysis of the steam generator should also be conducted. Unstable operation may be possible under some transient or accident conditions.

Flow oscillations during a LOCA are also of some concern. The flow pattern instability possible in the ECCS injection line has been described in Sec. 3.4.5. Flow oscillations have also been observed during the bottom reflooding of a core after a LOCA.²⁵² White and Duffey²⁵³ have proposed a simple model in which the liquid in the downcomer and the two-phase mixture around the fuel are regarded as a piston trapping a volume of steam, which is leaking out through an outlet orifice. Their model predicted oscillation frequencies that depended on the steam generation rate and that were in rough agreement with experiment. Since the steam generation dynamics depend strongly on the quench front, this is a unique phenomenon.

3.4.7 Recommended Methods of Analysis

The onset of static instability in a natural or forced circulation loop can be determined by the Ledinegg criterion²¹⁸ with a lumped pressure drop.

The avoidance of dynamic and parallel channel instability remains the major concern of the designer. The early preference for the linearization and Laplace transform approach for analysis of the dynamic instability was clearly based on the relatively short computer times required. However, with the very large increase in computer power that has become available, the longer computing times of the nonlinear approach are now not much of a problem. The nonlinear techniques allow a better representation of the problem as well as the examination of conditional stability. Further, most of the widely used accident analysis programs can be easily adapted to stability analysis.^{250,254}

Momentum integration remains a very convenient method for examining parallel channel instabilities. However, a conservative design philosophy would dictate that the final (reference design) stability analysis should include the effect of reactivity feedback (also true for vessel cores even though such effects are likely to be quite small). It may then be easier to use a nonlinear approach.

The onset of flow pattern instabilities cannot be adequately predicted in the general case. While the instability seen in the cold leg during emergency coolant injection was predicted after the fact, it was not expected prior to

the operation of the experiment in which it was observed. Experimental studies are recommended.

3.5 LIQUID-VAPOR SEPARATION

Liquid entrainment during evaporation, vapor carry-under in a natural circulation loop, and steam separation are closely related because they are all controlled by buoyancy forces. Although they are usually considered to be in the province of the boiling water reactor, they must be understood for steam generator design and system behavior during a LOCA. Although not discussed here, note that entrainment phenomena can be significant in severe accidents. Aerosol generation and molten cladding and fuel relocation models include entrainment as a transport mechanism.

3.5.1 Entrainment and Carry-Over

3.5.1.1 *Entrainment and Carry-Over from Horizontal Surfaces*

Early studies of entrainment from a horizontal liquid surface were carried out by Stermann,²⁵⁵ Newitt *et al.*²⁵⁶ and Mitsuishi *et al.*²⁵⁷ Mitsuishi *et al.* noted that at low gas superficial velocities most droplets injected into the vapor stream fell back into the liquid and only the smallest droplets were carried off by the vapor. However, above a superficial velocity of ~ 6 ft/s at atmospheric pressure, entrainment sharply increased and even small droplets were carried along with the vapor. Stermann found that in small size vessels the amount of entrainment depends on the vessel size. Stermann²⁵⁵ provides an expression for determining minimum diameter D at which entrainment is independent of vessel size:

$$\frac{D[\rho_v/(\rho_l - \rho_v)]^{-0.2}}{[\sigma'/(\rho_l - \rho_v)]^{1/2}} \geq 260 \quad (3.241)$$

where

σ' = surface tension, lb/ft

ρ_l = liquid density, lb/ft³

ρ_v = vapor density, lb/ft³

D = diameter, ft.

We may define carry-over as those entrained liquid droplets which are carried along by the vapor and do not fall back into the liquid. Davis²⁵⁸ examined water entrainment and carry-over at high pressures. He found that the maximum height to which droplets are carried by rising steam increases with increasing steam velocity. This is clearly seen in the insert of Fig. 3.29. The height at which the water droplets are observed gradually increases until a critical carry-over velocity is reached and surface separa-

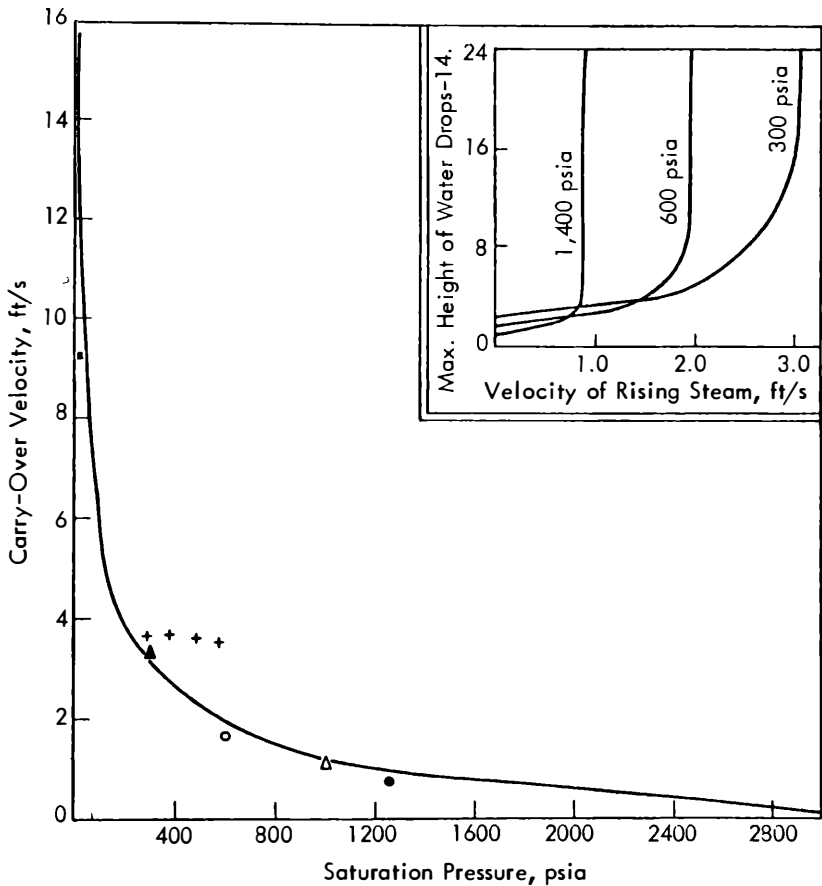


Fig. 3.29 Surface disengagement carry-over velocities. *Notation:* Curves from Ref. 258
 + Allis Chalmers, 33-in. sep. ht.
 Δ General Electric, 32-in. sep. ht.
 ▲ Combustion Engineering, 50-in. sep. ht.
 ○ Combustion Engineering, 30-in. sep. ht.
 ● Combustion Engineering, 24-in. sep. ht.
 ■ N. Mitsubishi *et al.*, AEC-TR-4225, U.S. Atomic Energy Commission, Washington, D.C. (1961).

tion can no longer be achieved. At high pressures, the carry-over occurs more suddenly and at a much lower steam velocity. The carry-over velocity computed by Davis as a function of pressure is shown in Fig. 3.29. Also shown are data obtained in various U.S. government-sponsored programs. Note that the data are in reasonable agreement with the Davis curve.

The Davis curve applies to carry-over from large size vessels. Wilson and McDermott's²⁵⁹ data can be used in evaluating carry-over in small flow channels with cold walls.

As indicated in Chapter 1, surface separation is used in the horizontal steam generators included in the Russian VVER system. All of these steam generators contain a horizontal perforated plate located slightly below the normal liquid level. It is found that such a plate substantially reduces the entrainment rate.²⁶⁰

Significant carry-over would be encountered during the reflood phase of a hypothetical LOCA. In this period, the core would be partly refilled, but there would be substantial steam generation due to the decay heat released by the fuel rods. When the average steam velocity exceeds the carry-over velocity (Fig. 3.29), a portion of the water being added to reflood the core is removed by entrainment. The amount of this entrainment is generally estimated from empirical correlations of test data. The rate of entrainment, $\dot{M}_E$, is sometimes given in terms of the carry-over fraction. That is,

$$\dot{M}_E = (\text{CRF})(\dot{M}_f) - \dot{M}_{sg} \quad (3.242)$$

where

$\dot{M}_f$ = rate at which water is added to core, mass/time

CRF = carry-over fraction

$\dot{M}_{sg}$ = rate of steam generation in the core, mass/time.

The carry-over fraction (rate of total core exit flow rate to core inlet flow rate) was correlated by²⁶¹

$$\begin{aligned} \text{CRF} = & 0.9553 \exp[-0.0013935(P/60)^2][1 - \exp[-4.3127(q'/1.24)]] \\ & \times \{1 - \exp[-16.62(V/6)]\} \exp[-0.037161(\Delta T/140)^2] \\ & \times \frac{1 + \sin 3.1416[-0.5 + (Z/6)^{0.025733(6/V) + 0.13553(\Delta T/140) + 0.0656(P/60)}]}{2} \end{aligned} \quad (3.243)$$

where

P = pressure, psia

q' = peak linear heat rate, kW/ft

V = flooding rate, in./s

ΔT = coolant subcooling at inlet, °F

Z = quench front level, ft.

Note that Eq. (3.243) is an empirical correlation of data for PWR ECCS evaluation during a hypothetical LOCA. It should not be used for other conditions.

Entrainment rates from horizontal surfaces are also of concern to the process industries and more general correlations are available from this

source. Azbel and Liapes²⁶² provide an entrainment rate correlation in terms of dimensionless groups. Based on data from several systems, they conclude:

$$G_e/G_g = 3.17 \times 10^{13} A^{1.4} B^{1.8} \quad (3.244)$$

where

G_e = superficial mass flux of entrained liquid, mass/area time

G_g = superficial mass flux of vapor, mass/area time

$$A = \frac{v_l^2 V_g \rho_g}{g^2 [\sigma' / (g \Delta \rho)]^{3/2} L_s (\Delta \rho)}$$

$$B = \left[\frac{\sigma'}{g(\Delta \rho)} \right]^{1/2} / L_s$$

σ' = surface tension

v_l = kinematic viscosity of liquid

$\Delta \rho = \rho_l - \rho_g$

V_g = superficial velocity of vapor

L_s = available height above separating surface

and other symbols have their previous meaning. Note that Eq. (3.244) would predict a near zero carry-over if a long disengagement height is provided.

3.5.1.2 Entrainment in Annular Flow

In adiabatic annular flow, liquid droplets are continuously entrained from the liquid film on the tube surface. At the same time, some of the liquid droplets suspended in the vapor core are deposited on the liquid film. For long unheated tubes, the rate of deposition is in equilibrium with the rate of entrainment, and the film thickness remains constant.

Modeling of annular flow is usually based on procedures developed at Harwell.²⁶³ The deposition rate, R (mass/area time), is determined from the product of a mass transfer or deposition coefficient K_d and droplet concentration C ; that is,

$$R = K_d C \quad (3.245a)$$

where C is determined from

$$C = W_E / [(W_E / \rho_l) + (W_G / \rho_g)] \quad (3.245b)$$

where

W_E = mass flow rate of entrained liquid

W_G = mass flow rate of gas

ρ_g = gas density

ρ_l = liquid density.

Values of K_d are based on correlations of experimental measurements. The entrainment rate is based on the observation that under equilibrium conditions during adiabatic operation, the entrainment rate, E' , and deposition rate are equal. Since under these conditions $E' = R$ and $R = K_d C$,

$$E' = K_d C_E \quad (3.246)$$

where C_E is the entrained droplet concentration under adiabatic equilibrium conditions. An estimate of K_d may be obtained from²⁶⁴

$$K_d = 0.023 V_g \left(\frac{D V_g \rho_g}{\mu_g} \right)^{-0.2} \left(\frac{C_E}{\rho_g} \right) \text{Pr}_g \quad (3.247)$$

where V_g = vapor velocity and Pr_g = gas phase Prandtl number.

In the Harwell approach C_E is taken as a function of the dimensionless group $(\tau\delta/\sigma')$, where τ = interfacial shear stress, δ = average liquid film thickness, and σ' = surface tension.

Calculation of the entrainment rate then requires a knowledge of the interfacial shear stress and film thickness. Use is made of the so-called triangular relationship, which allows W_F , δ , and dp/dz to be related in a way that knowledge of any two allows the third to be calculated. The relationship used is

$$4\delta/D = [(dp/dz)_{LF} / dp/dz]^{1/2} \quad (3.248)$$

where

D = tube diameter

dp/dz = two-phase pressure gradient

$(dp/dz)_{LF}$ = pressure gradient for single-phase liquid with same mass flow rate as in liquid film.

The value of $(dp/dz)_{LF}$ can be obtained for any assumed value of the liquid film flow rate, W_F . The two-phase pressure gradient is then obtained by determining the pressure gradient produced in the droplet-laden gas core by the friction at the core-liquid film interface. The interfacial shear stress is calculated using the Wallis²⁶⁴ relationship between film thickness and interfacial friction defined on the basis of a homogeneous gas core. That is,

$$\tau = C_d \rho_h (V_g - V_l)^2 / (2g_c) \quad (3.249)$$

where

ρ_h = homogeneous density of vapor-droplet core, mass/vol

$C_d = f/4(1 + 300\delta/D)$

f = single-phase friction factor based on mass flux and density of vapor-liquid core.

Instead of correlating the entrainment rate, E' with C_E , Sugawara²⁶⁵ correlated it directly with three dimensionless groups. Based on available data, he obtained

$$E' = 1.07 \left(\frac{\tau \Delta Z}{\sigma'} \right) \left(\frac{V_g \mu_l}{\sigma'} \right) \left(\frac{\rho_l}{\rho_g} \right)^{0.4} \quad (3.250)$$

where

E' = entrainment rate, kg/s m²

ΔZ = height of waves on liquid film,

and for $Re_g > 1 \times 10^5$

$$\Delta Z = 0.57\delta + 21.73 \times 10^3 \delta^2 - 38.8 \times 10^6 \delta^3 + 55.68 \times 10^9 \delta^4$$

δ = mean liquid film thickness, m

Re_g = Reynolds number based on vapor properties and flow rate.

Below a critical vapor velocity in annular flow, the entrainment rate is negligible. A simple criterion for the absence of entrainment,²⁶⁶ which is consistent with the previous annular flow entrainment rate discussion, is that the dimensionless group $(\tau\delta/\sigma') \leq 0.01$. The values of τ and δ for the condition of zero entrainment may be obtained by simultaneous solution of Eqs. (3.248) and (3.249) with the assumption that all liquid is in the liquid film. If the foregoing inequality is satisfied with the τ and δ values so obtained, then entrainment rate calculations are unnecessary.

When entrained droplets are produced from the liquid film, the droplets will have a range of sizes. Those droplets larger than a critical size are subject to further breakup as they interact with the gas core. The droplet diameter at which breakup begins²⁶⁷ is determined by the critical Weber number, We_c , where

$$We_c = (\rho_g V_r^2 d_d) / \sigma \quad (3.251)$$

d_d is the critical droplet diameter and V_r is the relative velocity between droplet and vapor. Values of We_c for room-temperature water are in the range of 12 to 22 (Ref. 267).

The mean drop size can also be expressed in terms of the Weber number. For LOCA calculations, it has been proposed²⁶⁸ that the mean drop size be

estimated using a Weber number of 2.7. Whenever V_r falls below the average fluid velocity, $[\alpha V_g + (1 - \alpha)V_l]$, the Weber number is calculated using the average fluid velocity in place of V .

3.5.2 De-Entrainment

De-entrainment of liquid from a vapor stream bearing droplets is observed when the mixture flows past an obstruction in the flow stream. De-entrainment is required to remove droplets carried over from the centrifugal separators in vertical U-tube steam generators and from the surface separation in VVER steam generators. In addition, de-entrainment would be observed during the outward flow of the droplet-laden steam produced during the reflood phase of a LOCA.

The steam demisters or driers in vertical U-tube steam generators have generally consisted of arrays of louvers with projections and corners on the plates. The VVER driers have generally been of the chevron type. As can be seen in Fig. 3.30(a) these driers consist of an array of parallel passages. The sharp corners of the chevrons have been rounded to improve separation and reduce pressure drop. Liquid collected on the surfaces runs downward and is returned.

Two mechanisms are important in droplet separation: diffusion and inertial impaction. In impaction separation, the larger droplets do not follow the streamlines around the surface but follow their initial trajectories and impact directly on the separating surface. The smaller droplets tend to follow the streamlines around the surface but turbulent diffusion brings them in contact with the collecting surface where they are intercepted. This latter mechanism is essentially the same as that accounting for droplet deposition in cocurrent annular flow. The droplet diffusion mechanism would be more important for chevron-type separators than for the louver type.

To achieve low residual moisture, the superficial vapor velocity across the drier must be sufficiently low so that the separated liquid is not re-entrained. There is a critical velocity above which entrainment rates rise sharply. Although data for steam-water systems are not available in the open literature, Styrikovitch *et al.*²⁶⁰ provide data for air-water systems. Figure 3.30(b) shows the data for the first five stacks of the chevron separator illustrated in Fig. 3.30(a). The critical velocity depends on the percentage of liquid in the air flow. Styrikovitch *et al.*²⁶⁰ suggest that these data may be extrapolated to steam-water systems by assuming that similar performance is obtained when the Kutateladze number for the steam-water system is the same as for the air-water system containing the same percentage of liquid. The Kutateladze number, Ku , is defined as

$$Ku = u_v \rho_g^{1/2} / [g g_c \sigma' (\rho_l - \rho_g)]^{1/4}, \quad (3.252)$$

where u_v = superficial velocity of vapor (L/t).

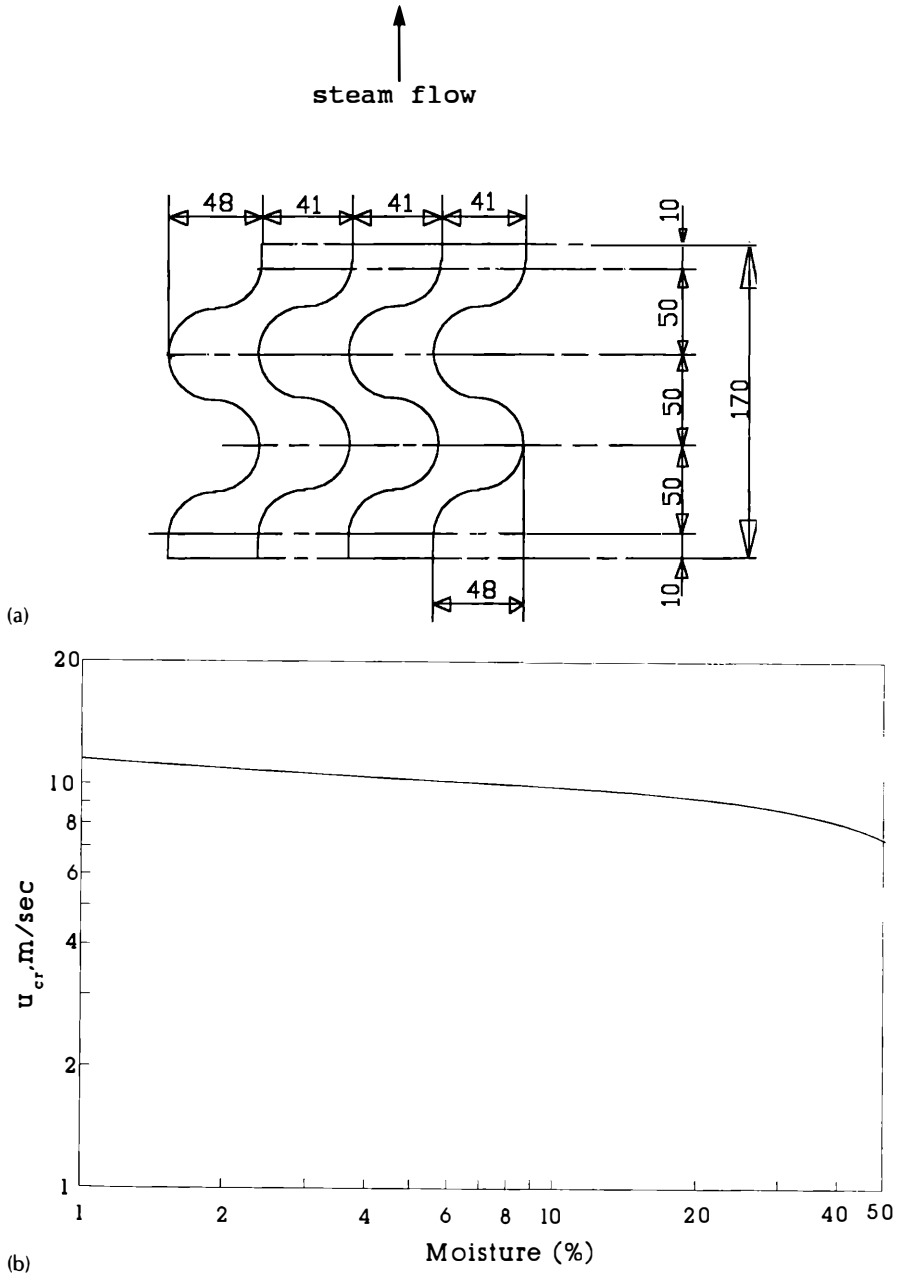


Fig. 3.30 Chevron drier performance: (a) Arrangement of best of driers (dimensions in millimeters) and (b) critical velocity for best of tested chevron driers [data obtained from M. A. Styrkovich *et al.*, *Two-Phase Cooling and Corrosion in Nuclear Power Plants*, Hemisphere Publishing, Washington, D.C. (1987)].

The effectiveness of a moisture separator for particles of a given size is defined as the fraction of the particles of that size that is removed. Styrikovitch *et al.*²⁶⁰ indicate that by a Monte-Carlo analysis of drop trajectories it is possible to estimate efficiencies for various droplet sizes. The determination of the overall efficiency then requires a knowledge of the droplet size distribution. Since this generally requires experimental data, in most cases the efficiency of a given separator is based on experimental tests of the separator for conditions close to those of expected operation. Styrikovitch *et al.*²⁶⁰ note that a minimum distance must be maintained between the driers and the normal liquid level. Performance fell off sharply if the distance dropped below 55 cm. A submerged perforated plate keeps the liquid level in the desired range.

In the droplet de-entrainment seen in the reflood portion of a LOCA, the droplets are relatively large (several hundred microns). Inertial impaction on the control rod guide tubes and upper plenum structure is therefore the predominant removal mechanism. This removal is often modeled as removal by an array of circular cylinders with the steam droplet mixture in cross-flow across the bank. Dallman²⁶⁹ has proposed that the overall removal efficiency, η_l , be based on the removal efficiency of an isolated element, η_l . This latter quantity is correlated by

$$\eta_l = 0.20 - 0.51 \left(\frac{\rho_g u_v^2 D}{\sigma'} \right) \quad (3.253)$$

where D = cylinder diameter L and η_l = fraction of the droplets in a stream of width D that is removed by the cylinder.

The array efficiency η_l is then determined by

$$\eta_l = 1 - (1 - 0.55\eta_l)(1 - F_2\eta_2) \dots (1 - F_N\eta_N) \quad (3.254)$$

where η_N = efficiency of element in row N and F_N = fraction of vapor-droplet jet formed in gaps of row $(N-1)$ intercepted by elements in row N . The parameters η_N and F_N are correlated by

$$\eta_N = (1 + 4.5e_v^2)\eta_l$$

$$\frac{F_N}{1 - F_N} = k''/V_{av} \quad (3.255)$$

where

k'' = proportionality constant (approximately 2.7 m/s)

V_{av} = average mixture velocity in array

e_v = volume fraction occupied by cylinders in array.

3.5.3 Carry-Under

Carry-under, the entrainment of bubbles of vapor in the liquid stream leaving a separating surface, is the inverse of carry-over. Carry-under in both steam-water and air-water separations was studied by Petrick²⁷⁰ whose air-water data were obtained using riser radii of 3.25 and 5.5 in., respectively, and whose steam-water data were taken at pressures of 600, 1000, and 1500 psia. He also obtained data from the Experimental Boiling Water Reactor (EBWR) at 300 and 600 psia and suggested two carry-under equations: For $\chi_D/\chi_R = 3$ to 64,

$$\frac{\chi_D}{\chi_R} = -0.04 \log \left[\frac{\left(\frac{V_g}{V_e} \right) \left(\frac{\sigma^{2/3}}{G^2 \mu} \right) \left(\frac{\rho_l}{\rho_g} \right)^{1/2} \left(\sqrt{\frac{D}{Z}} + \sqrt{\frac{Z}{D}} \right)}{64} \right] \quad (3.256)$$

For $\chi_D/\chi_R = 0.1$ to 3,

$$\frac{\chi_D}{\chi_R} = -0.6 \log \left[\frac{\left(\frac{V_g}{V_e} \right) \left(\frac{\sigma^{2/3}}{G^2 \mu} \right) \left(\frac{\rho_l}{\rho_g} \right)^{1/2} \left(\sqrt{\frac{D}{Z}} + \sqrt{\frac{Z}{D}} \right)}{3.7} \right] \quad (3.257)$$

where

χ_D = steam quality in the downcomer

χ_R = steam quality in the riser

V_g = vapor velocity in the riser, ft/s

V_e = velocity of entrained gas in the downcomer, ft/s

D = riser diameter

Z = interface height

σ = surface tension, lb/ft

μ = viscosity of liquid, lb/(s ft)

G = liquid mass flux in the downcomer, lb/(s ft²).

Petrick²⁷⁰ also suggested a correlation for calculating the slip ratio in downflow

$$\frac{V_v}{V_l} = 0.63 \left(\frac{\bar{V}^2}{gD} \right)^{0.4} \left(\frac{\chi}{1-\chi} \frac{\rho_l}{\rho_g} \right)^{0.2} \quad (3.258)$$

where $\bar{V}$ is average velocity of the mixture.

In a more recent study of an air-water²⁷¹ simulation of a natural circulation system, it was observed that at low-void fraction ($\alpha < 0.3$) bubbly

flow, there was little separation and nearly all the air was carried under. However, at higher void fractions, the air bubbles combined to form easily separated slugs. The void fraction in the downcomer remained at a maximum of 0.2 with void fractions in the riser of up to 0.6.

3.5.4 Flooding

The flooding phenomenon, which can be considered somewhat related to the subject of carry-over, arises during the latter stages of blowdown following a large cold-leg break in a hypothesized LOCA. Steam generated in the core flows up the annulus between the core barrel and the pressure vessel on its way to the break. Simultaneously, emergency core cooling water flows toward the same region and attempts to flow toward the lower plenum. It is desired to refill the plenum region and then flood the core. When high upward steam velocities are reached in the annulus, part or all of the emergency cooling water is prevented from getting to the lower plenum. Excess water exits through the break with the steam. This limitation on the rate of liquid penetration is imposed by "flooding," which is the term generally used to describe the boundary of allowable liquid-gas countercurrent flow rates.

Flooding can also occur at the upper core plate during the reflooding phase of a LOCA. The water in the upper plenum may come from de-entrainment of droplets from the reactor core or direct injection of emergency cooling water into the upper plenum.

A comprehensive review of the flooding literature prior to 1986 is provided by Bankoff and Lee.²⁷²

Early experiments^{273,274} were all in very small scale models. Subsequent experiments^{275,276} used $2/15$ and $1/5$ scale models. Based on these tests, a number of approaches were proposed. The earliest approach, due to Wallis,²⁷⁷ was in terms of the dimensionless groups j_f^* and j_g^* where

$$j_f^* = (Q_l / A) \rho_l^{1/2} [Lg(\rho_l - \rho_g)]^{-(1/2)} \quad (3.259)$$

$$= (Q_g / A) \rho_g^{1/2} [Lg(\rho_l - \rho_g)]^{-} \quad (3.260)$$

and

A = cross-sectional flow area

Q_g, Q_l = water and vapor volumetric flow rates, respectively

ρ_g, ρ_l = gas and liquid densities, respectively

g = acceleration of gravity

L = characteristic dimension.

On the basis of tests with saturated cooling water, Block *et al.*²⁷⁸ correlated the limiting liquid flow rate by*

$$(j_g^*)^{1/2} + m(j_{f_{\text{down}}}^*)^{1/2} = C \quad (3.261)$$

where

$$C = 0.32$$

$$m = \exp[-5.6(j_{f_{\text{in}}}^*)^{0.6}]$$

$j_{f_{\text{in}}}^*$ = dimensionless group of Eq. (3.259) based on supplied liquid flow and defined with L = total gap circumference

$j_{f_{\text{down}}}^*$ = dimensionless group of Eq. (3.259) which indicates fluid actually reaching lower plenum and defined with L = total gap circumference.

Several investigators^{279,280} suggested a similar condition based on the Kutateladze number. These have the general form

$$(\text{Ku}_g)^{1/2} + b(\text{Ku}_f)^{1/2} = C' \quad (3.262)$$

where

b = experimentally determined parameter

C' = constant

Ku_g = Kutateladze number for gas phase [see Eq. (3.252)]

$\text{Ku}_f = u_l \rho_l^{1/2} / [gg_c \sigma'(\rho_l - \rho_g)]^{1/4}$ = Kutateladze number for liquid

u_l = superficial velocity of liquid.

Both the Kutateladze number and the j^* represent a ratio of inertial force to hydrostatic force with different length scales. In the j^* approach, the length scale is a characteristic diameter. In the Ku number, the length scale is a capillary wavelength, which is appropriate to large diameters.

Because of concerns that small-scale tests cannot be extrapolated reliably to reactor-size behavior, reactor-scale tests have been conducted in the Upper Plenum Test Facility (UPTF).²⁸¹ As a result of these tests, revised flooding expressions²⁸² have been developed for saturated water injection. These equations can be directly applied to reactor accident calculations. For the downcomer:

$$(\text{Ku}_g)^{1/2} F_0^{-1/2} + 0.011(\text{Ku}_f)^{1/2} = 0.0245 \quad (3.263)$$

*Flooding due to counterflow of vapor could prevent the entrance of the coolant used to reflood a Savannah River production reactor following a LOCA. Lee [*Nucl. Technol.*, **104**, 64 (1993)] found that the conditions for flooding in the narrow channels formed by the concentric-ring fuel can be represented by an equation having the same form as Eq. (3.261).

for $F_0 \geq 5400$ and where

$$F_0 = g^{1/3} L' / v_g^{2/3} = \text{Re}_g^{2/3} / \text{Fr}^{1/3}$$

v_g = kinematic viscosity of vapor

$$L' = 0.5\pi D_{\text{outer}} \sin^2(0.5\theta)$$

θ = maximum angle between ECC injection line and broken leg

Fr = Froude number.

For the upper core plate (tie plate):

$$(\text{Ku}_g)^{1/2} F_1^{-1/2} + 0.014 (\text{Ku}_f)^{1/2} = 0.027 \quad (3.264)$$

for $F_1 > 4400$ where

$$F_1 = g^{1/3} L'' / v_g^{2/3}$$

$$L'' = (4A_{tk} / \pi)^{1/2} \sin(0.5\theta) \text{ for } y > 0.5[D_p - (4A_{tk} / \pi)^{1/2}]$$

A_{tk} = tie plate flow cross section

D_p = diameter of upper plenum

y = radial distance of ECC downflow center through tie plate from hot-leg inlet to upper plenum.

For a given steam flow rate, Eqs. (3.263) and (3.264) indicate the maximum liquid flow rate that can penetrate into the flow passage. If a greater liquid flow is provided, the excess liquid will accumulate at the inlet or be entrained with the exiting steam.

Glaeser²⁸² concludes that the earlier flooding correlations [Eqs. (3.261) and (3.262)] apply to smaller scale systems. He suggests that Eq. (3.261) be limited to systems where the Bond number, Bo , is below 400 $\{\text{Bo} = \pi D_{\text{avg}} [g(\rho_l - \rho_g) / \sigma]^{1/2}\}$. He indicates that equations having the form of Eq. (3.262) should apply to Bond numbers above 400 and below the range of Eq. (3.263) or (3.264).

When subcooled water is used as the injection fluid, a portion of the upward steam flow is condensed. This reduces steam velocity and allows increased liquid penetration. For downcomer flooding, Block *et al.*²⁷⁸ suggest that a modified dimensionless steam flux, $j_{g\text{mod}}^*$, be computed from

$$j_{g\text{mod}}^* = j_g^* - (F / \text{Ja} \cdot j_{f\text{in}}^*) \quad (3.265)$$

where

$$\text{Ja} = \text{Jakob number} = (c_p / H_{fg}) \Delta T_{\text{sub}} (\rho_l / \rho_g)^{1/2}$$

= specific heat of liquid

ΔT_{sub} = saturation temperature less actual coolant temperature

$F = (P)^{1/4} (1 + b j_{f\text{in}}^*)^{-1}$ = condensation efficiency

P = pressure, atm

$b = 16$ for flat plate geometry, 30 for cylindrical geometry.

Since the vessel-scale flooding correlations are based on Ku rather than j^* , it would probably be appropriate to use a modified Ku_g in these equations when subcooled fluid is used for injection. That is,

$$(Ku_g)_{\text{mod}} = (Ku_g) - FJa(Ku_f)_i \quad (3.266)$$

Another variable that must be considered is the temperature of the core and vessel walls.²⁸³ These walls can be superheated during the latter stages of blowdown and therefore lead to additional steam generation. It is total steam flow that causes flooding and limits water penetration.*

3.5.5 Primary Steam Separation

3.5.5.1 Separation by Surface Disengagement

Surface disengagement is effective when the steam velocity is low enough for water droplets to return to the two-phase interface by gravitational force. In designing this type of steam separator, the data in Fig. 3.29 can be used. The increase in vapor density with pressure initially overcomes the decrease in escape velocity, so the allowable steam release rate reaches a maximum at 1750 psia. This allowable steam release rate determines the maximum power output of a boiler relying solely on surface disengagement for its steam separation.

Recall that surface separation is used in the horizontal steam generators for the Russian VVER reactors. As noted in Sec. 3.5.1.1, the presence of a perforated plate just under the froth level of these steam generators appears to allow superficial steam velocities slightly above those that would be predicted from Fig. 3.29.

3.5.5.2 Separation by Centrifugal Force

This mechanism is encountered in a radial- or vane-type steam separator. Figure 3.31 shows the arrangement and parameters typical of centrifugal separators used in PWR U-tube steam generators. The swirl vanes produce a vortex with a central vapor core and an outer liquid annulus. Note that in the design shown, removal of liquid from the annulus in the riser is accomplished by perforating the wall of riser region above the vane.

A simplified analysis is used to illustrate the vane separator behavior. In the range of interest, the effects of gravity force, slip, and turbulence can be neglected, and the dynamic equation of a bubble becomes

*The reader who is unfamiliar with the ECCS provided for a hypothetical LOCA is referred to Sec. 6.5.1 for a description of the system.

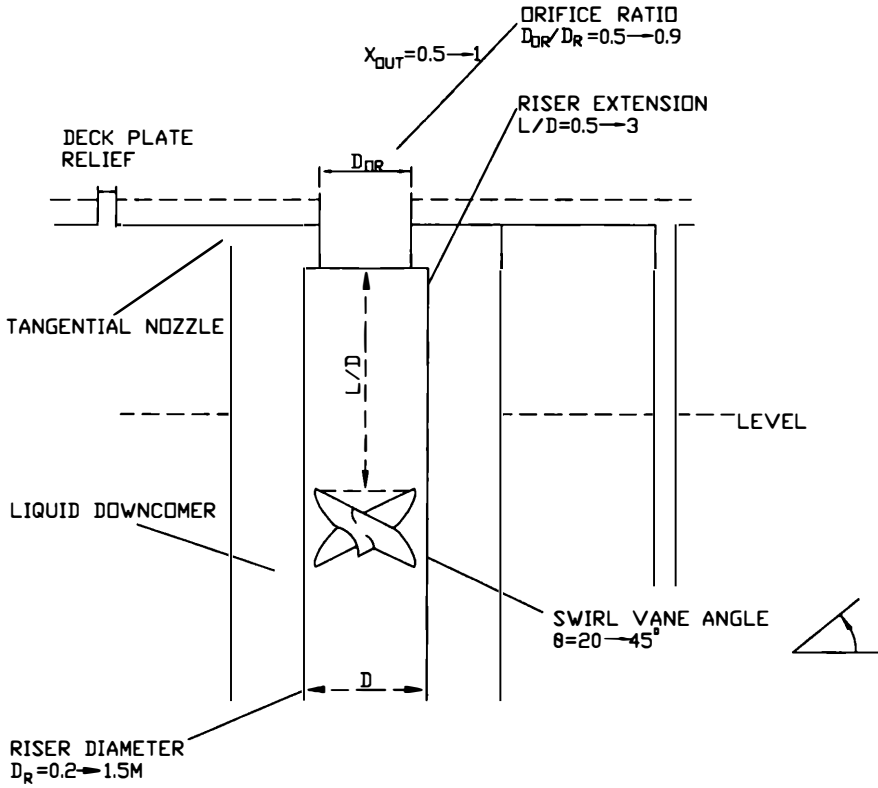


Fig. 3.31 Centrifugal separator arrangement [from Ref. 285].

$$\frac{C_d \pi D_b^2}{4} \rho_l \frac{u^2}{2g_c} = \frac{1}{6} \pi D_b^3 \frac{(\rho_l - \rho_g)}{g_c} \frac{V^2}{R} \quad (3.267)$$

where

C_d = drag coefficient, dimensionless

D_b = bubble diameter, ft

R = radius of separator, ft

u = radial velocity, ft/s

V = tangential velocity, ft/s.

By rearranging the above equation and assuming $[(\rho_l - \rho_g)/\rho_l] \approx 1.0$, we have

$$u^2 = \frac{4V^2 D_b}{3RC_d} \quad (3.268)$$

For a vane separator, design equations can be written by equating the time required for the bubble to separate

$$\frac{R\theta}{V} = \frac{b}{u} \quad (3.269)$$

where b is the water annulus thickness that the bubble travels and θ is the angle the bubble travels along the vane before separation. If we combine Eqs. (3.268) and (3.269), we get the condition of bubble separation²⁸⁴

$$\frac{b^2}{R\theta} = \frac{4D_b}{3C_d} \quad (3.270)$$

From Eq. (3.269), we see that maximum inlet velocity V (i.e., maximum mixture rate) can be increased by increasing R and by decreasing b .

The actual design of a vane-type separator will involve two steps: (1) evaluation of the radial stratification occurring in the vortex region and (2) evaluation of the skimming and draining of the stratified phases. To evaluate the progress of the radial stratification, Bouchter *et al.*²⁸⁵ examine a series of horizontal slices of a two-phase mixture moving upward at a constant axial velocity and swirling at a constant angular velocity. They rewrite Eq. (3.267) for axial elevation z_i as

$$u(r, z_i) = \left[\frac{4 \left(\frac{D_b}{C_d} \right) \left(\frac{\rho_l - \rho_g}{\rho_l} \right) V^2(r, z_i)}{r} \right]^{1/2} \quad (3.271)$$

where (D_b/C_d) is taken as a coefficient characteristic of the two-phase mixture and is allowed to be an adjustable parameter in the model. The actual tangential velocity distribution is approximated by

$$V(r, z_i) = A_{zi} r^n \quad (3.272)$$

where A_{zi} and n are based on experimental measurements. Once $V(r, z_i)$ is computed, the continuity equation and density distribution at z_{i-1} is used to get the radial density distribution at z_i .

The evaluation of the removal of the stratified phases is accomplished using a mechanical energy balance. The procedure proceeds iteratively by guessing how the stratified phases split between vapor and liquid flow paths. The calculation requires as input the results of the first phase calculations and the various empirically measured pressure drop coefficients for the components in the flow path.

Experimental tests of full-size separators have been performed using refrigerant 114 at a pressure such that steam-water density ratios are preserved. The steady-state tests²⁸⁵ indicated that phase separation was considerably improved by increasing the riser L/D from 0.5 to 2 or 3. Re-

lated observations were made by Paik *et al.*²⁸⁶ who considered separator behavior under off-normal conditions. They found that as long as the downcomer level remained low, separation was nearly complete. However, once the downcomer liquid level rose above a critical value, droplet entrainment increased rapidly. They note that the region of the downcomer over which the large change in separator performance occurs is small compared to the height of the steam generator. In view of the limited accuracy attainable in predicting water level during an accident, they suggest a single switch model. They suggest that the separator functions or fails as indicated by a critical liquid level.

3.5.6 Phase Separation in Branching Conduits

Experimental observations have shown that when a two-phase mixture enters the main line (run) of a tee, the quality of the mixtures exiting the branch and main line will differ. It appears that the degree of separation observed depends on both the flow pattern and the fraction of the flow exiting the branch.²⁸⁷ For dispersed bubbly flow, the branch quality is higher than the run quality. Since the vapor phase has less axial inertia than the liquid, it may be expected to turn the corner into the branch more easily than the liquid. In annular flow, similar behavior is observed when the ratio of branch flow to total flow is substantial. At low branch flows, the quality in the branch tends to be higher than that of the run. This occurs since the liquid traveling along the wall is preferentially withdrawn instead of the vapor in the interior of the conduit. This has been ascribed to the fact that there is a zone of influence in the conduit that depends on the fraction of the flow withdrawn through the branch. The larger the flow fraction in the branch, the greater the zone of influence and the more vapor withdrawn.

Data for churn flow through tees²⁸⁸ show that in essentially all cases the branch flow quality was less than the run quality. A good explanation for this behavior is not (1993) available.

In contrast to the situation where flow enters the run, little phase separation occurs if the flow enters the branch. If the flow in each run is between 15 to 85% of the total flow, the low-pressure data available indicate equal quality in the exit lines.²⁸⁹

The complete description of the behavior at a flow junction requires eight parameters to be determined: G_1 , G_2 , G_3 , x_1 , x_2 , x_3 , ΔP_{1-2} , ΔP_{1-3} where

G_1 , G_2 = inlet run and outlet run mass flux, respectively

G_3 = branch mass flux

x_1 , x_2 = inlet run and outrun quality, respectively

x_3 = branch quality

ΔP_{1-2} = pressure drop from inlet run to outlet run

ΔP_{1-3} = pressure drop from inlet run to branch.

Three of these qualities are specified as boundary conditions (e.g., x_1 , G_1 , ΔP_{1-3}) and the remainder must be computed. Typically, this is accomplished via (1) mixture continuity equation, (2) vapor continuity equation, (3) run pressure drop equation, (4) a run to branch pressure change equation, and (5) closure relationship. The difficulty in the calculation is the proper specification of the closure relationship, which relates the degree of phase separation to the flow and geometric parameters.

For annular flow in vertical lines or in horizontal lines at high velocity, Whalley and Azzopardi²⁹⁰ have developed the following closure relationship:

$$x_3/x_1 = \frac{1}{2\pi}(\theta - \sin\theta)/(w_3/w_1) \quad (3.273)$$

where

θ = angle defining sector from which the liquid film is

$$\text{removed} = 360^\circ \left\{ \frac{w_3(1-x_3)}{w_1(1-x_1)(1-E_1)} \right\} [1.2(D_3/D_1)^{0.4}]$$

w_1, w_3 = mass flow rates in inlet run and branch, respectively, mass/time

E_1 = fraction of liquid entrained as droplets in inlet run, dimensionless

D_1, D_3 = diameters of run and branch, respectively.

For flow patterns other than annular flow, the closure relationship of Saba and Lahey²⁹¹ may be used. These investigators chose the vapor-phase linear momentum equation for the branch as the required fifth relationship. This is given by

$$\Delta P_{1-3} = \frac{K_1 w_1^2}{2g_c \rho_l A_1^2} \phi_{l0}^2 + \rho_l \frac{g}{g_c} L_1 \sin\beta_1 + \Delta P_j + \frac{K_3 w_3^2}{2g_c \rho_l A_3^2} \phi_{l0}$$

where

$$\begin{aligned} \Delta P_j = & \frac{3}{4} \frac{\rho_l}{g_c} \left(\frac{C_D}{D_1} \right) V_{gl}^2 L_3 + \frac{\rho_g}{2g_c} \left(\frac{w_3^2 x_3^2}{\rho_g^2 A_3^2 \alpha_3^2} - \frac{w_1^2 x_1^2}{\rho_g^2 A_1^2 \alpha_1^2} \right) \\ & + \frac{K_{1-3}}{2g_c} \left(\frac{w_1^2 x_1^2}{\rho_g A_1^2 \alpha_1^2} \right) + [g/g_c \rho_g] (L_j \sin\beta_2) , \end{aligned} \quad (3.274)$$

V_{gl} = relative velocity of vapor = $V_g - V_l$

- L_j = length of vapor streamline
 $2.81D_3 \left\{ \exp[-0.12A] \left[\left(\frac{w_3}{w_1} \right)^{(1-x_1)^3} \right] (1-x_3)^3 \right\}$
 L_1, L_3 lengths of branch and run lines of tee, respectively
 $A = [(1-x_1)/x_1]^{0.15} (\rho_g/\rho_l)^{0.5}$
 A_1, A_3 = flow areas of inlet and outlet runs, respectively
 β_1 = angle of inclination of run to horizontal
 β_2 = angle between branch and run
 K_1, K_3 single-phase pressure loss coefficients for inlet and outlet segments of run, respectively
 K_{1-3} = single-phase pressure loss coefficient between run and branch.

Both Eqs. (3.273) and (3.274) should be used with caution because they have been verified only with low-pressure data. They should not be used for horizontal tees containing stratified (separated) flow. In such tees, the phase or phases exiting from the branch will depend primarily on the level of liquid in the run compared to the level at the entrance to the branch. In the case of a vertical upward branch, only vapor will flow through the branch for low liquid levels and low branch flow. However, at sufficiently high branch flows some liquid will be entrained and exit through the branch. Similarly, for the case of vertical downward branch, only liquid will exit through the branch. At a sufficiently high branch flow, vapor will be pulled through with the liquid. When a horizontal branch is present, liquid entrainment is possible when the liquid level is below the branch, and vapor pull through when the level is above the branch.

For tees with horizontal branches which contain stratified flow, the beginning of liquid entrainment has been predicted in terms of L , the distance between the top of the liquid level and the branch centerline.^{292,293} If the level is below the branch, liquid entrainment begins when

$$|L_e| = K \{ (w_g^2)_3 / [g \rho_g (\rho_l - \rho_g)] \}^{0.2} \quad (3.275)$$

where

- $(w_g)_3$ = gas flow rate through branch, kg/s
 ρ_l, ρ_g = liquid and gas densities, kg/m³
 $|L_e|$ = absolute value of distance between branch centerline and liquid when entrainment begins, m
 $K = 0.69$ for horizontal tee.

Similarly, the level at which gas pull-through begins, $(L)_g$, is predicted by

$$(L)_g = K \{ (w_l^2)_3 / [g \rho_b (\rho_l - \rho_g)] \}^{0.2} , \quad (3.276)$$

where $(w_l)_3$ is the liquid flow rate through branch (kg/s). To compute branch line quality, x_3 , Maciaszek and Micaelli²⁹³ propose

$$x_3 = \alpha_i / [\alpha_i + \sqrt{\rho_l / \rho_g} (1 - \alpha_i)] \quad (3.277)$$

where

$$\alpha_i = 1 - 0.5(1 - L/L_b)^n \text{ for } -L < L_b$$

$$\alpha_i = 0.5(1 - L/L_b)^n \text{ for } L < L_b$$

$$K = 0.69 \text{ for horizontal tee}$$

$$L_b = 0.69[(w_3)_t / [g\rho_m(\rho_l - \rho_g)]]^{0.2}$$

$$(w_3)_t = \text{total branch flow rate, kg/s}$$

$$\rho_m = \text{homogeneous mixture density in branch, kg/m}^3$$

$$n = \text{experimental coefficient (two for horizontal tee).}$$

For a vertical downward branch, the onset of gas entrainment can be computed from Eq. (3.277) if K is revised.²⁹² In this case

$$K = (1 - R_1^{0.2}) \quad (3.278)$$

where $R_1 = (V_l^2)_2 / (V_l^2)_3$ and $(V_l)_2, (V_l)_3$ are the liquid velocities in exit run and branch, respectively. Branch quality is computed via Eq. (3.277) with α_i being obtained from

$$\alpha_i = (1 - L/L_e) \quad (3.279)$$

For a vertical upward branch line, Reimann and Smoglie²⁹⁴ correlate the available data for the onset of entrainment by Eq. (3.275) with $K = 1.67$. Although other investigators have used a more complicated model, the complication does not appear warranted in view of the data scatter.

The phenomena governing the rate of entrainment are complex and three regions may be recognized. The CATHARE program uses the following simple model:

$$\begin{aligned} x_3 &= 1 & L_e < L \\ 0.98 \leq x_3 \leq 1 & & L_{\text{lim}} < L < L_e \end{aligned} \quad (3.280)$$

$$x_3 = 0.98\alpha_i / [\alpha_i + (\rho_l / \rho_g)^{1/2}(1 - \alpha_i)] \quad L < L_{\text{li}}$$

where

$$L_{\text{lim}} = 2.5(D_3/D_1)^{0.666}L_e(1 - R_2^{0.15})$$

$$R_2 = \rho_g(V_g^2)_2 / [(G_3)^2 / (\rho_3)_m]$$

$$(V_g)_2 = \text{gas velocity in exit run, kg/s}$$

$$G_3 = \text{mass flux in branch, kg/s m}^2$$

$$(\rho_3)_m = \text{homogeneous density of mixture in branch.}$$

The ability to determine the quality of the fluid exiting a branch is particularly important during a small-break LOCA. In most instances, flow through such a break can be considered as being similar to flow through the branch of a tee. The rate at which mass is lost by the system then depends on the critical flow out the branch of the tee. This in turn is strongly dependent on the quality of the stream entering the branch.

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CHAPTER FOUR

Heat Transfer and Transport

The reactor coolant removes heat from the core by transferring heat from the fuel elements through convection or boiling, and then transporting the energy to the steam generator. Since the rate of heat transfer depends on the enthalpy of the fluid as well as on the heat flux, we first examine the determination of fluid enthalpy assuming one-dimensional flow. Analytical expressions for steady-state conditions are developed to provide preliminary estimates of reactor core behavior at operating conditions. The one-dimensional flow approach is then extended to system transients using the two-phase flow models developed in Sec. 3.3.

An accurate determination of core behavior requires consideration of the mass and energy interchange between adjacent channels. The methods for accomplishing this are described for both steady-state and transient conditions. These methods may be used in conjunction with one-dimensional system computations.

The various heat transfer mechanisms that apply during single- and two-phase flow are presented and appropriate models or correlations are given. Heat transfer to reactor system components and core structure and moderator cooling are also examined.

Extensive tables of water and steam properties are available elsewhere^{1,2} and are not repeated here.

4.1 HEAT TRANSPORT ASSUMING ONE-DIMENSIONAL BEHAVIOR

In a pressure tube reactor, there is no fluid interchange with surrounding channels, and the average enthalpy rise for the entire tube can be obtained

by a simple heat balance. In a vessel-type reactor, mixing and cross flow affect the behavior of a single subchannel. However, when the average enthalpy rise along an entire assembly is sought, these effects are of lesser importance. Thus, a rough approximation of the average enthalpy rise along a coolant channel is often obtained considering the channel to be closed. Such an approach was taken in some of the computer codes that reactor operators used to evaluate steady-state core performance.³

In the programs used for accident and transient analysis, the need to reduce the complexity of the problem usually leads to modeling the reactor system as a series of linearly interconnected nodes. It is therefore appropriate to begin the consideration of reactor heat transport by examining the behavior in systems in which conditions vary only in the axial direction.

4.1.1 Steady-State Enthalpy Rise Along a Closed Coolant Channel

The heat absorbed by the coolant at location Z can be evaluated by taking a heat balance around a differential element and then integrating along the core. When the interchange with the surrounding region can be ignored, we get

$$(A/p')G(\Delta H) = \int_{-L/2}^Z q''(z)dz \quad (4.1)$$

where

$q''(z)$ assembly-average heat flux at axial location z , Btu/(hr ft²)

A = flow area of assembly, ft²

p' = total heated perimeter of assembly, ft

ΔH = enthalpy at location z less inlet enthalpy, Btu/lb

G = mass flow rate, lb/hr ft²

L core length.

In a large assembly, $(A/p') = (De/4)$.

The heat flux at a given axial location can be expressed in terms of the hot-channel factor, $F_{xy}(z)$. We then have

$$(A/p')G(\Delta H) = \int_{-L/2}^Z F_{xy}(z)q''_{av} dz \quad (4.2)$$

where q''_{av} is the average heat flux in the core. In many cases, no analytical expressions are available for $F_{xy}(z)$, and enthalpy rise is approximated from

$$(A/p')G(\Delta H) = q''_{av} \sum_j [F_{xy}(z)]_j (\Delta z)_j \quad (4.3)$$

where $F_{xy}(z)_j$ = average value of $F_{xy}(z)$ in axial segment j and $(\Delta z)_j$ = length of axial segment j .

4.1.1.1 Behavior with Simple Axial Flux Shapes

The axial power distribution of an unperturbed cylindrical reactor core was shown in Chapter 1 to follow a cosine curve; therefore, local flux q'' can be expressed as a function of $q''_{\max}$, the peak flux.

As shown in Fig. 4.1, we define

L_0 = length between $\theta = 0$ and $\theta = \pi$ (half of cosine cycle length)

$L = L_0 + L'$ = actual core length

L' = difference between actual core length and half of cosine length (negative for a chopped cosine)

z = axial distance from center line.

When $F_Z^N \leq 1.57$, we have

$$q'' = q''_{\max} \cos \theta = q''_{\max} \cos(\pi z / L_0) \quad (4.4)$$

If we replace $q''(z)$ in Eq. (4.1) by its equivalent from Eq. (4.4), for the enthalpy rise we obtain

$$\Delta H = (p / AG) \int_{-L/2}^Z q''_{\max} \cos(\pi z / L_0) dz \quad (4.5)$$

If the cumulative fraction F of heat absorbed by the coolant is defined as the ratio of heat absorbed at location Z to the total heat generated in the channel, then F can be evaluated by

$$F = \frac{\int_{-L/2}^Z q''_{\max} \cos(\pi z / L_0) dz}{\int_{-L/2}^{+L/2} q''_{\max} \cos(\pi z / L_0) dz} \quad (4.6)$$

providing the axial distribution is cosinusoidal. With this definition of F , the enthalpy of fluid H at location Z is given by

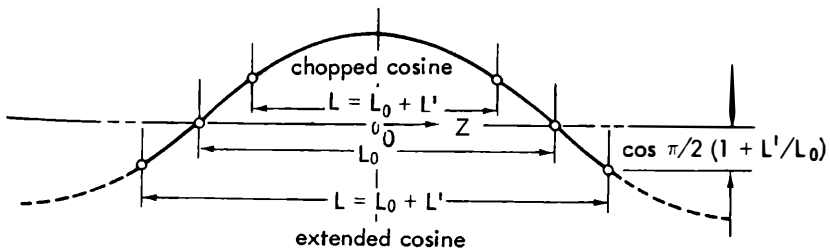


Fig. 4.1 Approximate axial flux distribution within thermal reactor cores.

$$H = H_{in} + [4q''_{max} L / (F_Z^N DeG)] F \quad (4.7)$$

where H_{in} is inlet enthalpy. Values of F at various distances along a reactor channel are plotted in Fig. 4.2 for several values of L/L_0 .

Under most circumstances, values of L_0 are not given, but values of F_Z^N are stated. However, by using our previous definition of axial hot-channel factor F_Z^N and referring to Fig. 4.2, we express F_Z^N in terms of L' and L_0 . For a chopped cosine distribution, we obtain

$$F_Z^N = \frac{1}{\frac{2}{L} \int_0^{L/2} \cos \frac{\pi z}{L_0} dz} = \frac{\pi \left(1 + \frac{L'}{L_0}\right)}{2 \sin \left[\frac{\pi}{2} \left(1 + \frac{L'}{L_0}\right) \right]} \quad (4.8)$$

where $-L/2 \leq z \leq L/2$ and L' is negative.

For cosine heat flux distribution with reported axial hot-channel factors in excess of $1.57(\pi/2)$, L' becomes positive. The flux is assigned a value of zero at $Z = \pm L/2$. We then have

$$q'' = q''_{max} \frac{\cos \frac{\pi z}{L_0} - \cos \left[\frac{\pi}{2} \left(1 + \frac{L'}{L_0}\right) \right]}{1 - \cos \left[\frac{\pi}{2} \left(1 + \frac{L'}{L_0}\right) \right]} \quad (4.9)$$

where $-L/2 \leq z \leq L/2$. Hence, for an extended cosine distribution,

$$\begin{aligned} F_Z^N &= \frac{1 - \cos \left[\frac{\pi}{2} \left(1 + \frac{L'}{L_0}\right) \right]}{\frac{2}{L} \int_0^{L/2} \left\{ \cos \frac{\pi z}{L_0} - \cos \left[\frac{\pi}{2} \left(1 + \frac{L'}{L_0}\right) \right] \right\} dz} \\ &= \frac{1 - \cos \frac{\pi}{2} \left(1 + \frac{L'}{L_0}\right)}{\frac{2}{\pi \left(1 + \frac{L'}{L_0}\right)} \sin \left[\frac{\pi}{2} \left(1 + \frac{L'}{L_0}\right) \right] - \cos \left[\frac{\pi}{2} \left(1 + \frac{L'}{L_0}\right) \right]} \end{aligned} \quad (4.10)$$

The values of L'/L_0 corresponding to the assigned value of F_Z^N are plotted in Fig. 4.3. Note that since two separate equations are used for defining F_Z^N , there is a change in slope at $L'/L_0 = 0$.

As noted in Sec. 1.2.3, a better approximation of an axial flux distribution that has been skewed by rod insertion is sometimes given by

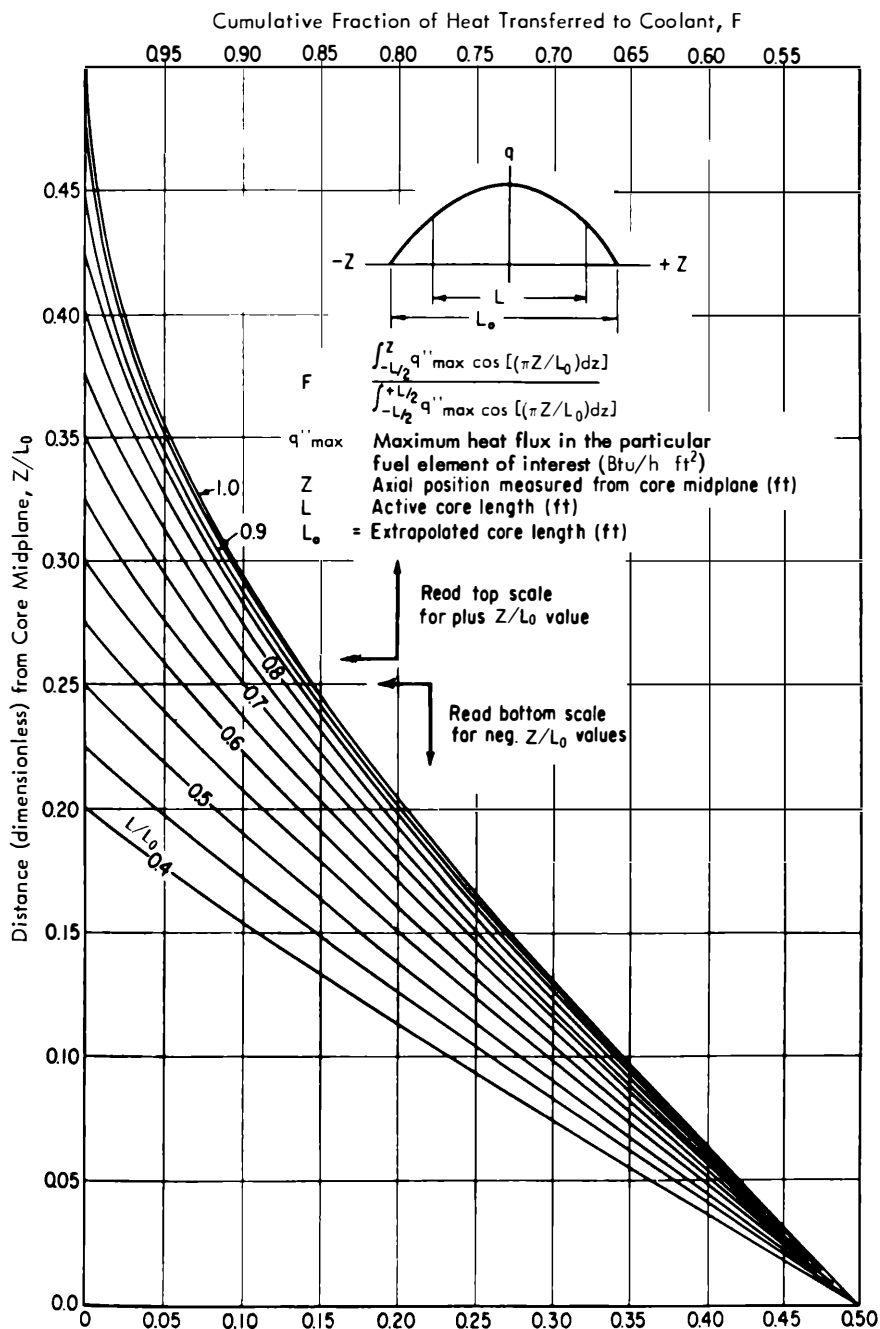


Fig. 4.2 Heat absorbed by coolant along reactor channel [from *Nucleonics*, 18, 11, 170 (1960)].

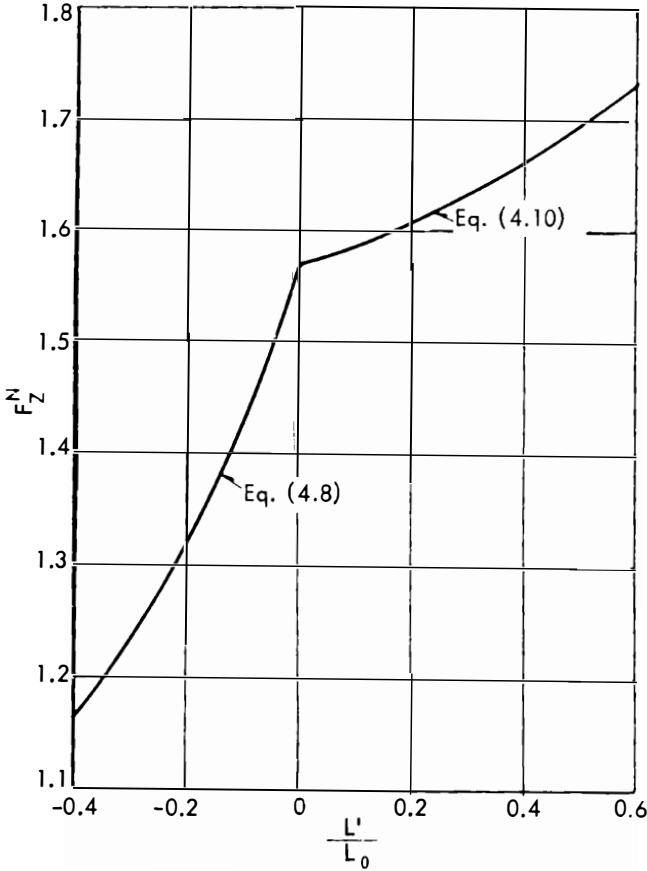


Fig. 4.3 Plot of F_Z^N versus L'/L_0 .

$$q'' = [A + (Bz/L)] \cos(\alpha_0 z/L) \quad (4.11)$$

The heat flux is assumed to go to zero at $z/L = \pi/2\alpha_0$. By assuming that the slope of the flux curve is also zero at this value of z/L , constants A and B can be obtained in terms of $q''_{\max}$, and we have

$$q'' = 0.54954 q''_{\max} \left(\frac{\pi}{2} - \frac{2\alpha_0 z}{L} \right) \cos\left(\frac{\alpha_0 z}{L}\right) \quad (4.12)$$

and

$$F_Z^N = (1.1585)\alpha_0 / \sin\alpha_0 \quad (4.13)$$

Equation (4.13) can be used to determine α_0 from the value of F_Z^N . If a distribution skewed toward the top of the core is being considered, the

above relations apply with each value of z/L replaced by its negative value. The foregoing applies as long as F_Z^N is < 1.82 . At higher values, negative fluxes at the core ends will be encountered.

4.1.2 Transient Heat Transport

For a single closed channel, the partial differential equation describing the change in coolant enthalpy H with position Z and time t is again readily obtained by an energy balance around a differential control volume. For constant ρ , this yields

$$\rho \frac{\partial E}{\partial t} + G \frac{\partial H}{\partial z} = \frac{q'' p'}{A} \quad (4.14)$$

where

q'' = heat flux, Btu/(hr ft²)

A = channel flow area, ft²

p' = heated perimeter of the channel, ft

ρ = fluid density, lb/ft³

E = internal energy at position Z , Btu/(lb ft³).

When the system pressure remains constant, Eq. (4.14) can be simplified to

$$\rho \frac{\partial H}{\partial t} + G \frac{\partial H}{\partial z} = \frac{q'' p'}{A} \quad (4.15)$$

For most situations, ρ varies significantly with H , and an analytic solution is not feasible. The usual procedure is to express Eq. (4.14) or (4.15) in finite difference form to obtain a numerical solution. When the coolant remains substantially subcooled, density changes can be negligible and an analytical solution for flow transients is then possible using the method of "characteristics."

4.1.2.1 Simple Flow and Power Transients

In Sec. 3.3.4.5, the concept of "characteristic directions" was introduced. It was observed that first-order linear partial differential equations become total differential equations along the characteristic directions. Consider the simple first-order linear partial differential equation given by:

$$S \frac{\partial z}{\partial x} + Q \frac{\partial z}{\partial y} = R \quad (4.16)$$

where S , Q , and R are functions of x and y . It can be shown that the general solution of Eq. (4.16) is of the form

$$F(U_1, U_2) = 0, \quad (4.17)$$

where $U_1(x,y,z)=C_1$ and $U_2(x,y,z)=C_2$ are any two independent solutions of

$$\frac{dx}{S} = \frac{dy}{Q} = \frac{dz}{R} \quad (4.18)$$

The intersection of U_1 and U_2 is a characteristic curve whose tangent has the direction numbers $S:Q:R$.

We consider a flow transient where mass velocity G varies in accordance with

$$G = G_0 / (1+t) \quad t > 0 \quad (4.19)$$

where G_0 is a constant, and both heat flux and inlet enthalpy remain constant with time t . The basic differential equation is given by Eq. (4.15). Therefore, our auxiliary equation, which is analogous to Eq. (4.18), is

$$\frac{dt}{\rho} = \frac{dz}{G} = dH / (q''p' / A) \quad (4.20)$$

There are two solutions to this differential equation. We first consider the behavior of a packet of fluid within the reactor when the transient began. It can be described in terms of position Z_0 , which it had when the transient began. The subsequent positions such a given packet (e.g., Z_{01}) will hold are shown by the appropriate lines of region I in Fig. 4.4. A packet of fluid that had not yet entered the reactor when the transient began can be described in terms of time t_0 , measured from the beginning of the transient when the packet enters the reactor. The position versus time history of a designated packet is shown by a line within region II or Fig. 4.4. The equations describing the solutions for regions I and II can be expected to differ. The two regions are separated by the limiting characteristic corresponding to the history of the fluid packet that was just at the reactor inlet when the transient began. We find the solution for region I by first integrating

$$dt / \rho = dz / G \quad (4.21)$$

to obtain

$$z - Z_0 = \int_0^t \frac{G_0}{\rho(1+t)} dt = \frac{G_0}{\rho} \ln(1+t) \quad (4.22)$$

where Z_0 is the axial position of the fluid packet at time zero. From integrating $dz/G = dH/(q''p'/A)$, we obtain

$$H - H_0(Z_0) = \int_{Z_0}^Z \frac{(1+t)}{G_0} \left(\frac{q''p'}{A} \right) dz \quad (4.23)$$

where $H_0(Z_0)$ is coolant enthalpy at zero time and axial position Z_0 . We evaluate this quantity from steady-state conditions. By substituting into Eq. (4.23), we obtain

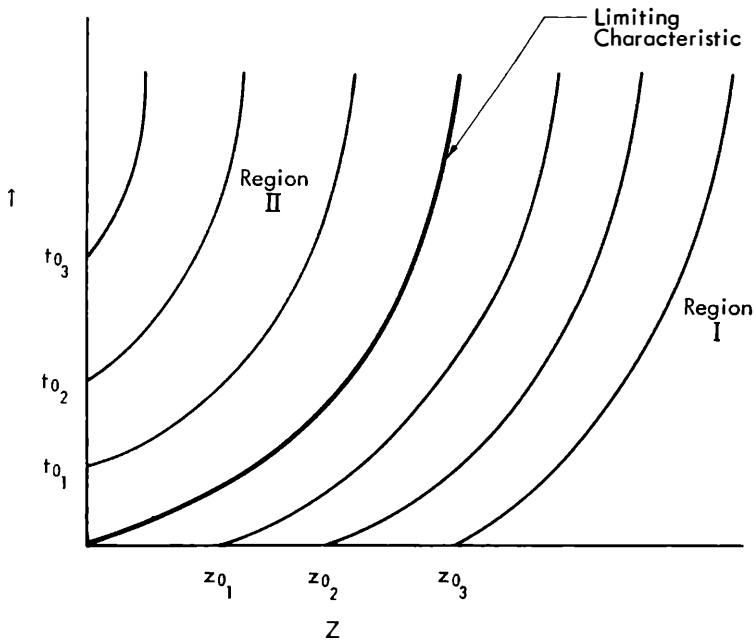


Fig. 4.4 Time-distance relationships during a hypothetical flow transient.

$$H_{(Z)} = H_{in} + \frac{p'}{AG_0} \int_0^{Z_0} q'' dz + \frac{p'}{AG_0} \int_{Z_0}^Z (1+t)q'' dz \quad (4.24)$$

where H_{in} is inlet enthalpy. From Eq. (4.22), we have

$$(1+t) = \exp[\rho(Z-Z_0)/G_0] \quad (4.25)$$

Therefore,

$$H_{(Z)} = H_{in} + \frac{p'}{AG_0} \int_0^{Z_0} q'' dz + \frac{p'}{AG_0} \int_{Z_0}^Z q'' \exp[\rho(Z-Z_0)/G_0] dz \quad (4.26)$$

Integrating the equation for the desired flux shape and substituting the value of Z obtained from Eq. (4.22) provides the desired solution for region I.

To obtain the solution for region II, we again begin by integrating Eq. (4.21), but now we consider the fluid that first entered the reactor after the transient began. Since t_0 is the time at which a given packet enters the reactor, we have

$$z = (G_0/\rho) \int_{t_0}^t dt/(1+t) = (G/\rho) \ln[(1+t)/(1+t_0)] , \quad (4.27)$$

and by rearrangement,

$$(1+t) = (1+t_0) \exp(\rho z/G_0) \quad (4.28)$$

Now, from

$$dz/G = dH/(q''p'/A) \quad (4.29)$$

$$H_{(z)} - H_{in} = [p'/(AG_0)] \int_0^z q''(1+t) dz \quad (4.30)$$

and by substituting for $(1+t)$, we obtain

$$H_{(z)} = H_{in} + \frac{p'}{G_0 A} (1+t_0) \int_0^z \exp(\rho z/G_0) q'' dz \quad (4.31)$$

The limiting characteristic is obtained by replacing G in terms of G_0 and t in Eq. (4.21) and integrating the resulting equation for both z and t :

$$\int_0^T \frac{dt}{1+t} = \int_0^Z \frac{\rho}{G_0} dz$$

$$\ln(1+T) = \frac{\rho Z}{G_0} \quad (4.32)$$

For $t \leq T$, the region I solution, Eq. (4.26), holds. For $t \geq T$, the region II solution, Eq. (4.31), holds.

4.1.2.2 Use of the Method of Characteristics for Complex Transients

Application of the previous approach requires a knowledge of the coolant flow as a function of time. This presupposes transient conditions such that the flow through the system is unaffected by the fluid properties. In many transient situations, particularly in a loss-of-coolant accident (LOCA), we cannot assume this decoupling and must simultaneously consider the conservation of energy, momentum, and mass.

If we assume one-dimensional, homogeneous flow, we can write the conservation-of-mass equation as

$$\rho \frac{\partial V}{\partial z} + \frac{\partial \rho}{\partial t} + V \frac{\partial \rho}{\partial z} = 0 \quad (4.33)$$

and the conservation-of-momentum equation as

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial z} + \frac{g_c}{\rho} \frac{\partial P}{\partial z} + gF = 0, \quad (4.34)$$

where V = fluid velocity, ft/s, F = frictional head loss per foot of length, ft/ft, P = pressure, lb/ft², and the other symbols are as previously defined. The gravitational effect (elevation head) has been omitted from Eq. (4.34).

The $\partial P/\partial z$ can be replaced by recalling that the velocity of sound, a_1 , can be obtained from

$$a_1^2 = g_c \partial P / \partial \rho \quad (4.35)$$

For the momentum balance, we then have

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial z} + \frac{a_1^2}{\rho} \frac{\partial \rho}{\partial z} + gF = 0 \quad (4.36)$$

In adiabatic single-phase flow, we would have $a_1^2 = \rho^{\gamma'-1}$, where $\gamma' = c_p/c_v$. In actuality, two-phase flow is encountered under nonadiabatic conditions. At any given pressure, a_1 is a function of ρ ; therefore, over a narrow pressure and enthalpy range, we can make the approximation

$$a_1^2 = \rho^K \quad (4.37)$$

where K is constant. We then can transform the conservation-of-mass and conservation-of-momentum equations to

$$a_1 \frac{\partial V}{\partial z} + \frac{2}{K} \left(\frac{\partial a_1}{\partial t} + V \frac{\partial a_1}{\partial z} \right) = 0 \quad (4.38)$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial z} + \frac{2a_1}{K} \left(\frac{\partial a_1}{\partial z} \right) + gF = 0 \quad (4.39)$$

Our energy equation is now more complex than previously, since ρ can no longer be assumed constant, and we obtain

$$\frac{q''p'}{A} = \frac{\partial}{\partial z} \left[G \left(H + \frac{v^2}{2g} \right) \right] + \frac{\partial}{\partial t} \left[\rho \left(E + \frac{v^2}{2g} \right) \right] \quad (4.40)$$

If we replace E in terms of H , ρ , and P and use the conservation-of-mass equation, Eq. (4.40) can be transformed to

$$\frac{q''p'}{A\rho} - \frac{1}{J\rho} \left(\frac{\partial P}{\partial t} \right) = \frac{\partial H}{\partial t} + V \frac{\partial H}{\partial z} + \left[\frac{V}{2g_c} \left(\frac{\partial V^2}{\partial z} \right) + \frac{1}{2g_c} \left(\frac{\partial V^2}{\partial t} \right) \right] \quad (4.41)$$

where J equals the mechanical equivalent of heat, Btu/ft lb. If the work due to friction is considered negligible in relation to the other terms in Eq. (4.41), the sum within the brackets at the right can be shown to be zero. Therefore,

$$\frac{q''p'}{A\rho} - \frac{1}{J\rho} \left(\frac{\partial P}{\partial t} \right) \approx \frac{\partial H}{\partial t} + V \frac{\partial H}{\partial z} \quad (4.42)$$

We observe that the three conservation equations are quasilinear, partial-differential equations similar to those for which we found the method of

characteristics applicable. Indeed, Courant and Friedrichs⁴ have shown how the notion of characteristic directions can be extended to n quasilinear, partial-differential equations in two independent variables. Although an analytical solution is not possible under general conditions, Lister⁵ showed how the method of characteristics can be used to obtain numerical solutions for flow transients, and Fabric⁶ used the method for analysis of LOCAs.

Let us follow Lister⁵ and consider Eq. (4.38), designated L_1 , and Eq. (4.39), designated L_2 , assuming a value for K is known. We form the linear combination L , where

$$L = \lambda_1 L_1 + \lambda_2 L_2 \quad (4.43)$$

Therefore,

$$L = \frac{\partial V}{\partial z}(a_1 \lambda_1 + V \lambda_2) + \frac{\partial V}{\partial t}(\lambda_2) + \frac{\partial a_1}{\partial z}(V \lambda_1 + a_1 \lambda_2)(2/K) + \frac{\partial a_1}{\partial t}\left(\frac{2}{K} \lambda_1\right) + gF \lambda_2 = 0 \quad (4.44)$$

We now choose values for λ_1 and λ_2 such that

$$\frac{dt}{dz} = \frac{\lambda_2}{a_1 \lambda_1 + V \lambda_2} = \frac{\lambda_1}{V \lambda_1 + a_1 \lambda_2} \quad (4.45)$$

Next, we multiply both sides of Eq. (4.44) by dt

$$L dt = (\lambda_2) \left(\frac{\partial V}{\partial z} dz + \frac{\partial V}{\partial t} dt \right) + \left(\frac{2}{K} \lambda_1 \right) \left(\frac{\partial a_1}{\partial z} dz + \frac{\partial a_1}{\partial t} dt \right) + gF \lambda_1 dt \quad (4.46)$$

Since $V = V(Z, t)$ and $a_1 = a_1(z, t)$,

$$dV = \frac{\partial V}{\partial z} dz + \frac{\partial V}{\partial t} dt, \quad da_1 = \frac{\partial a_1}{\partial z} dz + \frac{\partial a_1}{\partial t} dt \quad (4.47)$$

therefore,

$$L dt = \lambda_2 dV + [(2/K) \lambda_1] da_1 + gF \lambda_1 dt \quad (4.48)$$

In the expression for L in Eq. (4.44), the derivatives of V and a have been combined to be in the same direction. This direction (dz/dt) is called the "characteristic direction" or ζ . Equation (4.46) can be solved for the ratio λ_1/λ_2

$$-\frac{\lambda_1}{\lambda_2} = \frac{dz - V dt}{-a_1 dt} = \frac{-a_1 dt}{dz - V dt} \quad (4.49)$$

Hence,

$$dz^2 - 2V dz dt + (V^2 - a_1^2) dt^2 = 0 \quad (4.50)$$

There are two real solutions for dz/dt ,

$$\frac{dz}{dt} = \zeta_+ = (V + a_1), \quad \frac{dz}{dt} = \zeta_- = (V - a_1) \quad (4.51)$$

Equation (4.51) defines two families of characteristic curves (designated C_+ and C_-) in the z,t plane. Since ζ_+ and ζ_- are functions only of z and t , we can substitute these into Eq. (4.49) and combine the results with Eq. (4.44) to obtain

$$dV + (2/K) da_1 - gF dt = 0 \quad (4.52)$$

and

$$-dV + (2/K) da_1 - gF dt = 0 \quad (4.53)$$

as the two characteristic equations.

The energy equation must also be written as a total differential equation. We rewrite Eq. (4.42) to eliminate the pressure term, and obtain

$$\frac{q''p'}{A\rho} - \frac{a_1^2}{J\rho g_c} \left(\frac{\partial \rho}{\partial t} \right) = \frac{\partial H}{\partial t} + V \frac{\partial H}{\partial z} \quad (4.54)$$

If we were to temporarily assume that $(\partial \rho / \partial t)$ is a constant, Eq. (4.54) would be in the form of Eq. (4.16), and we then make use of Eq. (4.18) to obtain

$$dt = dz/V \quad (4.55)$$

or

$$V = dz/dt \quad (4.56)$$

Substituting for V in Eq. (4.54) yields

$$\left[\frac{q''p'}{A\rho} - \frac{a_1^2}{J\rho g_c} \left(\frac{\partial \rho}{\partial t} \right) \right] = \frac{\partial H}{\partial t} + \left(\frac{dz}{dt} \right) \frac{\partial H}{\partial z} \quad (4.57)$$

On multiplication by dt , we find the right side to be the total differential, dH

$$\left[\frac{q''p'}{A\rho} - \frac{a_1^2}{J\rho g_c} \left(\frac{\partial \rho}{\partial t} \right) \right] dt = dH \quad (4.58)$$

This is considered a third characteristic equation whose characteristic direction is equal to V

We noted in Sec. 3.3.4.5 that characteristic equations can also be written when the homogeneous flow assumption is removed, providing some simplifications are made. Tentner and Weisman⁷ indicate one approach that can be used. The conservation equations are written in terms of liquid velocity, V . Gas velocity, V_g , is taken as SV where S is the slip ratio. For simplicity of notation, they write

$$\frac{d\rho_g}{dp} = \dot{a}^2 \quad \frac{dH_g}{dp} = n^2 \quad \frac{dH_l}{dp} = m^2 \quad (4.59)$$

where ρ_g = gas density, P = pressure, and H_g , H_l = gas and liquid enthalpies, respectively. They then make the substitutions

$$\begin{aligned}\frac{\partial \rho_g}{\partial t} &= \dot{a}^2 \frac{\partial P}{\partial t}; & \frac{\partial \rho_g}{\partial z} &= \dot{a}^2 \frac{\partial P}{\partial z}; & \frac{\partial H_g}{\partial t} &= n^2 \frac{\partial P}{\partial t} \\ \frac{\partial H_g}{\partial z} &= n^2 \frac{\partial P}{\partial z}; & \frac{\partial H_l}{\partial t} &= m^2 \frac{\partial P}{\partial t} & \frac{\partial H_l}{\partial z} &= m^2 \frac{\partial P}{\partial z}\end{aligned}\quad (4.60)$$

where z is axial distance and t is time.

With the further assumption that $\partial \rho_l / \partial t$, $\partial \rho_l / \partial z$, $\partial S / \partial t$, $\partial S / \partial z$ are sufficiently small to be ignored, on rearrangement the conservation equations become

$$\begin{aligned}(\alpha \dot{a}^2) \frac{\partial P}{\partial t} + (\alpha S \dot{a}^2) \frac{\partial P}{\partial z} + 0 \frac{\partial V}{\partial t} + [\alpha \rho_g S + (1 - \alpha) \rho_l] \frac{\partial V}{\partial z} \\ + (\rho_g - \rho_l) \frac{\partial \alpha}{\partial t} + (\rho_g S V - \rho_l V) \frac{\partial \alpha}{\partial z} = 0\end{aligned}\quad (4.61)$$

$$\begin{aligned}(\alpha S V \dot{a}^2) \frac{\partial P}{\partial t} + (1 + \alpha S^2 V^2 \dot{a}^2) \frac{\partial P}{\partial z} + [\alpha \rho_g S + (1 - \alpha) \rho_l] \frac{\partial V}{\partial t} + [2 \alpha \rho_g S^2 V + \\ 2(1 - \alpha) \rho_l V] \frac{\partial V}{\partial z} + (\rho_g S V - \rho_l V) \frac{\partial \alpha}{\partial t} + (\rho_g S^2 V^2 - \rho_l V^2) \frac{\partial \alpha}{\partial z} = -\frac{\tau'}{A}\end{aligned}\quad (4.62)$$

$$\begin{aligned}[\alpha \dot{a} E_g - 1 + n^2 \alpha \rho_g + m^2 (1 - \alpha) \rho_l] \frac{\partial P}{\partial t} + [\alpha S E_g \dot{a}^2 + n^2 \alpha \rho_g V S \\ + m^2 (1 - \alpha) \rho_l V] \frac{\partial P}{\partial z} + [\alpha \rho_g S^2 V + (1 - \alpha) \rho_l V] \frac{\partial V}{\partial t} \\ + [\alpha \rho_g S E_g + \alpha \rho_g V^2 S^3 + (1 - \alpha) \rho_l E_l + (1 - \alpha) \rho_l V^2] \frac{\partial V}{\partial z} \\ + (\rho_g E_g - \rho_l E_l) \frac{\partial \alpha}{\partial t} + (\rho_g V S E_g - \rho_l V E_l) \frac{\partial \alpha}{\partial z} = \frac{q'}{A}\end{aligned}\quad (4.63)$$

where

α = void fraction

ρ_l = liquid density

A = cross-sectional area

$$E_g = H_g + \frac{S^2 V^2}{2}$$

$$E_l = H_l + \frac{V^2}{2}$$

τ' = wall shear stress.

Each of the conservation equations is now in the general form

$$A_{i,1} \frac{\partial P}{\partial t} + A_{i,2} \frac{\partial P}{\partial z} + A_{i,3} \frac{\partial V}{\partial t} + A_{i,4} \frac{\partial V}{\partial z} + A_{i,5} \frac{\partial \alpha}{\partial t} + A_{i,6} \frac{\partial \alpha}{\partial z} + A_{i,7} = 0 \quad (4.64)$$

for $i=1, 2, 3$, which is suitable for solution by the method of characteristics.

It was previously observed (see Sec. 3.3.5.2) that Tentner and Weisman⁷ always obtained real characteristics as long as slip ratios approaching unity were used at very high mass fluxes. This would appear to be another confirmation that a two-phase mixture behaves as an essentially homogeneous fluid at very high mass fluxes.

The characteristic directions obtained with a five-equation drift-flux model (see Sec. 3.3.5.5) may also be used to obtain solutions to transient problems. Early versions of the THOR code⁸ used this approach.

4.1.2.3 Numerical Solution Procedure for Characteristic Equations

Lister⁵ indicates that numerical solutions can be obtained by using a grid of characteristics or by the method of specified time intervals. The latter method appears more easily adaptable to the complex geometries encountered in any real problem. We assume that the homogeneous flow assumption applies, that the system is divided into a series of axial segments of length ΔZ , and that we know the values of V , H , ρ , and a_1 at the grid locations at time t (points A , B , and C of Fig. 4.5). We want to find the

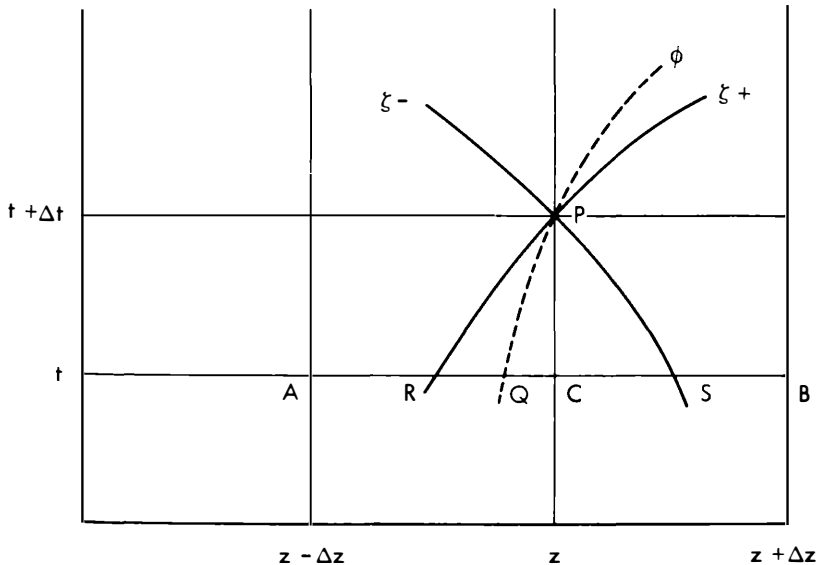


Fig. 4.5 Characteristic curves with grid at specified time intervals.

values of the problem parameters at a given location and time, $t + \Delta t$ (point P of Fig. 4.5). Curves ζ_+ and ζ_- are two of the characteristic curves through P . Then, if R and S are the intersections of the characteristics with the horizontal line through time t , from Eq. (4.51) we have

$$(Z_p - Z_R) = \zeta_+ (\Delta t) \quad (4.65)$$

$$(Z_p - Z_S) = \zeta_- (\Delta t) \quad (4.66)$$

We need the values of V , a_1 , (Z_R, t) and (Z_S, t) , which can be estimated by linear interpolation from the values at A , C , and B . We then use a finite difference form of Eqs. (4.52) and (4.53) to obtain V and a_1 at point P :

$$(V_p - V_R) + (2/K)(a_{1p} - a_{1R}) - gF(t_p - t_R) = 0 \quad (4.67)$$

$$(V_p - V_S) + (2/K)(a_{1p} - a_{1R}) - gF(t_p - t_S) = 0 \quad (4.68)$$

The value of a_{1p} is then used to obtain an updated value for ρ from Eq. (4.37), $\Delta\rho/\Delta t$, and $\Delta P/\Delta t$.

Our energy equation, Eq. (4.58), is the third characteristic ϕ through point P . Position Z_q at which this curve intersects the line through t is obtained from the finite difference form of Eq. (4.56).

$$Z_p - Z_q = V(\Delta t) \quad (4.69)$$

The enthalpy at point Q is obtained by linear interpolation and used to obtain H_p ,

$$H_p = \left[\frac{q''p'}{A\rho} - \frac{a_1^2}{J\rho g_c} \left(\frac{\Delta\rho}{\Delta t} \right) \right] \Delta t + H_Q \quad (4.70)$$

New values of ρ , P , and H can then be used to update K and, if necessary, the calculation can be repeated using an improved average value of K . If the change has been small, the calculation proceeds to the next grid point to be evaluated.

The characteristic curves have a direct physical interpretation. The ζ curves can be thought of as compression and rarefaction waves traveling through the system at near sonic velocities; the ϕ curve represents the transport of thermal energy through the system, which occurs at a rate determined by fluid velocity. Once we have divided the system into a series of increments, we are no longer completely free in our choice of time step Δt , and it must be such that

$$\Delta t \leq \Delta Z / (a_{\max}) \quad (4.71)$$

where $a_{\max}$ represents the maximum value the acoustic velocity has anywhere in the system. Without this restriction, we could not obtain the properties on the characteristic curve at time t by interpolation between the properties at adjacent nodes. In this interpolation, we assumed that the

characteristic curves are linear, while in reality they are not. For improved accuracy, quadratic interpolation procedures can be used⁵ for determining conditions at points R , Q , and S (see Fig. 4.5).

Solutions based on the method of characteristics are particularly useful in transients where there are substantial pressure pulses moving through the system. Thus, the method of characteristics finds use in the very early portion of a large LOCA. At that time, it is desirable to be able to determine the pressure forces to which internal components are subject. However, once a significant portion of the system is in the two-phase region, pressure pulses are greatly diminished and the need to track them disappears. Less complex and time-consuming approaches are therefore used for the bulk of LOCA calculations and most transient simulations.

4.1.2.4 Numerical Solution Procedures Via Finite Difference Methods

Basic Techniques. Although a variety of techniques can be used to solve transient fluid dynamic problems (see Ref. 9), all of the major programs now (1994) used for analysis of PWR transients use an approach based on finite difference procedures. In contrast to the method of characteristics, the finite difference procedure allows the solution values to be determined at evenly spaced points in the solution plane.

Basic conservation equations all express the time rate of change of one variable in terms of spatial changes in the others. Generally, those can be written as first-order equations of the form

$$\partial u_i / \partial t = \sum_k [A_k (\partial u_k / \partial x) + B_k u_k] \quad (4.72)$$

where u_k are variables other than time, t , and space, x , and A_k and B_k depend on x , t , and u_k . When expressed in finite difference form, the time rate of change can be approximated by the spatial change observed in the previous timestep. That is, for a particular variable we write

$$\frac{u_i^{j,n+1} - u_i^{j,n}}{\Delta t} = \sum_k \left[A_k \left(\frac{u_k^{j+1,n} - u_k^{j-1,n}}{\Delta x} \right) + B_k (u_k^{j+1,n} - u_k^{j-1,n}) \right] \quad (4.73)$$

where notation $u_k^{j,n}$ indicates the value of variable u_k at spatial location j and timestep n . With this approach, the values of independent variables at time period $(n+1)$ can be directly determined from the values at time n . This approximation procedure requires that the timestep be restricted so that

$$\Delta t \leq \Delta x / (a_1 + |V|) \quad (4.74)$$

where V is fluid velocity and a_1 is sonic velocity. The general conditions for stability and convergence of finite difference schemes of this nature are discussed by Fox¹⁰ and Wulff.⁹

When acoustic effects are unimportant, the timestep may be appreciably lengthened. When sonic effects are ignored, the timestep restriction

$$\Delta t \leq \Delta x / |V| \quad (4.75)$$

applies. This is considerably less restrictive than Eq. (4.74). However, a reduced solution accuracy may be expected.

The timestep restriction imposed on the previously described explicit method of Eq. (4.73) can lead to long computing times. This difficulty can be circumvented by going to a fully implicit method. (See Sec. 2.6.1.5 for discussion of explicit and implicit solution procedures.) Here spatial changes are written in terms of variable values at the current timestep. That is, we write equations of the form

$$\frac{u_i^{j,n+1} - u_i^{j,n}}{\Delta t} = \sum_k \left[A_k \left(\frac{u_k^{j-1,n+1} - u_k^{j,n+1}}{\Delta x} \right) + B_k(u_k^{j,n+1}) \right] \quad (4.76)$$

This approach requires the solution of a set of simultaneous equations covering all the variables at all spatial locations. However, the timestep limitation of Eq. (4.74) is removed. Procedures have been developed for minimizing the effort involved in solving these simultaneous equations.¹¹ It should again be noted that as the timestep grows, the accuracy of the solution decreases.

The large number of equations that must be solved simultaneously in a fully implicit solution can pose formidable difficulties. The difficulties are most severe if the derivatives of products are fully expanded before being expressed in finite difference form. A common method for avoiding this difficulty is the approach in which all variables but one in a product are written as the known variables at time interval n . The scheme is generally termed a *semi-implicit* formulation.

Although Eqs. (4.73) and (4.76) have been written in linear form, the equations are really nonlinear since the values of the A_k and B_k vary with time. If very short timesteps are taken [e.g., those of Eq. (4.74)], the linearity assumption may be justified. However, when appreciably larger timesteps are used, the assumption of constant values for the A_k and B_k is invalid. An accurate solution of Eq. (4.76) can generally be achieved only by an iteration in which the coefficients are updated.

Modeling Approaches. The earliest of the computer programs used for LOCA analysis was relatively simple. This simplicity was due in part to the use of the *node and branch* approach.

In electrical and mechanical systems, it is usual to represent a distributed parameter system by a series of nodes connected to each other through mechanical or electrical resistances. When applied to fluid dynamics, it is

assumed that we have a series of spatial elements (nodes) with a capability for energy and mass storage only. The nodes are connected by branches that contain flow resistance and inertance. Thus, the conservation-of-momentum equation is written for each branch, while the conservation-of-energy and -continuity equations are written for each node. Nahavandi¹² and Murphy *et al.*¹³ described computational procedures of this type. In the FLASH codes,¹³ the conservation equations are written in the following forms:

Conservation of nodal energy:

$$\frac{E_i(t + \Delta t) - E_i(t)}{\Delta t} = \sum_j w_{ij} H_{ij} + Q_i \quad (4.77)$$

Conservation of nodal mass:

$$\frac{M_i(t + \Delta t) - M_i(t)}{\Delta t} = \sum_j w_{ij} \quad (4.78)$$

Conservation of flowpath (branch) momentum:

$$(1/A)(\Delta w_{ij}/\Delta t) = (g_c/L_j) \left(P_{i+1} - P_i - \frac{C_j |w_{ij}| w_{ij}}{2g_c A^2 \rho} + \rho Z_j \right) \quad (4.79)$$

where

A = flowpath area

H_{ij} = enthalpy associated with flow w_{ij} , Btu/lb

C_j = pressure loss coefficient for branch j

Q_i = total heat generation rate in node i , Btu/s

M_i = total mass of node i

L_j = length of flowpath (branch) j

P_i = pressure at node i , lb_f/ft²

E_i = total fluid internal energy in control volume i , Btu

w_{ij} = flow into node i through path j , lb/s

Z_j = difference in elevation head along branch j .

Note that the spatial differences are taken between adjacent nodes. Further, the flow in a given branch is taken as constant along that branch and the effects of spatial acceleration thus are not incorporated in the model.*

The early versions of the RELAP codes,¹⁴ which were based on FLASH, used essentially the same basic equations. Early versions of FLASH¹³ used

*The basic equations used in the FLASH code are derived from the early work of J. E. Meyer, "Conservation Laws in One-Dimensional Hydrodynamics," Westinghouse Report WAPD-BT-20, Bettis Atomic Power Laboratory (1960).

an explicit integration procedure, while later versions used the implicit approach. All of the node and branch applications used single fluid models.

Many current transient computer programs have abandoned the node and branch approach and are based on the use of control volumes for system modeling. Node and branch techniques made sense when the system was modeled by a few large nodes, representing major components, and branches representing the pipes connecting these components. However, the need for a more accurate description of system behavior now requires that the system be modeled by a much larger number of segments. It is convenient to do this with the control-volume-based models, which can account for spatial acceleration.

In the usual approach, the system is represented by a set of linearly interconnected segments or control volumes. All of the conservation equations, in finite difference form, are written for each control volume. In most cases, a staggered mesh system is used. That is, mass and energy balances are written for the control volume center and the momentum balances are written for cell edges for which they produce velocities. Properties within the control volume are usually taken as uniform and the properties of the fluid leaving a control volume are taken as those at the volume center.

The earliest PWR system analysis codes using the control volume approach were based on single fluid models. Finite difference approximation of equations similar to Eqs. (3.125) through (3.127) formed the basis of many of these models. Computer programs of this type are still (1993) used for a number of licensing applications.

More recent computer programs have been based on models using from four to six conservation equations. Table 4.I illustrates a typical set of finite difference equations for a five-conservation-equation model used with the control volume approach.

The RETRAN program,¹⁵ which is widely used by U.S. utilities for system analysis, has a five-equation model in the -02 version.¹⁶ The program has three equations for mixture-mass, momentum, and energy conservation. In addition there is one equation for vapor mass conservation and one for the time rate of change of the liquid velocity relative to the vapor velocity (dynamic slip equation). The equations are linearized and, after algebraic substitution, a system of linear equations is obtained for the change in mixture mass flux, dynamic slip velocity, and mixture energy. These are solved by matrix inversion and the results used to obtain mixture and vapor mass by back-substitution.

A somewhat related approach is used in RELAP 5¹⁷ where, after linearization and algebraic manipulation, a single set of n equations is obtained in terms of the n unknown pressures in the control volumes. After an implicit solution of these equations, the other variables are computed by back-substitution.

A considerably more elaborate procedure is used in the TRAC code.¹⁸ TRAC uses a two-fluid model with six conservation equations. The numer-

TABLE 4.I(a)
Finite Difference Form of Five-Conservation-Equation Model Applied to Control Volume Approach

Differential Equations	Finite Difference Equations
Vapor mass: $\frac{\partial}{\partial t}(\alpha \rho_v) + \frac{\partial}{\partial z}(\alpha \rho_v v_m) + \frac{\partial}{\partial z} \left[\frac{\alpha \rho_v (1 - \alpha) \rho_l v_r}{\rho_m} \right] = \Gamma$	$\frac{(\alpha \rho_v)_i^{n+1} - (\alpha \rho_v)_i^n}{\Delta t} + \frac{(\alpha \rho_v)_{i+1/2}^n (v_m)_{i+1/2}^{n+1} - (\alpha \rho_v)_{i-1/2}^n (v_m)_{i-1/2}^{n+1}}{\Delta z}$ $+ \frac{1}{\Delta z} \left\{ \left[\frac{(1 - \alpha) \rho_l \alpha \rho_v v_r}{\rho_m} \right]_{i+1/2}^n - \left[\frac{(1 - \alpha) \rho_l \alpha \rho_v v_r}{\rho_m} \right]_{i-1/2}^n \right\} = \Gamma^{n+1}$
Mixture mass: $\frac{\partial \rho_m}{\partial t} + \frac{\partial}{\partial z}(\rho_m v_m) = 0$	$\frac{(\rho_m)_i^{n+1} - (\rho_m)_i^n}{\Delta t} + \frac{(\rho_m)_{i+1/2}^n (v_m)_{i+1/2}^{n+1} - (\rho_m)_{i-1/2}^n (v_m)_{i-1/2}^{n+1}}{\Delta z} = 0$
Mixture momentum: $\frac{\partial}{\partial t}(\rho_m v_m) + \frac{\partial}{\partial z}(\rho_m v_m^2) + \frac{\partial}{\partial z} \left[\frac{(1 - \alpha) \rho_l \alpha \rho_v v_r^2}{\rho_m} \right] + \frac{\partial P}{\partial z} + \rho_m g_z + \tau = 0$	$\frac{(v_m)_{i+1/2}^{n+1} - (v_m)_i^n}{\Delta t} + \frac{(v_m)_{i+1/2}^n (v_m)_{i+1/2}^{n+1} - (v_m)_i^n (v_m)_i^{n+1}}{\Delta z}$ $+ \frac{1}{(\rho_m)_{i+1/2}^n \Delta z} \left\{ \left[\frac{(1 - \alpha) \rho_l \alpha \rho_v v_r^2}{\rho_m} \right]_{i+1}^n \right.$ $\left. - \left[\frac{(1 - \alpha) \rho_l \alpha \rho_v v_r^2}{\rho_m} \right]_i^n \right\} + \frac{1}{(\rho_m)_{i+1/2}^n} \left(\frac{P_{i+1}^{n+1} - P_i^{n+1}}{\Delta z} \right) + (g_z)_{i+1/2} + \left(\frac{\tau}{\rho_m} \right) = 0$

Continued

TABLE 4.1(a) (Continued)

Differential Equations	Finite Difference Equations
Vapor specific internal energy: $\frac{\partial}{\partial t} (\alpha \rho_v E_v) + \frac{\partial}{\partial z} (\alpha \rho_v E_v v_m) + \frac{\partial}{\partial z} \left[\frac{\alpha \rho_v (1 - \alpha) \rho_l E_v v_r}{\rho_m} \right]$ $+ P \frac{\partial}{\partial z} (\alpha v_m) + P \frac{\partial}{\partial z} \frac{\alpha (1 - \alpha) \rho_l v_r}{\rho_m} = q_v'' + q_{lv}''$	$\left[\frac{(\alpha \rho_v E_v)^{n+1} - (\alpha \rho_v E_v)^n}{\Delta t} + \frac{1}{\Delta z} \left\{ (\alpha \rho_v E_v)^{n+1}_{i+1/2} - (\alpha \rho_v E_v)^n_{i-1/2} \right\} \right]$ $- \left[(\alpha \rho_v E_v)^{n+1}_{i+1/2} - (\alpha \rho_v E_v)^n_{i-1/2} \right]$ $+ \frac{1}{\Delta z} \left\{ \left[\frac{\alpha \rho_v (1 - \alpha) \rho_l E_v v_r}{\rho_m} \right]^{n+1}_{i+1/2} - \left[\frac{\alpha \rho_v (1 - \alpha) \rho_l E_v v_r}{\rho_m} \right]^n_{i-1/2} \right\}$ $+ P^{n+1}_i \left[\frac{\alpha^n (v_m)^{n+1}_{i+1/2} - \alpha^n (v_m)^{n+1}_{i-1/2}}{\Delta z} + \frac{P^n_i}{\Delta z} \left\{ \left[\frac{\alpha (1 - \alpha) \rho_l v_r}{\rho_m} \right]^n_{i+1/2} - \left[\frac{\alpha (1 - \alpha) \rho_l v_r}{\rho_m} \right]^n_{i-1/2} \right\} \right] = q_v'' + q_{lv}''$
Mixture specific internal energy: $\frac{\partial}{\partial t} (\rho_m E_m) + \frac{\partial}{\partial z} (\rho_m E_m v_m) + \frac{\partial}{\partial z} \left[\frac{\alpha (1 - \alpha) \rho \alpha \rho_v v_r (E_v - E_l)}{\rho_m} \right]$ $+ \rho \frac{\partial v_m}{\partial z} + P \frac{\partial}{\partial z} \left[\frac{\alpha (1 - \alpha) v_r (\rho_l - \rho_v)}{\rho_m} \right] = q''$	$\left[\frac{(\rho_m E_m)^{n+1}_i - (\rho_m E_m)^n_i}{\Delta t} + \frac{1}{\Delta z} \left\{ (\rho_m E_m)^{n+1}_{i+1/2} (v_m)^{n+1}_{i+1/2} - (\rho_m E_m)^n_{i-1/2} (v_m)^n_{i-1/2} \right\} \right]$ $+ \frac{1}{\Delta z} \left\{ \left[\frac{(1 - \alpha) \rho \alpha \rho_v (E_v - E_l)}{\rho_m} \right]^n_{i+1/2} v_r - \left[\frac{(1 - \alpha) \rho \alpha \rho_v (E_v - E_l)}{\rho_m} \right]^n_{i-1/2} v_r \right\}$ $+ P^{n+1}_i \left[\frac{(v_m)^{n+1}_{i+1/2} - (v_m)^{n+1}_{i-1/2}}{\Delta z} + \frac{P^n_i}{\Delta z} \left\{ \left[\frac{\alpha (1 - \alpha) (\rho_l - \rho_v)}{\rho_m} v_r \right]^n_{i+1/2} - \left[\frac{\alpha (1 - \alpha) (\rho_l - \rho_v)}{\rho_m} v_r \right]^n_{i-1/2} \right\} \right] = q''$

TABLE 4.I(b)
Definition of Symbols Used in Table 4.I(a)

α	= vapor volume fraction
ρ_v	= vapor microscopic density
ρ_l	= liquid microscopic density
ρ_m	= mixture density
v_m	= mixture velocity
v_r	= relative velocity ($v_v - v_l$)
v_v	= vapor velocity
v_l	= liquid velocity
P	= pressure
g_z	= gravitational force
τ	= wall shear
E_v	= vapor specific internal energy
E_l	= liquid specific internal energy
E_m	= mixture specific internal energy
q_v''	= heat flux to vapor from wall
q''	= heat flux to mixture from wall
Γ	= vapor production rate due to flashing
q_{lv}''	= interfacial heat flux from liquid to vapor

In Table 4.I(a) the superscripts n and $n+1$ refer to the time levels, i and $i+1/2$ are space points and Δt and Δz are mesh spacings in time and space, respectively.

ical technique used is based on the SETS procedure described by Mahaffey.¹⁹ The SETS procedure may be considered a method that attempts to deal with the inherent nonlinearity of the basic conservation equations. In the SETS approach, the momentum balance equations are first solved using the pressure field from the previous timestep. In the second step, the equations of momentum are again solved but using a first approximation of the new time pressure field. The resulting revised velocities are substituted into the mass and energy conservation equations. The four mass and energy balances, together with the equations of state, are combined to form four nonlinear equations in terms of temperatures, void fraction, and pressure. These are solved by Newton-Raphson iteration. Finally, the mass and energy balance equations are rewritten using the latest pressure and temperature estimates. These equations are then solved for new time values of density and energy.

Numerical procedures continue to evolve and it may be expected that newer program versions will have improved solution schemes that differ from those described here.

Accelerated Solution Techniques. All the foregoing techniques require considerable computer time for most transients. A number of approaches have been taken to reduce the time needed for calculating system behavior in slowly varying transients where acoustic effects are unimportant.

One approach to slowly varying system transients (e.g., pump coast-down) is to assume that flow in the system can be obtained from a knowledge of pump behavior (e.g., use of a pump coastdown curve) and then to solve only the conservation-of-energy and -mass equations for each control volume. Elimination of the momentum equation results in a substantial decrease in computing time. The DYNODE code is an example of this technique.

Alternatively, the momentum balance equation may be retained but sonic effects can be ignored by omitting the $\partial p / \partial t$ terms from the momentum equation.²⁰ An explicit solution technique with expanded timesteps based on Eq. (4.75) can then provide fairly rapid solution times.

4.1.3 Heat Transport Via Natural Convection

At the termination of the flow coastdown during a loss-of-flow accident (LOFA), core cooling occurs only by means of natural circulation. The driving head for natural circulation is due to the density differences between the upflow and downflow sections of the loop. The fluid flowing through the core is heated and hence has its density reduced (particularly if boiling occurs). The warm fluid leaving the core is cooled in the elevated steam generator and its density increases. The net elevation head is balanced by the frictional losses in the loop. That is,

$$\int_0^0 \frac{g}{g_c} \rho \, dz - \int_z^0 \frac{g}{g_c} \rho \, dz = \int_0^0 C_f \frac{\rho V^2}{2g_c} \, dz + \int_z^0 C_f \frac{\rho V^2}{2g_c} \, dz \quad (4.80)$$

where C_f = frictional loss coefficient. This equation is commonly used in finite-difference form using the average density, ρ_j , for each loop segment. If each side of the loop is divided into n segments of height Δz ,

$$\sum_{j=1}^n \frac{g}{g_c} \rho_j \Delta z - \sum_{j=n+1}^{2n} \frac{g}{g_c} \rho_j \Delta z = \sum_{j=1}^{2n} \frac{C_{fj} \rho_j V_j^2}{2g_c} \quad (4.81)$$

An iterative procedure is required to estimate the total flow. We may assume a flow, calculate the appropriate densities, and then recalculate the flow. A new flow is chosen and the process completed when the assumed and calculated flow are equal (within acceptable tolerance).

Natural circulation cooling is also of significance during a small-break loss-of-coolant accident (SBLOCA). U.S. licensing regulations require that the primary coolant pumps be tripped during such an occurrence. As the

primary system inventory is depleted, the system moves from single-phase natural convection to two-phase cocurrent circulation to reflux condensation.

The onset of two-phase natural circulation causes an increase in the natural circulation flow rate due to the greater density difference between hot and cold legs. Based on tests in the Semiscale,²¹ a peak in the flow occurs at system mass inventories between 86 and 91%. This appears to be due to steam bubbles from the hot side of the steam generator tubes spilling over the downflow side and thus causing a reduction in the density difference. When the primary system mass inventory decreases to below 70% of its initial value, the steam generator tubes and hot leg are nearly empty of liquid and the reflux condenser mode begins. The steam generated in the core is then condensed in the steam generator tubes. Condensation occurs in both upflow and downflow sides.

Tests in both the Semiscale²¹ and LOBI²² facilities indicate that flow instabilities in the primary system are induced at low secondary system levels. However, these instabilities had little net effect on core cooling. The origin of the instabilities is believed to be a sort of syphon condensation occurring in the U-tube of the steam generators. Flow oscillations of the same magnitude and frequency of those observed are predicted by RELAP5.²²

Natural circulation is used for residual heat removed in the advanced AP600 design.²³ The passive residual heat removal system transfers decay heat from the reactor coolant system to the containment by heating and boiling the water in the in-containment refueling water storage tank. The steam produced transfers heat to the atmosphere by condensing on the inside surface of the containment shell.

4.2 HEAT TRANSPORT IN THE PRESENCE OF INTERACTING CHANNELS

4.2.1 Subchannel Analysis

4.2.1.1 *Basic Models for Steady-State Conditions*

While the assumption of essentially zero interchange with adjacent channels can, under some circumstances, be considered approximately correct when the average behavior of a fuel assembly is determined, the assumption is not appropriate when we consider the channels defined by four neighboring rods in a fuel assembly. Here interchannel mixing and cross-flow should always be considered. In this section, the basic flow models developed in Chapter 3 are extended to the interacting channels in a rod bundle geometry.

Adjacent subchannels are open to each other through the gap between two neighboring fuel rods; flow in one channel mixes with that of the other.

In addition, as previously observed, there is also cross-flow between channels because of the pressure gradient. Local turbulent mixing reduces the enthalpy rise of the hot channel. On the other hand, flow leaving the hot channel increases its enthalpy rise. Calculation of the net result is complicated although the equation describing enthalpy rise is easily written.

We shall assume, as is done in nearly all subchannel analyses, that a single fluid model is adequate to describe the fluid behavior. This is certainly true at the high mass fluxes characteristic of normal PWR operation. Under LOCA conditions, this assumption may be poor.

For simplicity, we consider the case of two adjacent subchannels that are only linked with each other. This situation is represented in Fig. 4.6 for a segment of length ΔZ . Quantities H , V , ρ , and P represent coolant enthalpy, velocity, density, and static pressure, respectively; A is channel flow area; W_{mn} is cross-flow between channels; and w' is the flow exchange of diffusion mixing. The numbered subscripts refer to two different axial elevations in the core. Writing mass, energy, and momentum balance equations for channel m between elevations 1 and 2 results in the following relationships where W_{mn} has been considered positive in the direction from channel n into channel m :

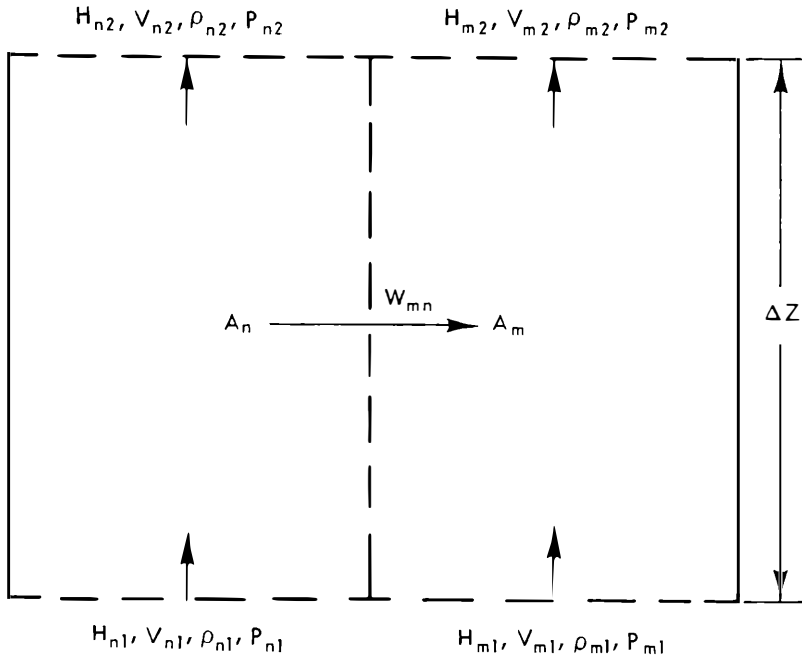


Fig. 4.6 Mathematical representation of flow redistribution.

Conservation of mass:

$$A_m V_{m1} \rho_{m1} + W_{mn} = A_m V_{m2} \rho_m \quad (4.82)$$

Conservation of thermal energy:

$$A_m V_{m1} \rho_{m1} H_{m1} + Q_{mz} + W_{mn} \bar{H}_n + w'(H_n - H_m) \Delta Z = A_m V_{m2} \rho_{m2} H_m \quad (4.83)$$

Conservation of axial momentum:

$$\begin{aligned} A_m P_{m1} + A_m \rho_{m1} V_{m1}^2 / g_c + W_{mn} \bar{V}_n / g_c \\ = A_m P_{m2} + A_m \rho_{m2} V_{m2}^2 / g_c + K_{mz} A_m \bar{\rho}_m V_m^2 / 2g_c + \bar{\rho}_m \Delta Z g / g_c \end{aligned} \quad (4.84)$$

where

Q_{mz} heat input into channel m in interval ΔZ , energy/time

W_{mn} = cross flow from channel n to channel m , mass/time

$\bar{H}, \bar{\rho}, \bar{V}$ mean values in ΔZ of enthalpy, density, and velocity

w' = turbulent flow exchange rate per unit length, mass/time length

K_{mz} = pressure loss coefficient for channel m in interval ΔZ .

Evaluation of ρ and K_{mz} requires the determination of void fraction and two-phase pressure drop as described in Chapter 3. Cross-flow is determined from the appropriate lateral momentum balance equation (see Sec. 4.2.1.2). The interchange due to mixing, represented by w' is determined by a suitable correlation (see Sec. 4.2.1.3). The additional terms arising from the coupling of channel m to other surrounding channels are readily deduced.

A similar set of equations can be written for every other channel in the region studied. A simultaneous solution of all of these equations is required to determine fluid conditions at the exit of the length interval. The complexity of the calculational procedure requires a computer solution; a number of computer codes have been written for this purpose. In all of these codes, the cross section of the region of interest is divided into a series of subchannels, while the length is divided into a series of axial segments. Thus, a set of connecting control volumes is provided.

All of the calculational procedures assume that conditions are uniform within a given control volume. Gradients exist only across control volume boundaries. Further, all lateral flow entering the control volume loses its direction. Since properties and velocities are homogenized within each control volume, subchannel analysis may be considered a subclass of the lumped parameter approach.

Subchannel analysis derives its name from the fact that the basic control volume used is the small channel, or subchannel, between adjacent fuel rods. As shown in Fig. 4.7, the subchannel considered for a square lattice is the flow area (shaded region) contained within the lines drawn through

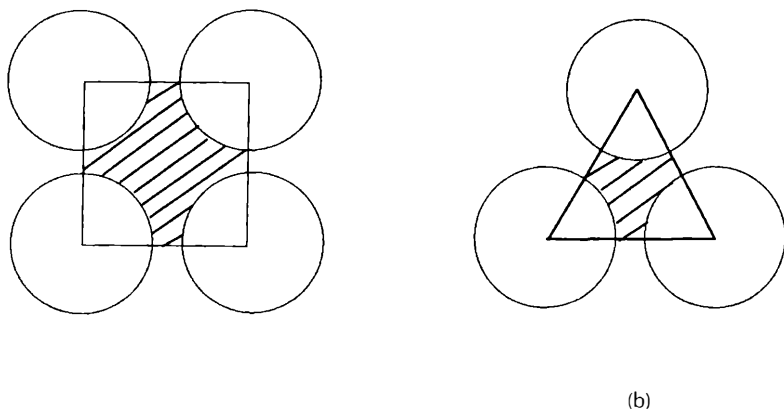


Fig. 4.7 Subchannel configurations: (a) square lattice and (b) triangular lattice.

the centers of four adjacent rods. For a triangular lattice, the subchannel is defined by the lines drawn through the center of three adjacent rods. The fuel rod segments contained within the defining lines are assumed to transfer all their thermal energy to the shaded flow area. The enthalpies and flows determined for each axial segment of each subchannel are used with appropriate heat transfer correlations to determine the margin between operating conditions and design limits.

Individual subchannels of the configurations shown in Fig. 4.7 are generally used to determine the flow and enthalpy distribution within the hottest fuel assemblies of the reactor core. However, the large number of such subchannels in an entire core, or symmetrical section of a reactor core, makes it impractical to use such subchannels for an analysis of corewide behavior. When a core-wide analysis is performed, it is usual to take an entire assembly (sometimes one-quarter of an assembly) as a channel. The analysis then determines the average enthalpy and flow within the assembly. It differs from a closed-channel analysis in that interchange with adjacent assemblies is considered.

4.2.1.2 *Flow Redistribution*

The rod bundle subchannels in a reactor core are open to each other through the gap between two neighboring fuel rods. The flow in one channel mixes with that of the others. In the presence of a pressure differential between the channels, there is also flow redistribution between channels. Local fluid mixing, caused by turbulence, reduces the enthalpy rise of a hot channel. On the other hand, flow leaving the hot channel increases its enthalpy rise.

Variation in hydraulic conditions among the various subchannels leads to differing axial pressure drops. Hence, at any given axial level there will be pressure gradients leading to lateral flow. In early subchannel computations, it was simply assumed that lateral flow velocity u could be obtained from

$$\Delta P = \frac{K_1 \rho u |u|}{2g_c} \quad (4.85)$$

where ΔP = pressure difference between two adjacent assemblies, K_1 = lateral frictional resistance, and ρ = fluid density. However, it was recognized that the above formulation was inadequate since the inertial effect, due to high axial velocity, was neglected. Chelemer *et al.*²⁴ recognized the effect of axial velocity and, on the basis of single-phase data for flow through slots, computed K_1 from

$$(K_1/K_\infty - 1)K_1 = \gamma'(u/V)^{-} \quad (4.86)$$

where γ' = constant, V = axial velocity, and K_∞ = value of K_1 as (V/u) approaches zero. Subsequently, Kim and Stoudt²⁵ used a similar approach to correlate cross-flow between bundles containing spacer grids. They proposed the average interbundle cross-flow velocity, u , could be correlated by an equation of the form

$$u = (2K_s V_1 + B) - [(2K_s V_1 + B)^2 - 4K_s C]^{1/2} / (2K_s \Delta x A_c / A_x) \quad (4.87)$$

where

$$B = 2(V_1 + V_2) + \Delta x(f_1 V_1 + f_2 V_2)/D$$

$$C = K_s V_1^2 - 2g \Delta P / \rho + \Delta x(f_1 V_1^2 - f_2 V_2^2)/D$$

A_c, A_x = cross-sectional flow area in the transverse and axial direction, respectively

D = hydraulic diameter of the rod bundle

f = friction factor

g = gravitational constant

K_s = empirical cross-flow correlation constant

ΔP = interbundle differential pressure

V_1, V_2 = axial velocity of water at the beginning and end of an increment

Δx = calculational increment in axial direction

ρ = fluid density.

Weisman²⁶ showed that the single-phase data indicating K to be a function of (u/V) could be explained by using a lateral momentum balance and recognizing that only a portion of the duct or bundle area is affected by

cross flow. Weisman proposed that the pressure difference between channels i and j be obtained from

$$\left(\frac{A''}{a}\right)_2 a_2 u_2 = \frac{g_c (a_2 - a_1)(\bar{P}_i - \bar{P}_j)}{\rho(\bar{V}_i + \bar{V}_j)} + \left(\frac{A''}{a}\right)_1 a_1 u_1 \quad (4.88)$$

where A'' = effective axial flow area over which the lateral velocity can be taken as u ; a_k = lateral flow area from inlet to location k , where $k=1$ or 2 ; and P_i, P_j = pressures in adjacent channels at the edge of areas affected by cross flow. Subscripts 1 and 2 indicate the inlet and outlet of the length step being considered.

The ratio (A''/a) was found to be a function of the length of the connecting region (slot length). The relationship found is shown in Fig. 4.8. For large slot lengths, A'' is considered the entire flow area, A , of the assembly. Equation (4.88) then simplifies to

$$(u_2 - u_1)A = \frac{g_c (a_2 - a_1)(\bar{P}_i - \bar{P}_j)}{\rho(\bar{V}_i + \bar{V}_j)} \quad (4.89)$$

Note that when $A'' = A$, $\bar{P}_i$ and $\bar{P}_j$ become the pressures at the outer edges of the two adjacent assemblies. Thus, if ΔP_l is the lateral pressure drop across one assembly, $\bar{P}_i - \bar{P}_j \approx 2\Delta P_l$. After rearrangement, Eq. (4.89) can be rewritten as

$$(\Delta u / \Delta x)(\rho V_{av}) + g_c (\Delta P_l / \Delta y) = 0 \quad (4.90)$$

where Δx is axial distance and Δy is lateral distance. This form is more convenient for estimating the lateral flow across a single assembly at an axial position removed from the core inlet.

Equation (4.90) holds only when the lateral frictional resistance is negligible. When this is not the case, we have the more general expression

$$\left(\frac{\Delta u}{\Delta x}\right)(\rho V_{av}) + g_c (\Delta P_l / \Delta y) = \frac{K_1^1}{2} \rho u |u| \quad (4.91)$$

where K_1^1 = lateral frictional resistance per unit length.

The effective area concept is designed for use in determining flows between fuel assemblies. Computing cross-flow between the subchannels of an assembly requires a different formulation of the lateral momentum balance. Rowe²⁷ writes the steady-state lateral momentum balance for the small gap between fuel rods (see Fig. 4.9) as

$$\frac{1}{g_c} \frac{\partial(Vw)}{\partial x} = \frac{s}{l} (P_i - P_j) - F \quad (4.92)$$

where x = axial distance in feet, F = frictional and form pressure loss, and w = diversion cross-flow between channels i and j , lb/s ft. Other terms are defined in Fig. 4.9.

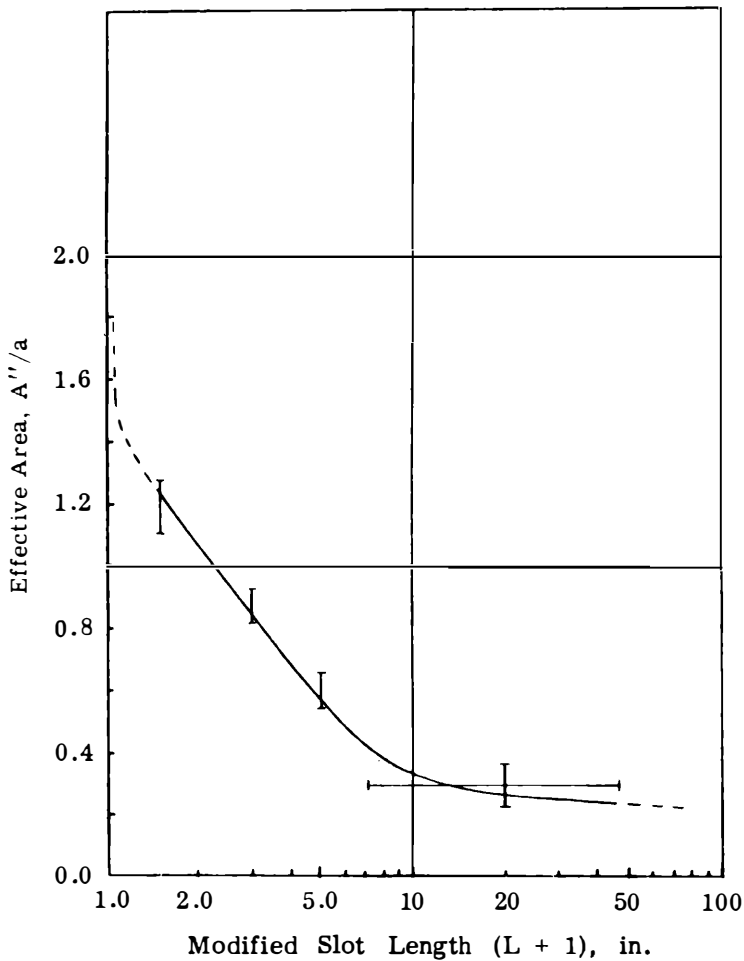


Fig. 4.8 Effective area for assignment of lateral velocity in rod bundles [from *Nucl. Technol.* 15, 467 (1972)].

Rouhani²⁸ pointed out that Eqs. (4.88) through (4.92) are simplifications of the general transverse momentum equation. Rouhani indicates that these simplifications are justified only when similar channels, in the absence of blockages, are considered. When blockages are present, Brown *et al.*²⁹ suggest that Eq. (4.92) can still be retained, providing a control volume of variable size is used. That is, length l is replaced by Δy , where Δy is a function of the distance above the blockage. With this modification, Brown *et al.*²⁹ were able to fit available data on lateral flow in the vicinity of a blockage placed in a simple two-channel geometry.

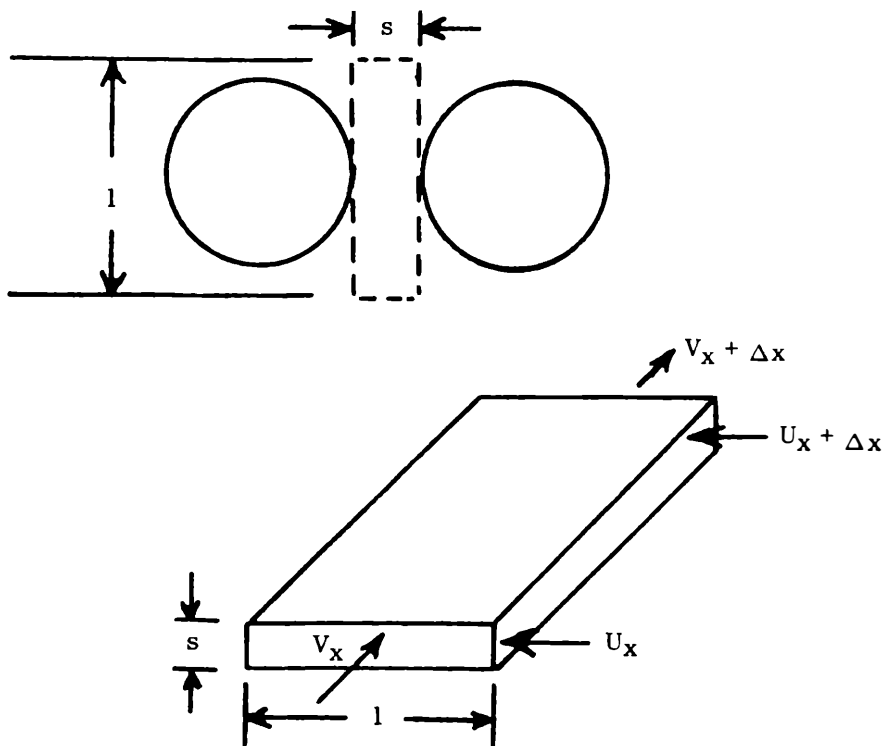


Fig. 4.9 Control volume for lateral momentum balance [from *Nucl. Technol.*, **15**, 466 (1972)].

For flow blockage problems, Sha *et al.*³⁰ suggest the transverse momentum equation be expanded. This requires addition of the term $(1/g_c) [\partial(vw)/\partial x]$, where v is lateral velocity at right angles to w , to the left side of Eq. (4.92).

The lateral momentum balances of Eqs. (4.88) through (4.92) implicitly assume that the two-phase mixture present in the core can be treated as a homogeneous fluid. Madden³¹ experimentally measured lateral two-phase flow of air and water flow through slots. Madden defined separate discharge coefficients for gas and liquid streams. However, when these data were recalculated³² in terms of a single discharge coefficient using an average density of the cross-flow stream, the data followed the curve, with some scatter, that Weisman³² predicted for a 4-in.-long slot, giving some confirmation to homogeneous treatment.

Evidence^{31,33} that the quality of the cross-flow stream may be somewhat higher than that of the main stream is available. However, quality variation is perhaps at most 0.1. Weisman³² concluded that in view of the small size

of the cross flow under most circumstances, such a variation generally would not lead to major error in the enthalpy of the main stream. Therefore, the homogeneous flow approximation almost universally used in subchannel calculations appears to be reasonable.

When considering small subchannels defined by individual rods (see Fig. 4.7), the assumption that the enthalpy and axial momentum carried by the lateral flow corresponds to the average value of the three parameters in the donor channel is reasonable. However, when the channel considered consists of a substantial number of rods (e.g., entire fuel assembly), then a substantial error may be involved if this assumption is followed. The actual exchange is proportional to the property gradient across the boundary rather than the property difference. This difficulty may be dealt with by defining a transport coefficient, N , where

$$N = \frac{\bar{u}_n - \bar{u}_m}{u_n^* - u_m^*} \quad (4.93)$$

and $\bar{u}_m, \bar{u}_n$ = average value of parameter at given axial location in channels m and n , respectively; and u_m^*, u_n^* = parameter values at given axial location at boundaries of channels m and n , respectively. Transport coefficients for velocity, N_u , enthalpy, N_H , and pressure, N_p , may be defined by Eq. (4.93). However, only the transport coefficient for enthalpy has any significant effect on the calculated results.³⁴ If we take $N_u = N_p = 1.0$, then only Eq. (4.83) for conservation of thermal energy needs to be rewritten. This becomes

$$\begin{aligned} A_m V_{m1} \rho_{m1} H_{m1} + Q_{mz} + W_{mn} H_n^* + w'(H_n^* - H_m^*) \Delta Z \\ = A_m V_{mz} \rho_{mz} H_{mz} \end{aligned} \quad (4.94)$$

where

$$\begin{aligned} H_n^* - H_m^* &= (\bar{H}_n - \bar{H}_m) / N_H \\ H_n^* &= \frac{1}{2} (\bar{H}_n + \bar{H}_m) + \frac{1}{2} (\bar{H}_n - \bar{H}_m) / N_H \end{aligned}$$

and H_n^*, H_m^* = enthalpy of fluid at boundaries of channels n and m , respectively.

Chiu³⁵ has proposed a numerical approach for evaluation of N_H . If values of $H_n^*, H_m^*, \bar{H}_n$, and $\bar{H}_m$ are known at a given axial elevation, Chiu³⁵ suggests that approximate values of these quantities may be calculated at the elevation above by assuming no cross-flow. The approximate values so obtained are then used to calculate N_H for the next elevation via Eq. (4.93). This approach is used in the TORC computer code.³⁶

Chiu's³⁵ approach adds to the time required to complete a core-wide analysis. Moreno and Leira³⁷ conducted extensive numerical and analytical studies of core behavior under a variety of circumstances. On the basis of

these studies, they were able to develop two simple correlations for N_H applicable when assembly-size control volumes are used: For $R_p > 1.1$,

$$N_H = 1.0 + a \left(\frac{\beta}{0.005} \right)^{0.6} R_p^{-0.15} \left(\frac{L}{144} \right)^{0.5} \left(\frac{n}{17} \right)^{0.15} \quad (4.95a)$$

and for $R_p < 1.1$,

$$N_H = 1.0 + b \left(\frac{\beta}{0.005} \right)^{0.72} R_p^{-6.6} \left(\frac{L}{144} \right)^{0.5} \quad (4.95b)$$

where

R_p = ratio of powers in two assemblies of interest (high power/lower power)

L = core length, in.

β = mixing parameter ($w' = \beta sG$)

n = square root of number of fuel rods in assembly

a = 0.62 for sinusoidal or quasisinusoidal axial power distribution;
0.70 for uniform axial power distribution

b = 1.0 for sinusoidal or quasisinusoidal axial distribution; 1.12 for uniform axial power distribution

G = axial mass flux (mass/time area)

s = width of flow passage between assemblies.

The computation of lateral flows between control volumes is generally the most time-consuming portion of a subchannel analysis because the lateral flows are sensitive to the pressure differences between control volumes.³⁸ Since changes in lateral flow also change the axial flow and pressure drop, an iterative solution procedure is usually required. At each stage of the iteration, the lateral momentum equations must be solved simultaneously. A more rapid solution is possible when channel flows are determined without the use of a lateral momentum balance. In this scheme, originally used in THINC II,²⁴ it is assumed that the pressure at any axial elevation is completely uniform. This implies that the axial pressure drop in each control volume must be identical. The axial flows are redistributed so that this is the case. In the approach described by Weisman *et al.*,^{39,40} the axial momentum balance equations are all linearized and placed in the form

$$B_j + C_j \Delta G_j = \Delta P_j \quad (4.96)$$

where

B_j, C_j = constants for channel j at given point in the iteration

ΔG_j = change in mass flux in channel j over given axial segment

ΔP_j = axial pressure drop across given segment in channel j .

Since the ΔP_j are all equal and the sum of all lateral flows must be zero ($\sum_j \Delta G_j A_j = 0$), we may solve for ΔP as

$$\Delta P = \frac{\sum_j A_j B_j / C_j}{\sum_j A_j / C_j} \quad (4.97)$$

where A_j = axial flow area of channel j . The values of ΔG_j are then determined by using the value of ΔP in Eq. (4.96) for each channel. Revised values of B_j and C_j are then computed and the iteration repeated until convergence. Note that this procedure avoids the necessity for repeated solution of the simultaneous lateral momentum balance equations.

Since the lateral momentum equation is not used, the source of the flow entering a channel is unknown. Hence, the enthalpy and axial velocity carried by the flow must be determined by a heuristic rule. Experience⁴¹ has shown the following approach to work well:

$$u_j^* = \frac{1}{2} \sum_i [|X_{ij}|(u_i + u_j) + X_{ij}(u_i - u_j)] / \sum_i |X_{ij}| \quad (4.98)$$

where

u_j^* = value of velocity or enthalpy carried by flow entering channel

i = subscript representing the four nodes surrounding assembly j

X_{ij} = a weighting factor representing the net gain or loss of flow in bundle j (1.0 if j gains flow; -1.0 if j loses flow to surroundings).

Wu and Weisman⁴² compared channel enthalpy rises and exit velocities obtained with the zero lateral pressure gradient assumption to those computed using a lateral momentum balance. They found generally good agreement (enthalpy rises within 2%). The zero gradient results were slightly conservative in that they predicted slightly higher enthalpies in the hottest channels.

4.2.1.3 Interchannel Mixing

The subchannel analysis approach is concerned only with the mixing that occurs across the rod gap at the subchannel boundary. In the discussion of mixing in rod bundles of Sec. 3.2.1.3, it was observed that w' , the average value of the turbulent fluctuating mass flow at the gap, could be obtained from

$$w' = \rho \bar{e} (l / De) = \rho \epsilon \quad (4.99)$$

where w' = average fluctuating mass flow at gas per unit length (mass/time length), $\bar{e}$ = eddy diffusivity (area/time), ϵ = average mixing coefficient in gap area (area/time), and l = Prandtl mixing length.

A variety of empirical approaches have been used to correlate w' and ϵ . Some investigators have preferred to correlate their data on the basis of a modified Peclet number, Pe , where

$$Pe = VDe/\epsilon \quad (4.100)$$

and De is equivalent diameter. For bundles of bare rods, Bell and Le Tourneau⁴³ reported

$$1/Pe = \epsilon/(VDe) = 0.003; \quad 10^4 < Re < 5 \times 10^4 \quad (4.101)$$

They believed the data indicated that the correlation would be valid up to $Re = 2 \times 10^5$. For rectangular channels, Sembler⁴⁴ reports

$$\epsilon/(VDe) = 0.0015; \quad Re > 10^5 \quad (4.102)$$

Sugawara *et al.*⁴⁵ prefer to base their definition of the Peclet number on the mixing length, l_m . Based on data from several sources, they propose:

$$\epsilon/(l_m V_i) = \lambda Re^{-0.1} \left[1 + \left(\frac{De_j}{De_i} \right)^{1.5} \right] \left(\frac{De_i}{D} \right) \quad (4.103)$$

where

$$\lambda = 0.0058(s/D)^{-1.46}$$

D = rod diameter

De_i, De_j = equivalent diameter of interconnecting channels i and j

l_m = mixing length $\approx De/4$

V_i = axial velocity in channel i

s = gap between rods.

Reynolds number and velocity are based on properties of the two-phase mixture.

Other investigators represented the degree of mixing in terms of w' , the turbulent transverse fluctuating flow rate per foot of axial length [lb/(hr ft)]. In one approach, it is assumed that

$$w' = \beta sG \quad (4.104)$$

where $\beta = \epsilon\rho/(sG)$ = thermal diffusion coefficient (dimensionless) and G = axial mass flux (mass/time area). Waters⁴⁶ reported on the mixing obtained in wire-wrapped bundles. He found that β increased from 0.1 to 1 as the wrap pitch changed from 0.7 to 3 wraps/ft.

Based on studies of small size rod bundles, Weisman *et al.*⁴¹ reported $\beta \approx 0.076$ for a rod bundle containing 0.4222-in.-diameter rods on a 0.535 in square pitch. They found α to be essentially independent of mass velocity [G up to 2.75×10^6 lb/(hr ft²)] and quality. Since their grids contained small mixing vanes, a lower value for β would be seen in bundles without vanes.

Kelley *et al.*⁴⁷ report that β values of the order of 0.01 are typically used for bundles with grids lacking vanes. This value is conservative and a value of 0.02 is probably closer to a best estimate value for bare rod bundles with typical PWR geometry and flow.

Equation (4.104) implies that w' will decrease directly with gap width. It was observed in Sec. 3.2.1.2 that the average eddy diffusivity tended to increase as gap width decreases. Hence, β should be a function of gap width.

Rowe and Angle⁴⁸ correlated their data by

$$w' = cGDeRe^{-0.1} \quad (4.105)$$

where G = fluid mass velocity, lb/(hr ft²), Re = Reynolds number, and c = Constant = 0.0062. Rodgers and Rosehart⁴⁹ analyzed the data of a number of experimenters and found they could use Eq. (4.105) to represent the data if $c = 0.004$.

More recently, Rodgers and Rosehart⁵⁰ modified their correlation. For square pitch rods (adjacent channels with the same De), they propose

$$w' = 0.005sG\left(\frac{De}{D}\right)Re^{-0.1}\left(\frac{s}{D}\right)^{-0.894} \quad (4.106)$$

where G = mass velocity = m/A , lb/(hr ft²); s = gap between rods, ft; and D = rod diameter, ft. This latter correlation indicates a slight effect of rod spacing in contrast to Eq. (4.104), which shows no such direct effect. However, in both Eqs. (4.105) and (4.106) decreasing the rod spacing decreases De , which leads to lower values of w' . The more recent work of Petrunik⁵¹ and Singh⁵² helps to elucidate the effect of rod spacing. For p/D ratios in excess of 1.05, there is only a slight effect of gap spacing. At slightly lower ratios, they saw an increase in mixing with rod gap. Petrunik⁵¹ believes there must be a region where there is a sharp decrease in w' since w' must go to zero as $b \rightarrow 0$. These conclusions are in agreement with the theoretical analysis of Van der Ros and Bogaart⁵³ and the results of Galbraith and Knudson.⁵⁴ For the range of p/D ratios used in reactor design, it appears that the effect of gap spacing on turbulent mixing (w') is small. This conclusion is supported by the later fundamental mixing studies of Renksizbulut and Hadaller.⁵⁵

Russian workers have preferred to correlate interchannel mixing in terms of the thermal diffusion coefficient, β , which they call the Stanton number, St_w , for mass transfer. Based on a number of studies, Zhukov *et al.*⁵⁶ recommend for single-phase flow

$$St_w = \left[1.0744 + \frac{0.1864}{(p/D) - 1} \right] (10^{-2}/Re^{0.1}) \quad (4.107)$$

for $1.25 \leq p/D \leq 1.35$ and $6.5 \times 10^4 \leq Re \leq 18 \times 10^4$ where p = rod pitch (center-to-center spacing) and St_w = Stanton number for turbulent mass exchange = $w' / s(V\rho) = \beta$. Observe that Eq. (4.107) predicts increased values of St as the pitch decreases. However, the decreased value of s means that there is little change in w' . It is unfortunate that the upper limit on Eq. (4.106) is at a Reynolds number below that attained under the usual PWR operating conditions.

All of the foregoing expressions for interchannel mixing, except for Eq. (4.103), represent data for bare rod bundles. All current fuel bundles in vessel-type PWRs are supported by grids and in most cases these grids contain mixing vanes. As previously indicated in Sec. 3.2.1.3, mixing vanes can increase mixing rates by a factor of 2 to 8. Mixing is highest immediately downstream of the grid and then falls off exponentially with distance from the grids. Very limited data are available on actual PWR bundles since most organizations consider these data to be proprietary. However, Raymond *et al.*⁵⁷ provide limited data on bundles typical of French PWRs. They correlate their data on the basis of the ratio of turbulent viscosity or conductivity to its molecular counterpart. For typical PWR bundles they model the single-phase heat transfer between channels i and j , q_{ij} , due to thermal conductivity and diffusion as

$$q_{ij} = (k_l + k_t) \frac{1}{c_{pl}} \frac{H_j - H_i}{L_{ij}} \quad (4.108)$$

where

k_l, k_t = liquid and turbulent thermal conductivities

c_{pl} = specific heat of liquid

H_j, H_i = fluid enthalpy in channels i and j , respectively

L_{ij} = distance between channels i and j .

The turbulent parameters are obtained from

$$\mu_t / \mu_l = k_t / k_l = K_t (Re - 3000)^b \quad (4.109)$$

where μ_l, μ_t = liquid and turbulent viscosities, respectively, and K_t, b = empirical constants. The effect of the grids on mixing is modeled by having K_t vary with distance from the nearest grid:

$$K_t = K_1 \exp[-K_2(z - z_g)] \quad (4.110)$$

where K_1, K_2 = empirical constants that vary with grid design, and z, z_g = axial location of interest and axial location of grid, respectively.

For a simple grid without mixing vanes, Raymond *et al.* indicate $K_1 = 0.015$ and $K_2 = 0.020$. For the typical French PWR arrangement with mixing vanes, $K_1 = 0.35$ and $K_2 = 0.10$. Over a 40-cm span between grids, the average value of K_t was 0.01 for the simple grid and 0.08 for the mixing

vane grid. Both of these values are considerably higher than for a rod bundle without any grids where $K_t = 0.003$.

Although the actual mixing varies significantly with distance when mixing vanes are present, calculations of fluid conditions just downstream of the support grids yield approximately the same result if Eq. (4.110) or an average value of K_t is used. Because limiting heat transfer conditions are generally encountered at this location, the complications introduced by equations of the form of Eq. (4.110) may not be warranted.

Since boiling occurs in the hotter channels of most PWRs, the effect of two-phase conditions on mixing must also be considered. Modeling two-phase mixing in most computer codes used for rod bundle analysis is based on a single fluid model. That mixing can be visualized as the exchange of two pockets of fluid between adjacent channels. In the equi-mass model, the packets are of equal mass and there is no net transfer of mass, only of heat and momentum. Enthalpy interchange due to mixing can be written as

$$w'(H_i - H_j) \quad (4.111)$$

where H_i , H_j = mean enthalpy of channels i and j , respectively, and w' = mass interchange between subchannels, lb/(hr ft).

In the equi-volume model, the packets are of equal volume. When densities in the two subchannels are different, the mixing process transfers mass as well as heat and momentum. Hence, cross-flow is not the only mechanism transferring mass and its definition and magnitude are changed accordingly.

Fundamental data on two-phase mixing under reactor conditions appear to be unavailable. The most nearly representative data are those of Rowe and Angle.⁵⁸ They used lithium, deuterium, and tritium tracers to measure steam-water mixing in the presence of boiling at 750 and 400 psia. Their data differ substantially from single-phase observations. For void fractions below ~65 to 70%, mixing was enhanced as the void fraction increased. Overall mixing enhancement was greatest at the lowest velocity and largest gap spacing. The maximum mixing observed was ~10 times that observed in single-phase operation. Results with and without spacers were similar.

Mixing coefficients for low-pressure air-water systems were measured by Rudzinski *et al.*⁵⁹ and Gonzalez-Santalo and Griffith,⁶⁰ among others. All these investigators saw trends similar to those observed by Rowe and Angle.⁵⁸

At high void fractions, all investigators report an inverse relationship between quality (or void fraction) and mixing rate. Rudzinski *et al.*⁵⁹ observed that their peak mixing, as well as Rowe and Angle's,⁵⁸ occurred prior to the slug-annular flow transition. Singh and St. Pierre's⁶¹ investigation mixing in the annular flow region confirms the inverse relationship with void fraction in that region.

Beus⁶² devised a correlation that predicts the general form of the observed two-phase mixing data. He assumes that in the dispersed or churn flow regimes, turbulent mixing is dominated by the churning motion of slugs and bubbles. In this regime, w'_l , the exchange flow per unit length, is given by

$$w'_l = w'_L + B_1 \left(\frac{AG}{De} \right) \left(\frac{\rho_l}{\rho_g} \right) \left(\frac{S-1}{S} \right) \chi \quad (4.112)$$

where

w'_L exchange flow per unit length for pure liquid

A = channel flow area

S = slip ratio

χ quality

ρ_l, ρ_g = liquid and vapor densities, respectively

B_1 = empirical constant.

Peak mixing, w'_c , is assumed to take place at the annular transition at quality χ_c , given by the transition relationship as

$$\chi_c = \frac{\frac{0.4}{G} [g \rho_l De (\rho_l - \rho_g)]^{1/2} + 0.6}{0.6 + (\rho_l / \rho_g)^{1/2}} \quad (4.113)$$

The peak mixing rate is determined by solving Eq. (4.112) when $\chi = \chi_c$.

The transition from peak mixing to mixing, w'_g , obtained in the presence of pure vapor, is given by an empirical equation that has the form

$$w'_{II} = w'_g + (w'_c - w'_g) \left(\frac{1-K}{\frac{\chi}{\chi_c} - K} \right) \quad (4.114)$$

where K is the empirical coefficient. Beus found that he could reasonably fit the available air-water and the moderate pressure steam-water mixing data by setting

$$B_1 = 0.04 \left(\frac{s}{De} \right)^{1.5} \quad (4.115)$$

and

$$K = 0.57 \text{Re}^{0.0417} \quad (4.116)$$

Despite the limited database for the correlation of Beus,⁶² it has been used in FIDAS subchannel analysis computer program.⁴⁶

Zhukov *et al.*⁵⁶ treat two-phase mixing effects in a manner very similar to that of Beus.⁶² However, in the FLICA program, Raymond *et al.*⁵⁷ calculate two-phase mixing by simply multiplying the single-phase value by the ratio of two-phase to single-phase wall friction.

In view of the uncertain effect of void fraction on mixing, most reactor computations use empirical mixing parameters. While mixing parameters can be calculated from the equations of Beus, it is not certain whether these equations properly account for pressure effects. Mixing coefficients are more generally estimated by fitting coefficients to data from rod bundle heat transfer tests.

It is usually assumed that a single mixing parameter can be used for vapor and liquid. This is reasonable only if the mixing quality (quality of w') equals subchannel quality. Values of mixing quality were reported by Singh and St. Pierre.⁶¹ For gap spacings of 40 mils and above, the mixing quality was close to the subchannel quality. At the lowest gap spacing, the mixing quality fell significantly below the subchannel quality. Since this effect occurs at a gap spacing considerably below that of reactor interest, the use of a single mixing coefficient for liquid and vapor does not appear unreasonable.

Under some conditions, it has not been possible to model the results of rod bundle tests satisfactorily using eddy diffusivity mixing alone. In some of their rod bundle tests, Lahey and Schraub⁶³ found that the quality in a particular corner channel was lower than that of the center channel, even when there was corner flux peaking. They proposed superposition of a void drift on eddy diffusivity mixing, with the drift proportional to the mass velocity gradient. By considering turbulent mixing to be on a volume-to-volume basis, they obtained m'' the net mass flux from channel i to j ,

$$m'' = \frac{e(\rho_i - \rho_j)}{l} + e_{VD} \frac{(G_i - G_j)}{l} \quad (4.117)$$

where

m'' = transverse mass flux due to mixing and void drift, lb/(hr ft²)

l = characteristic mixing length, ft

e = eddy diffusivity, ft²/hr

e_{VD} = void drift diffusivity coefficient, ft

ρ_i, ρ_j = average density in channels i and j , respectively

G_i, G_j = average mass velocities in channels i and j , respectively, lb/(hr ft²).

A somewhat similar point of view has been put forward by Gonzalez-Santalo and Griffith.⁶⁰

The formulation of Lahey and Schraub⁶³ and Gonzalez-Santalo and Griffith⁶⁰ are both approximate methods for dealing with annular flow. Annular

flow occurs at void fractions in excess of those normally encountered in PWRs. Consideration of void drift effects is usually not needed for PWR modeling at operating conditions. However, void drift needs to be considered under some accident conditions.

4.2.1.4 Transient Analysis Methods

The most common approach to analysis of reactor core behavior during a transient is based on the assumption that the time variation in reactor power and flow to the core is provided by a systems analysis program. The thermal analysis is further simplified if we assume that a single fluid model is adequate and sonic velocity propagation may be ignored. This approach is used by the COBRA codes²⁰ for examining core behavior during a transient. Rowe²⁰ writes the basic conservation equations for an axial segment (length Δz) of subchannel i connected to channel j as:

Continuity:

$$A_i(\partial \rho_i / \partial t) + (\partial m_i / \partial z) = -w_{ij} \quad (4.118)$$

Energy:

$$\begin{aligned} (1/V'_i)(\partial H_i / \partial t) + (\partial H_i / \partial z) &= (q'_i / m_i) - (H_i - H_j)(w'_{ij} / m_i) \\ &\quad - (T_i - T_j)(k_{ij} / m_i) + (H_i - H^*)(w_{ij} / m_i) \end{aligned} \quad (4.119)$$

Axial momentum:

$$\begin{aligned} (1/A_i)(\partial m_i / \partial t) - 2V_i(\partial \rho_i / \partial t) + (\partial P_i / \partial z) \\ = -(m_i / A_i)^2 \left[\frac{f_i \phi^2}{2\rho_i De} + \frac{A_i \partial(1/\rho_i A_i)}{\partial z} \right] - \rho_i g \cos \theta \\ - (f_i / A_i)(V_i - V_j)w'_{ij} + (1/A_i)(2V_i - V^*)w_{ij} \end{aligned} \quad (4.120)$$

Lateral momentum:

$$(\partial w_{ij} / \partial t) + \frac{\partial(V_{ij}^* w_{ij})}{\partial z} + (s/l)(C_{ij} w_{ij}^2) = (s/l)(P_i - P_j) \quad (4.121)$$

where

A_i = flow area of channel i

C_{ij} = loss coefficient for transverse flow

De = equivalent diameter

f_i = friction factor based on all liquid flow

- f_t = turbulent momentum factor
 g = gravitational constant
 H_i, H_j = enthalpy of fluid in subchannels i and j , respectively
 H^* = enthalpy carried by cross flow
 k_{ij} = thermal conductivity
 l = effective length over which lateral momentum transfer takes place (see Fig. 4.9)
 m_i = flow rate in channel i , mass/time
 P_i, P_j = pressure in channels i and j , respectively
 q'_i = heat addition per unit length in channel i
 s = gap between adjacent rods
 t = time
 T_i, T_j = temperature of fluids in channels i and j , respectively
 V'_i = effective enthalpy transport velocity in channel i , where $V'_i = V_i$ for homogeneous flow
 V_i, V_j = axial velocities in channels i and j , respectively
 V^* = effective axial velocity carried by cross-flow
 w_{ij} = net cross-flow between channels i and j (mass/time length)
 w'_{ij} = turbulent cross flow between channels i and j (accounts for mixing, no net transfer of mass)
 ϕ^2 = two-phase friction multiplier
 ρ_l, ρ = densities of liquid and mixture, respectively.

Note that Eq. (4.121) is based on the assumption that transverse momentum flux terms can be ignored.

Total flow to the core is assumed given. Since flow distribution between the channels is to be determined, axial and lateral momentum balances are required. The omission of a $\partial p / \partial t$ term from the momentum balance is consistent with the assumption that sonic velocity propagation effects are ignored. Therefore, there is no requirement that very small timesteps be used with an explicit solution procedure.

Equations (4.118) through (4.121) are appropriate for use only at high mass fluxes where the mixture tends to behave as a single fluid. Deviations from this behavior are most noticeable in horizontal rod bundles where vapor tends to accumulate in the upper channels once mass fluxes drop significantly below those of normal operation. A single fluid model allowing for relative vapor motion (modified drift flux model) is probably the simplest way of dealing with this problem. This approach was taken by Carver *et al.*⁶⁴ who used mass and energy balances similar to Eqs. (4.118) and (4.119) but revised the axial and lateral momentum balances for channel i connected to channel j as follows. For axial momentum conservation:

$$\begin{aligned} & \frac{\partial m}{\partial t} + \frac{\partial}{\partial z} \left(\frac{m^2}{\rho A_i} \right) + \frac{\partial}{\partial z} \left[A \frac{\rho_g \rho_l}{\rho} \alpha (1 - \alpha) V_r^2 \right] \\ & + V^* w_{ij} + \frac{s \rho_g \rho_l}{\rho^*} \alpha^* (1 - \alpha^*) V_r U_r + A_i \frac{\partial P}{\partial z} = - (K' \rho V^2 + g \rho \cos \theta) A_i \end{aligned} \quad (4.122)$$

and for lateral momentum conservation:

$$\begin{aligned} & \frac{\partial w_{ij}}{\partial t} + \frac{\partial (V w_{ij})}{\partial z} + \frac{\partial}{\partial z} \left[\frac{s \rho_g \rho_l}{\rho} \alpha (1 - \alpha) V_r U_r \right] - (s/l) (\Delta P)_l \\ & = - \left(\frac{s}{l} \right) \frac{K \rho U^2}{2} - s g \rho \cos \phi \end{aligned} \quad (4.123)$$

where the asterisk indicates donor channel values and

l = distance over which lateral momentum balance evaluated

s = effective lateral spacing for cross flow = average channel area/ l

$(\Delta P)_l$ = pressure difference between channels i and j

V = average axial velocity = G/ρ

V_r = axial relative velocity = $V_g - V_l$

V_g, V_l = axial vapor and liquid velocities, respectively, in channel i

K, K' frictional pressure loss coefficients

$U = w_{ij}/\rho s$

U_r lateral relative velocity = $U_g - U_l$

U_g, U_l = lateral velocity of vapor and liquid, respectively

ϕ = angle of gap connection from the upward vector (gravity is a downward vector at 180°)

ρ = average density of mixture = $\alpha \rho_g + (1 - \alpha) \rho_l$.

Other symbols have their previous meaning.

The relative velocities are determined from the drift flux parameters. For axial flow, only C_0 is considered significant and

$$\bar{V}_r = V / \left\{ (1 - \alpha) \left[1 - \frac{\rho_l - \rho_g}{\rho_{av}} \alpha (C_0 - 1) \right] \right\} \quad (4.124)$$

In the lateral direction, the distribution effect is considered irrelevant and $C_0 = 1$. The value of U_r is then determined by void drift (diffusion) and gravity.

$$U_r = V_{gj} + \frac{(\epsilon/l)'}{1 - \alpha} \left[\frac{\Delta(\alpha_0 - \alpha)}{\Delta x} + \frac{\Delta(\alpha_0 - \alpha)}{\Delta y} \right] \quad (4.125)$$

where V_{gj} = drift flux parameter $\approx$ bubble rise velocity; $(\epsilon/l)'$ = empirically determined ratio of eddy diffusivity to mixing length (length/time); and

α_0 = equilibrium void fraction. This approach was used in early versions of the ASSERT program⁶⁴ used for analysis of CANDU assemblies. Later versions of ASSERT used a modified two-fluid model.⁶⁵

A full two-fluid model is used by VIPRE-02⁴⁷ for reactor core and vessel analysis. This program solves mass, momentum, and energy equations for each phase. The basic equations used, which may be considered as an extension of those of COBRA, are:

Mass continuity for each phase:

$$A \frac{\partial}{\partial t} (\langle \alpha_i \rho_i \rangle) + \frac{\partial}{\partial z} (\alpha_i \rho_i V_i) + \sum_{k \in m} \epsilon_{mk} \langle \alpha_i \rho_i U_i \rangle s = A \Gamma_v \quad (4.126)$$

Energy equation for vapor phase:

$$A \frac{\partial}{\partial t} (\langle \alpha_v \rho_v H_v \rangle) + \frac{\partial}{\partial z} (\alpha_v \rho_v H_v V_v) A + \sum_{k \in m} \epsilon_{mk} (\alpha_v \rho_v H_v U_v) s \\ \sum_{n \in m} p_w \phi_{nm} C_{wv} \langle q'' \rangle - \sum_{k \in m} \overline{\frac{\alpha_v \rho_v}{\rho_m}} w' \Delta H_v + (\Gamma_v H_{fg} + q_{lv}) A \quad (4.127)$$

Energy equation for liquid phase:

$$A \frac{\partial}{\partial t} (\langle \alpha_l \rho_l H_l \rangle) + \frac{\partial}{\partial z} (\alpha_l \rho_l H_l V_l) A + \sum_{k \in m} \epsilon_{mk} \langle \alpha_l \rho_l H_l U_l \rangle s \\ \sum_{n \in m} p_w \phi_{ln} (1 - C_{wv}) \langle q'' \rangle + \sum_{n \in m} C_q \phi_{nm} q' \\ \sum_{k \in m} \overline{\frac{\alpha_l \rho_l}{\rho_m}} w' \Delta H_l - (\Gamma_v H_{fg} - q_{ll}) A \quad (4.128)$$

Axial momentum balance equation for each phase:

$$A \frac{\partial}{\partial t} (\langle \alpha_i \rho_i V_i \rangle) + \frac{\partial}{\partial z} (\alpha_i \rho_i V_i^2) A + \sum_{k \in m} \epsilon_{mk} \langle \alpha_i \rho_i U_i V_i \rangle s \\ = A \langle \alpha_i \rangle \frac{\partial}{\partial z} (P) - \frac{1}{2} \left(\frac{f_i}{De} + K'_i \right) \langle \alpha_i \rho_i V_i^2 \rangle A - \frac{1}{2} K'_i (\alpha \rho)_i (V_i - V'_i)^2 A + A \Gamma_i U_i^* \\ - C_T \sum_{k \in m} \overline{\frac{\alpha_i \rho_i}{\rho_m}} w' \Delta U_i - A \langle \alpha_i \rho_i \rangle g \cos \theta. \quad (4.129)$$

Lateral momentum balance equation for each phase:

$$s \frac{\partial}{\partial t} (\langle \alpha_i \rho_i U_i \rangle) + \frac{\partial}{\partial z} \langle \alpha_i \rho_i V_i U_i \rangle s = -\frac{1}{2} \frac{s}{l} K'_i \langle \alpha_i \rho_i U_i^2 \rangle - \frac{1}{2} \frac{s}{l} K'_i (\alpha \rho)_i (U_i - U'_i)^2 \\ + s \Gamma_i U_i + \frac{s}{l} \langle \alpha_i \rangle (\langle P \rangle_{ii} - \langle P \rangle_{jj}) , \quad (4.130)$$

where

f_i = wall friction factor

K'_i = form loss factor

K''_I = interfacial friction term

q_{li} = energy deposited per unit length in phase i from heated surfaces (energy/time length)

C_q = fraction of local heater rod power which is direct energy deposition in the coolant

C_{wv} = fraction of heater rod power going to production of vapor

q'' = average surface heat flux (energy/area time)

p_w = rod perimeter (area/length)

V = axial velocity (length/time)

U = lateral velocity (length/time)

w' = turbulent cross-flow between adjacent channels (mass/time length)

α_i = volume fraction of phase i

Γ_v = vapor generation rate at phase interface (mass/time)

ϕ_{nj} = fraction of perimeter of rod n facing channel j

ϵ_{jk} = sign determinator for lateral velocity in gap k with respect to channel j

Subscripts:

I = interface

i = phase index

= other phase ($i' = I$ if $i = v$)

l = liquid

v = vapor.

Undefined terms have their previous meaning.

Observe that the lateral momentum balance has the simplified form used in COBRA. Additional constitutive equations are specified for evaluating interfacial transfer (K'_I and Γ_v). The constitutive equations used depend on the flow pattern, which is assumed to be determined by the void fraction.

Two-fluid codes are limited at high void fractions where annular flow prevails. Under these conditions, the liquid phase is divided between a liquid film on the fuel rod surfaces and droplets in the central vapor core. The newer subchannel codes for BWR analyses (e.g., FIDAS code⁴⁶) use a three-field model: continuous liquid (liquid film), entrained liquid drops, and vapor. Since annular flow prevails through much of the reflood phase following a LOCA, the most sophisticated of the programs designed for

LOCA analysis use a two-fluid, three-field model. The COBRA-TRAC code⁶⁶ and its modifications, are examples of this approach. In COBRA-TRAC,⁶⁷ the multifluid, three-dimensional reactor core analysis of COBRA-TF is linked to the one-dimensional, two-fluid TRAC code which computes overall system behavior.

COBRA-TF⁶⁶ considers three fields: continuous liquid, entrained liquid, and continuous vapor. Models are included for interfacial mass and momentum transfer, wall drag, entrainment, and de-entrainment.

Mass and momentum conservation equations are written for each field but the energy equation is written only for the liquid and vapor. The vector forms of these equations are:

Conservation of mass:

Rate of change
of mass + Rate of mass
efflux = Rate of mass
transfer to phase

$$\frac{\partial}{\partial t}(\alpha_v \rho_v) + \nabla \cdot (\alpha_v \rho_v \mathbf{U}_v) = \Gamma''' \quad (4.131a)$$

$$\frac{\partial}{\partial t}(\alpha_l \rho_l) + \nabla \cdot (\alpha_l \rho_l \mathbf{U}_l) = -\Gamma_l''' - E''' \quad (4.131b)$$

$$\frac{\partial}{\partial t}(\alpha_e \rho_l) + \nabla \cdot (\alpha_e \rho_l \mathbf{U}_e) = -\Gamma_e''' + E''' \quad (4.131c)$$

Conservation of momentum:

Rate of change of momentum + Rate of momentum efflux = Pressure gradient force + Gravity force + Viscous turbulent force + Interfacial drag + Momentum exchange due to mass transfer

$$\begin{aligned} \frac{\partial}{\partial t}(\alpha_v \rho_v \mathbf{U}_v) + \nabla \cdot (\alpha_v \rho_v \mathbf{U}_v \mathbf{U}_v) &= -\alpha_v \nabla P + \alpha_v \rho_v \mathbf{g} \\ &+ \nabla \cdot [\alpha_v (\boldsymbol{\sigma}_v + \mathbf{T}_v^T)] + \boldsymbol{\tau}_{wv}''' - \boldsymbol{\tau}_{vl}''' - \boldsymbol{\tau}_{lvl} + (\Gamma''' \mathbf{U}) \end{aligned} \quad (4.132a)$$

$$\begin{aligned} \frac{\partial}{\partial t}(\alpha_l \rho_l \mathbf{U}_l) + \nabla \cdot (\alpha_l \rho_l \mathbf{U}_l \mathbf{U}_l) &= -\alpha_l \nabla P + \alpha_e \rho_l \mathbf{g} \\ &+ \nabla \cdot [\alpha_e (\boldsymbol{\sigma}_l + \mathbf{T}_l^T)] + \boldsymbol{\tau}_{wl}''' - \boldsymbol{\tau}_{lwl} - (\Gamma_l''' \mathbf{U}) - (E''' \mathbf{U}) \end{aligned} \quad (4.132b)$$

$$\begin{aligned} \frac{\partial}{\partial t}(\alpha_e \rho_l \mathbf{U}_e) + \nabla \cdot (\alpha_e \rho_l \mathbf{U}_e \mathbf{U}_e) &= -\alpha_e \nabla P + \alpha_e \rho_l \mathbf{g} + \boldsymbol{\tau}_{we}''' + \boldsymbol{\tau}_{lve}''' - (\Gamma_e''' \mathbf{U}) + (E''' \mathbf{U}) \end{aligned} \quad (4.132c)$$

Conservation of energy:

Rate of change of enthalpy + Rate of enthalpy efflux = Conduction & turbulent heat flux + Energy exchange due to mass transfer + Interfacial heat transfer + Heat transfer from wall + Pressure derivative

$$\frac{\partial}{\partial t} (\alpha_v \rho_v H_v) + \nabla (\alpha \rho_v H_v \mathbf{U}_v) = - \nabla [\alpha_v \mathbf{q}_v + \mathbf{q}_v^T] + \Gamma''' H_g + q_{lv}''' + Q_{wv}''' + \alpha_v \frac{\partial P}{\partial t} \quad (4.133a)$$

$$\begin{aligned} & \frac{\partial}{\partial t} [(\alpha_l + \alpha_e) \rho_l H_l] + \nabla (\alpha_l \rho_l H_l \mathbf{U}_l) + \nabla (\alpha_e \rho_l H_e \mathbf{U}_e) \\ & = - \nabla [\alpha_l (\mathbf{q}_l + \mathbf{q}_l^T) - \Gamma''' H_f + q_{ll}''' + Q_{wl}''' + (\alpha_l + \alpha_e) \frac{\partial P}{\partial t}] \quad (4.133b) \end{aligned}$$

where

$\alpha_e, \alpha_l, \alpha_v$ = volume fractions of entrained liquid, continuous liquid, and vapor, respectively

Γ''' = average rate of vapor generation per unit volume

Γ_e''', Γ_l''' = rate of vapor generation per unit volume in entrained and continuous liquid, respectively

$\tau_{lv}''', \tau_{le}'''$ = drag force per unit volume between vapor and continuous liquid and entrained liquid, respectively

$\tau_{wv}''', \tau_{wl}'''$ = drag force per unit volume between wall and vapor and liquid, respectively

σ_v, σ_l = fluid-fluid stress for vapor and liquid, respectively

$\mathbf{T}_i^T$ = turbulent (Reynolds) stress tensor for phase i

$\mathbf{U}$ = average velocity

$\mathbf{U}_e, \mathbf{U}_l, \mathbf{U}_v$ = velocity of entrained liquid, continuous liquid, and vapor, respectively

E''' = net rate of entrainment per unit volume

$\mathbf{g}$ = acceleration of gravity

$\mathbf{q}_l, \mathbf{q}_v$ = liquid-to-liquid and vapor-to-vapor conduction heat transfer per unit volume, respectively

q_{ll}''', q_{lv}''' = interface heat transfer per unit volume to liquid and vapor, respectively

Q_{wl}''', Q_{wv}''' = conduction heat transfer from wall to liquid and vapor, respectively

H_e, H_l, H_v = enthalpy of entrained liquid, continuous liquid, and vapor, respectively.

Note that entrained and continuous liquid are included in the same energy equation.

Each of the foregoing vector momentum equations represents three component directions if Cartesian coordinates are used. Some simplification is possible if the subchannel approach is taken. For a subchannel analysis, only the vertical and transverse directions must be considered. One lateral equation is applied to all gaps regardless of direction. We then require only two momentum equations for each component equation. The component form of the vapor momentum equation is illustrative of the final equations used by the program

Vertical momentum:

$$\begin{aligned} \frac{\partial}{\partial t}(\alpha_v \rho_v u_v A) + \frac{\partial}{\partial z}(\alpha_v \rho_v v_v v_v A) + \sum_k (\alpha_v \rho_v v_v u_v s'_k) = -\alpha_v A \frac{\partial P}{\partial z} - \alpha_v \rho_v A g \\ + \tau_{wv}''' A - \tau_{lvz}''' A + (\Gamma''' U)_z A \end{aligned} \quad (4.134)$$

Lateral momentum:

$$\begin{aligned} \frac{\partial}{\partial t}(\alpha_v \rho_v A_k) + \frac{\partial}{\partial L}(\alpha_v \rho_v v_v u_v A_k) + \frac{\partial}{\partial z}(\alpha_v \rho_v u_v v_v A) + \sum_{nk} (\alpha_v \rho_v v_v v_{vk} s'_{nk}) \\ = -\alpha_v A_k \frac{\partial P}{\partial L} + \tau_{wvk}''' A_k - \tau_{lvk}''' A_k - \tau_{lve_k}''' A_k + (\Gamma''' v)_k A_k \end{aligned} \quad (4.135)$$

where

u, v = vertical and lateral velocities

A = vertical flow areas of subchannel

A_k = lateral area of gap

s' = orthogonal gap width

L = lateral coordinate

z = axial coordinate.

4.2.1.5 Subchannel Analysis Programs

All of the subchannel analysis models are sufficiently complex so that actual calculations must be via a computer code using finite difference approximations of the basic differential equations. The earliest codes, still used for analysis at operating or near operating conditions, are all based on single fluid models. The earlier of these computer codes, such as THINC II,^{34,38} COBRA II,⁶⁸ and HAMBO⁶⁹ use "marching procedures." If conditions at the inlet of a given axial segment are known, the governing equations can be solved iteratively to give the conditions at the exit of the segment. By proceeding stepwise along the channel length, a marching solution for the whole channel is obtained.

The iterative-interval calculation methods in COBRA-II and HAMBO are similar. A set of cross-flows between subchannels is "guessed." With this

set, the energy equation is solved by forward differencing in COBRA-II and central differencing in HAMBO. From the momentum equation, pressure drop in each subchannel is computed. From this calculation, a new set of cross-flow guesses that give a pressure balance is obtained by backward differencing. The iteration is continued until an acceptable pressure balance is obtained.

The boundary conditions to be satisfied are that the lateral pressure difference between subchannels should be zero at channel inlet and exit. Having passed once along the channel, this implies that iteration over the channel length may be necessary by using improved guesses of flow division between subchannels at the inlet. In practice, only one pass may be necessary, particularly for a hydraulic model in which lateral momentum transfer is neglected or only notionally included.

Rowe⁷⁰ has shown that for a single-pass solution to be stable and acceptable, the calculational increment of length must be greater than a critical value

$$\frac{2C|w|}{g(m,H)} \quad (4.136)$$

where

C = cross-flow resistance coefficient

w = cross flow

$g(m,H)$ = difference in axial pressure gradient caused by the cross-flow

m = subchannel flow rate

H = enthalpy.

For long enough increments, calculated exit conditions for an interval tend to compensate for errors in the assumed inlet conditions. This provides a self-correcting mechanism in the calculation and, conversely, means that large changes in assumed channel inlet conditions are required to affect calculated conditions and pressure balance at the channel exit.

The acceptability of a single-pass marching solution depends on coupling between subchannels. If this is weak (e.g., if the cross-flows are small), a single-pass marching solution technique is adequate; this follows from Rowe's criterion above. Upstream effects would be entirely confined to the preceding interval. From Rowe's criterion, stronger coupling (i.e., large cross-flows) could mean the intervals need to be impracticably large.

A multipass marching solution is used in COBRA IIIC.²⁰ The inlet flow division between subchannels is fixed as a boundary condition, and an iterated solution is obtained to satisfy the other boundary solution of zero pressure differential at channel exit. The procedure is to guess a pattern of subchannel boundary pressure differentials for all mesh points simultaneously and from this to compute, without further iteration, the correspond-

ing pattern of cross-flows using a marching technique up the channel. The pressure differentials are updated during each pass, and the overall channel iteration is completed when the fractional change in subchannel flows is less than a preset amount.

Pressure differentials at the exit of the length steps are used to calculate cross flows. Cross-flows are then used to calculate pressure differentials at the outlet of the previous (upstream) length steps. These pressure differentials are saved for use during the next iteration. At the exit of the last length step, the boundary condition of zero pressure differential is imposed and cross flows at the exit are calculated on this basis. The iterative procedure forces agreement with the assumed boundary condition.

The TORC code³⁶ is a modified version of COBRA IIIC. The basic numerics are those of COBRA IIIC, but TORC contains some additional features useful in overall core design [e.g., revised energy balance of Eq. (4.94)]. The Lynx program⁷¹ is another code generally based on COBRA.

Use of a marching solution to determine the behavior of individual subchannels in an assembly requires that the inlet flow to that assembly be known. The assumption that all assemblies in a core have the same inlet flow can be appreciably in error. Flow must be divided so that the core pressure drop remains essentially constant. Therefore, higher pressure loss coefficients in high power assemblies, due to the presence of significant exit quality, will lead to lower flows in these assemblies.

The THINC I code⁷² was the first calculational technique capable of satisfactorily assigning inlet flows to the assemblies within a semi-open core. In the THINC I approach, the inlet flows to each channel are successively perturbed by a small amount and the resulting changes in channel outlet pressures determined. By assuming a linear relationship between inlet velocity changes and outlet pressure changes, it is possible to determine a set of inlet velocities that should yield a uniform outlet pressure subject to a constant total flow. However, since the actual relationship is nonlinear, the newly assigned velocities will generally not provide a uniform outlet pressure. The process is then repeated until convergence is obtained. Unfortunately, convergence difficulties were often encountered and this approach is no longer used.

The convergence difficulty encountered when a marching solution is used to examine full core behavior is one of the motivating factors that led to the development of other solution procedures. An additional motivation was the desire to be able to handle recirculating flows that could develop under severe blockage conditions. Marching solutions cannot handle reverse flows.

Reverse flows can be successfully treated by numerical procedures that solve the conservation equations for the control volumes at all axial levels simultaneously. This approach is followed in THINC IV code.⁷³ Here, lateral velocity components are regarded as perturbed quantities much

smaller than axial flow velocity. The original governing equations are split into a perturbed and unperturbed system of equations. Perturbed momentum and continuity equations are then combined to form a field equation that is solved for the entire velocity field. Inlet velocities are determined such that a uniform outlet pressure is obtained. The initial solution obtained is used to update the properties and conditions assumed to exist in various control volumes. The iteration continues until assumed and calculated properties are in satisfactory agreement. The THINC IV code^{73,74} is written so it can also be used for determining behavior in a core region where the subchannels are defined by four neighboring fuel rods.

The COBRA IV code³⁸ is another program that can treat the core-wide problem successfully. As in COBRA IIIC,²⁰ a multiple-pass marching solution is used. However, inlet flows are not set as a boundary condition, and outlet pressure differentials are not set at zero when exit cross flows are computed. Instead, the requirement that outlet pressure differentials be zero is met by adjusting the inlet flows. That is, if flow $(w_i)_0$ to channel i leads to pressure drop ΔP_i across channel i , flow to channel i in the next iteration, $(w_i)_1$, is given by

$$(w_i)_1 = \left[1 - \frac{(\Delta P_s - \Delta P_i)}{2(\Delta P_i - \bar{h})} \right] (w_i)_0 \quad (4.137)$$

where P_s = specified pressure drop across core and $\bar{h}$ = average gravitational head of all channels.

Castellana and Bonilla⁷⁴ compared rod bundle pressure drop data with pressure drop calculations obtained from subchannel modeling codes. They conclude that estimates from subchannel codes are superior to those from mixed flow models, particularly in tightly spaced bundles at low exit qualities. Weisman and Bowring⁷⁵ provide a comprehensive review of modeling procedures and computational techniques for single fluid models.

In a revised version of COBRA IV, Stewart and Rowe⁷⁶ extended the ICE technique developed by Harlow and Amsden.⁷⁷ Stewart and Rowe⁷⁶ use an iterative procedure for solving the conservation equations in which error in the energy equation is used to give appropriately updated values of density, mass flow, and pressure. The scheme was successfully incorporated in a version of COBRA IV used to study a PWR core during reflooding oscillations. The approach is said to avoid the computational difficulties that arise for liquid-vapor mixtures with large density ratios. Further computational advances can be expected to lead to increased accuracy and flexibility in the types of problems handled, as well as decreased computation time.

Multifluid models generally require complex solution schemes. In the VIPRE-02 program,⁴⁷ the solution proceeds by solving the phasic energy equations first, and then the axial and lateral momentum equations. This is followed by an inner iteration where the phasic continuity equations are

solved. The program provides for the direct solution by Gaussian elimination or an iterative procedure. The direct solution is fast but it requires a large solution matrix and this may become impractical for some large problems.

New pressure and volume fractions are then computed. These are used to test for convergence of the continuity equations. If convergence is not attained, another iteration is performed. Figure 4.10 shows a flowchart for this VIPRE solution procedure.

Table 4.II lists some of the major subchannel analysis programs that have been written for PWR core analysis as of 1993. The program developers and key characteristics are also shown in this tabulation.

Under single-phase flow conditions, it is possible to determine the detailed temperature and velocity distributions within a subchannel using the boundary fitted coordinate transformation method.⁷⁸ In this approach, the complicated geometry within a small segment of a rod bundle is transformed into a rectangular coordinate mesh system. The governing equations are transformed into the new variables in the transformed space before being approximated by finite difference equations. Since the complications introduced by two-phase flow in the hot channels have prevented application of this approach to PWR design (as of 1994), programs utilizing this method are not shown in Table 4.II. Future developments may make such detailed examination of PWR subchannels feasible.

4.2.2 Porous Media Approach

4.2.2.1 Basic Model

The subchannel analysis approach is designed for the examination of behavior in rod bundles where the primary flow is in the axial direction. Since axial flow dominates, the transverse momentum equation is simplified, thus allowing more rapid computations. This simplification is not justified in the case of flow blockages in the bundle or in the analysis of the shell side of a steam generator where there is appreciable transverse flow in some regions. Similarly, the flow in the upper and lower plena of a reactor vessel would not be properly handled with a subchannel analysis approach. All of the foregoing situations, as well as those with recirculating flow, can be satisfactorily examined using the porous media approach.

In the porous media approach, the fluid is treated as a quasicontinuum and complete momentum balances are written for each of the three Cartesian directions (or in some cases for the θ and z directions). The presence of rods, or other obstructions to flow, are considered by replacing the real system by an ideal system in which the solid objects are uniformly distributed. To accomplish this, four qualities are defined:⁸⁰

γ_v = volume porosity, the ratio of volume occupied by fluid to the total computational cell volume

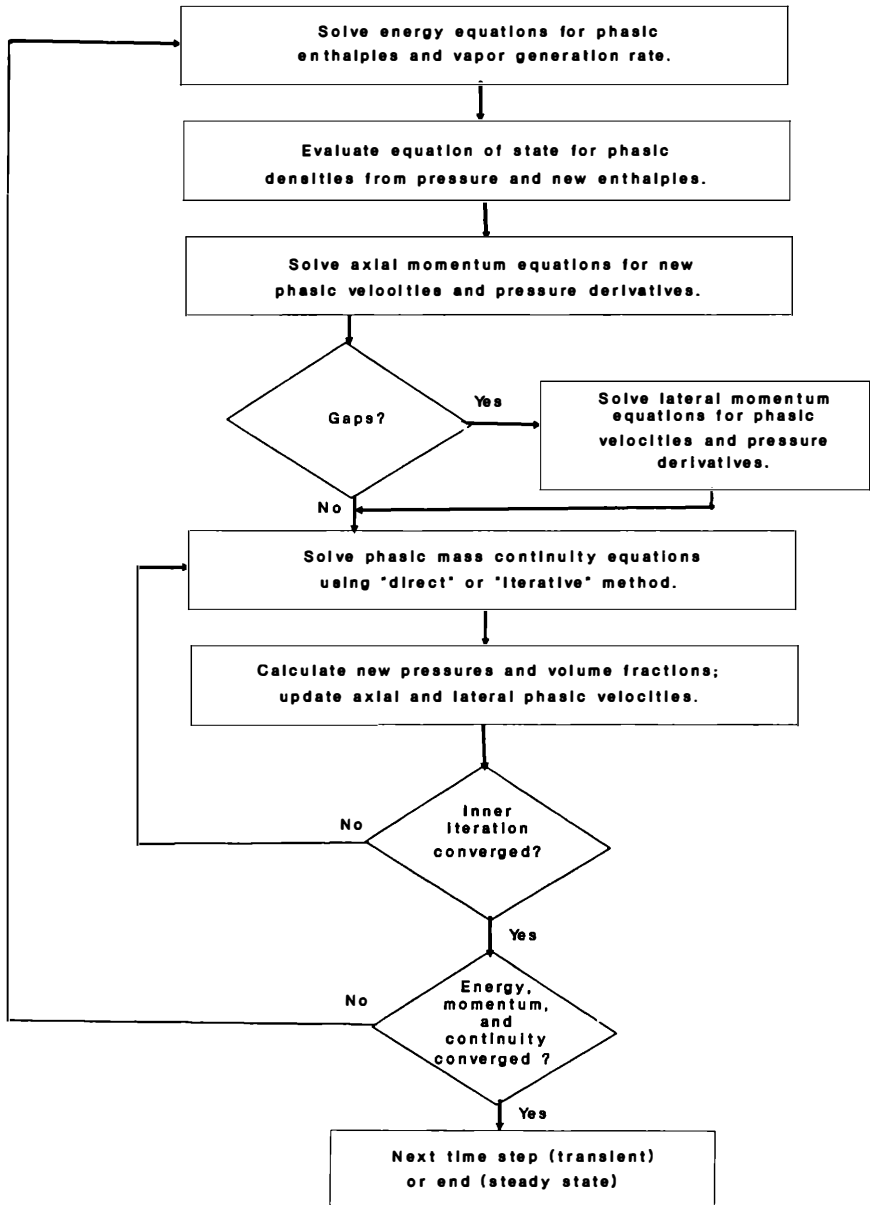


Fig. 4.10 Flowchart for two-fluid solution of VIPRE-02 [From *Nucl. Technol.*, **100**, 255 (1992)].

TABLE 4.II
Major Subchannel Analysis Programs for PWR's

Code Name	Reference	Developers	Geometry	Model
ASSERT	65	Atomic Energy of Canada Ltd. (AECL)	Horizontal rod bundles	Single fluid or drift flux
COBRA	20, 38, 68	Pacific Northwest Lab.	Vertical rod bundles	Single fluid
COBRA-TRAC	66 67	Pacific Northwest Lab. and Westinghouse	Vertical rod bundles	Two-fluid (three fields)
FLICA	57	Commissariat a l'Energie Atomique (France)	Vertical rod bundles	Single fluid
HAMBO	69	UK Atomic Energy Authorities, Winfrith	Vertical rod bundles	Single fluid
LYNX		Babcock & Wilcox	Vertical rod bundles	Single fluid
THINC	24, 74	Westinghouse	Vertical rod bundles	Single fluid
TORC	36	Combustion Engineering	Vertical rod bundles	Single fluid
VIPRE	47	Battelle Northwest and Electric Power Research Institute	Vertical rod bundles	Single fluid or two fluid
THERMIT	79	Massachusetts Institute of Technology (MIT)	Vertical rod bundles	Two-fluid

γ_x = surface permeability, the ratio of flow area to total area of the computational cell in the x direction

γ_y = surface permeability, the ratio of the flow area to total area of the computational cell in the y direction

γ_z = surface permeability, the ratio of flow area to total area of the computational cell in the z direction.

With the foregoing definitions, the single-fluid model conservation equations for a control volume having dimensions Δx , Δy , Δz (see Fig. 4.11) are written as:⁸⁰

Continuity:

$$\frac{\partial}{\partial t}(\rho\gamma_v) + \frac{\Delta(\rho u\gamma_x)}{\Delta x} + \frac{\Delta(\rho v\gamma_y)}{\Delta y} + \frac{\Delta(\rho w\gamma_z)}{\Delta z} = 0 \quad (4.138)$$

Momentum equation for x direction:

$$\begin{aligned} \frac{\partial}{\partial t}(\rho\gamma_v u) + \frac{\Delta(\rho\gamma_x u^2)}{\Delta x} + \frac{\Delta(\rho\gamma_y uv)}{\Delta y} + \frac{\Delta(\rho\gamma_z uw)}{\Delta z} = \frac{\Delta(P)}{\Delta x} \gamma_v \\ + \rho\gamma_v g_x - R_x + \frac{\Delta(\tau_{xx}\gamma_x)}{\Delta x} + \frac{\Delta(\tau_{xy}\gamma_y)}{\Delta y} + \frac{\Delta(\tau_{xz}\gamma_z)}{\Delta z} \end{aligned} \quad (4.139)$$

(Note: Momentum equations for y and z directions are analogous.)

Energy equation:

$$\begin{aligned} \gamma_v \frac{\partial(\rho H')}{\partial t} + \left[\frac{\Delta(\rho u \gamma_x H')}{\Delta x} + \frac{\Delta(\rho v \gamma_y H')}{\Delta y} + \frac{\Delta(\rho w \gamma_z H')}{\Delta z} \right] = \gamma_v \frac{\partial}{\partial t} (P) \\ + \frac{\Delta[\gamma_x (\tau_{xx} u + \tau_{xy} v + \tau_{xz} w)]}{\Delta x} + \frac{\Delta[\gamma_y (\tau_{xy} u + \tau_{yy} v + \tau_{yz} w)]}{\Delta y} \\ + \frac{\Delta[\gamma_z (\tau_{xz} u + \tau_{yz} v + \tau_{zz} w)]}{\Delta z} + [\rho\gamma_v (g_x u + g_y v + g_z w)] \\ + \left[\frac{\Delta\left(\gamma_x \eta \frac{\partial T}{\partial x}\right)}{\Delta x} + \frac{\Delta\left(\gamma_y \eta \frac{\partial T}{\partial y}\right)}{\Delta y} + \frac{\Delta\left(\gamma_z \eta \frac{\partial T}{\partial z}\right)}{\Delta z} \right] \\ + \dot{Q}_s + \gamma_v \dot{q} - [R_x u + R_y v + R_z w] \end{aligned} \quad (4.140)$$

where

H' = sum of kinetic energy and fluid enthalpy (energy/mass)

u, v, w = velocities in x, y , and z directions, respectively

ρ = fluid density (mass/time)

g_x, g_y, g_z = gravitational acceleration in x, y , and z directions, respectively

P = pressure (force/area)

R_x, R_y, R_z = frictional resistances

$\dot{Q}_s$ = volumetric heat source within fluid in control volume (energy/volume time)

$\dot{q}$ = volumetric heat source within *solids* located within control volume (energy/volume time)

η = combined molecular and turbulent thermal conductivity of fluid

$\tau_{xx}, \tau_{xy}, \tau_{xz}$ = viscous stresses acting on the surfaces of control volume (force/area).

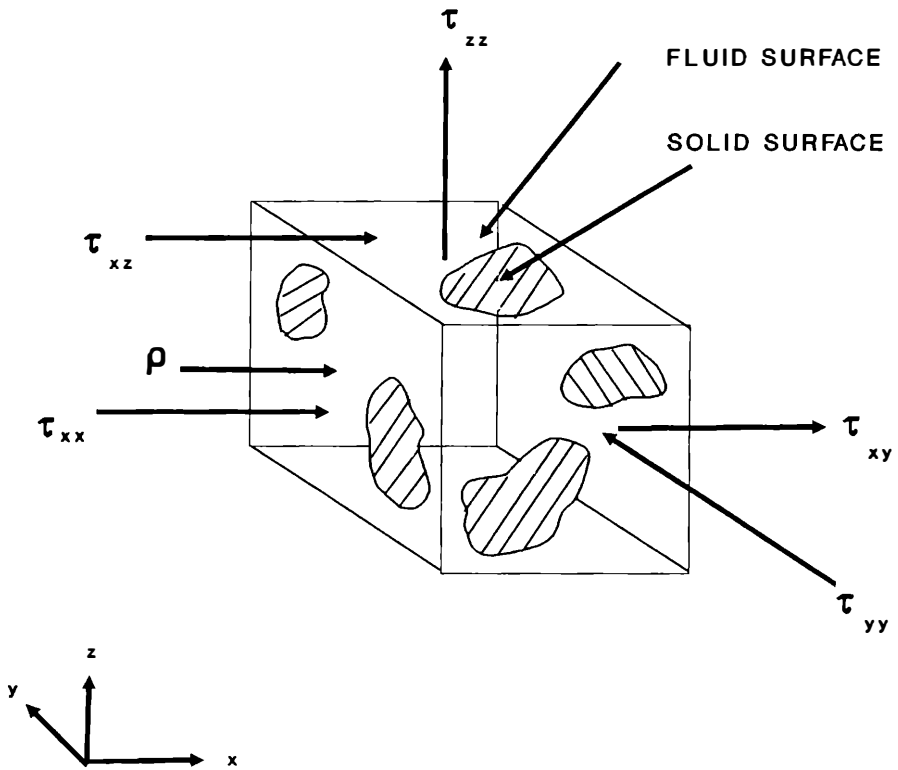


Fig. 4.11 Surface forces on control volume in quasicontinuum [from Ref. 79].

Numerical solution of the finite difference versions of the foregoing equations for large-size systems poses a number of difficulties. COMMIX-IB⁸¹ uses the so-called "mass rebalancing scheme" where the solution procedure focuses on the determination of the unknown pressure in the momentum equations. This pressure is determined from the continuity equation after the energy equation has been used to calculate temperature and density and the explicit terms of the momentum equation determined. Since an iterative procedure is used for solution, the continuity equations [Eq. (4.137)] for each control volume will not be equal to zero but each will equal some residual, $D'_{i,j,k}$. The value of $\partial D'_{i,j,k} / \partial P$ is determined for each control volume and then each pressure is readjusted by an amount $\Delta P_{i,j,k}$ where

$$\Delta P_{i,j,k} = \frac{C' D'_{i,j,k}}{(\partial D' / \partial P)_{\max}}, \quad (4.141)$$

where c' is the relaxation factor and

$$\left(\frac{\partial D'}{\partial P}\right)_{\max} = \text{maximum value of } \left(\frac{\partial D'_{i,j,k}}{\partial P}\right)$$

The iteration then continues by returning to the momentum and energy equations and then computing new residuals from the continuity equation.

To reduce the number of iterations required to obtain convergence, a coarse mesh domain is created by combining a number of finite difference cells. For each region, a single pressure correction equation is determined by summing the individual pressure equations for the component cells. After convergence is obtained for the coarse mesh, the program returns to the individual cells and continues the pressure iterations until individual cell convergence is obtained. The initial use of the coarse mesh reduces the total number of iterations needed.

4.2.2.2 Extension to Two-Phase Flow

A single-phase model can be useful in examining the flow in the lower plenum of a reactor vessel but it clearly will not be useful for the secondary side of a steam generator. The simplest two-phase model that is appropriate assumes thermodynamic equilibrium and treats the mixture as a single fluid. However, provision is allowed for slip between the phases. With these simplifying assumptions, the conservation equations [Eq. (4.138) to (4.140)] are modified slightly by replacing ρ by ρ_m and H' by H_m where

$$\bar{\rho} = \alpha \rho_g + (1 - \alpha) \rho_l \quad (4.142)$$

$$\bar{H} = x H'_g + (1 - x) H'_l$$

In addition, the following is added to the left-hand side of the momentum equation:

$$\frac{\Delta[x(1-x)\bar{\rho} V_R^2 \gamma_x]}{\Delta x} + \frac{\Delta[x(1-x)\bar{\rho} V_R^2 \gamma_y]}{\Delta y} + \frac{\Delta[x(1-x)\bar{\rho} V_R^2 \gamma_z]}{\Delta z} \quad (4.143)$$

where $V_R = \text{relative velocity} = (\text{velocity of vapor}) - (\text{velocity of liquid})$. No additional terms are added to the continuity equation. The value of the fluid enthalpy is used to obtain the quality, which, together with V_R , is used to determine the void fraction, α . The energy equation is also revised to reflect the relative velocity contribution and to replace the temperature gradients in terms of enthalpy gradients. We then have

$$\gamma_v \frac{\partial}{\partial t} (\bar{\rho} \bar{H}) + \left[\frac{\Delta(\bar{\rho} u \gamma_x \bar{H})}{\Delta x} + \frac{\Delta(\bar{\rho} v \gamma_y \bar{H})}{\Delta y} + \frac{\Delta(\bar{\rho} w \gamma_z \bar{H})}{\Delta z} \right] =$$

$$\begin{aligned}
& \gamma_v \frac{\partial}{\partial t} (P) + \frac{\Delta[\gamma_x(\tau_{xx}u + \tau_{xy}v + \tau_{xz}w)]}{\Delta x} \\
& + \frac{\Delta[\gamma_y(\tau_{xy}u + \tau_{yy}v + \tau_{yz}w)]}{\Delta y} + \frac{\Delta[\gamma_z(\tau_{xz}u + \tau_{yz}v + \tau_{zz}w)]}{\Delta z} \\
& + [\bar{\rho}\gamma_v(g_xu + g_yv + g_zw)] \\
& + \left\{ \frac{\Delta\left[\gamma_x\bar{\rho}\bar{e}(l/D)\frac{\partial\bar{H}}{\partial x}\right]}{\Delta x} + \frac{\Delta\left[\gamma_y\bar{\rho}\bar{e}(l/D)\frac{\partial\bar{H}}{\partial y}\right]}{\Delta y} + \frac{\Delta\left[\gamma_z\bar{\rho}\bar{e}(l/D)\frac{\partial\bar{H}}{\partial z}\right]}{\Delta z} \right\} \\
& - [R_xu + R_yv + R_zw] + \dot{Q}_s + \gamma_v\dot{q}
\end{aligned} \tag{4.144}$$

where

$\bar{e}$ = eddy diffusivity (area/time)

l = Prandtl mixing length

D = appropriate length scale (length)

s = effective distance for interchannel mixing.

No additional terms are added to the continuity equation. The value of the fluid enthalpy is used to obtain the quality which, together with $V_{R'}$, is used to determine the void fraction.

The COMMIX solution scheme⁸² for a two-phase system is somewhat similar to that used for single-phase fluid. An equation describing the pressure distribution is obtained by combining the energy equation with the three momentum equations. An iterative solution scheme is used to determine the void fraction and pressure distributions that will simultaneously satisfy the pressure and continuity equations.

Porous media programs based on a two-fluid model have also been devised.⁸² A full two-fluid model requires two continuity equations, two energy equations, and six momentum equations. In addition to the numerical complexity of such a model, a significant problem is the difficulty of appropriately describing the viscous dissipation terms under two-fluid conditions. However, a two-fluid model does allow the relative velocity of the phases to be determined from interfacial drag rather than a steady-state slip correlation.

4.2.2.3 Modeling of Turbulent Interaction

Both viscous dissipation and heat transfer due to turbulence are incorporated in the conservation equations used by the porous media approach. The viscous dissipation effects are represented by the terms containing the

viscous stresses, τ , while the heat transfer due to turbulence is represented by the terms containing the product of η and the temperature gradient.

In a rod bundle geometry, the viscous shear may be related to w' the turbulent interchange flow per unit length, which was previously described in the subchannel analysis discussion. We may write

$$\tau_{ij} a_{ij} = w'_{ij} (v_i - v_j) \quad (4.145)$$

where

a_{ij} = flow area per unit length between control volume i and j

w'_{ij} = turbulent exchange flow per unit length (mass/length time)

v_j = fluid velocity parallel to surface i, j in control volume i and j , respectively.

Similarly, we may use Eq. (3.50) to replace $\rho \bar{e}(l/D)$ by w' in the energy equation used with two-phase flow. The correlations for w' indicated as applicable in subchannel analysis may be applied here.

For analysis of the flow behavior in a reactor vessel plenum, the approach used for rod bundles is not applicable. It is then necessary to express τ in terms of $\bar{e}$, the eddy diffusivity,

$$\tau_{ij} = \frac{\rho \bar{e}(l/D)}{a_{ij}} (v_i - v_j) \quad (4.146)$$

The simplest, and very approximate, way of estimating $\bar{e}$ is via the correlation used in COMMIX-1B,⁸¹ which suggests

$$\bar{e} = 0.007 c' V_{\max} (0.4 D e) \quad (4.147)$$

where

$$= 0.1 \text{ for } \text{Re}_{\max} > 2000$$

$$V_{\max} = \max(u, v, w)$$

$$\text{Re}_{\max} = \max(\text{Re}_x, \text{Re}_y, \text{Re}_z).$$

When Eq. (4.140) is used, it is suggested⁸¹ that the turbulent thermal conductivity, η , be obtained from

$$\eta = (c_p \bar{e} \rho / 0.8) [1 - \exp(-6 \times 10^{-5} \text{Re} P_r^{1/3})] \quad (4.148)$$

Note that Eq. (4.148) is a gross simplification. Sophisticated turbulence modeling usually uses the so-called " $K-\epsilon$ " approach. The parameter K represents the turbulent kinetic energy and is defined by

$$K = \frac{1}{2} (\overline{u'^2 + v'^2 + w'^2}) , \quad (4.149)$$

where u', v', w' = the fluctuating turbulent velocities in the x , y , and z directions, respectively. The rate of dissipation of turbulent kinetic energy, ϵ , is defined by

$$\epsilon = \nu \overline{\frac{\partial u'_i}{\partial x_j} \frac{\partial u'_i}{\partial x_j}} \quad (4.150)$$

where ν = kinematic viscosity = μ/ρ , and the repeated indices (i, j) imply a sum over all the Cartesian directions.

Once the values of K and ϵ are computed, the eddy diffusivity is then obtained from

$$\bar{e} = 0.09 K^2 / \epsilon \quad (4.151)$$

Evaluation of K and ϵ is complicated but may be accomplished when values for $\partial u_i / \partial x_j$ are available. The simplest treatment for a two-phase mixture considers the mixture to be a pseudo-single-phase flow, which behaves as a homogeneous mixture. For flow in a round tube where the axial velocity of the mixture is $\bar{V}$ and the radial velocity in $\bar{U}$, Ferng *et al.*⁸³ suggest that turbulent kinetic energy be obtained from

$$\begin{aligned} \frac{\partial}{\partial t}(\bar{\rho} K) + \frac{\partial}{\partial z}(\bar{\rho} \bar{V} K) + \frac{1}{r} \frac{\partial}{\partial r}(r \bar{\rho} \bar{U} K) &= \frac{\partial}{\partial z} \left(a_1 \frac{\partial K}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(r a_1 \frac{\partial K}{\partial r} \right) \\ &+ \mu_t \left\{ 2 \left[\left(\frac{\partial \bar{V}}{\partial z} \right)^2 + \left(\frac{\partial \bar{U}}{\partial r} \right)^2 + \left(\frac{\bar{U}}{r} \right)^2 \right] + \left(\frac{\partial \bar{V}}{\partial r} + \frac{\partial \bar{U}}{\partial z} \right)^2 \right\} - \bar{\rho} \epsilon \end{aligned} \quad (4.152a)$$

and turbulent energy dissipation from

$$\begin{aligned} \frac{\partial}{\partial t}(\bar{\rho} \epsilon) + \frac{\partial}{\partial z}(\bar{\rho} \bar{V} \epsilon) + \frac{1}{r} \frac{\partial}{\partial r}(r \bar{\rho} \bar{U} \epsilon) &= \frac{\partial}{\partial z} \left(a_2 \frac{\partial \epsilon}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(r a_2 \frac{\partial \epsilon}{\partial r} \right) \\ &+ \frac{\epsilon}{K} C_1 \mu_t \left\{ 2 \left[\left(\frac{\partial \bar{V}}{\partial z} \right)^2 + \left(\frac{\partial \bar{U}}{\partial r} \right)^2 + \left(\frac{\bar{U}}{r} \right)^2 \right] + \left(\frac{\partial \bar{V}}{\partial r} + \frac{\partial \bar{U}}{\partial z} \right)^2 \right\} - \bar{\rho} C_2 \epsilon \end{aligned} \quad (4.152b)$$

where

r = radius of tube

$\mu_t = 0.09 \bar{\rho} K^2 / \epsilon$ = turbulent viscosity

$a_1 = \mu_t + \mu_l / 0.9$

$a_2 = (\mu_t / 1.3) + \mu_l = 0.9$

μ_l = liquid viscosity

$C_2 = 1.92$

$1/\bar{\rho} = (x/\rho_g) + [(1-x)/\rho_l]$

Analogous equations may be written for three-dimensional flow.

The use of Eq. (4.152) will obviously considerably complicate solution procedures. In addition, obtaining the partial derivatives with reasonable

accuracy requires the use of control volumes that are considerably smaller than those generally used in porous-media computations.

4.3 FORCED CONVECTION HEAT TRANSFER IN SINGLE-PHASE FLOW

Since we established the means for determining fluid enthalpy along the reactor core, we are now able to determine existing heat-transfer regimes. Single-phase forced convection heat transfer is encountered in the inlet and low-power regions of a PWR core where fluid enthalpy is below that of the saturated liquid. It is also encountered on the tube side of the steam generators. Conditions for the onset of boiling are described in detail Sec. 4.4.

4.3.1 Empirical Equations for Single-Phase Heat Transfer

While laminar flow ($Re < 2000$) will never be encountered during any normal operating situation, some calculations of hypothetical accident conditions indicate that this regime can be found. We can then follow Rohsenow and Choi's⁸⁴ modification of the theoretical equation and obtain the heat transfer coefficient in a round tube from

$$Nu = hDe/k = 4.0 \quad (4.153)$$

In laminar flow parallel to an array of circular rods, the Nusselt number increases as the pitch to diameter ratio (p/D) increases.⁸⁵ At a p/D of 1.1, Nu is about 4.5, while at a p/D of 1.2 it has increased to about 7.5, and at $p/D = 1.9$, $Nu = 15$.

Despite marked progress in understanding turbulence, the limitations and complexities of theoretical approaches have led to the use of empirical correlations in turbulent flow.* For turbulent flow of water inside smooth conduits and annuli, the most extensively used correlation is that of Dittus and Boelter⁸⁶:

$$Nu = \left(\frac{hDe}{k} \right)_b = C \left(\frac{DeG}{\mu} \right)_b^{0.8} \left(\frac{c_p \mu}{k} \right)_b^n \quad (4.154)$$

*Since turbulence in rod bundles appears gradually, a gradual transition from laminar to turbulent heat transfer coefficients is needed. Based on the work of Halder *et al.* *Proc. First ISHMT/ASME Heat & Mass Transfer Conf.* p. 921, Bombay, India (1994), the transition region heat transfer can be represented by

$$Nu = C_1 + C_2 Re$$

The constants C_1 and C_2 are determined by requiring that Nu equal the laminar value at $Re = 800$ and Nu equal the fully turbulent value at $Re = 10^4$.

where

- h = heat transfer coefficient, Btu/(hr ft² °F)
 De = equivalent diameter, ft
 k = thermal conductivity of fluid, Btu/(hr ft °F)
 c_p = specific heat of fluid, Btu/lb
 G = mass velocity, lb/(hr ft²)
 μ = fluid viscosity, lb/(hr ft)
 C = 0.023
 n = 0.4
 $(DeG/\mu) > 10,000$ and $L/De > 60$
 b = bulk conditions.

The Dittus-Boelter correlation is also widely used for water flow parallel to tube banks. Available experimental data indicate that the coefficient of Eq. (4.153) varies with the pitch-to-diameter ratio. The h values obtained from the Dittus-Boelter correlation are slightly conservative for the pitch-to-diameter ratios of interest to reactor designers. Weisman⁸⁷ correlated the available data for triangular-pitch lattices using Eq. (4.153) with

$$C = 0.026(s/D) - 0.006 \quad (4.155a)$$

and square-pitch lattice data with

$$C = 0.042(s/D) - 0.024 \quad (4.155b)$$

where s = tube pitch and D = tube diameter. The results for both types of lattices can be expressed as

$$C = 0.0333E_1 + 0.0127 \quad (4.155c)$$

where E_1 is the fraction of the total cross-sectional area in an infinite array taken up by the fluid. Variations in the heat coefficients around the periphery of a rod have been found negligible for s/D ratios > 1.2 (Ref. 88).

While all present PWR cores are designed with flow parallel to the fuel elements, single-phase cross-flow, which provides higher heat transfer coefficients than attainable at the same mass flow rate in parallel flow, is encountered on the shell side of a once-through steam generator. The heat transfer data for flow of liquids normal to banks of un baffled tubes have also been correlated by Weisman.⁸⁹ For staggered tube banks, he found

$$\left(\frac{hD}{k}\right)E_1^{\Phi b} = 0.38 \left(\frac{DG_s}{\mu E_1}\right)_f^{0.61} \left(\frac{c_p \mu}{k}\right)_f^{0.4} \quad 300 < \left(\frac{DG_s}{\mu E_1}\right) < 40,000 \quad (4.156a)$$

$$\left(\frac{hD}{k}\right)E_1^{\Phi b} = 0.051 \left(\frac{DG_s}{\mu E_1}\right)_f^{0.8} \left(\frac{c_p \mu}{k}\right)_f^{0.52}, \quad 80,000 < \left(\frac{DG_s}{\mu E_1}\right) < 800,000, \quad (4.156b)$$

where f refers to film conditions, G_s is the superficial mass velocity based on total cross-section area,

$$b = 0.175(DG_s / \mu E_1)$$

and ϕ , which depends on s_t/s_l , the ratio of the tube pitch transverse to the flow to the pitch parallel to the flow direction, is obtained from Fig. 4.12. For in-line tube banks, Weisman obtained

$$\left(\frac{hD}{k}\right) E_1^{\phi b} = 0.101 \left(\frac{DG_s}{\mu E_1}\right)_f^{0.745} \left(\frac{c_p \mu}{k}\right)_f^{0.4} \quad 2000 < \left(\frac{DG_s}{\mu E_1}\right) < 30,000 \quad (4.157)$$

Again, ϕ is obtained from Fig. 4.12. The use of Fig. 4.12 is inconvenient if the computations are to be incorporated in a computer program. In that

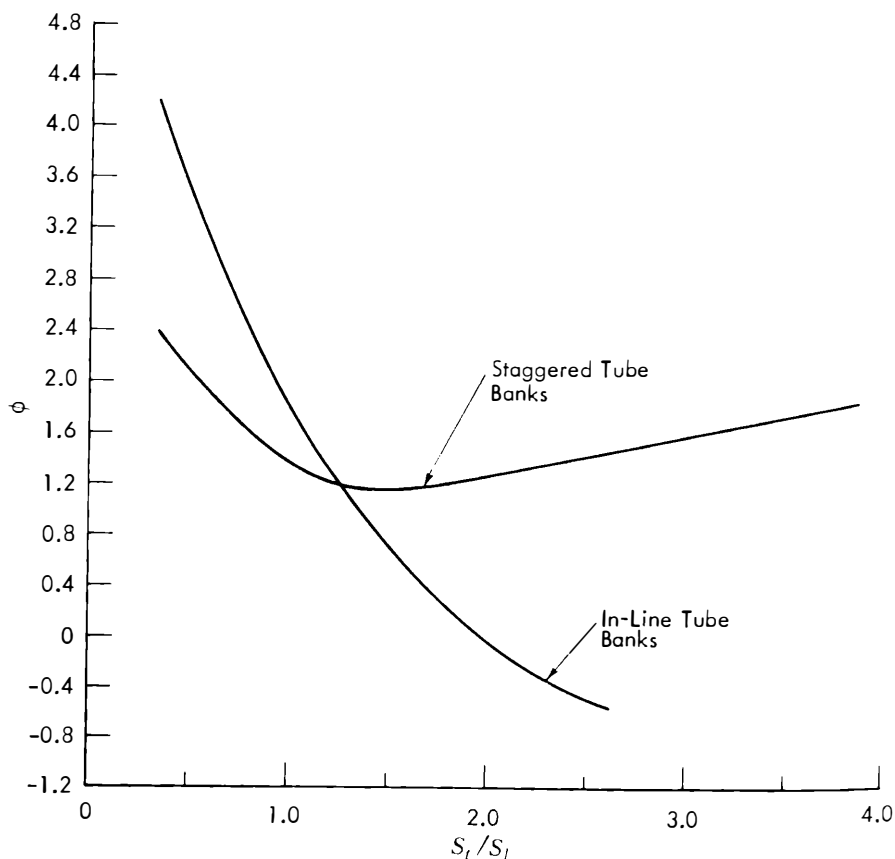


Fig. 4.12 Arrangement coefficients for heat transfer to fluids flowing normal to tube banks [from *AIChE. J.*, **1**, 342 (1955)].

TABLE 4.III
Mean Nusselt Number Correlations for In-Line and Staggered Tube Bundles

Range of Re	Recommended Correlations			
	In-Line Tube Bundles			
$10^0\text{--}10^2$	$Nu = 0.9$	$Re^{0.4}$	$Pr^{0.36}$	$(Pr/Pr_w)^{0.25}$
$10^2\text{--}10^3$	$Nu = 0.52$	$Re^{0.5}$	$Pr^{0.36}$	$(Pr/Pr_w)^{0.25}$
$10^3\text{--}2 \times 10^5$	$Nu = 0.27$	$Re^{0.63}$	$Pr^{0.36}$	$(Pr/Pr_w)^{0.25}$
$2 \times 10^5\text{--}2 \times 10^6$	$Nu = 0.033$	Re^0	Pr^0	$(Pr/Pr_w)^{0.25}$
	Staggered Tube Bundles			
$10^0\text{--}5 \times 10^2$	$Nu = 1.04$	$Re^{0.4}$	$Pr^{0.36}$	$(Pr/Pr_w)^0$
$5 \times 10^2\text{--}10^3$	$Nu = 0.71$	$Re^{0.5}$	$Pr^{0.36}$	$(Pr/Pr_w)^{0.25}$
$10^3\text{--}2 \times 10^5$	$Nu = 0.35$	$(s_t/s_l)^{0.2}$	$Re^{0.6}$	$Pr^{0.36} (Pr/Pr_w)^{0.25}$
$2 \times 10^5\text{--}2 \times 10^6$	$Nu = 0.031$	(s_t/s_l)	$Re^{0.8}$	$Pr^{0.36} (Pr/Pr_w)^{0.25}$

Nu , Re , and Pr are evaluated at the bulk temperature; Pr_w is evaluated at the wall temperature.

case it may be more convenient to obtain cross-flow heat transfer coefficients from the set of correlations devised by Zukauskas⁹⁰ and listed in Table 4.III. It will be observed from the table that Zukauskas includes no (s_l/s_l) effect for in-line banks. Therefore, use of these correlations for in-line banks should probably be restricted to geometries where (s_l/s_l) is close to unity.

Based on data obtained in conduits and annuli, for steam at moderately high pressures and Reynolds numbers in parallel flow situations, Bishop *et al.*⁹¹ recommend

$$\left(\frac{hDe}{k}\right)_f = 0.0073 \left(\frac{DeG}{\mu}\right)_f^{0.886} \left(\frac{c_p \mu}{k}\right)_f^{0.61} \left(1 + \frac{2.76}{L_h/De}\right) \quad (4.158)$$

where

L_h = heated length = 30 – 385 ft

P 1000 – 3190 psi

DeG/μ 100,000 – 0.6×10^6

$c_p \mu/k$ = 0.88 – 2.38

f = refers to film conditions [$T = (T + T_b)/2$].

However, Swenson *et al.*⁹² recommend the use of Eq. (4.154) for tubes with high L/D ratios, with C as a function of pressure. For $P = 2600$ psia, $C = 0.05$; for $P = 3000$ psia, $C = 0.076$; for $P = 3128$ psia, $C = 0.08$.

For water flowing in a conduit under supercritical conditions, the Bishop *et al.* equation⁹³ is recommended:

$$\left(\frac{hDe}{k}\right)_b = 0.0069 \left(\frac{DeG}{\mu}\right)_b^{0.90} \left(\frac{\bar{c}_p \mu}{k}\right)_b \left(\frac{\rho_{tw}}{\rho_b}\right)^{0.43} \left(1 + \frac{2.4}{L/De}\right), \quad (4.159)$$

where $\bar{c}_p = (H_w - H_b)/(T_w - T_b)$ and b and w refer to bulk and wall conditions, respectively. This correlation was developed from data in the following range:

$$q'' = 0.1 \times 10^6 \text{ to } 1.1 \times 10^6 \text{ Btu}/(\text{hr ft}^2)$$

$$G = 0.5 \times 10^6 \text{ to } 2.7 \times 10^6 \text{ lb}(\text{hr ft}^2)$$

$$De = 0.10 \text{ to } 0.20 \text{ in.}$$

$$T_b = 561 \text{ to } 976^\circ\text{F}$$

$$T_w = 666 \text{ to } 1172^\circ\text{F.}$$

4.3.2 Surface Roughness Effects on Heat Transfer

Increasing surface roughness causes an increase in single-phase heat transfer coefficients. Most of the reported surface roughness effects on the heat transfer are empirical. However, Kolar⁹⁴ analyzed the mechanism of heat transfer in rough tubes by recognizing that the change in resistance is due to the production of local vortices. Since a vortex will contact the heating surface only momentarily, the transient conduction between the vortex element and solid wall can be calculated from

$$\partial T/(\partial t) = \alpha' \partial^2 T/(\partial x^2) \quad (4.160)$$

where α' is thermal diffusivity. The solution of the above equation for sudden temperature change at the surface of an infinitely thick plate is

$$\theta = [2/(\pi)^{1/2}] \theta_i \int_0^u \exp(-u_1^2) dw \quad (4.161)$$

where

$$u_1 = x/[2(\alpha't)^{1/2}] \quad \text{and} \quad \theta_i = (T_{\text{wall}} - T_{\text{vortex}})_{t=0}$$

The heat flux at interface ($x=0$) is then given by

$$q'' = -k \frac{\partial \theta}{\partial x} = -k \frac{\partial \theta}{\partial u} \frac{\partial u_1}{\partial x} = -\frac{k \theta_i \exp[-x^2/(4\alpha't)]}{(\pi\alpha't)^{1/2}} = \frac{k \theta_i}{(\pi\alpha't)^{1/2}} \quad (4.162)$$

The heat transfer coefficient at time τ is thus

$$h = q''/\theta_i = k/(\pi\alpha'\tau)^{1/2} \quad (4.163)$$

where $\tau = \lambda_0/v_\lambda$, in which λ_0 is the size of the vortex and is called the local degree of turbulence and v_λ is the velocity in fluctuation. At maximum energy dissipation ($v_\lambda = v/\lambda_0$), where v is kinematic viscosity μ/ρ , Eq. (4.163) can be rewritten as

$$h = k(v/\alpha')^{1/2}/(\pi^{1/2}\lambda_0) \quad (4.164)$$

According to Kolmogorov⁹⁵ the value of λ_0 can be evaluated from

$$\lambda_0 = 28 \nu / u^* \quad (4.165)$$

where $\nu = \mu / \rho$, $u^* = V(f/8)^{1/2}$, and V = average fluid velocity.

By substituting Eq. (4.165) into Eq. (4.164) and rearranging, we get

$$hD/k = 0.02(u^*D/\nu)(c_p\mu/k)^{0.5} \quad (4.166)$$

Note that the above equation is derived for a certain time, τ , corresponding to that at which maximum energy dissipation occurs. The average heat-transfer coefficient can be determined from the data of Kolar,⁹⁴ who studied the effect of surface roughness by heating air and water in 33- to 26-mm-diameter tubes with roughness ratios (tube radius/projection height) of 26.39, 13.5, and 9.15, as well as in a smooth tube of the same diameter. The roughness was obtained by machining with a 60-deg triangular thread on the surface. The Reynolds number varied from 4.5×10^3 to 1.45×10^5 ; The Prandtl number varied from 0.71 to 5.52. Based on his experimental data, Kolar⁹⁴ obtained the following equation with an uncertainty of $<4\%$ in the Re range from 2×10^3 to 1×10^5 ,

$$(hD/k)_f = 0.0517(u^*D/\nu)_f(c_p\mu/k)_f^{0.5} \quad (4.167)$$

where subscript f refers to film temperature and

$$u^* = \bar{u}(f_f/8)^{1/2}$$

$$f_f = \text{friction factor} = 0.515(e_a/D)^{0.63} \text{ for } \text{Re}_f > 3 \times 10^4$$

$$e_a = \text{height of projections above surface.}$$

Enhanced heat transfer obtained by using roughened surfaces, grooves, wire coils, etc., has become of significant industrial importance. Summaries of the various approaches and available empirical correlations are given by Bergles⁹⁶ and Webb.⁹⁷

4.3.3 Turbulence Promoter and Grid Spacer Effects

Although higher coolant mixing rates and heat transfer coefficients are induced by grids due to the promotion of turbulence, hot spots generally occur at grids or spacers. Wilkie and White⁹⁸ reported that in air flow, a 25% reduction in the local heat transfer coefficient is introduced by grids; the hot spot occurs immediately downstream of the grid. Reduction in the heat transfer coefficient is caused by the local retardation of flow over the heating surface. Because of the complicated configuration of grid spacers, the local velocity profile is usually experimentally determined. The net benefit of a turbulence promoter or a grid spacer depends on the balance of the following benefits and penalties:

1. Benefit from increased flow mixing between adjacent channels
2. Benefit from the increase of the average heat transfer coefficient

3. Benefit from the increase in the critical heat flux by wiping off the bubble layer
4. Penalty of having a local hot spot
5. Penalty of a higher pumping power
6. Penalty of forming a flow stagnation point
7. Penalty of creating additional crud collection locations
8. Penalty of increased lift forces on fuel assembly.

Therefore, the design of turbulence promoters should vary with geometry and flow conditions; no general design criterion can now be established or followed. However, designs incorporating small mixing vanes on the upper edges of the spacer grids have been successful in significantly increasing the critical heat flux and interchannel mixing.

4.4 BOILING HEAT TRANSFER

Although the earliest core designs were based on the assumption that surface boiling could not be allowed, this limitation was soon discarded, and boiling heat transfer is now one of the steady-state heat transfer mechanisms in the PWR core. Note that the nonboiling limitation may still be desirable in a maritime reactor.

4.4.1 Flow Boiling Heat Transfer

Heating a liquid at a very high heat flux brings the heater wall temperature above that of the liquid's saturation point. The liquid adjacent to the wall is then superheated and nucleation sites are activated. Bubbles are generated in patches, while forced convection persists in the remaining area. This heat transfer region is referred to as "partial nucleate boiling." If the heat flux is increased, bubbles are generated over a larger part of the surface until at "fully developed nucleate boiling," bubbles are generated over the entire surface. If the bulk of the liquid is subcooled, the nucleate boiling is called local boiling and the bubbles formed condense locally. If the liquid is saturated, the bubbles do not collapse; this is called "bulk boiling."

In fully developed nucleate boiling with saturated liquid, the wall temperature primarily is determined by heat flux and pressure, and only slightly affected by liquid velocity. This essential independence of liquid velocity is illustrated in Fig. 4.13. For subcooled water at pressures between 30 and 90 psia, McAdams *et al.*⁹⁹ correlated the available data by

$$q'' = 0.074(T_w - T_{\text{sat}})^{3.86} \quad (4.168)$$

where q'' = heat flux, Btu/(hr ft²); T_w = wall temperature, °F; T_{sat} = saturation temperature, °F.

For pressure between 500 and 2000 psia, Jens and Lottes¹⁰⁰ correlated subcooled boiling data by

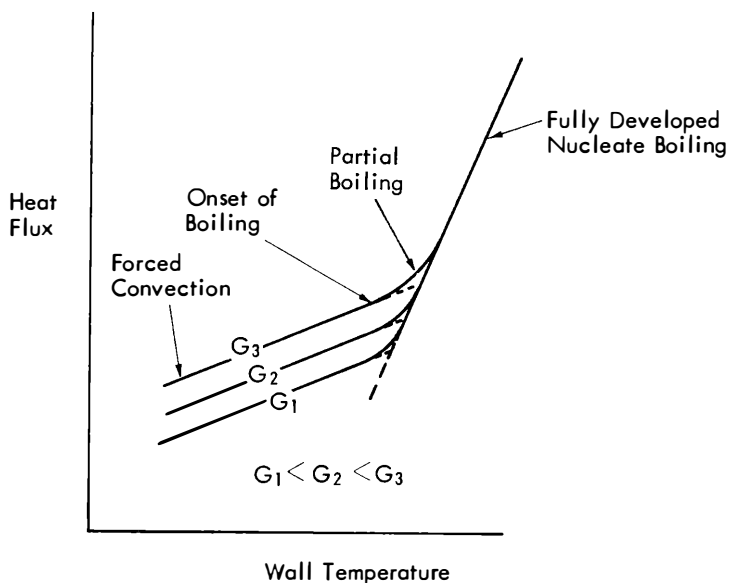


Fig. 4.13 Onset of boiling heat transfer.

$$(T_w - T_{\text{sat}}) = 60(q''/10^6)^{1/4} / [\exp(P/900)] \quad (4.169)$$

where P is pressure in psia. The correlation appears to hold for all geometries and for both local and bulk boiling.

There has been concern that the temperature differences predicted by the Jens-Lottes correlation are too low at high pressures. Thom *et al.*¹⁰¹ concluded that their extensive data at pressures from 750 to 2000 psia were best correlated by

$$(T_w - T_{\text{sat}}) = 0.072(q'')^{1/2} / [\exp(P/1260)] \quad (4.170)$$

The temperature differences predicted by this correlation tend to be higher than those obtained from Eq. (4.169). However, much of Thom's data were obtained at relatively low heat fluxes (perhaps in the region of the bend of the curves in Fig. 4.13). It has been argued that this is the reason for the lower exponent in Eq. (4.170).

Although some investigations¹⁰² have suggested that forced convection effects should be added to the heat transfer rates during fully developed saturated boiling, the more recent information appears to indicate that this is not required. However, during subcooled boiling this is clearly necessary because boiling heat transfer rates considerably in excess of those of Eq. (4.169) or Eq. (4.170) may be observed. A relatively simple correlation of

subcooled boiling heat transfer has been proposed by Gungor and Winterton.¹⁰³ Their approach may be represented by

$$q''_{\text{total}} = h_L(T_w - T_b) + Sq''_b \quad (4.171)$$

where

h_L = single-phase forced convection coefficient evaluated for liquid at the bulk temperature

T_b, T_w = bulk coolant temperature and wall temperature, respectively

q''_b = fully developed saturated boiling heat flux evaluated at T_w

S = boiling suppression factor = $[1 + 1.15F^2 \text{Re}_L^{1.17} \times 10^{-6}]^{-1}$

$F = [1 + 24,000(q''_b / (Gh_{fg}))^{1.16}]^{-1}$

Kutateladze¹⁰⁴ proposed a relationship between temperature and heat flux for the partial nucleate boiling region. However, the region is small and, for most design purposes, it is adequate to take the onset of nucleate boiling as the condition where the forced convection wall temperature equals the fully developed nucleate boiling temperature.

At high vapor fractions, the flow pattern in tubes is such that a vapor core exists surrounded by an annulus of water. The velocity of vapor in the core can be so high that the very high turbulence at the vapor-liquid interface causes the heat transfer mechanism to change character. Evaporation now occurs at the liquid-vapor core interface and, as characteristic of nonboiling heat transfer, the heat transfer coefficient varies strongly with flow. This heat transfer region has been referred to as "forced convection vaporization."

Suppression of nucleate boiling occurs at high values of the liquid Reynolds numbers, Re , and $1/X_{tt}$, where X_{tt} is the Lockhart-Martinelli parameter,

$$1/X_{tt} = [\chi / (1 - \chi)]^{0.9} [\rho_l / (\rho_v)]^{0.5} (\mu_v / \mu_l)^{0.1} \quad (4.172)$$

and χ = steam quality, ρ_l, ρ_v = density of liquid and vapor, and μ_l, μ_v = viscosity of liquid and vapor.

Chen¹⁰⁵ proposed a correlation where the heat transfer coefficient, h , in this region is the sum of a nucleate boiling component and a forced convection component; thus,

$$h = S(0.00122) \frac{k_l^{0.79} c_l^{0.45} \rho_l^{0.49} g_c^{0.25} \Delta T^{0.24} \Delta P^{0.75}}{\sigma^{0.5} \mu_l^{0.29} H_{fg}^{0.24} \rho_v^{0.24}} + F(0.023) \text{Re}_l^{0.8} \text{Pr}_l^{0.4} k_l / De \quad (4.173)$$

and

c_l = specific heat of liquid

De = equivalent diameter

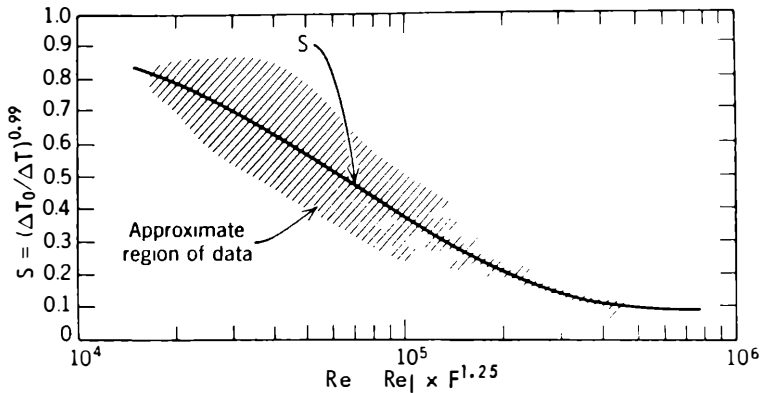


Fig. 4.14 Suppression factor S for Chen correlation [from L. S. Tong, *Boiling Heat Transfer and Two-Phase Flow*, John Wiley & Sons, New York (1965)].

g_c = gravitational conversion factor

ΔP = difference in saturation pressures corresponding to wall superheat

H_{fg} = heat of vaporization

Re_l, Pr_l = Reynolds and Prandtl numbers, respectively, based on liquid properties

k_l = conductivity of liquid

$\Delta T = (T_{\text{wall}} - T_{\text{sat}})$, wall superheat

σ = surface tension.

The term on the left represents the nucleate boiling component, while S , given by Fig. 4.14, is the nucleate boiling suppression factor. The term on the right is the forced convection component and varies with F , which is a function of $1/X_{tt}$ (Fig. 4.15).

More recent investigations have found deviations from the Chen correlation and revisions have been proposed. Gungor and Winterton¹⁰³ use a correlation of the same general form

$$h = Fh_c + Sh_b \quad (4.174)$$

where h_c = forced convection coefficient determined using the Dittus-Boelter equation and liquid phase properties, and $h_b = q_b'' / (T_w - T_{\text{sat}})$. The parameters F and S have their previous meaning. The parameter S is still defined as required by Eq. (4.171) but F is given by

$$F = 1 + 24,000(q_b'' / GH_{fg})^{1.16} + 1.37(X_{tt})^{-0.75} \quad (4.175)$$

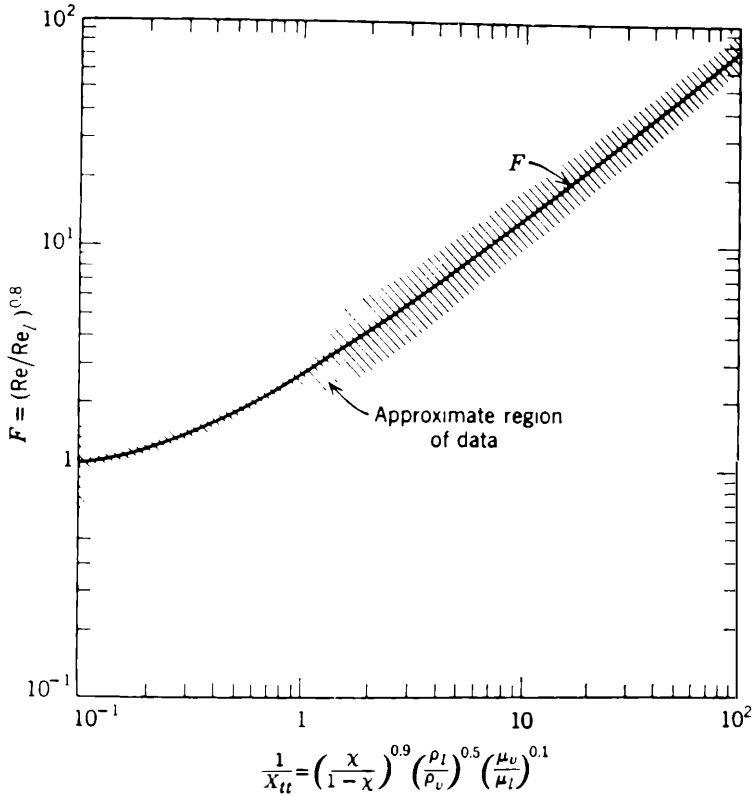


Fig. 4.15 Reynolds number factor F for Chen correlation [from L. S. Tong, *Boiling Heat Transfer and Two-Phase Flow*, John Wiley & Sons, New York (1965)].

If the tube is horizontal and $Fr_L < 0.05$, then F should be multiplied by F_2 where

$$F_2 = Fr_L^{0.1} \quad (4.176)$$

and S is multiplied by S_2 where

$$S_2 = \sqrt{Fr_L} \quad (4.177)$$

$$Fr_L = G^2 / (\rho_l^2 g D)$$

Hassan *et al.*¹⁰⁶ note that when using the boiling suppression approach to predict behavior in rod bundles, improved results are obtained when h_i is predicted using Eq. (4.155) instead of the Dittus-Boelter equation.

4.4.2 The Boiling Crisis or Critical Heat Flux

The nucleate boiling heat flux cannot be increased indefinitely. At some critical flux, the steam produced forms an insulating layer over the surface

and raises surface temperature. This is the “boiling crisis.” Immediately after the critical heat flux (CHF) has been reached, boiling is unstable and partial film boiling or transition boiling occurs. Here the surface is successively covered by a vapor film and liquid layer. Surface temperature finally increases enough to cause the formation of a stable vapor layer that produces “stable film boiling.”

The term boiling crisis is not in universal use. The phenomenon is also referred to as burnout since early tests detected the crisis by the physical failure of electrically heated test elements. The boiling crisis can be classified as *departure from nucleate boiling* (DNB) in the subcooled or low-quality region and dryout in the high-quality region. The CHF terminology applies to both regions.

The behavior of the boiling crisis depends on flow conditions. In the subcooled or low-quality region, the crisis occurs at a relatively high heat flux and appears to be associated with the cloud of bubbles, adjacent to the surface, which reduces the amount of incoming water. When this crisis occurs, surface temperature rapidly rises to a high value.

In the high-quality region, the crisis occurs at a lower heat flux. The flow pattern is usually annular, and the surface is normally covered by a liquid layer. When the evaporation rate is high enough, the liquid layer can develop a dry patch when the boiling crisis occurs. Since the velocity in the vapor core is high, post-CHF heat transfer is much better than for low-quality cases; wall temperature rises are lower and less rapid.

4.4.2.1 Theoretical and Phenomenological Approaches to CHF in Round Tubes

Behavior at High Quality. Various attempts have been made to develop analytical approaches for predicting the boiling crisis. The first promising attempts considered the boiling crisis in high-quality annular flow.^{107–109} Here the models are based on predicting a dryout of the liquid film on the wall.

Models of dryout in the annular flow regime postulate that, under diabatic conditions, the liquid film flow in a round tube is determined by an integral balance between entrainment, evaporation, and deposition. The overall balance on the liquid film flow rate W_F from the inlet $z(i)$ to location z is given by

$$W_F = W_{z(i)} - (W_E)_{z(A)} - \pi D \left\{ \int_{z(i)}^z \left(\frac{q''}{H_{fg}} \right) dz - \int_{z(A)}^z [E'(z) - R(z)] dz \right\} \quad (4.178)$$

where

D = tube diameter

$W_{z(i)}$ = total mass flow rate at inlet (m/t)

W_E = mass flow rate of entrained liquid drops (m/t)

R = rate of deposition of droplets onto liquid (m/Lt)

E' = rate of entrainment of drops from liquid (m/Lt)

$z(b)$ location at which boiling begins

$z(A)$ = location at which annular flow begins

q'' heat flux

H_{fg} latent heat of evaporation.

Since W_F should be zero at the dryout location $z(D)$, we have

$$\int_{z(A)}^{z(D)} \left(\frac{q''}{H_{fg}} \right) dz = \frac{W_{z(i)} - (W_E)_{z(A)}}{\pi D} - \int_{z(A)}^{z(D)} [E'(z) - R(z)] dz \quad (4.179)$$

Studies by workers at Harwell¹¹⁰ have developed prediction procedures for vertical flow, round-tube dryout to the point where generally good agreement with experimental data is obtained. In the Harwell modeling used in Whalley's¹¹¹ dryout predictions, the differential mass balances on the film flow and entrained liquid flow are written as

$$dW_E/dz = \pi D(E' - R) \quad (4.180)$$

and

$$\frac{dW_{LF}}{dz} = \pi D \left(R - E' - \frac{q''}{H_{fg}} \right) \quad (4.181)$$

where W_{LF} is the mass flow rate of liquid film and the other symbols have their previous meanings. Integration of the foregoing equations requires appropriate procedures for determination of the entrainment and deposition rates. The Harwell approach to this calculation has been described in Sec. 3.5.1.2 [Eqs. (3.245) to (3.249)].

Levy *et al.*¹¹² used a model based on the Harwell approach, but they related the liquid film mass flux G_f to a nondimensional radius r^+ and film thickness y^+ as follows:

$$G_f/\rho_L = (\tau_w/\rho_L)^{1/2} = \frac{2}{(r^+)^2} \phi(y_f^+, r^+) \quad (4.182)$$

where

$\phi(y^+, r^+) =$ functional relationships for various ranges of y^+

$$= r(\tau_w/\rho_L)^{1/2} \rho_L/\mu_L$$

$$y^+ = \rho_L y_f (\tau_w/\rho_L)^{1/2} / \mu_L$$

tube radius

y_f = film thickness

ρ_L liquid density

μ_L = liquid viscosity.

The film thicknesses are determined by assuming a value for y_f and calculating τ_w . Revised values of r^+ and y^+ are calculated, and the process continues iteratively until the assumed value of y_f satisfies Eq. (4.182).

A considerably more elaborate approach has been used by Sugawara and Miyamoto¹¹³ and by Saito *et al.*¹¹⁴ These investigators formulated a multi-fluid model of annular flow. In a round tube, the liquid film on the wall, the vapor core, and the entrained droplets are each considered to be a separate fluid. The differential equations for continuity, momentum, and energy conservation are written for each fluid. Constitutive equations for estimating the fluid-fluid interfacial areas, interfacial drag, and interfacial mass transfer (deposition and entrainment) are also required. By starting at the channel inlet, the numerical solution may be obtained by a standard initial value, ordinary differential equation solution method.

In the Saito *et al.*¹¹⁴ approach, the entrainment and deposition constitutive models used differ only slightly from those of Harwell. Deposition rates are still obtained from Eq. (3.247), but K is a function of droplet concentration. Entrainment mass flux is obtained directly from a correlation between E' and $(\tau\delta/\sigma')$ (see Sec. 3.5.1.2).

The Saito *et al.*¹¹⁴ model appears to provide reasonable prediction of dryout in both round tubes and annuli (in annuli, the liquid on the inner and outer walls are separate fluids). However, the round tube predictions do not appear to be noticeably superior to those obtained by the simpler approach of Whalley.¹¹¹

Behavior in the Subcooled and Low Void Regions. In a PWR, hypothetical transient conditions generally lead to critical heat fluxes in the subcooled or low-quality region. Some success was obtained in relating critical heat flux in the subcooled region to the two-phase friction factor.¹¹⁵ Subsequently, Tong¹¹⁶ developed a phenomenological approach that can be applied to both the subcooled and low void regions. His equation was given in terms of reduced pressure, P_r , mixture Reynolds number, Re_m , and modified Jakob number, Ja .

More recent work has focused on the behavior associated with the thin layer containing a high concentration of bubbles (bubbly layer), which exists adjacent to the heated wall prior to CHF. Katto¹¹⁷ has categorized the various approaches taken into the following groups:

1. Critical liquid superheat: DNB is assumed to occur when the thin liquid sublayer (beneath the bubbly layer) reaches a critical superheat (see Tong *et al.*¹¹⁸).
2. Inhibition of enthalpy transport to the bubbly layer due to separation of the boundary layer from the wall (see Kutateladze and Leontev¹¹⁹).
3. Blockage of the liquid flow to the wall by flow of vapor from the wall (see Smogalev¹²⁰).

4. Dryout of the liquid sublayer underneath vapor clots flowing along the wall (Lee and Mudawar¹²¹ and Katto¹¹⁷).
5. Bubble coalescence in the bubbly layer brought about by inadequate vapor removal rate at the edge of the bubbly layer (see Hebel *et al.*¹²² and Weisman and Pei¹²³).

The best success appears to have been obtained with the sublayer dryout (item 4) and the bubble coalescence theories (item 5).

The Weisman-Pei¹²³ version of the bubble coalescence model assumes the following:

1. During subcooled and low-quality boiling, the bubbly layer builds up along the channel until it fills the region near the wall where the turbulent eddies are too small to transport bubbles radially. At the CHF site, the bubbly layer is assumed to be at this maximum thickness.
2. CHF occurs when the volume fraction of steam in the bubbly layer just exceeds the volume fraction (critical void fraction) at which an array of slightly flattened ellipsoidal bubbles can be maintained without significant contact between the bubbles.
3. The volume fraction of steam in the bubbly layer is determined by a balance between the outward flow of vapor and the inward flow of liquid at the bubbly layer-core interface.

The model used may be understood by reference to Fig. 4.16, which shows the transport between the core and bubbly layer. In addition to axial flow in and out of the bubbly layer control volume, there is radial interchange between the bubbly layer and core regions. If $\dot{m}_3$ represents the total flow rate from core to bubbly layer and $\dot{m}_4$ the total flow rate from bubbly layer to core, then the total mass balance on the bubbly layer is

$$\dot{m}_3 = \Delta \dot{m}_2 + \dot{m}_4 \quad (4.183)$$

By letting x_1 represent the quality in the core and x_2 the quality in the bubbly layer, a mass balance on the liquid in the bubbly layer yields

$$\dot{m}_3(1 - x_1) = \frac{q_b''(2\pi r \Delta z)}{H_{fg}} - \dot{m}_2(\Delta x_2) + \Delta \dot{m}_2(1 - x_2) + \dot{m}_4(1 - x_2) \quad (4.184)$$

when second-order terms are ignored and q_b'' is taken as the heat flux going toward vapor production. By combining these mass balances, one obtains

$$\dot{m}_3(x_2 - x_1) = \frac{q_b''(2\pi r \Delta z)}{H_{fg}} - \dot{m}_2(\Delta x_2) \quad (4.185)$$

This may be rewritten by replacing $\dot{m}_3$ by $2\pi G'(R - S)\Delta z$ to obtain

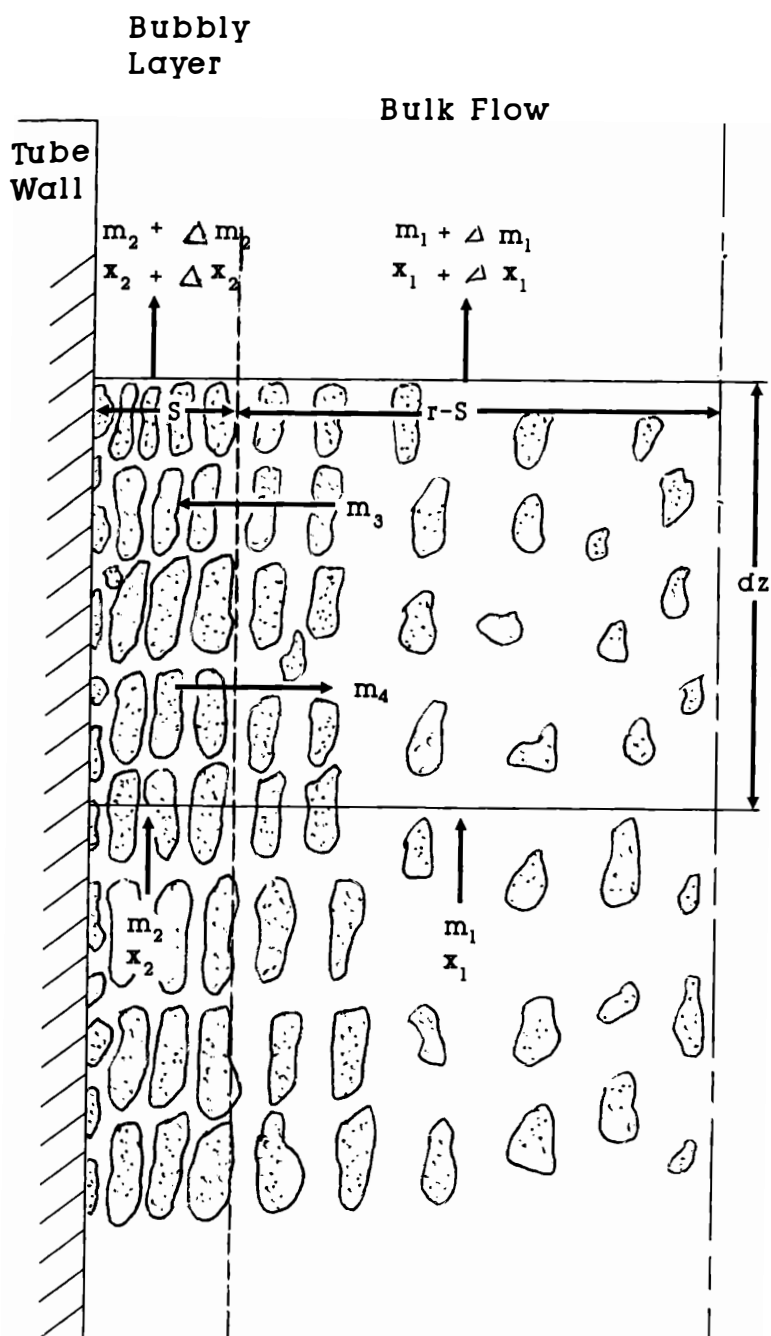


Fig. 4.16 Schematic of transport between core and bubbly layer [from B. S. Pei, Ph.D. Thesis, University of Cincinnati (1981)].

$$G'(x_2 - x_1) = \frac{q_b''}{H_{fg}} \left(\frac{r}{r - S} \right) - \frac{\dot{m}_2}{2\pi(r - S)} \left(\frac{\Delta x_2}{\Delta z} \right) \quad (4.186)$$

This is simplified by recognizing that the ratio $[r/(r - S)]$ is close to unity and by assuming that at CHF conditions the term $\{[\dot{m}_2/2\pi(r - S)](\Delta x_2/\Delta z)\}$ is negligible. Then,

$$G'(x_2 - x_1) = q_b''/H_{fg} \quad (4.187)$$

In Pei and Weisman's original work, the value of q_b'' is related to q'' by

$$q_b'' = Fq'' = [(H_1 - H_{ld})/(H_f - H_{ld})]q'' \quad (4.188)$$

where H_{ld} = enthalpy at bubble detachment point [calculated via Eq. (3.80)], H_f = enthalpy of saturated liquid, and H_l = enthalpy of liquid at given axial location. This allows Eq. (4.187) to be rewritten in dimensionless form as

$$q_{\text{DNB}}''/(H_{fg}G') = (x_2 - x_1) \frac{(H_f - H_{ld})}{(H_l - H_{ld})} \quad (4.189)$$

Note that in Eq. (4.189), x_1 may not be assigned a value below zero.

The quantities x_1 and x_2 represent the vapor qualities in the core region and bubbly layer, respectively, at DNB. These are actual values and not thermodynamic equilibrium qualities. The value of x_2 is calculated as that quality that corresponds to the maximum void fraction that is possible in a bubbly layer of independent bubbles just prior to agglomeration. For slightly flattened elliptically shaped bubbles with a length-to-diameter ratio of 3/1, this void fraction is estimated as 0.82. The quality of the core fluid, x_1 , is based on the average fluid quality calculated via Eq. (3.82). Note that only locations beyond the bubble detachment point, where x_1 is positive, can be considered.

The quantity G' represents the mass flow rate into the bubbly layer. This mass flow rate is determined by the turbulent velocity fluctuations at the bubbly layer edge. The distance from the edge of the bubbly layer to the wall is taken as the distance at which the size of the turbulent eddies is k times the average bubble diameter.

Only a fraction of the turbulent velocity fluctuations produced are assumed to be effective in reaching the wall. The effective velocity fluctuations are those in which the velocity exceeds the average vapor velocity away from the wall produced by the vapor being generated at the wall. At the bubbly layer-core interface, the effective mass flux toward the wall is

$$G' = \Psi i_b G \quad (4.190)$$

where G is the total axial mass flux. The product $i_b G$ represents the flux toward the wall in the absence of outward flow. The parameter i_b , representing the turbulent intensity at the bubbly layer-core interface, is calcu-

lated as the product of the single-phase turbulent intensity at the bubbly layer edge and a two-phase enhancement factor. This results in

$$i_b = 0.462(k')(\text{Re})^{-0.1}(D_p/D)^{0.6}[1 + a(\rho_l - \rho_g)/\rho_g] \quad (4.191)$$

where

k' = empirical constant = 1.71

a = empirical constant that varies with G

D = tube diameter

D_p = bubble diameter = $0.015 (4\bar{\rho}g_c\sigma De/fG^2)$

Re = Reynolds number

ρ_g = vapor density

ρ_l = liquid density

$\bar{\rho}$ = average density of core fluid.

The use of a two-phase enhancement factor, $[1 - a(\rho_l - \rho_g)/\rho_g]$, is supported by the measurements of turbulence in two-phase bubbly flow by Serizawa and Kataoka.¹²⁴ They found that although bubbles might suppress turbulence in the central region of the tube, there was a substantial enhancement in the region near the wall.

Weisman and Pei¹²³ assumed that the velocity fluctuations are normally distributed with a mean of zero and standard deviation $\sigma_{v'}$. The parameter ψ , which represents the velocity fluctuations that are effective in reaching the wall, was then computed to be

$$\psi = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(v_{11}/\sigma_{v'})^2\right] - \left(\frac{1}{2}\right)(v_{11}/\sigma_{v'}) \text{erfc}[v_{11}/(\sqrt{2}\sigma_{v'})] \quad (4.192)$$

where v_{11} = radial velocity produced by vapor generation = $q_b''/(\rho_g H_{fg})$, and $\sigma_{v'}$ = standard deviation of radial fluctuating velocity = $(G/\bar{\rho})i_b$. The two empirical coefficients k and a required by the foregoing formulation were determined by fitting a large number of uniform heat flux CHF experiments.

Ying and Weisman¹²⁵ examined higher qualities and lower velocities than those originally considered. They were able to extend the void fraction range over which the model applied by recognizing that at void fractions above 0.6, the void fraction in the core fluid is not uniform in the radial direction. The quality of the core fluid adjacent to the bubbly layer is below the core average quality. Therefore, when $\alpha > 0.6$, x_1 in Eq. (4.189) is replaced by $(x_1)_{\text{eff}}$, which is appropriately below x_1 . In addition, it is necessary to reduce a at low mass velocities. Ying and Weisman¹²⁵ suggested that at low velocities, a should be a function of the superficial liquid velocity at the inlet. More recent work indicated that a should be a function of the

ratio of actual mass flux to the mass flux at the onset of dispersed bubble flow. Best results are obtained with

$$a = \begin{cases} 0.135 + 1.51(1 - G/G_c)^2 - 0.25(1 - G/G_c) & G < G_c \\ 0.135 & G_c < G < 9.7 \times 10^6 \text{ kg}/(\text{hr m}^2) \\ 0.135(G/9.7 \times 10^6)^{-0.3} & G \geq 9.7 \times 10^6 \text{ kg}/(\text{hr m}^2) \end{cases} \quad (4.193)$$

where G_c is the mass flux at onset of dispersed flow as determined from Fig. 3.14 and Table 3.III. To avoid the possibility of an increase in the CHF value with decreasing G at low pressures, the product $i_b G$, for $G < G_c$, may not exceed the value of the product at $G = G_c$. With these modifications, the prediction procedure has been shown to apply over the following range:

$$\begin{aligned} 0.004 &\leq \rho_g / \rho_l \leq 0.41 \\ 0.25(G/G_c) &\leq G \leq 150 \times 10^6 \text{ kg}/(\text{hr m}^2) \\ 3.5 \text{ cm} &\leq L \leq 360 \text{ cm} \\ 0.115 &\leq D \leq 3.75 \text{ cm} \\ \alpha_{\text{CHF}} &\leq 0.8. \end{aligned}$$

The Weisman-Pei model is able to predict CHF conditions for water, refrigerant 11, refrigerant 12, refrigerant 113, several of the newer HFC refrigerants, anhydrous ammonia, and liquid helium.

Weisman and Ileslamlou¹²⁶ considered the highly subcooled region where the original predictive procedure tended to overpredict the available data somewhat. They concluded that the overpredictions were the result of using Eq. (4.188) to predict the boiling heat flux in a region where the liquid enthalpy H_l was close to the enthalpy of bubble detachment H_{ld} . To avoid this difficulty, they used the basic energy balance for the bubbly layer to compute the CHF; that is,

$$q''_{\text{DNB}} = G \psi i_b (H_2 - H_1) \quad (4.194)$$

where H_2 = enthalpy of fluid in bubbly layer when α_2 is 0.82 = $H_f(1 - x_2) + H_g x_2$, and H_1 = enthalpy of fluid in core. The fraction F of the total energy going to produce vapor was obtained by simultaneous solution of the energy balances [Eq. (4.194)] and the mass balance [Eq. (4.187)]. This allows determination of v_{11} from

$$v_{11} = F q'' / (\rho_g H_{fg}) \quad (4.195)$$

and hence enables the computation of ψ . With these revisions, they were able to predict satisfactorily highly subcooled DNB for water, refrigerant 113, and liquid nitrogen at thermodynamic equilibrium qualities as low as -0.68 . They suggested that this revised procedure be used whenever the thermodynamic equilibrium quality is below -0.12 . In this highly subcooled region, the actual vapor quality in the core region is so low that it was taken as zero.

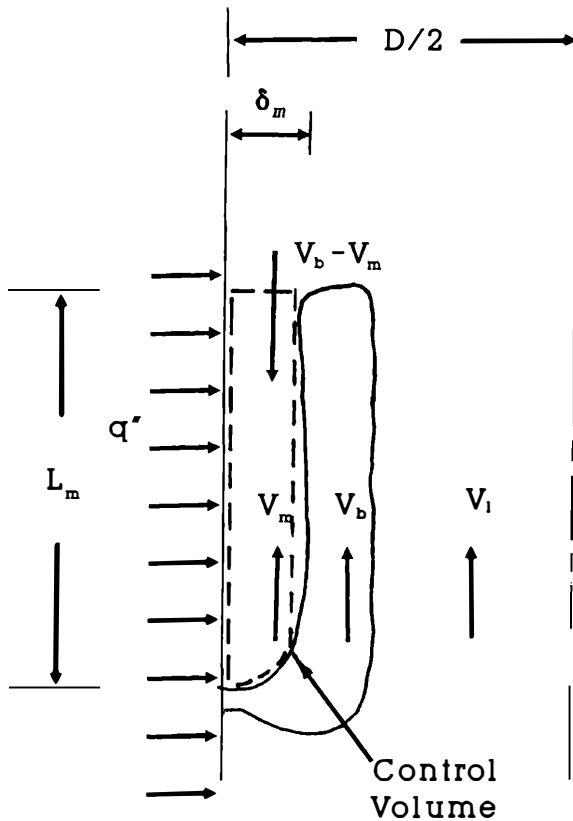


Fig. 4.17 Schematic representation of sublayer dryout according to Lee and Mudawar [from Ref. 121].

Chang and Lee¹²⁷ who used the same general approach as Weisman and Ileslamlou,¹²⁶ determined the mass flux entering the bubbly layer from a momentum balance model. However, their approach appears to be somewhat less accurate than the original model and to be limited to water.

An alternative model for subcooled and low-quality DNB was originated by Lee and Mudawwar.¹²¹ They explained the onset of DNB in terms of the dryout of a very thin liquid sublayer that lies below the vapor layer that forms at DNB (see Fig. 4.17). Their energy balance on this sublayer was written as

$$q''_{\text{DNB}} = G_m \delta_m [H_{fg} + a_1 (H_f - H_m)] / L_m \quad (4.196)$$

where

a_1 = empirical constant

G_m = liquid mass flux flowing into the sublayer

δ_m = thickness of liquid sublayer

L_m = length of vapor clot

H_f = saturated liquid enthalpy

H_m liquid enthalpy flowing into the sublayer.

Helmholtz instability was assumed to govern the release of the vapor clot. Hence, the length L_m of the vapor clot was assumed equal to the Helmholtz critical wavelength, and after simplification,

$$L_m = 2\pi\sigma'(\rho_l + \rho_g)/(\rho_l\rho_g V_b^2) \quad (4.197)$$

where V_b = velocity of vapor clot or blanket, and σ' = surface tension. The value of G_m is simply obtained from

$$G_m = \rho_l V_b \quad (4.198)$$

where ρ_l is the density of saturated liquid and V_b is determined by a balance of the buoyancy and drag forces acting on the vapor. The sublayer thickness δ_m is based on a force balance on the vapor clot in the radial direction. The rate of momentum produced by sublayer evaporation into the vapor is balanced by a lateral force caused by the rotation of the blanket due to the relative velocity between the two phases.

Since the accuracy of the original Lee and Mudawwar¹²¹ model was somewhat less than desired, Lin *et al.*¹²⁸ developed a modified version of this predictive procedure. The major changes included in their revision were as follows:

1. Use of the homogeneous two-phase flow model for fluid properties instead of single-phase fluid properties
2. Use of a liquid enthalpy flowing into the liquid sublayer under the clot, which is equal to the bulk liquid enthalpy below saturation and to saturation enthalpy above saturation.

With these modifications, good accuracy was obtained.

Although Lin *et al.*'s revision¹²⁸ of the Lee and Mudawwar prediction achieves good accuracy with water data, it has not been shown to apply to any other fluid. Katto¹¹⁷ has presented an alternative revision to the Lee and Mudawwar approach, which can be applied to other fluids. Katto's approach differs from the Lee and Mudawwar¹²¹ computations in the manner in which they determine the sublayer thickness δ_m and the velocity of the vapor blanket. The sublayer thickness is predicted using nearly the same equation as that used by Haramura and Katto¹²⁹ for the liquid sublayer in saturated pool boiling. The velocity of the vapor blanket V_b is taken as an empirically determined fraction of the fluid velocity at a distance δ_m from the wall; that is,

$$V_b = k'' V_\delta \quad (4.199)$$

where V_δ is the velocity at distance δ_m from the wall. With these modifications, they were able to obtain moderate agreement for subcooled boiling of water and other fluids. More recently, Katto¹³⁰ improved his earlier model by presenting a revised empirical equation for k'' . He suggests that

$$k'' = \frac{242[1 + K''(0.335 - \alpha)]\text{Re}^{-0.8}}{\left[0.0197 + \left(\frac{\rho_g}{\rho_l}\right)^{0.733}\right] \left[1 + 90.3\left(\frac{\rho_g}{\rho_l}\right)^{3.68}\right]} \quad (4.200)$$

where

$$k'' = \begin{cases} 0 & \text{for } \alpha > 0.355 \\ 3.76 & \text{for } \alpha < 0.355 \end{cases}$$

With this modification, improved agreement was obtained for both the water and nonwater data. However, the revised Katto theory is still less accurate than that of Weisman and Pei, particularly for refrigerants.¹³¹ It should also be noted that the use of Eq. (4.200), which is not theoretically based, makes the Katto¹³⁰ approach substantially more empirical than the other phenomenologically based approaches discussed here.

It would seem somewhat surprising that two rather different phenomenological approaches to DNB are both capable of providing good predictions of observed behavior. It is possible that both approaches are basically correct. The bubble crowding and coalescence theory focuses on conditions just before DNB; that is, just before the formation of a vapor clot (point A on Fig. 4.18). The sublayer dryout theory considers the behavior just after a clot has been formed (CHF point; point B on Fig. 4.18). Since the heat fluxes at the DNB and CHF points (points A and B) are close to each other, either location should be satisfactory as a basis for DNB prediction.

Before leaving the subject of phenomenological modeling it may be useful to use the Pei-Weisman model [Eq. (4.189)] to point out some of the data trends seen with subcooled and low-quality CHF. In the subcooled region, the ratio $(H_f - H_{ld})/(H_l - H_{ld})$, representing the reciprocal of the fraction of the heat producing vapor, increases with increased subcooling. Thus CHF increases with increasing subcooling. Once the liquid is saturated, this ratio remains at unity but $(x_2 - x_1)$ decreases as the fluid quality increases. Hence, CHF decreases with increasing quality. The CHF value will also decrease with decreasing values of G but at less than a linear rate. This arises since G' is primarily determined by the product $i_b G$, and i_b increases slightly with decreasing G . We may also observe that $H_{f,g}$ goes toward zero as the pressure approaches the critical pressure. Therefore, CHF will also approach zero. All of the foregoing trends are observed experimentally. We may thus maximize DNB heat fluxes by using high subcooling, high mass fluxes, and low pressures.

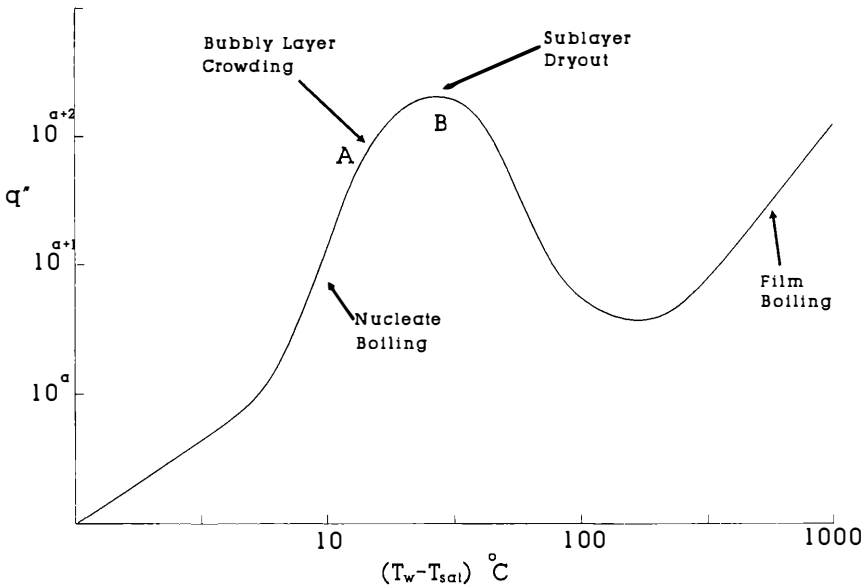


Fig. 4.18 Typical forced convection boiling curve showing suggested relationship between CHF theories.

4.4.2.2 Empirical Correlations for Uniformly Heated Round Tubes

Despite progress in developing analytical and phenomenological predictive schemes, most reactor design is accomplished using empirical, dimensional correlations carefully tested by application to experimental data obtained under conditions similar to those of PWR operation. Early correlations were based on data obtained in round tubes and assumed to apply to rod bundles. More recently, correlations were devised using rod bundle data directly or from round tube or annulus correlations modified to specifically account for rod bundle effects.

For dryout conditions, the usual prediction method is the so-called critical quality boiling length approach. This procedure, originally suggested by Bertoletti *et al.*¹³² for correlation of data from test sections with axially uniform heating, states that there is a critical quality x_c at dryout that depends on the boiling length (distance from the point at which fluid reaches saturation enthalpy); that is,

$$x_c = \frac{aL_b}{b + L_b} \quad (4.201)$$

where L_b = boiling length, and a, b = parameters depending on mass flux, pressure, and hydraulic diameter.

It can be shown¹³¹ that for relatively long boiling lengths, entrainment-deposition theory leads to a similar relationship for test sections with uni-

form axial heating. Hewitt and Hall-Taylor¹³³ point out that Eq. (4.181) may be simplified for long test sections near the dryout location. For long sections, the change in liquid film flow rate per unit length dW_{LF}/dz becomes small and may be neglected. Furthermore, Hewitt and Hall-Taylor indicate that near the dryout location, the liquid film on the surface is so thin that the entrainment rate E' is also negligible. Equation (4.181) then simplifies to

$$q''/H_{fg} = R \quad (4.202)$$

After noting that at the CHF location, all of the liquid is entrained, we have

$$R = K_d(1 - x_c)\rho_{avg} \quad (4.203)$$

where K_d = deposition coefficient. For a uniformly heated test section, the average heat flux can be expressed by

$$q'' = \frac{x_c GDH_{fg}}{4L_b} \quad (4.204)$$

where D is the diameter of the test section. After replacing R and q'' in Eq. (4.202), we have

$$x_c = \frac{4L_b K_d \rho_{avg}}{4L_b K_d \rho_{avg} + GD} = \frac{L_b}{GD/(4K_d \rho_{avg}) + L_b} \quad (4.205)$$

which is in the same form as the empirical relationship of Eq. (4.201). With an appropriate deposition coefficient correlation, reasonable agreement with experimental data is obtained.¹³¹

The simple form of Eq. (4.205) is useful in understanding some of the observed experimental trends for CHF at high qualities. Since the mass flux is in the denominator, the critical quality decreases with increasing G . However, the critical quality decreases at a rate that is less than linear in G . Thus, in the usual flow ranges encountered in nuclear reactors, the total channel power to CHF, which is proportional to the product of x and G , still increases with G .

For subcooled and low to moderate quality DNB, predictions of the boiling crisis focus on the critical heat flux rather than on a critical exit quality. One of the earliest successful and widely used DNB correlations for PWR conditions is the so-called W-3 correlation due to Tong.¹³⁴ For uniformly heated round tubes

$$\begin{aligned} \frac{q''_{crit,EU}}{10^6} = & \{(2.022 - 0.0004302P) + (0.1722 - 0.0000984P) \\ & \times \exp[(18.177 - 0.004129P)\chi]\} \\ & \times [(0.1484 - 1.596\chi + 0.1729\chi|\chi|)G/10^6 + 1.037] \\ & \times (1.157 - 0.869\chi) \times [0.2664 + 0.8357 \exp(-3.151De)] \\ & \times [0.8258 + 0.000794(H_{sat} - H_{in})] \end{aligned} \quad (4.206)$$

where

$$P = 1000 \text{ to } 2300, \text{ psia}$$

$$G = 1.0 \times 10^6 \text{ to } 5.0 \times 10^6, \text{ lb}/(\text{hr ft}^2)$$

$$De = 0.2 \text{ to } 0.7, \text{ in.}$$

$$X_{\text{loc}} \leq 0.15$$

$$H_{\text{in}} \geq 400 \text{ Btu/lb}$$

$$L = 10 \text{ to } 144, \text{ in.}$$

$$\frac{\text{heated perimeter}}{\text{wetted perimeter}} = 0.88 \text{ to } 1.00$$

$$q''_{\text{crit}, EU} = \text{critical heat flux for uniformly heated tubes, Btu}/(\text{hr ft}^2).$$

The Bettis Atomic Power Laboratory has developed a design equation for water flowing in the narrow (~ 0.1 in.) rectangular channels used in the first Shippingport core. This critical heat (Btu/hr ft²) is given by

$$q''_{\text{crit}}/10^6 = K[(H' - H)/(H' - H_0)]^{1/2} \quad (4.207)$$

where

$$K = 0.84\{1 + [(2000 - P)/800]^2\}$$

$$H_0 = 655 - 0.004(2000 - P)^{1.63}$$

$$H' = H_v - 0.275H_{fg} - 0.725H_{fg}(300/H_{fg})^{10^6/}$$

$$H_{fg} \text{ heat of vaporization, Btu/lb}$$

$$H_v = \text{enthalpy of saturated vapor, Btu/lb}$$

$$P = \text{pressure, psia.}$$

Some investigators have taken an entirely different approach to CHF prediction.^{136,137} Rather than provide a correlating equation, the predicted values are presented in the form of look-up tables where the predicted critical heat flux is tabulated as a function of pressure, local quality (quality at CHF location), and mass flux. The data are presented for a uniformly heated tube of given diameter, which is sufficiently long so that (L/D) effects are not significant. Table 4.IV presents a portion of the CHF look-up table devised by Groenveld *et al.*¹³⁷ for the range of 5 to 17 MPa (50 to 170 bar). The data are presented for a tube diameter of 8 mm.

To use these predictions for other tube diameters or short lengths, the table values are multiplied by correction factors. Groenveld *et al.*¹³⁷ indicate that in order to correct for other tube diameters the values in the look-up table should be multiplied by K_1 where

$$K_1 = \begin{cases} \left(\frac{8}{De}\right)^{1/3} & \text{for } 2 \text{ mm} < De < 16 \text{ mm} \\ 0.79 & \text{for } De > 16 \text{ mm} \end{cases} \quad (4.208)$$

and De = equivalent diameter (mm). To correct for (L/D) effects, Groenveld *et al.*¹³⁷ suggest that the results be multiplied by a second correction factor, K_2 , where

$$K_2 = \exp\left(\frac{De}{L} e^{2\alpha}\right) \quad (4.209)$$

and α = void fraction evaluated from homogeneous model. Inclusion of the void fraction in Eq. (4.209) predicts a diminishing length effect at subcooled and low-quality conditions.

Both the CHF look-up table and the empirical CHF correlations of this section have all been based on data obtained with vertical tubes. It has been observed by several investigators^{138,139} that at high mass fluxes there is essentially no difference between vertical and horizontal CHF data. However, as the mass flux is reduced, lower critical heat fluxes are seen. This is generally attributed to vertical stratification, which results in less liquid in the upper portion of the tube and hence an earlier CHF in the upper region. On the basis of the examination of the available data for horizontal bundles, Wong *et al.*¹⁴⁰ recommend that the CHF flux from the AECL look-up table be modified by

$$(q''_{Cr})_H = (q''_{Cr})_V \left\{ 1 - \exp\left[-\left(\frac{T'}{3}\right)^{0.5}\right] \right\} \quad (4.210)$$

where $(q''_{Cr})_H, (q''_{Cr})_V$ = critical heat flux for horizontal and vertical tubes, respectively,

$$T' = 0.046 \text{ Re}_L^{-0.2} \left(\frac{1-x}{1-\alpha}\right)^2 \left(\frac{G^2}{gD\rho_l(\rho_l - \rho_g)\alpha^{0.5}}\right)$$

and Re_L = Reynolds number based on all liquid flow.

4.4.2.3 CHF in Uniformly Heated Rod Bundles

In BWRs, bundle size is small and, hence, velocity and enthalpy gradients do not tend to be severe. Bundle average parameters are often used for this situation. The early Janssen-Levy¹⁴¹ correlation as well as the later CISE-GE correlation¹⁴² are of this type. The CISE-GE correlation¹⁴² is a modification of the Bertoletti *et al.*¹³² correlation for round tubes [Eq. (4.201)] and the quality at CHF, x_{Cr} , is given by

TABLE 4.IV
Critical Heat Flux (in kW · m⁻²) Look-Up Table for 8-mm Tubes

Pressure (kPa)	Mass Flux (kg m ⁻² s ⁻¹)	CHF Quality											
		-0.1	-0.05	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	
5000	1500	6859	6707	6441	5779	5317	4899	4530	4074	3623	3337	2983	
5000	2000	6944	6593	6110	5262	4779	4405	3984	3610	3206	2865	2557	
5000	2500	7195	6565	5849	4915	4515	3981	3594	3401	3067	2474	1861	
5000	3000	7353	6543	5664	4750	4321	3782	3428	3268	2855	2024	1406	
5000	3500	7551	6527	5421	4581	4144	3693	3380	3109	2510	1688	1195	
5000	4000	7764	6476	5139	4421	3916	3540	3317	2945	2221	1437	1140	
5000	4500	7980	6502	5044	4317	3784	3457	3260	2799	2059	1425	1247	
5000	5000	8443	6655	4986	4255	3723	3454	3251	2745	1990	1470	1385	
7000	1500	5886	5603	5145	4673	4301	3874	3486	3189	2964	2735	2523	
7000	2000	6142	5684	4952	4275	3785	3407	3122	2890	2731	2451	2023	
7000	2500	6690	5806	4876	4104	3537	3147	2922	2723	2445	1983	1367	
7000	3000	6953	5816	4724	3981	3369	2940	2714	2491	2133	1570	967	
7000	3500	7135	5848	4567	3834	3199	2786	2565	2294	1896	1323	782	
7000	4000	7398	5938	4372	3469	2928	2645	2490	2201	1730	1228	840	
7000	4500	7399	6061	4410	3347	2743	2515	2418	2139	1640	1200	930	
7000	5000	7427	6225	4552	3378	2696	2458	2380	2105	1591	1193	1010	
9000	1500	4958	4693	4467	4189	3730	3227	2850	2712	2495	2034	1429	
9000	2000	5299	4835	4196	3569	3097	2746	2448	2312	1931	1428	1027	
9000	2500	6025	5004	4119	3339	2797	2521	2241	1942	1373	1028	809	
9000	3000	6399	5097	4045	3240	2604	2289	2038	1724	1354	967	708	
9000	3500	6453	5140	3995	3155	2487	2124	1896	1643	1276	952	650	
9000	4000	6674	5266	3983	3066	2349	1931	1750	1604	1375	982	664	
9000	4500	6936	5469	4047	3026	2245	1827	1719	1631	1385	1037	771	
9000	5000	7305	5645	4183	3080	2249	1836	1755	1663	1416	1152	908	
11000	1500	4389	4171	3806	3390	2982	2564	2201	1934	1567	1212	675	
11000	2000	4675	4221	3695	3122	2619	2195	1772	1484	972	738	554	
11000	2500	5371	4332	3676	3005	2461	1970	1537	1265	818	529	399	
11000	3000	5785	4446	3668	2920	2257	1695	1311	1109	802	457	322	
11000	3500	5939	4491	3619	2843	2127	1570	1251	1101	820	524	407	
11000	4000	6242	4605	3503	2761	1952	1471	1266	1160	962	733	572	
11000	4500	6551	4757	3566	2765	1999	1476	1336	1256	1133	922	699	
11000	5000	6753	4892	3707	2890	2004	1540	1491	1440	1299	1057	804	
13000	1500	3594	3201	2766	2390	2013	1697	1396	1182	937	699	476	
13000	2000	3820	3363	2852	2361	1983	1637	1277	962	638	455	387	
13000	2500	4126	3471	2940	2403	1965	1567	1149	797	504	362	290	
13000	3000	4509	3631	3022	2449	1962	1434	1031	777	562	385	309	
13000	3500	4775	3802	3091	2491	1978	1413	1065	874	681	462	364	
13000	4000	5113	4060	3223	2622	2024	1493	1222	1050	882	695	586	
13000	4500	5464	4230	3342	2784	2106	1604	1362	1188	1023	866	712	
13000	5000	5834	4420	3429	2879	2232	1789	1532	1324	1139	950	769	

TABLE 4.IV (Continued)

Pressure (kPa)	Mass Flux (kg m ⁻² s ⁻¹)	CHF Quality											
		-0.1	-0.05	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	
15000	1500	3155	2674	2312	1955	1649	1367	1179	1025	850	595	368	
15000	2000	3351	2833	2464	2071	1843	1501	1178	918	664	424	281	
15000	2500	3686	3083	2705	2224	1912	1559	1171	891	618	395	272	
15000	3000	4010	3336	2854	2322	1974	1595	1225	954	675	461	344	
15000	3500	4226	3459	2904	2340	2055	1670	1339	1062	769	557	454	
15000	4000	4502	3517	2916	2615	2270	1815	1483	1217	939	750	673	
15000	4500	4808	3552	3057	2953	2472	1951	1607	1355	1103	934	841	
15000	5000	5383	3789	3255	3188	2657	2131	1774	1495	1250	1083	944	
17000	1500	2320	2043	1753	1526	1305	1187	1020	859	685	527	361	
17000	2000	2568	2288	2018	1757	1533	1448	1223	993	780	567	405	
17000	2500	2911	2529	2283	1991	1790	1637	1408	1148	885	618	462	
17000	3000	3197	2726	2415	2218	2081	1832	1562	1289	1006	719	568	
17000	3500	3398	2844	2588	2526	2337	1993	1685	1399	1114	831	694	
17000	4000	3528	2931	2765	2746	2519	2134	1807	1533	1246	978	832	
17000	4500	3694	3017	2902	2863	2619	2245	1935	1681	1380	1120	947	
17000	5000	3933	3247	3128	3110	2821	2406	2092	1833	1539	1273	1048	

Note: 100 kPa = 1 bar.

Source: Groeneveld and L. Leung, Atomic Energy of Canada Ltd., Chalk River, Ontario, personal commun. (1994).

$$x_{Cr} = \frac{AL_B}{B + L_B} \left(\frac{1.24}{R_f} \right)^{1/2} \quad (4.211)$$

where

$$A = 1.055 - 0.013 \left[\frac{(P - 4.137 \times 10^6)}{2.758 \times 10^6} \right]^2 - 1.233(7.37 \times 10^{-4}G) \\ + 0.907(7.37 \times 10^{-4}G)^2 - 0.205(7.37 \times 10^{-4}G)^3$$

and

$$B = 0.457 + 2.003(7.37 \times 10^{-4}G)^2 \text{ for } 100 < G(\text{kg/m}^2 \text{ s}) \leq 1400$$

G = mass flux (kg/m² s)

R_f = radial peaking factor in fuel assembly

P = pressure (Pa).

Bundle-average or mixed-flow approaches are not normally used in analyzing PWR behavior. Subchannel analysis codes, such as THINC⁷³ or COBRA,²⁷ are used to determine flow and enthalpy variation in the hot channel. These conditions are then used in an appropriate critical heat flux correlation to determine the critical heat flux as a function of axial position. The validity of this approach has been demonstrated by several investigators^{4,70,71} who have successfully correlated rod bundle data in this manner.

The first DNB correlations used for rod bundles in conjunction with sub-channel analysis programs were adaptations of those developed for round tubes. The W-3 correlation [Eq. (4.206)] was adapted for rod bundle use by multiplying it by F_s , a grid or spacer factor. This factor is intended to account for bundle effects such as those introduced by the presence of spacer grids. One formulation for this factor is¹⁴³

$$F_s = (P/225.896)^{0.5} (1.445 - 0.0371L) \{ \exp[(\chi + 0.2)^2] - 0.73 \} + K_s (G/10^6)(\alpha/0.019)^{0.35} \quad (4.212)$$

where

P system pressure, psia

L total heated core length, ft

χ = quality

K_s = axial grid spacing coefficient with the following values:

Grid Spacing (in.)	K_s
32	0.027
26	0.046
20	0.066

Note the increase in F_s and, hence, in q''_{crit} as the grid spacing is decreased. This spacer factor relationship [Eq. (4.212)] should not be used for grids without mixing vanes.

Groenveld *et al.*¹³⁷ indicate that the CHF values from the round-tube look-up table may also be adapted to rod bundles by replacing D by De and multiplying the resulting round-tube prediction by both a grid spacer factor and a bundle factor. For the grid spacer factor, F_s , they suggest

$$F_s = 1 + A' \exp(-B' L_{sp}/De) \quad (4.213a)$$

and for the bundle factor

$$K_3 = \min[0.8, 0.8 \exp(-0.5x^{1/3})] \quad (4.213b)$$

where

x quality at CHF location

L_{sp} = distance between grid spacers

$A' = 1.5K_p^{0.5}(G/1000)^{0.2}$

$B' = 0.10$

G = mass flux ($\text{kg}/\text{m}^2 \text{ s}$)

K_p = grid pressure loss coefficient.

Groenveld *et al.* note that more accurate results are obtained if A' and B' are separately determined for each grid spacer since K_p is not an entirely adequate way of correlating the grid effect.

Most design correlations have been directly derived from rod bundle data. For example, based on tests of uniformly heated rod bundles, Gallerstedt *et al.*¹⁴⁴ proposed

$$q''_{\text{crit,EU}} = \frac{(a - bD_e)A_1(A_2G)^{A_3 + \frac{-2000}{P}} - A_9G\chi H_{fg}}{A_5(A_6G)^{A_7 + A_8(P - 2000)}} \quad (4.214)$$

where

H_{fg} = heat of vaporization, Btu/lb

G = mass velocity, lb/hr ft²

χ = quality

$a = 1.15509$

$b = 0.40703$

$A_1 = 0.3702 \times 10^8$

$A_2 = 0.59137 \times 10^{-6}$

$A_3 = 0.8304$

$A_4 = 0.68479 \times 10^{-}$

$A_5 = 12.71$

$A_6 = 0.30545 \times 10^{-}$

$A_7 = 0.71186$

$A_8 = 0.20729 \times 10^{-}$

$A_9 = 0.15208.$

This correlation replaces an earlier one devised by Wilson and Ferrell.¹⁴⁵ Current (1995) proprietary constants differ from those shown above.

A widely used generally applicable DNB correlation is that developed by EPRI.¹⁴⁶ It is applicable to both PWR and BWR conditions and has the form

$$q''_{\text{Cr}} = \frac{A_o - x_{in}}{C_o F_s F_{nu} + \left(\frac{x_L - x_{in}}{q''_L} \right)} \quad (4.215)$$

where

q''_{Cr} = critical heat flux (Btu/hr ft²)

x_L, x_i local and inlet qualities

q''_L local heat flux

F_s = grid spacer factor = $1.3 - 0.3K_p$

F_{nu} = nonuniform axial flux factor (see Sec. 4.2.2)

$$A_o = a_1 P_R^{a_2} G^{(a_5 + a_7 P_R)}$$

$$C_o = a_3 P_R^{a_4} G^{(a_6 + a_8 P_R)}$$

P_R = reduced pressure (P/P_{critical})

G = mass flux (10^6 lb/hr ft²)

K_p = grid pressure loss coefficient.

The coefficients a_1 through a_8 are 0.5328, 0.1212, 1.6151, 1.4066, -0.304 , 0.4843 , -0.3285 , and -2.0749 .

The correlation holds for

$$200 \leq P \leq 2450 \text{ psia}$$

$$0.2 \times 10^6 \text{ lb/hr ft}^2 < G \leq 4.1 \times 10^6 \text{ lb/hr ft}^2$$

$$-0.25 < x_L < 0.75$$

$$30_{in} \leq L \leq 168 \text{ in.}$$

$$0.35_{in} < De \leq 0.55 \text{ in.}$$

In deriving the EPRI correlation, the local thermal hydraulic conditions in the bundle were determined using COBRA with a mixing coefficient held constant at a value appropriate for simple grids. The grid factor used with Eq. (4.215) may, therefore, in part, reflect the improved interchannel mixing with high-pressure-drop grids. Although the EPRI correlation has a very wide range, it has the undesirable feature of containing the heat flux on both sides of the equation.

When PWR rod bundles were changed to a 17×17 array from the previously common 15×15 array, it was observed that the rod bundle correlation available at that time [e.g., Eqs. (4.214) and (4.206) modified by Eq. (4.212)] failed to predict the behavior of the new bundles adequately. This has led some reactor vendors away from the use of generalized correlations supposedly capable of predicting all available rod bundle data. Instead, vendors have developed correlations geared to their rod bundle designs. A common general correlation form is used but at least some of the constants are varied with bundle design. This is the approach taken with the WRB-1 correlation.¹⁴⁷ This correlation has the form

$$q''_{Cr} \times 10^{-6} = PF + A_1 + \frac{B_3 G}{10^6} - \frac{B_4 G}{10^6} x \quad (4.216)$$

where G and x refer to local conditions and

q''_{Cr} = critical heat flux (Btu/hr ft²)

PF = performance factor depending on rod diameter and mixing vane design

A_1 = function depending on P , G , H , L , grid spacing, De , heated equivalent diameter and distance from last grid

$B_3 = \text{constant}$

$B_4 = \text{function depending on } P, G, \text{ and } L.$

The WRB-1 correlation is restricted to $1440 \leq P \leq 2490$ psia, $0.9 \times 10^6 \leq G \leq 3.7 \times 10^6$ lb/hr ft², and $-0.2 \leq x \leq 0.3$. The correlation is stated to provide an appreciably better fit of rod bundle data than the previously used W-3 correlation. A WRB-2 correlation, which is similarly formulated, is used for later fuel assembly designs. Unfortunately, the specific parameters needed to use these correlations are considered proprietary. Other recent reactor vendor correlations have also been treated as proprietary.

All of the foregoing rod bundle correlations have been devised for vertical rod bundles using light water as the coolant. As noted previously, vertical flow correlations should apply to horizontal flow providing the mass flux is above that required for the onset of dispersed flow. However, the effect of heavy water properties needs to be included for CANDU reactor calculations. The U-1 correlation,¹⁴⁸ which applies to D₂O in horizontal CANDU bundles is

$$q''_{Cr} = \frac{1.0184 \times 10^{-4} P^{0.21249} G^{1.93244} - 1.2264 \times 10^{-11} P^{1.65613} G^{2.3235} x}{1.0 + (5.3917) \times 10^{-6} P^{-0.17342} G^{1.77740}} \quad (4.217)$$

where

q''_{Cr} = bundle average critical heat flux (kW/m²)

P = pressure (kPa)

G = bundle average mass flux (kg/m² s)

x = bundle average quality.

The correlation holds for $6.6 < P < 10.5$ MPa, $1500 < G < 5000$ (kg/m² s) and $0.18 \leq x_{\text{exit}} < 0.46$. The relatively low value for the upper limit on P is satisfactory because CANDU reactors operate at ~ 9.6 MPa. The high-quality range of the correlation reflects the long length of CANDU elements (6 m) and hence the higher x at CHF conditions. Note that in contrast to the correlations used for the larger vertical assemblies, which are based on local conditions obtained using subchannel analysis, bundle average conditions are used for the smaller CANDU assemblies.

4.4.2.4 Application of Phenomenological CHF Models to Rod Bundles

Both the Weisman and Pei¹²³ and the Lin *et al.*¹²⁸ prediction procedures have been applied to rod bundles at subcooled to moderate quality conditions at the bundle exit. When used with an appropriate subchannel analysis program, both correlations were capable of predicting DNB in uniformly heated rod bundles. Weisman,¹³¹ who had access to vendor data on mixing coefficients, was able to obtain good accuracy without the aid of a grid factor or any revision of the correlation so long as local conditions

were used. Lin *et al.*,¹⁴⁹ who did not have access to mixing data, assumed a constant mixing factor for all grids and found it necessary to use a grid correction factor.

The success of these round-tube models with rod bundle DNB indicates that (1) rod bundle DNB behavior is indeed similar to that of round tubes and (2) that the supposition that low-quality DNB is primarily controlled by local conditions applies to both round tubes and rod bundles.

Entrainment-deposition models have been successfully applied to rod bundle dryout at high qualities. The success of the modeling depends on the accuracy of the entrainment and deposition modeling within the sub-channels of the rod bundle. The most comprehensive of the computer programs devised for this application¹¹³ consider redistribution of the liquid film around the rods and interchannel mixing.

Although phenomenologically based modeling has been successful in achieving good rod bundle CHF predictions, most reactor vendors believe that their empirical correlations provide more accurate predictions than those attainable with current (1995) phenomenological models.

4.4.2.5 Effect of NonUniform Axial Heat Flux

At BWR conditions, most designers have concluded that dryout is determined primarily by the total heat input to the channel. Dryout correlations based on the boiling length concept are found to apply to both uniform and nonuniform axial heat fluxes. Thus Eqs. (4.201) and (4.211) may be applied to nonuniform flux shapes without revision. Dryout at BWR qualities will occur close to the tube exit even though the heat flux is considerably higher near the central region of the tube or bundle.

At PWR conditions (subcooled or low quality), DNB will not occur at the tube exit with a cosinusoidal or skewed cosinusoidal flux distribution. Here DNB is in the high flux region and is governed primarily by local conditions. However, it has been found by most investigators that a correction factor must be applied to empirical correlations that have been derived on the basis of data from tubes or bundles with a uniform axial heat flux. Most correlations based on uniform heating slightly overpredict the data from nonuniformly heated channels at PWR conditions.¹⁵⁰ At these conditions, this overprediction is of the order of 5%.

The common approach for PWR design was developed by Tong *et al.*¹⁵¹ They were able to extend Tong's¹³⁴ design correlation [Eq. (4.206)] to channels with a nonuniform axial flux distribution by

$$q''_{\text{crit},N} = q''_{\text{crit},EU} / F \quad (4.218)$$

where $q''_{\text{crit},N}$ = DNB heat flux for the nonuniformly heated channel; $q''_{\text{crit},EU}$ = equivalent uniform DNB flux from Eq. (4.206);

$$F = \frac{C}{q''_{\text{local}} [1 - \exp(-Cl_{\text{DNB}, EU})]} \int_{l_{OB}}^{l_{\text{DNB}, N}} q''(z) \exp[-C(l_{\text{DNB}, N} - z)] dz$$

$$C = 0.44 \frac{(1 - x_{\text{DNB}})^{7.9}}{(G/10^6)^{1.72}} \quad [\text{in.}^{-1}] \quad (4.219)$$

and

$l_{\text{DNB}, EU}$ = axial location at which DNB occurs for uniform heat flux, in.

$l_{\text{DNB}, N}$ = axial location at which DNB occurs for nonuniform heat flux, in.

l_{OB} = axial location at which nucleate boiling begins

x_{DNB} = quality at DNB location under nonuniform heat flux conditions

G = mass flux (lb/hr ft²).

This approach successfully correlates a wide variety of nonuniform flux distribution data.

Tong¹⁵² notes there is theoretical justification for this approach since the flow, particularly in the boundary-layer region, coming from the upstream region carries superheat and bubbles with it when it contacts the downstream surface. Thus, upstream heat flux distributions affect the boundary layer at the DNB position. This "memory effect" is conveyed by the F factor. In the subcooled region, where the value of C in Eq. (4.219) is large, the F factor is small and local heat flux determines the boiling crisis. At high qualities, C is small, the memory effect is high, and the average heat flux, or enthalpy rise, primarily determines the boiling crisis.

To use Eq. (4.219), F must be evaluated as a function of axial position. A position past the midplane is chosen and DNB is assumed to occur at this location. Hence, l_{DNB} is the axial distance at the position chosen. The quality at the location is taken as x_{DNB} and C is evaluated. The value of x_{DNB} is then used in Eq. (4.206) or its equivalent to evaluate $q''_{\text{crit}, EU}$. The length that must be heated at $q''_{\text{crit}, EU}$ to achieve x_{DNB} is $l_{\text{DNB}, EU}$. The value of F is then determined by numerically integrating Eq. (4.219) from the onset of boiling to $l_{\text{DNB}, N}$; $q''_{\text{crit}, N}$ is calculated from Eq. (4.219). The process is then repeated at a number of other axial locations, and variation of $q''_{\text{crit}, N}$ is established as a function of axial position.

Based on Tong *et al.*'s shape factor equation [Eq. (2.219)], slightly revised forms of the F factor have been devised.^{153,149} The form used by Lin *et al.*¹⁴⁹ is

$$F = \frac{C}{q''_{c, n} [1 - \exp(-Cl_{\text{DNB}, N})]} \int_{l_{OB}}^{l_{\text{DNB}, n}} q''(z) \exp[-C(l_{\text{DNB}, N} - z)] dz, \quad (4.220)$$

where

$$C = \frac{0.15(1 - x_{\text{DNB}})^{4.31}}{G^{0.478}} \quad [\text{in.}^{-1}]$$

and the mass flux G is in units of $\text{Mlb}_m/(\text{ft}^2 \text{ hr})$. This form is slightly more convenient than Eq. (4.219) because it does not require use of $l_{\text{DNB}, \text{EU}}$.

In the EPRI correlation, an even simpler form of the nonuniform factor is used.¹⁴⁶ As seen in Eq. (4.215), the factor F_{nu} appears in the denominator of the correlation. It is expressed as

$$F_{nu} = 1 + (Y + 1)/(1 + G) \quad (4.221)$$

where

$$Y = \frac{1}{l_{\text{DNB}}} \int_0^{l_{\text{DNB}}} q''(z)/q''_{l_{\text{DNB}}} dz$$

and G is again in units of $\text{Mlb}_m/(\text{ft}^2 \text{ hr})$.

Under subcooled conditions, the Groenveld *et al.*¹³⁷ look-up table apparently requires no flux distribution correction. In the annular flow region, the table value is multiplied by the ratio of the average heat flux existing over the boiling length to the local heat flux. At low qualities, the F factor of Eq. (4.220) can apparently be used to provide a conservative estimate of the CHF value. Lee and Liao¹⁵⁰ recommend that, when the table is used for axially nonuniform rod bundles, the bundle factor be revised. However, Lee and Liao¹⁵⁰ evaluated CHF using a one-dimensional systems code and did not conduct a subchannel analysis.

In contrast to most other investigators, Weisman and coworkers¹⁵⁴ found that no correction was needed when the phenomenologically based model of Weisman and Pei¹²³ was used (at subcooled and low-quality conditions) with axially varying heat fluxes in rod bundles of reactor length. Yang and Weisman¹⁵⁵ noted that the derivation of Weisman and Pei considered the term accounting for the increase in bubbly layer enthalpy with distance to be negligible. For the uniform axial flux conditions for which that theory was derived, Weisman and Pei concluded this term accounted for about 1% of the total heat being supplied. The constants used in the correlation would therefore have been adjusted slightly upward to allow for this effect. With an axially varying heat flux leading to DNB in the central region of the tube, the bubbly enthalpy reaches a maximum and (dH_2/dz) is zero at the DNB location. The measured heat flux would then be slightly less than predicted. However, the small difference would not be sufficient to be observed with the usual data scatter. Yang and Weisman's¹⁵⁵ round-tube CHF measurements agreed with this conclusion until very sharp flux variations (cosine over less than 0.5 m) were considered. For these conditions, the measurements were about 6% below predictions.

4.4.2.6 Cold Wall Effects

In a channel containing an unheated wall (e.g., subchannel adjacent to RCC tube), liquid film builds up along the cold wall. This fluid is not effective in cooling the heated surface and the fluid cooling the hot surface is effectively at a higher enthalpy than calculated by an energy balance. Thus, the critical heat flux in a channel with an unheated wall is generally lower than that in a channel with all sides heated and at the same bulk exit enthalpy.

Tong¹³⁵ extended his design correlation [Eq. (4.206)] to the case of a channel with an unheated wall by defining a cold wall factor (CWF) such that

$$q''_{\text{crit},EU,CW} = (q''_{\text{crit},EU,D_h})\text{CWF} \quad (4.222)$$

where

q''_{crit,EU,D_h} = critical heat flux evaluated from Eq. (4.206) with D_h replacing De

D_h = equivalent diameter based on heated perimeter, in.

$q''_{\text{crit},EU,CW}$ = critical heat flux in presence of cold wall

$$\text{CWF} = 1 - \text{Ra}[13.76 - 1.372 \exp(1.78\chi) - 4.732(G/10^6)^{-0.0535} - 0.0619(P/10^3)^{0.14} - 8.509D_h^{0.107}]$$

$$\text{Ra} = 1 - (De/D_h).$$

The correlation was devised for

$$\chi_{\text{DNB}} \leq 0.1$$

$$1.0 \times 10^6 \leq G \leq 5.0 \times 10^6 \text{ lb}/(\text{h ft}^2)$$

$$1000 \text{ psia} \leq P \leq 2300 \text{ psia}$$

$$L = \text{heated length} \geq 10 \text{ in.}$$

$$\text{gap} \geq 0.1 \text{ in.}$$

When the axial heat flux is nonuniform, the predicted critical heat flux

$$q''_{\text{crit},N,CW} = \frac{(q''_{\text{crit},EU,D_h})\text{CWF}}{F} \quad (4.223)$$

where F is defined in Eq. (4.219).

An alternative approach to the use of a cold-wall factor is to use an effective local quality or enthalpy directly in the CHF prediction procedure.^{149,154} Lin *et al.*¹⁴⁹ concluded that the local enthalpy in Eq. (4.196) should be computed using an effective quality, x_e , given by

$$x_e = x(p_w/p_h)^{0.7(1)} \quad (4.224)$$

where x = local quality calculated by subchannel analysis, and (p_w/p_h) = ratio of wetted to heated perimeter.

4.4.2.7 Effects of Lattice Spacing, Rod Bowing, and Surface Condition

Early investigators appear to have concluded that rod spacing has little effect on critical heat flux (CHF). However, some of these earlier tests may have been less precise than more recent measurements. It was reported¹³³ that a change in CHF was observed when going from the previously common 15×15 array to the closer spacing of the later 17×17 array. In making this change, it was necessary to multiply the spacer factor of Eq. (4.212) by 0.88 to obtain agreement with experimental data. This amounts to an $\sim 7\%$ decrease in CHF since the unmodified correlation of Eq. (4.206) predicts an $\sim 5\%$ increase in CHF. Similar behavior was seen with Eq. (4.214).

Recent vendor calculations account for spacing and grid effects by incorporating a performance factor, or similar quantity, directly in the correlation [e.g., PF in Eq. (4.216)]. More recent general correlations and prediction procedures [e.g., Eqs. (4.189) (4.196), (4.215)], which considered data with rod spacing equivalent to both 15×15 and 17×17 bundles, can predict the behavior for either spacing without revision.

A more severe test of the ability of a correlation to model spacing effects is the ability to model DNB in the very tight lattices that would be used in a high conversion PWR. While separate correlations have been proposed for tight lattices supported by spacer grids,¹⁵⁶ Akiyama *et al.*¹⁵⁷ conclude that, when multiplied by a factor of 1.025, the EPRI correlation [Eq. (4.215)] can reasonably predict the behavior of such bundles when grid spacers are used. The W-3 correlation, however, performed poorly.

On the basis of limited data, Dalle Donne and Hame¹⁵⁸ used a modification of an empirical CHF correlation for standard bundles to correlate CHF in closely spaced bundles with wire wraps as spacers. Later data by Erbacher *et al.*¹⁵⁹ indicate that wire wraps show better performance than bundles with spacers at high negative qualities and the reverse is true with near zero or positive exit qualities. Erbacher *et al.*¹⁵⁹ conclude that correlations developed for gridded bundles should not be applied to bundles with wire wraps.

After considering the data on surface roughness effects of several investigators, Tong¹³⁵ concludes that when surface projections exceed the width of the bubbly layer, subcooled CHF is increased. In high-quality flow, the reverse effect appears to be obtained.

Hill *et al.*¹⁶⁰ examined the effect of rod bowing on CHF. They found that with rods bowed to contact, there was an adverse effect above a threshold heat flux. They defined the quantity δ_{bow}

$$\delta_{\text{bow}} = \frac{q''_{\text{crit,unbow}} - q''_{\text{crit,bow}}}{q''_{\text{crit,u}}} \quad (4.225)$$

where the subscripts unbow and bow refer to conditions with unbowed (straight) and bowed rods, respectively. With this definition, δ_{bow} has a

positive value whenever rod bowing reduces the CHF. Based on their experimental measurements, Hill *et al.*¹⁶⁰ concluded that

$$\delta_{\text{bow}} = 0 \quad \text{when} \quad q''_{\text{av}} / 10^6 \leq (b_1 - b_2 P) \quad (4.226)$$

and at higher heat fluxes,

$$\delta_{\text{bow}} = (a_1 - a_2 P) \left[\frac{q''_{\text{av}}}{10^6} - (b_1 - b_2 P) \right]$$

where

q''_{av} = average heat flux on hot rod, Btu/hr ft²

P = system pressure, psia

$a_1 = 5.66$

$a_2 = 2.065 \times 10^{-7}$

$b_1 = 1.51$

$b_2 = 0.53 \times 10^{-7}$

The experimental data covered a pressure range of 1500 to 2400 psi, mass velocities between 1.4 to 3.5×10^6 Btu/hr ft², and inlet temperatures from 396 to 604°F. Note that at 2100 psia the threshold average heat flux is over 350,000 Btu/hr ft² which is above the values usually encountered in PWR design. Lin *et al.*¹⁴⁹ reexamined the data considered by Hill *et al.*¹⁶⁰ and came to very similar conclusions.

Akiyama *et al.*¹⁵⁷ examined the effect of rod bow in a tightly spaced lattice typical of a high conversion core. Based on refrigerant tests, they concluded that rod bowing effects on CHF were small and not deleterious.

4.4.2.8 Critical Heat Flux Under Off-Normal Conditions

Estimation of CHF under conditions resulting from various postulated accidents is required for the safeguard analysis performed for all plants. Since the range of most CHF design equations is limited to conditions close to those of normal operation, CHF correlations covering off-normal conditions are required. At flows somewhat below normal (2×10^5 lb/hr ft² < G) and pressures down to 200 psia, the EPRI correlation [Eq. (4.215)] may be used. At lower flows, and pressures somewhat below normal, the correlation of Hao *et al.*¹⁶¹ may be used. These investigators correlated their data from tubular test sections by

$$q''_{\text{crit}} = \frac{G(H_g - H_{in})}{C_1(G/10^6)^{C_2}(1 + C_3H_{fg} + C_4H_{fg}^2) + 4L/De} \quad (4.227)$$

where

G = mass velocity, lb/hr ft²

H_g = enthalpy of saturated vapor, Btu/lb

- H_{in} = enthalpy of inlet coolant, Btu/lb
 H_{fg} = latent heat of evaporation, Btu/lb
 De = hydraulic diameter, ft
 L = heated length, ft
 $C_1 = 3.362$
 $C_2 = 0.7546$
 $C_3 = 0.8799$
 $C_4 = -0.001159$.

The range of parameters covered was

$$\begin{aligned}
 800 \text{ psia} &\leq P \leq 1600 \text{ psia} \\
 10^4 \text{ lb/hr ft}^2 &\leq G \leq 0.5 \times 10^6 \text{ lb/hr ft}^2 \\
 8 &\leq L/D \leq 120.
 \end{aligned}$$

Under the conditions of a hypothetical LOCA, the flow at some point may reverse. This means there would be a brief period of zero, or near zero, flow. Hao *et al.*¹⁶¹ state that their correlation applies to both up and downflow. However, the limits on Eq. (4.227) indicate it should not be used for $G \leq 10^4 \text{ lb/hr ft}^2$. Furthermore, any equation in which q''_{crit} is proportional to G leads to a zero CHF at zero flow. Avedesian and Griffith¹⁶² show this is not the case. Based on their studies with Freon, they concluded that CHF at very low flow is a function of void fraction, α . They expressed their results (Fig. 4.19) as a ratio of the observed CHF to the CHF in pool boiling on a horizontal surface. Limited low pressure (near atmospheric pressure) water data of Ramu and Weisman¹⁶³ and high pressure data reported by Plummer *et al.*¹⁶⁴ appear to be in agreement with this approach. This approach appears to apply for G less than $\sim 50,000 \text{ lb/hr ft}^2$.

Since use of a curve is awkward in a computer program, Fig. 4.19 is often replaced by a simple straight line, which is represented by the product of $(1 - \alpha)$ and a modified pool boiling CHF prediction.¹⁶⁵ That is:

$$q''_{\text{crit}} = (1 - \alpha) \left\{ 0.118 \left[\frac{\sigma' g (\rho_l - \rho_g)}{\rho_g^2} \right]^{1/4} H_{fg} \rho_g + q''_{\text{sub}} \right\} \quad (4.228)$$

$$\text{where } q''_{\text{sub}} = \frac{2k_l(T_{\text{sat}} - T_l)[\sigma' g \Delta \rho / \rho_g^2]^{1/8}}{\left(\frac{2.65 \pi k_l}{\rho C_{Pl}} \right)^{1/2} \left[\frac{\sigma'}{g(\rho_l - \rho_g)} \right]^{1/4}}$$

The subscript l indicates liquid properties and g represents vapor properties. The quantity q''_{sub} represents a convection contribution that applies only when the liquid temperature is below the saturation temperature. Leung¹⁶⁶ concludes that pool boiling CHF [Eq. (4.228)] applies for upflows below

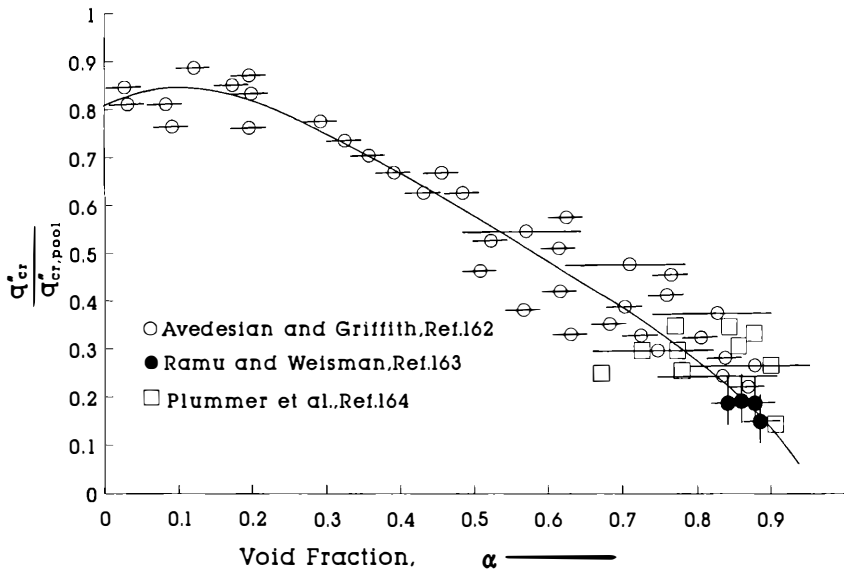


Fig. 4.19 Effect of void fraction on maximum heat flux at low flow rates [from Ref. 163].

80,000 lb/hr ft² and downflow below 90,000 lb/hr ft². However, based on an examination of available data, Weisman *et al.*¹⁶⁷ conclude that pool boiling CHF can be used for upflows up to 150,000 lb/hr ft² providing a mass flux correction is applied above 50,000 lb/hr ft². They recommend

$$q''_{crit,s} = q''_{crit,p} (G/50,000)^{0.33} \quad (4.229)$$

where

$q''_{crit,p}$ = pool boiling critical heat flux from first term of Eq. (4.228)

$q''_{crit,s}$ = critical heat flux with saturated liquid flowing at mass flux G

G = mass flux, lb/hr ft².

If the liquid is subcooled, q''_{sub} from Eq. (4.228) would be added to $q''_{crit,p}$.

Since no one CHF correlation covers the entire range of flow, quality, and pressure encountered during a LOCA, a computer analysis of the accident requires the use of a series of correlations. For a small-break LOCA, where the pressure declines slowly, one might use the EPRI [Eq. (4.215)] correlation, which covers a wide P , x , and G range, for the first portion of the accident. This could then be followed by the Hao correlation [Eq. (4.227)] and finally at the lowest flow and pressure, by a pool-boiling correlation [Eq. (4.228)]. Linear interpolation ranges are required to obtain a smooth transition between correlations.

In a large-break LOCA, where pressures fall rapidly, the EPRI correlation might be followed directly by the pool boiling correlation. A linear interpolation scheme would be required both to assure a smooth transition between correlations and to cover the mass fluxes that lie between the ranges of the two correlations.

Because of its wide range, the early Biasi correlation¹⁶⁸ has been used as one of the series of correlations called on by some computer analyses. However, Abel-Larsen *et al.*¹⁶⁹ indicate that the Biasi correlation performs poorly with full-scale rod bundle data.

4.4.2.9 Transient Effects

The investigations on which the previous correlations are based have all been steady state. The application of steady-state correlations to the prediction of CHF during a LOCA obviously implies that the steady-state correlations are valid during transients.

Cermak *et al.*¹⁷⁰ examined the applicability of steady-state data to simulated blowdown runs. Their study was conducted using a 21-rod bundle and covered pressures between 750 and 1500 psia, inlet temperatures from 480 to 540°F, and mass velocities from 1×10^6 to 3×10^6 lb/(hr ft²). They found their transient and steady-state data to be in good agreement, provided comparisons were made using transient enthalpy, flow, and pressure. They concluded that predictions based on steady-state data provided conservative estimates of the transient observations since, in most cases, their data were equal to or slightly greater than the predictions obtained from a steady-state correlation.

Leung's more recent study¹⁶⁶ of rod bundle blowdown data confirmed that steady-state CHF design correlations may be used for transient predictions provided that actual transient local conditions are used and the CHF correlation is within its range of applicability. Similar conclusions were reached by Celata *et al.*¹⁷¹ for round tubes and by Szabados¹⁷² for triangular-pitch assemblies of the VVER design. Leung also noted that in the low-flow range, a pool-boiling CHF correlation similar to Eq. (4.228) should be used.

Since CHF during blowdown often occurs outside the range of most design correlations, Hsu and Beckner¹⁷³ devised a correlation which they believe incorporates the transient effects introduced by rapid depressurization and velocity changes. Based on CHF data from various blowdown tests, they proposed that the CHF be obtained from

$$q''_{\text{crit}} = (q''_{W-3})_0 [1.76(0.96 - \alpha)]^{1/2} + q''_{\text{conv}} \quad (4.230)$$

where

$(q''_{W-3})_0$ = CHF obtained from W-3 correlation [Eq. (4.206)] with quality equal to zero

α = average void fraction

q''_{conv} = heat flux that would be transferred to steam flowing at the same total mass velocity as test fluid.

Theoretical approaches to prediction of transient CHF have been limited. Pasamehmetoglu *et al.*¹⁷⁴ have developed a model based on the idea that, during a transient, when the steady-state CHF is exceeded, surface overheating is not instantaneous. A finite time must elapse before the liquid layer on the surface (under bubbly layer) can evaporate. In a power transient, the power would continue to increase and the temperature rise would be seen at a somewhat higher flux than predicted by steady state correlations. This approach has been compared to only a very limited set of data.

4.4.3 Effects of Crud Deposition

During the operation of any water system at high temperatures, corrosion products are generated. The metallic ions rapidly hydrolyze to form insoluble hydroxides that partially decompose to oxides. This insoluble material is referred to as "crud." Most crud is removed by the purification system, but a portion is deposited on loop and core surfaces. There is some evidence that it is deposited preferentially on the surfaces with the highest heat flux. The effect of crud deposition on core heat transfer naturally concerns the thermal designer.

A comparison of convective and nucleate boiling heat transfer from clean and crudded vertical surfaces at atmospheric pressure is illustrated in Fig. 4.20. The data were obtained from crudded surfaces formed under conditions believed to simulate those of a PWR.¹⁷⁵ Note that a decrease in convective heat transfer was observed and nucleate boiling began at lower wall temperatures. The results obtained are expected to depend on conditions under which the crud was deposited. Weisman *et al.*⁴¹ observed that a crud deposit can form an insulating layer on heater rods; however, the crud contained an appreciable quantity of magnesium oxide from a ruptured preheater and was deposited on rods bearing an electric potential. Such conditions are not representative of those in a reactor core.

The effect of crud deposition on the boiling crisis was tested by Cohen and coworkers¹⁷⁶ using a 45-in.-long, 0.375-in. electrically heated tube. To establish a basis for comparison, tests were also conducted before application of crud. Although the effects of crud were small, the results indicated a slight increase in DNB heat flux.

Goldstein¹⁷⁷ reported on the results of high-pressure boiler tests in which contaminants were added to water. He observed that under some conditions, deposition of contaminants resulted in DNB where nucleate boiling was obtained with clean surfaces. The effect was found to be temporary with conditions returning to normal over a period of several hours after the contaminant was added.

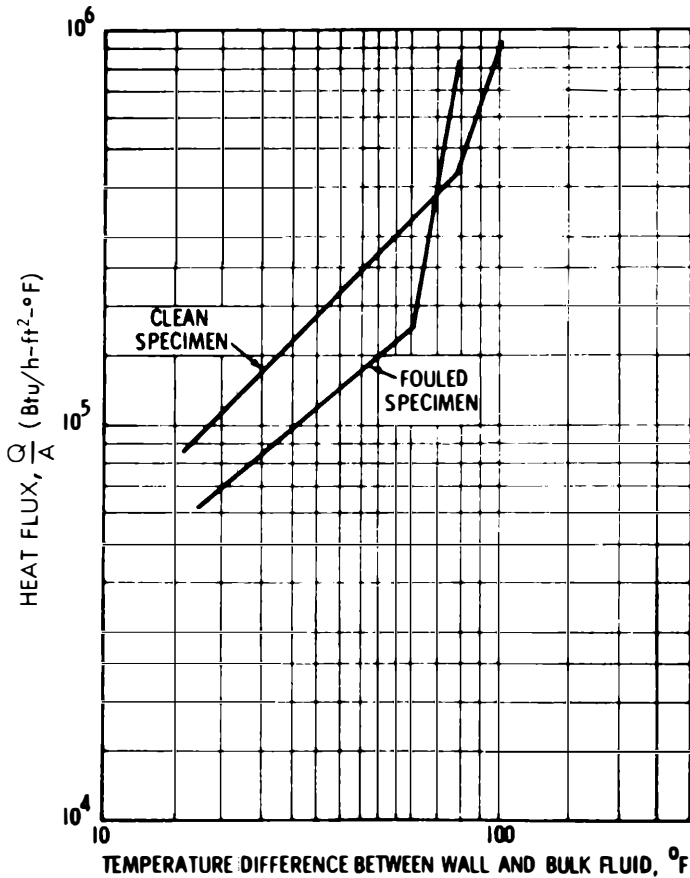


Fig. 4.20 Effect of subcooling on heat transfer from clean and crudded electrically heated cylindrical specimens [from Ref. 175].

Styrikovitch *et al.*¹⁷⁸ summarize later Russian work on the effect of crud deposits on CHF. The Russian investigators found that crud deposits clearly reduced the critical heat flux and that the reduction increased as the crud thickness increased. This is in accord with the general observation that CHF in porous structures arises at significantly lower heat fluxes than on an impermeable surface. Based on the experiments of Balabanov,¹⁷⁹ the effect of crud deposits on q''_{cr} is given as

$$q''_{crit(d)} / q''_{crit(o)} = 1 - \delta^{0.323}(0.065) \quad (4.231)$$

where $q''_{crit(d)}$, $q''_{crit(o)}$ = critical heat flux with deposit and with clean surface, respectively, and δ = thickness of crud deposit (μm). Styrikovitch *et al.* note that the effect of crud thickness varies with the structure and composition

of the crud deposit. Data by Rikhter¹⁸⁰ with a different crud composition show the same general trends as the data of Balabanov but the change in CHF with crud thickness is different.

Macbeth *et al.*¹⁸¹ also found crud deposits to decrease CHF. However, the decrease varied little with crud thickness. Balabanov believes that lack of a thickness effect is due to the presence of "steam-chimneys" in the Macbeth deposit while Balabanov deposits lacked clearly defined channels.

The effect of crud deposition on the flow distribution through the core is probably of most concern. After loading a new core, observation of the CVTR reactor¹⁸² indicated a small decrease in core flow attributed to crud deposition. If highly preferential crud deposition were to take place in a core with very narrow passages (e.g., plate-type cores), flows to some core channels could be significantly reduced. Coolant conditions must be maintained to prevent this from occurring.

4.4.4 Post-CHF Heat Transfer

If a heat flux in excess of the critical heat flux is maintained, the film boiling regime is entered rapidly. If this were to occur at full reactor power, low film-boiling heat-transfer coefficients would cause the fuel cladding temperature to rise rapidly to its melting point. Since every precaution is taken to ensure that the design conditions are well below those at which critical heat flux occurs, the onset of film boiling at normal power is very unlikely.

In the analysis of accident situations, particularly the LOCA, the possibility of film boiling must be considered. Since the reactor is shut down promptly after initiation of the accident, the power level of the reactor can be significantly reduced when critical flux is exceeded. Film-boiling heat-transfer coefficients must be computed to estimate the temperature rise of the cladding. Under proper conditions, the reactor heat flux can be sufficiently reduced so that nucleate boiling conditions can be restored before damage to the fuel occurs. This phenomenon is referred to as *return to nucleate boiling* (RNB).

It was previously noted that before the onset of stable film boiling, an unstable mix of nucleate and film boiling exists. This is referred to as *partial film boiling* or *transition boiling*. Some early designers conservatively assumed that stable film boiling begins as soon as the critical heat flux is exceeded. In more realistic current approaches, behavior in the transition region has been modeled.

4.4.4.1 Transition Boiling

Transition boiling may be encountered as a result of a reactivity insertion accident or during the blowdown and reflood phases of a LOCA. Although transition boiling is still poorly understood, it is now recognized that the

type of transition boiling behavior observed depends on whether the transition boiling has followed DNB in at low qualities or dryout at high void fractions. A reasonable criterion for the dividing line is the onset of annular flow determined from an appropriate flow pattern transition map or correlation (see flow pattern maps, Figs. 3.14 and 3.15).

In the high void fraction region, it is common to assume that total heat transferred, q'' has a convective component, q_c'' , and a boiling component, q_b'' . That is,

$$q'' = q_c'' + q_b'' \quad (4.232)$$

The convective component is obtained from an appropriate correlation of stable film boiling data. The boiling component is obtained from a correlation such as that suggested by Tong and Young,¹⁸³ where

$$q_b'' = q_{nb}'' \exp \left[-0.001 \frac{\chi_c \exp(2/3)}{(d\chi_c/dL)} (\Delta T/100)^{(1 + 0.0016\Delta T)} \right] \quad (4.233)$$

where

q_{nb}'' = nucleate boiling heat flux, Btu/hr ft² °F

ΔT = wall temperature minus saturation temperature, °F

$d\chi_c/dL$ = change in equilibrium quality per unit length.

The above correlation was based on data where $0 \leq \chi_c \leq 1.45$, $0 \leq \Delta T \leq 300^\circ\text{F}$, and $P \geq 1000$ psi.

Ramu and Weisman¹⁶³ attempted to correlate both high- and low-pressure data. They assume, as do Tong and Young, that the total heat flux is the sum of a convective component, derived from stable film boiling data, and a boiling component [Eq. (4.232)] applies. Their correlation is given in terms of h_m , the maximum heat transfer coefficient seen near the CHF. Based on high-pressure data from several sources, they proposed that the boiling heat transfer coefficient, h_b , be obtained from

$$h_b/h_m = 0.5\{\exp[-0.0078(\Delta T - \Delta T_m)] + \exp[-0.0698(\Delta T - \Delta T_m)]\} \quad (4.234)$$

where ΔT = wall temperature minus saturation temperature, °F; and $\Delta T_m = \Delta T$ at conditions of maximum h . At high mass velocities and pressures ($P \geq 700$ psi), they suggest that ΔT_m can be taken as approximately equal to the maximum ΔT seen under pool boiling conditions. The value of h_m is then obtained by using this value of ΔT in Chen's¹⁰⁵ equation [Eq. (4.173)] for boiling heat transfer coefficients. At low pressures and at mass velocities below 30,000 lb/hr ft², they suggest the maximum heat flux be estimated from Avedesian and Griffith's data (Fig. 4.20). The heat flux so obtained is then used in McAdams⁹⁹ equation [Eq. (4.168)] to determine ΔT_m and h_m .

France *et al.*¹⁸⁴ combined convective and boiling heat fluxes in a single term using a very simple transition boiling correlation. They were able to

correlate transition boiling in the high void region when operating at high pressure. This correlation may be written as

$$q''/q''_{\text{crit}} = \exp[-C(\Delta T - \Delta T_{\text{CHF}})] \quad (4.235)$$

where

$$C = 0.027 - 0.00556G$$

$$G = \text{mass flux (Mg/m}^2 \text{ s)}$$

$$\Delta T = T_w - T_{\text{bulk}} \text{ (K)}$$

$$\Delta T_{\text{CHF}} = T_w - T_{\text{CHF}}$$

$$T_w = \text{wall temperature (K)}$$

$$T_{\text{CHF}} = \text{wall temperature at CHF (K)}.$$

The range of this correlation is $7 \text{ (MPa)} \leq P < 15.3 \text{ (MPa)}$ and $0.7 \leq G \leq 3.2 \text{ (Mg/m}^2 \text{ s)}$.

Kao and Weisman¹⁸⁵ examined steady-state transition boiling in a temperature controlled system at pressures of 1 to 4 bar. With annular flow conditions at the CHF point, they found that the transition boiling region was characterized by a thin film on the heater surface that oscillated in the vertical direction. They correlated the transition boiling heat flux, q'' by

$$q'' = f q''_{\text{wet}} + (1 - f) h_c (\Delta T_{\text{sat}}) \quad (4.236)$$

where f = fraction of area wetted (function of ΔT_{sat} and heat flux), h_c = convective heat transfer to steam, and $(\Delta T)_{\text{sat}}$ = wall temperature – saturation temperature.

The value of q''_{wet} , the heat flux to the wetted area, was obtained from

$$q''_{\text{wet}} = q''_{\text{crit}} \exp[-1.6 \times 10^{-3} (\Delta T_{\text{sat}})] \quad (4.237)$$

where q''_{crit} = critical heat flux estimated from Fig. 4.19 and (ΔT_{sat}) is °C and q'' is (W/m²).

Observation of rewetting transients, which simulated behavior at low reflooding rates (high void) after a LOCA, showed very little oscillation of the liquid film as it climbed up the heater. Weisman and Kao¹⁸⁶ therefore recommended that the very minor oscillation of the rewetting liquid boundary be ignored during rewetting transients. They found that q''_{wet} from Eq. (4.237) could be used to approximate the heat flux in the transition boiling region. They concluded that this region extended for T_{crit} to $(T_{\text{crit}} + 100^\circ\text{C})$ at low pressures. The value of T_{crit} is obtained by using q''_{crit} in an appropriate boiling heat transfer correlation.

In the transition boiling region following CHF at low qualities (continuous liquid phase; void fractions below that of annular flow as indicated by flow maps of Figs. 3.14 or 3.15) we have the beginning of inverted annular flow where there is a vapor film adjacent to the heater or fuel rod.

During transition boiling, the steam film is not fully formed and there is intermittent contact with the water and the surface.

Experimental observation of transition boiling heat transfer indicates that the heat flux initially falls off rapidly as the wall temperature is increased above the temperature at which CHF occurs. The flux then falls more slowly as the wall temperature approaches the film boiling region. A simple approach to modeling this behavior is to assume that the heat flux falls off exponentially with temperature between the critical heat flux temperature and T_{MFB} , the minimum film boiling temperature (lowest temperature at which film boiling will occur under pool boiling conditions). We then have

$$q_b'' = q_{\text{crit}}'' \exp[-a(T_w - T_{\text{crit}})] \quad (4.238)$$

where

T wall temperature

T_{crit} = wall temperature at onset of critical heat flux

$a = \ln(q_{\text{crit}}''/q_{\text{MFB}}'')/(T_{\text{MFB}} - T_{\text{crit}})$

q_{MFB}'' = film boiling heat flux at T_{MFB}

T_{MFB} = value of wall temperature at q_{MFB}'' obtained from an appropriate film boiling correlation.

On the basis of Taylor-Helmholtz instability considerations, Berenson¹⁸⁷ computed q_{MFB}'' for pool boiling as

$$q_{\text{MFB}}'' = 0.09 H_{fg} \rho_{gf} \left[\frac{\sigma' g (\rho_l - \rho_g)}{(\rho_l + \rho_g)^2} \right]^{1/4} \quad (4.239)$$

where ρ_{gf} = vapor density at film temperature, and σ' = surface tension. It is, however, not clear that the minimum film boiling heat flux determined for pool boiling conditions is appropriate for use under forced convection conditions. Note that Eq. (4.238) implies that whatever reduces CHF will also reduce the average transition boiling heat flux.

Nelson and Unal¹⁸⁸ recommend an alternative approach for transition boiling encountered when ECCS water first contacts hot fuel surfaces during the reflood phase of a LOCA. Nelson and Unal¹⁸⁸ assume that, under these conditions, the heat flux falls off exponentially with distance from the CHF location, z_{CHF} . That is

$$q'' = q_{\text{CHF}}'' \exp[-B(z - z_{\text{CHF}})] \quad (4.240)$$

where B = constant depending on Re and Ca , and Ca = Capillary number = $(V_l \mu_l / \sigma')$. At present (1993) the appropriate value for B remains in question.

Conservative design programs for LOCA analysis more commonly base their transition boiling predictions on transient tests that use electrically

heated rods to simulate the core.¹⁸⁹ Based on such data, Goldemund¹⁹⁰ suggests that the transition-boiling heat-transfer coefficients observed can be fitted by a modified pool boiling correlation at high values of ΔT :

$$h = \left(\frac{k_g^3 \rho_g \rho_l H_{fg} g}{12 \mu_g \Delta T L} \right)^{0.25} (1/x_e)^{0.33} \sqrt{V_{in}} \quad (4.241)$$

where

L = heated length, ft

k_g thermal conductivity of vapor, Btu/hr ft °F

h total heat transfer coefficient, Btu/hr ft² °F

g = gravitational acceleration, ft/hr²

ρ_g, ρ_l = density of saturated vapor and liquid, respectively, lb/ft³

H_{fg} = latent heat of vaporization, Btu/lb

μ_g = viscosity of saturated vapor, lb_m/ft hr

ΔT = wall temperature minus saturation temperature, °F

x_e equilibrium steam quality

V_{in} = core inlet velocity, in./s.

An alternative correlation of rod bundle transition boiling data with a continuous liquid phase has been proposed by Hsu.¹⁹¹ For values of α that are clearly below that α value required for annular flow (see flow pattern maps of Figs. 3.14 and 3.15), Hsu¹⁹¹ proposes that

$$h_b = 1456 P^{0.558} \exp(-0.003758 P^{0.1733} \Delta T_{sat}) \quad (4.242)$$

where h_b = convective boiling component of heat transfer coefficient; P = pressure, psia; and $\Delta T_{sat} = T_w - T_{sat}$, °F. Note that this correlation is based only on data taken at pressures below 100 psia. Hsu finds that Eq. (4.242) overpredicts data at high void fractions where the liquid phase may no longer be continuous.

4.4.4.2 Film Boiling in the Presence of a Continuous Steam Phase

Film boiling in the presence of a continuous steam phase arises following (1) CHF induced by high-quality dryout or (2) above the quench front in the reflood phase of a LOCA. In either case, heat transfer will be to steam containing entrained droplets. We may assume that we have film boiling with a continuous steam phase (dispersed droplet regime) if the steam quality is high enough for annular flow to exist at the CHF or rewet point and the film boiling heat transfer coefficient is higher than the applicable transition boiling heat transfer coefficient.

When dealing with rewetting which occurs at low quality (high reflood-ing rates), there is a region of inverted annular flow and agitated (or churn)

flow below the dispersed flow region. Nelson and Unal¹⁸⁸ suggest that the dispersed droplet regime begins when the void fraction exceeds that at which a froth can be maintained. Unfortunately, this void fraction is not now (1993) well defined. Alternatively, the onset of dispersed flow may be taken as occurring when the wall temperature falls below the quench temperature. The quench or rewet temperature is defined as the temperature below which there is a significant improvement in the heat transfer coefficient.

In a mixture of steam and liquid drops at fairly low wall temperatures, it is possible for the entrained droplets to wet the surface. At the *Leidenfrost point*, the surface becomes so hot that rapidly evaporating steam between the liquid droplet and heater forms a vapor cushion that supports the drops and keeps the liquid away from the surface.

If the droplets can wet the wall, heat transfer is enhanced. Thus, Parker and Grosh¹⁹² noted that with a steam-water mixture at 30 psia and a wall superheat of $< 50^{\circ}\text{F}$, the heat transfer coefficient was three to six times the value for dry steam flowing at the same conditions. Above 50°F superheat, they observed heat transfer coefficients almost identical to those for pure steam. From these data and those of Bennet *et al.*¹⁹³ and Bertoletti *et al.*,¹⁹⁴ it appears that the Leidenfrost point increases from $\sim 300^{\circ}\text{F}$ at 30 psia to $\sim 560^{\circ}\text{F}$ at 250 psia and to 750 to 800°F at 1000 psia. Thus, the allowable wall superheat increases from $\sim 50^{\circ}\text{F}$ at 30 psia to 160°F at 250 psia and to $\sim 250^{\circ}\text{F}$ at 1000 psia.

Other investigators¹⁹⁵ have argued that the rewet and Leidenfrost temperatures are not identical. Various empirical estimates of rewet temperature, as distinct from the Leidenfrost temperature, have therefore been used (see Sec. 6.5.3.2).

Most of the attempts to correlate dispersed flow film boiling are based on assuming forced convection to the vapor. Therefore, the correlating equations are forced convection correlations that have been modified to account for steam flow and properties. Groenveld¹⁹⁶ points out that the simplest modification possible is that cooling is by forced convection to the vapor only and that the liquid is in thermal equilibrium with the vapor. Hence,

$$\text{Nu} = a(DeGx_c / \alpha \mu_v)^b (\text{Pr}_v)^c \quad (4.243)$$

where

- Nu Nusselt number = hDe / k_v
- Pr_v Prandtl number for vapor
 thermodynamic equilibrium quality
- μ_v viscosity of vapor
- a, b, c constants
- α = void fraction.

The correlations that have been developed may be divided into two groups:

1. Correlations for film boiling at high pressure under conditions that might follow CHF at high pressure
2. Correlations applicable to the low pressures and other conditions that are found during the reflood phase of a LOCA.

In the former group, we find that thermal equilibrium is generally assumed. For high pressures and temperatures generally below the Leidenfrost point, the following correlation of Bishop *et al.*¹⁹⁷ can be used over the range of data correlated:

$$\left(\frac{hDe}{k}\right)_f = 0.0193 \left(\frac{DeG}{\mu}\right)_f^{(0.80)} \left(\frac{c_p \mu}{k}\right)_f^{1.23} \left(\frac{\rho_v}{\rho_{\text{bulk}}}\right)^{0.68} \left(\frac{\rho_v}{\rho_{l,\text{sat}}}\right) \quad (4.244)$$

where f refers to vapor properties at the film temperature, which equals $(T_w + T_b)/2$; refers to the vapor phase; and l_{sat} refers to the saturated liquid. This correlation was developed from data in the following range..

$$q'' = 0.11 \times 10^6 \text{ to } 0.61 \times 10^6 \text{ Btu}/(\text{hr ft}^2)$$

$$G = 0.88 \times 10^6 \text{ to } 2.5 \times 10^6 \text{ lb}/(\text{hr ft}^2)$$

$$P = 580 \text{ to } 3190 \text{ psia}$$

$$De = 0.10 \text{ to } 0.32 \text{ in.}$$

$$T_b = 483 \text{ to } 705^\circ\text{F}$$

$$T_w = 658 \text{ to } 1100^\circ\text{F.}$$

Morris *et al.*¹⁹⁸ examined the predictive ability of a number of high void, film boiling correlations using rod bundle data over the ranges

$$q'' = 0.3 \times 10^5 \text{ to } 0.3 \times 10^6 \text{ Btu}/\text{hr ft}^2$$

$$G = 0.3 \times 10^5 \text{ to } 0.6 \times 10^6 \text{ lbs}/\text{hr ft}^2$$

$$P = 60 \text{ to } 1900 \text{ psi.}$$

They found that a modified version of a round-tube correlation originally due to Condie and Bengston provided the best predictions. The modified correlation is

$$h = \{0.00141 k_v^{0.4593} \text{Pr}_w^{2.2598} / [De^{0.095} (1+x)^{2.0514}]\} \text{Re}_v^{[0.6249 + 0.204 \ln(x+1)]} \quad (4.245)$$

where the subscript v indicates properties and flows of the bulk vapor and subscript w indicates vapor properties at the wall temperature. The round-tube data from which the unmodified correlation was developed included data for pressures down to 60 psia and mass fluxes up to 3.8×10^6 . Hence, the correlation should apply over a wide range.

The more recent correlations for film boiling during the high void regime of a LOCA reflood have generally considered the lack of thermal equilibrium. Groenveld¹⁹⁶ observes that the assumption of thermal equilibrium

between vapor and liquid holds only at very high mass flows where liquid-vapor heat exchange is efficient, near the critical heat flux location, and in inverse annular flow (vapor on wall, liquid in center). Under other conditions, appreciable superheating of the vapor will occur. Chen *et al.*,¹⁹⁹ Tong and Young,¹⁸³ and Groenveld and Delorme²⁰⁰ among others have developed film boiling heat transfer correlations that allow this superheating. Groenveld and Delorme assume the liquid phase does not participate in cooling of the heated surface, radiative heat transfer is negligible, and the flow is homogeneous. They estimate actual enthalpy H_{va} of the superheated steam by

$$(H_{va} - H_{ve})/H_{fg} = \exp(-\tan\psi) \exp[-(3\alpha)^{-4}] \quad (4.246)$$

where the nonequilibrium factor ψ is given by

$$\psi = a_1 \text{Pr}^{a_2} \text{Re}_h^{a_3} (q'' \text{Dec}_p / k_{ve} H_{fg})^{a_4} \sum_{i=0}^{i=2} b_i (\chi_e)^i \quad (4.247)$$

and

$$a_1 = 0.13864$$

$$a_2 = 0.2031$$

$$a_3 = 0.20006$$

$$a_4 = -0.09230$$

$$b_0 = 1.3072$$

$$b_1 = -1.0833$$

$$b_2 = 0.8455$$

α = void fraction calculated assuming homogeneous equilibrium flow

c_p = specific heat

H_{ve} = enthalpy of vapor calculated assuming thermodynamic equilibrium

H_{fg} = enthalpy of evaporation

Pr = Prandtl number

q'' = surface heat flux

Re_h = $G D_e \chi_1 / \mu_{ve}$

χ_e = equilibrium vapor quality, may be above 1.0

χ_1 = equilibrium vapor quality with maximum of 1.0 ($\chi_1 = \chi_e$ if $\chi_e \leq 1.0$)

k_{ve} = thermal conductivity of vapor at thermodynamic equilibrium

D_e = equivalent diameter

μ_{ve} = viscosity of vapor at equilibrium conditions.

When $\alpha \leq 0.33$, Eq. (4.246) predicts the nonequilibrium effect to disappear and $H_{va} = H_{ve}$. Similarly, at very high Reynolds numbers, the nonequilibrium effects become unimportant.

The value of H_{va} is used to obtain the actual steam temperature, T_{va} , from the steam table. In turn, this temperature is used to compute the heat flux from

$$q'' = h(T_{\text{wall}} - T_{va}) \quad (4.248)$$

where h is obtained from an appropriate superheated steam correlation. Groenveld and Delorme²⁰⁰ recommend Hadaller and Banerjee's²⁰¹ equation modified for two-phase flow:

$$(hD/k)_f = 0.0008348 \{ (GD/\mu_f) [\chi_a + (\rho_v/\rho_l)(1 - \chi_a)] \}_f^{0.8774} \text{Pr}_f^{0.6112} \quad (4.249)$$

where $\chi_a = H_{fg} \chi_e / (H_{va} - H_l)$, H_l = enthalpy of saturated liquid, and f indicates that steam properties are evaluated at film temperature. Groenveld and Delorme were able to fit more than 1800 data points with a standard deviation of $< 7\%$.

Webb and Chen²⁰² have developed a nonequilibrium film boiling model that has been able to predict rod bundle data and meets the requirements of the two-fluid model used in the TRAC programs. Webb and Chen predict the heat transfer coefficient by:

$$h = h_c (1 + F_s) [1 + 0.8(L/D)^{-1}] \quad (4.250)$$

where F_s = sink function accounting for effect of entrained droplets:

$$F_s = 250(P/P_c)^{0.69} \left(\frac{1 - x_a}{x_a} \right)^{0.49} \text{Re}_v^{0.55} \quad (4.251)$$

and L = distance from CHF location; P, P_c = actual and critical pressure, respectively; and Re_v = Reynolds number based on vapor flow and properties. The value of the convective heat transfer coefficient, h_c , is obtained from

$$h_c = \frac{f}{2} (c_p)_v G x_a (\text{Pr}_v)^{-2/3} \quad (4.252)$$

where f = Moody friction factor and the subscript v indicates vapor properties.

A correlation of the vapor generation rate based on a phenomenological model is used for determination of x_a . The vapor generation rate (mass/time vol), Γ , is defined as

$$\Gamma = G \, dx_a / dz \quad (4.253)$$

Webb and Chen postulate that in the region close to the quench front (near field) there is significant vapor generation due to direct contact between the liquid and the heated wall. At greater distances (far field), direct vapor

contact is negligible and evaporation is due solely to heat transfer from the vapor to the entrained droplets. The total evaporation rate, Γ , is then given by

$$\Gamma = \Gamma_{NF} + \Gamma_{FF} \quad (4.254)$$

where Γ_{NF} , Γ_{FF} represent evaporation rates in near field and far field, respectively ($\text{kg}/\text{m}^2 \text{ s } ^\circ\text{C}$).

For the near field, the model obtains

$$\Gamma_{NF} = \frac{4q''}{DH_{fg}} \exp \left[\frac{-0.0016(T_w - T_{sat})(L/D)}{G(1 - x_a)} \right] \quad (4.255)$$

The revised value¹⁸⁸ of the far-field vapor generation rate is given by

$$\begin{aligned} \Gamma_{FF} = & 0.3154 \exp[-10.89(P/P_c)] \left(\frac{Gx_a}{\alpha''} \right)^{0.55} \\ & \times \left\{ \frac{k_v \text{Pr}_v^{0.33} [g(\rho_l - \rho_g)]^{0.725} (1 - \alpha'')(T_{va} - T_w)}{\mu_v^{0.55} (2\sigma')^{0.725} N_\mu^{0.433} H_{fg}} \right\} \end{aligned} \quad (4.256)$$

where

$$N_\mu = \mu_v / [\rho_g \sigma' \sqrt{\sigma' / g(\rho_l - \rho_g)}]^{1/2} \quad (\text{dimensionless}) \quad (4.257)$$

and α'' = void fraction based on homogeneous flow, T_{va} = actual vapor temperature (obtained from vapor phase energy balance), and σ' = surface tension.

An iterative process is required to evaluate h in the nonequilibrium approaches. Thus, in Groenveld and Delorme's method,²⁰⁰ a value for q'' is needed before ψ can be calculated, but a value for ψ is needed before a correct value for q'' can be obtained. The procedure is tedious; many designers prefer to use an empirical equation that assumes steam is at the saturation temperature and that has been fitted to data points in the range of interest. The modified Condie-Bengston correlation [Eq. (4.245)] is of this type and has been shown to apply under reflood conditions. It has been used on the RELAP5/Mod 1 accident analysis program.

Yoder *et al.*²⁰³ point out that the presence of grids increases the flow film boiling heat transfer seen in rod bundles relative to that seen in a round tube with equivalent De . The enhancement is greatest just downstream of the grid and is completely dissipated in traveling 20 to 30 rod diameters downstream. The 10% enhancement factor included in the modified coefficient of Eq. (4.245) may be attributed to this enhancement.

4.4.4.3 Film Boiling in the Presence of a Continuous Liquid Phase

Film boiling in the presence of a continuous liquid phase (void fraction below that required for annular flow as indicated by flow pattern maps of

Figs. 3.14 or 3.15) will be seen following subcooled or low-quality DNB. It also arises when the core is recovered after a LOCA using a high reflooding rate, which leads to a low-quality mixture at the quench front. In both cases, the film boiling region is initially characterized by a thin film of vapor adjacent to the hot rod and a continuous liquid phase, with some vapor bubbles, in the remainder of the channel. This flow pattern is usually described as *inverted annular flow*.

Behavior in this region is often considered to be the same as that of pool-film boiling. At pressures significantly above atmospheric, the following laminar film boiling correlation of Borishanski and Fokin²⁰⁴ can be used:

$$h = (F k_v / \delta_v) \{ (k_v g \delta_v^3 / \nu_v) [(\rho_l - \rho_v) / \rho_v] \}^n \quad (4.258)$$

where

k_v = thermal conductivity of vapor

ρ_l = density of liquid

ρ_v = density of vapor

ν_v = kinematic viscosity of vapor

δ_v = mean vapor film thickness = $31[\sigma/(\gamma'_l - \gamma'_v)]^{0.5}$
 $[q''\mu_v/(H_{fg}\gamma_v\sigma')]^{0.53}$

H_{fg} = latent heat of vaporization

σ' = surface tension

μ_v = viscosity of vapor

q'' = heating rate

$F = 2.8$
 $n = 0.33$ } for $2 \times 10^4 < \left[\frac{g \delta_v^3}{\nu_v} \left(\frac{\rho_l - \rho_v}{\rho_v} \right) \right] < 1.4 \times 10^6$

$F = 0.0094$
 $n = 0.57$ } for $1.4 \times 10^6 < \left[\frac{g \delta_v^3}{\nu_v} \left(\frac{\rho_l - \rho_v}{\rho_v} \right) \right] < 1.5 \times 10^7$

γ'_l, γ'_v = specific gravity of liquid and vapor, respectively.

Bromley²⁰⁵ correlated the data for pool boiling at low pressures by

$$h = 0.62 \{ k_v^3 \rho_v (\rho_l - \rho_v) g H_{fg} / (L \Delta T \mu_v) \}^{0.25} \quad (4.259)$$

where

H_{fg} = heat of vaporization, Btu/lb

L = heated length, ft

$\Delta T = T_{\text{wall}} - T_{\text{bulk}}$

and the subscripts are v = vapor property and l = liquid property.

Hsu and Westwater²⁰⁶ indicated that the coefficient 0.62 should be replaced by 0.7 for vertical surfaces. Griffith and Kirchner²⁰⁷ point out that observations of reflooding conditions show that a smooth vapor film exists

for only a short distance above the quench front. Subsequently, waves are present at the liquid-vapor interface and vapor varicosities occur at regular intervals. They conclude that the instability wavelength governs the flow field, and the flow effectively restarts at each varicosity. They suggest that L in Eq. (4.259) be replaced by a characteristic wavelength that they consider to be $\sim \pi D_h$.

Pomerantz²⁰⁸ also concluded that the characteristic length in Eq. (4.259) should be that of the unstable wavelength for a disturbance. His calculations, based on a Helmholtz instability analysis, indicate that Eq. (4.259) should be modified to

$$h = 0.6(De/\lambda)^{0.172} \{ [k_v^3 \rho_v (\rho_l - \rho_v) g H_{fg}] / (De \Delta T \mu_v) \}^{0.25} \quad (4.260)$$

where

$$\lambda = 2\pi \left[\frac{g_c \sigma'}{g(\rho_l - \rho_v)} \right]^{1/2}$$

and σ' is surface tension.

More recent approaches to modeling of film boiling with a continuous liquid phase have been based on a more complex model, which assigns several flow patterns to this region. Based on the flow map of Obat and Ishii,²⁰⁹ Nelson and Unal's TRAC code model¹⁸⁸ assigns the flow patterns shown in Fig. 4.21. Pattern transition distances are determined in terms of the Capillary number ($Ca = V_l \mu_l / \sigma'$) as shown in Fig. 4.21.

The transition boiling region is immediately followed by smooth inverted annular flow. In this region, all heat transfer is across the smooth vapor film to the liquid. A heat transfer correlation due to Denham²¹⁰ is used here

$$h = 0.4472 k_v \left[\frac{g(\rho_l - \rho_v)}{\mu_v V} \right]^{1/2} \quad (4.261)$$

where V_v = vapor velocity and other symbols have their previous meaning.

In the rough-wavy inverted annular film boiling region that follows, the coefficient for heat transfer to the liquid is obtained by an interpolation between the h values of Eq. (4.261) and a modified version of Eq. (4.259):

$$h = h_D \left(1 - \frac{z - z_{sm}}{z_{rw} - z_{sm}} \right)^{0.5} + h_{MB} \left(\frac{z - z_{sm}}{z_{rw} - z_{sm}} \right)^{0.9} \quad (4.262)$$

where

h_D = heat transfer coefficient of Eq. (4.261)

$h_{MB} = h_{Br} F(Re_a)$

h_{Br} = heat transfer coefficient from Eq. (4.259) with L replaced by λ

$F(Re_a) = -0.09567 + 4.8644 \times 10^{-4} Re_a$

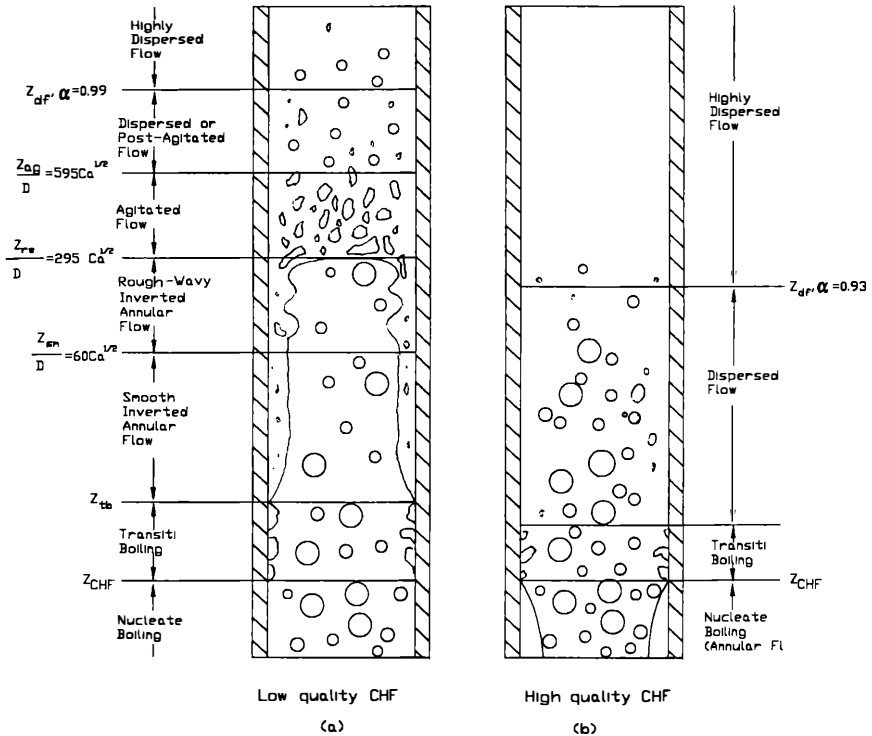


Fig. 4.21 Flow patterns in reflood heat transfer [from Ref. 188].

$$n = 1 + (1 - 0.95) \left[\frac{P - 2 \times 10^5}{(10 - 2)10^5} \right]$$

P = pressure (Pa)

z = axial location of interest

z_{sm} = axial location at which smooth inverted annular flow begins

z_{rw} = axial location at which rough-wavy annular flow ends.

In addition to the heat transferred directly to the liquid, Nelson and Unal conclude that some heat is also transferred directly to the vapor. This heat to the vapor is obtained using 120% of the Webb-Chen heat transfer coefficient [Eq. (4.250)].

The rough-wavy inverted annular flow region is succeeded by the agitated and postagitated inverted-annular flow patterns where the heat transfer coefficient to the liquid is estimated from

$$h = h_{MB} \left(\frac{0.75 - \alpha}{0.75 - \alpha_2} \right)^{F_1(Re_a)} \quad (4.263)$$

where

$$F_1(\text{Re}_a) = 6.18 - 9.37(10^{-3})\text{Re}_a + 5.36 \times 10^{-6}\text{Re}_a^2 - 1.0 \times 10^{-9}\text{Re}_a^3$$

and α = void fraction, α_2 = void fraction at which postagitated regime assumed to terminate ($0.45 \leq \alpha_2 \leq .75$).

Heat transfer to the vapor in this region is obtained in the same manner as in rough-wavy inverted annular flow.

The Nelson-Unal model of Eqs. (4.261) to (4.263) contains a number of incompletely verified features that are likely to change.

4.4.4.4 Radiative Heat Transfer

For film boiling conditions, the surface temperature may rise to the point where radiative heat transfer is significant. The foregoing equations describe the convective heat transfer only, but the radiation coefficient appropriate to the geometry should be included. For this purpose, Hottell's empirical method²¹¹ is recommended.

For the simple geometry of a circular pipe, we write

$$q''_{\text{total}} = h_c(T_w - T_{\text{sat}}) + \epsilon 1.714 \times 10^{-9}(T_w^4 - T_{\text{sat}}^4) \quad (4.264)$$

where

h_c = appropriate convective heat transfer coefficient, Btu/(h ft² °R)

$$\epsilon = \text{effective emissivity} = \left\{ \frac{1}{\epsilon_w} + \left[\frac{1}{\epsilon_v \alpha + (1 - \alpha)\epsilon_l} \right] - 1 \right\}^{-1}$$

α = void fraction

ϵ_w = emissivity of wall

ϵ_v = emissivity of steam

ϵ_l = emissivity of liquid

T_w, T_{sat} absolute temperatures of wall and saturated liquid, °R.

The emissivity of steam depends on the temperature and the product of pressure, P , and mean beam length, L (see Ref. 212). To obtain an appropriate ϵ_v , we write

$$\epsilon_v = \epsilon_v(T_w, PL)(T_b/T_w)^{0.5} \quad (4.265)$$

where T_b, T_w = bulk and wall temperature, respectively, °R.

4.4.5 Recommended Approaches to Boiling Heat Transfer Estimation

Estimation of boiling heat transfer coefficients during normal operation presents no problem. Boiling heat transfer coefficients are high and the Jens-Lottes¹⁰⁰ correlation [Eq. (4.169)] produces conservative values in the subcooled and very low quality range at normal operating conditions. When

the coolant has any appreciable quality, Chen's boiling suppression approach [Eq. (4.173)] had been found to provide the most reliable predictions.¹³⁶ However, the more recent Gunger-Winterton revision of this approach [Eq. (4.174) through Eq. (4.177)] appears to be an improvement.

Once we are outside the nucleate boiling area, there is much less agreement on the most appropriate correlation. This is clearly the case in predicting the critical heat flux in a reactor core. For determination of the safety margin at normal or near-normal conditions, the fuel manufacturer's proprietary correlation for the particular fuel design will probably provide the most accurate predictions. The prediction should be used in conjunction with an appropriate subchannel analysis code to which measured mixing factors have been supplied.

If the fuel vendor's CHF correlation is not available, reasonably reliable CHF predictions can be obtained from either the EPRI rod bundle correlation¹⁴⁶ [Eq. (4.215)] or the Weisman-Pei phenomenological model.¹²³ Both of these approaches should be used in conjunction with a subchannel analysis program for determination of local conditions. Good results with the Weisman-Pei model¹²³ require the use of an appropriate mixing coefficient in the subchannel analysis (the EPRI correlation uses a grid factor correction). If the critical heat flux is evaluated without the use of a subchannel analysis (e.g., evaluations included in a one-dimensional systems analysis program), best results appear to be obtained¹⁵⁰ with the Groenveld *et al.*¹³⁶ CHF look-up table (Table 4.IV).

In the absence of general models, prediction of boiling heat transfer under off-normal conditions is highly dependent on the accident situation being examined. During the initial depressurization (blowdown) following a LOCA, the Hsu and Beckner¹⁷³ CHF correlation [Eq. (4.230)], which was fitted to blowdown data, applies. The transition boiling that follows CHF may be predicted by the Tong and Young¹⁸³ correlation provided the pressure flow and quality are within the range of the correlation. This correlation appears to fit high-pressure transition boiling data.¹³⁶ At lower pressures and qualities, the exponential decay of Eq. (4.238) appears to provide the most reasonable predictions.¹³⁶

In the film boiling region encountered at the end of blowdown, the Bishop *et al.*¹⁹⁷ correlation [Eq. (4.244)] can be used providing high-pressure conditions hold. For pressures below about 500 psi, the modified Condie-Bengston correlation [Eq. (4.245)] appears to provide the most reasonable results.¹⁹⁸ Table 6.II summarizes the heat transfer recommendations in the blowdown region.

The predictive procedures to be followed during the reflooding phase of a LOCA depend on the reflooding rate. At low reflooding rates, the nucleate boiling region is succeeded by a narrow transition boiling region and then by heat transfer to droplet laden steam. The narrow transition boiling region can be ignored with little error. If a single-fluid model system analysis

code is used, then the modified Condie-Bengston correlation [Eq. (4.245)] should provide reasonable predictions of the overall heat transfer. If the systems program is based on a two-fluid model, it is necessary to compute separately the heat transferred to the liquid and the vapor. The Webb-Chen correlation²⁰² [Eq. (4.250)] is suggested here.

At high reflooding rates, inverted annular film boiling, transition boiling, and transition to droplet laden steam must be considered. With a systems code based on single fluid modeling, it is adequate to use a modified Bromley film boiling prediction [Eq. (4.260)], an exponential transition boiling relationship [Eq. (4.238)], and the modified Condi-Bengston correlation [Eq. (4.245)] for the steam-region heat transfer. When a two-fluid system model is used, the separate heat transfer to vapor and liquid is again required. The Nelson and Unal¹⁸⁸ model [Eqs. (4.261) through (4.263) plus (4.250); model summarized in Table 6.III] would appear to be the best currently (1994) available. Note that the two-fluid model also requires interfacial drag and heat transfer rates (see Tables 6.IV and 6.V).

4.5 CONDENSATION

4.5.1 Basic Models

Condensation heat transfer plays a significant role in pressurizer operation as well as in various accident situations.

The simplest form of condensation to analyze is condensation on a vertical plate in the absence of any noncondensable gas. The average heat transfer coefficient for a plate of length L , based on a modification of Nusselt's analysis of condensate flow acted on by gravity and wall friction, is given by

$$(h_c)_{\text{avg}} = 1.13 \left[\frac{\rho_f (\rho_f - \rho_g) g H_{fg} k_f^3}{\mu_f L (T_w - T_{\text{sat}})} \right]^{1/4} \quad (4.266)$$

where the subscript f indicates saturated liquid and g saturated vapor. The local coefficient at length L is 75% of h_{avg} . The foregoing holds providing upward vapor flow is low and $Re_l \leq 1800$ where $Re_l = 4m/\mu_f$ and m = condensation flow rate at L [mass/(tL)]. Some investigators replace H_{fg} in Eq. (4.266) by an effective value, H'_{fg} :

$$H'_{fg} = H_{fg} + 0.68 c_{pf} (T_b - T_w) \quad (4.267)$$

where T_b , T_w = primary fluid (vapor) and wall temperature, respectively.

The situation is somewhat more complicated when there is a noncondensable gas present. As vapor condenses on the cold surface, the noncondensable gas is left behind. The gas mixture near the cold surface becomes rich in the noncondensable gas and the condensable vapor (steam) must

diffuse across this noncondensable gas-rich layer. The condensation mass flux, w_g , is then determined by

$$w_g = K_g M_g (P_{g0} - P_{gi}) \quad (4.268)$$

where K_g = mass transfer coefficient [moles/(time area atm)]; M_g = molecular weight of condensable gas; and P_{g0} , P_{gi} = partial pressure of condensable gas in bulk steam and vapor-liquid interface, respectively (atm).

When the vapor is stagnant or moving at a very low velocity, the value of K_g is determined from the thickness, δ , of the noncondensable rich film near the liquid surface.

$$K_g = \left[\frac{D_v}{RT\delta} \frac{P}{P_m} \right] \quad (4.269)$$

where

R = ideal gas constant [atm vol/(mol K)]

P = total pressure (atm)

P_m = log mean of partial pressure of noncondensable gas at interface and in bulk mixture

D_v = vapor diffusivity (area/time).

(Diffusivity at 1 atm and 0°C; water vapor in air = 0.22 cm²/s, water vapor in hydrogen = 0.75 cm²/s.) Collier²¹³ recommends that under stagnant conditions δ be obtained from

$$L/\delta = C_1 \left[\frac{gL^3 \rho_b}{\mu_v D_v} \left(\frac{\rho_b}{\rho_i} - 1 \right) \right]^{0.373} \quad (4.270)$$

where $C_1 = 1.02$ for vertical surface, 0.62 for horizontal surface, and ρ_b , ρ_i = density of vapor mixture at bulk and interface temperatures, respectively. When there is significant flow parallel to the surface, the mass transfer coefficient is obtained from h_g , the gas phase heat transfer coefficient by use of the heat and mass transfer analogy. That is,

$$K_g = \left[\frac{h_g}{c_{p,g} P_m M_g} \right] \left(\frac{Pr}{Sc} \right)^{2/3} \quad (4.271)$$

and Sc = Schmidt number = $\mu/\rho D_v$, and Pr = Prandtl number.

The total heat transfer rate, q'' , is obtained from the sum of the heat transferred by condensation plus the heat transferred by convection to the liquid film

$$q'' = h_g (T_b - T_{gi}) + K_g M_g H_{fg} (P_{gb} - P_{gi}) \quad (4.272)$$

where P_{gb} , P_{gi} = pressure of condensable gas in bulk steam and interface, respectively, and T_b , T_{gi} = temperature of condensable gas in the bulk stream and at liquid-gas interface, respectively. When the vapor is stagnant only the second term on the right is significant.

In the presence of a noncondensable gas, the total resistance to heat transfer between the vapor and the surface is the sum of the resistance of the gas film and the liquid layer on the surface. Due to the maintenance of a hydrogen overpressure in the pressurizer of a PWR, the effect of noncondensable gas must be considered for most situations. In many cases, the resistance of the gas film will be controlling and the calculation greatly simplified by ignoring the liquid film resistance.

4.5.2 Reflux Condensation

It was previously noted that, after a small LOCA, the system level may decrease to the point where the steam generator tubes and hot leg are drained of liquid and the steam produced in the core is condensed in the steam generator tubes. It is generally appropriate to assume that the hydrogen overpressure, as well as gamma-ray initiated water decomposition, will lead to a significant presence of noncondensable gas. Since the gas flow in the steam generator tubes is very low, the steam may be considered stagnant and Eqs. (4.268) and (4.269) may be used to obtain the condensation rate. The gas diffusivity used should be corrected to the temperature and pressure of the expected operation by²¹⁴

$$D_v(P,T) = (D_v)_{\text{std}} \left(\frac{T}{273} \right)^{3/2} \left(\frac{1}{P} \right) \quad (4.273)$$

where $(D_v)_{\text{std}}$ = diffusivity at 0°C and 1 atm, T = temperature (K), and P = pressure (atm). Both the gas phase and liquid phase resistances need to be considered and thus the interface temperature T_i is unknown. The interface temperature will correspond to the saturation temperature at partial pressure P_{gi} . This temperature is then the saturation temperature appearing in Nusselt's equation [Eq. (4.266)]. The interface temperature, T_i , may then be determined by the simultaneous solution of Eq. (4.272) and

$$q''_i = \frac{T_{\text{sec}} - T_i}{\left[\frac{1}{h_c} + \frac{l}{k} \left(\frac{D_{\text{in}}}{D_{\text{av}}} \right) + \frac{1}{h_{\text{sec}}} \left(\frac{D_{\text{in}}}{D_{\text{out}}} \right) \right]} \quad (4.274)$$

where

T_{sec} temperature of secondary system fluid

h_c liquid film condensation coefficient

h_{sec} = secondary system heat transfer coefficient

l = tube wall thickness

k = tube wall conductivity

D_{av} , D_{in} , D_{out} = average, inner, and outer tube diameter, respectively.

Since both K_g and h_c vary with length, an appropriate averaging procedure needs to be used. As noted earlier, if the gas phase resistance is substantially greater than that of the liquid film, the calculation can be greatly simplified by ignoring the liquid film.

4.5.3 Spray Condensation

Condensation on a liquid spray occurs when liquid is sprayed into the pressurizer of a PWR in order to counteract a pressure surge. The efficiency of the condensation achieved may be described by the parameter R_T where

$$R_T = \frac{H_{fg} W_c}{(c_p)_L (T_{sat} - T_f) W_f} \quad (4.275)$$

and

$(c_p)_L$ = specific heat of liquid

T_{sat} , T_f = saturation temperature and inlet temperature of spray liquid, respectively

W_f = spray liquid flow rate (mass/time)

W_c = rate of steam condensation on spray (mass/time).

If no condensation occurs, R_T is zero. The maximum condensation that can be achieved occurs when all the exiting spray fluid is heated to the saturation temperature ($R_T = 1.0$). Some pressurizer models²¹⁵ assume that pressurizer operation is such that $R_T = 1.0$ although this somewhat overestimates condensation rates.

More accurate representations of pressurizer behavior recognize that equilibrium may not be achieved. Spray condensation rates, W_c , may then be obtained by²¹⁶

$$W_c = (UA)_c (T_v - T_f) / (H_v - H_f) \quad (4.276)$$

where $(UA)_c$ = product of effective heat transfer coefficient and effective heat transfer area of spray, T_v = temperature of vapor, and H_v , H_f = enthalpy of vapor and spray liquid, respectively.

Experimental values for $(UA)_c$ are required if this approach is used. Because of the hydrogen presence in the pressurizer, $(UA)_c$ will be appreciably lower than the value applicable in the presence of steam alone.

Condensation by water sprayed into the vapor container after a LOCA also involves spray condensation. An approach similar to that of Eq. (4.276) is desirable.

4.5.4 Direct Contact Condensation on Flowing Liquid Streams

Direct contact condensation on a flowing liquid is of particular importance during a LOCA. When the emergency core cooling water flows into the primary system, steam will condense on (1) the ECCS liquid in the bottom of the line into which it is injected and (2) on the liquid flowing down the wall of the downcomer. In both cases stratified flow is expected. Condensation in the downcomer was considered in the discussion of flooding in Sec. 3.5.4. Equation (3.266) may be used to estimate the effect of the cold injection water on the steam discharge rate.

Direct condensation on a stratified liquid flow in a horizontal pipeline has been examined by a number of investigators. During the early phases of a LOCA, the effect of any dissolved noncondensable gas is negligible because of the large steam discharge flow. Condensation is thus from an essentially pure condensable vapor and hence the heat transfer resistance is entirely within the liquid phase. A variety of empirical correlations had been proposed for this situation. The correlation of Lim *et al.*²¹⁷ provides a convenient way of computing condensation rates in cocurrent flow. For smooth interfaces

$$\text{Nu}_z = 0.534 \text{Re}_{Gz}^{0.58} \text{Re}_{Lz}^{0.09} \text{Pr}_L^{0.3} \quad (4.277a)$$

and for wavy interfaces

$$\text{Nu}_z = 0.0291 \text{Re}_{Gz}^{0.58} \text{Re}_{Lz}^{0.42} \text{Pr}_L^{0.3} \quad (4.277b)$$

where

$$\text{Re}_{Lz} = zG_L / (\mu_L)_{av}$$

$$\text{Re}_{Gz} = zG_G / \mu_G$$

$$\text{Nu}_z = \bar{h}z / (k_L)_x$$

z = axial distance from entrance

G_z, G_L = superficial mass velocity of liquid and vapor, respectively (mass/area time)

$\bar{h}$ = average heat transfer coefficient (energy/area time K)

$(k_L)_x$ = average of liquid thermal conductivity at entrance and distance z

$(\mu_L)_{av}$ = average of liquid viscosity at entrance and distance z

μ_G = gas viscosity.

Recent attempts at prediction of direct contact condensation have tended to examine the problem from a more fundamental basis using either $K-\epsilon$ turbulence models²¹⁸ or surface renewal theory.²¹⁹ These approaches are capable of predicting available experimental data and can be used as a basis for the derivation of design procedures.

4.5.5 Steam Condensation on Containment Surfaces

Condensation on containment surfaces requires the estimation of the condensation of steam in the presence of large amounts of air. The approach described in Sec. 4.3.1 for determining condensation rates in the presence of a noncondensable gas may be used here. However, the problem is complicated by the requirement for estimating the gas phase heat transfer coefficient in the presence of turbulent natural convection. Although the resistance of the liquid film is negligible in the presence of large quantities of noncondensable gas, Kim and Corradini²²⁰ show that the waviness of the liquid film significantly affects the gas phase resistance. In view of these complications, some investigators have therefore preferred to use an empirical correlation of experimental data. Dehbi *et al.*²²¹ correlate their steam condensation data by

$$\overline{h}_L = \frac{L^{0.05}[(3.7 + 28.7P) - (2438 + 458.3P) \log(y)]}{(T_\infty - T_w)^{0.25}} \quad (4.278)$$

where

$\overline{h}_L$ average condensation coefficient (W/m² °C)

P pressure (atm)

T_w, T_∞ = wall and bulk vapor temperatures, respectively (°C)

L = length of vertical condensing surface (0.3 m < L < 3.5 m)

y air mass fraction.

4.6 HEAT TRANSFER IN REACTOR SYSTEM COMPONENTS

4.6.1 Heat Transfer in Steam Generators

4.6.1.1 Heat Transfer Rate

Most PWR steam generators are natural circulation units. The heat transfer area for such units can be estimated using an overall heat transfer coefficient, U , and the log mean-temperature difference (LMTD). Lewis *et al.*²²² estimate that the normal range for U in a natural circulation boiler is between 600 and 900 Btu/(hr ft² °F). For a more precise area calculation, consideration should be given to the fact that secondary-side water entering the tube bundle is subcooled. Figure 4.22 shows the temperature change in primary and secondary fluid as a function of fluid enthalpies for a typical straight-through design. It is obvious that the use of an LMTD, taking the secondary fluid to be at saturation, is conservative. The situation can be handled by computing the LMTDs for the subcooled and bulk-boiling regions separately. In the case shown, areas for the two regions would be computed. When a once-through superheating unit is designed, the superheating section must be considered separately.

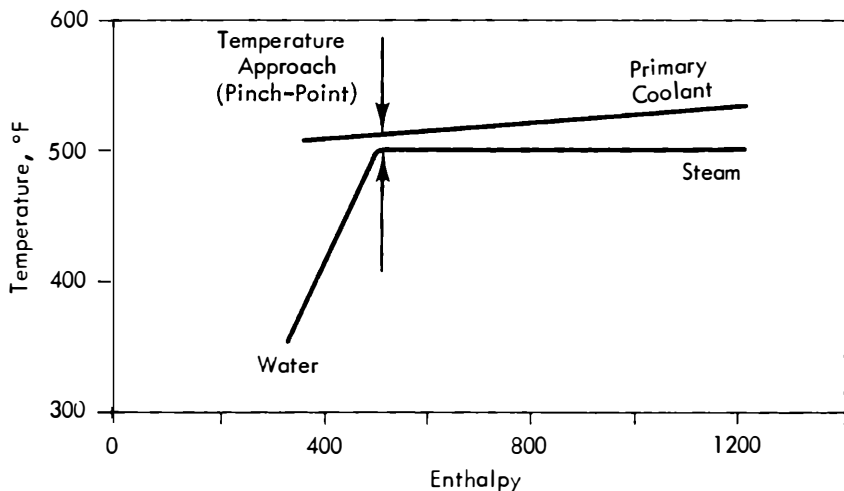


Fig. 4.22 Temperature variation in a straight-through steam generator.

The U-tube recirculating steam generator (see Fig. 1.9) really contains one shell-side pass and two tube-side passes. This can reduce the effective temperature difference and, in most heat exchangers of this configuration, the LMTDs would be multiplied by an appropriate F factor to account for this. (See any basic heat transfer texts for graphic presentation of F factors.) However, since steam temperature does not change, the F factor becomes 1.0 and the LMTD can be used without correction.

Some recent recirculating U-tube designs use an integral preheater on the cold leg of the tube bundle to decrease the effective temperature difference in the boiling region and produce higher steam pressures. This design is illustrated in Fig. 4.23.

In the integral preheater design,²²³ feedwater is introduced directly into the preheater in the lower shell. Feedwater is directed in a counterflow arrangement with a baffling zone adjacent to the tube plate. The preheater heats the feedwater to a temperature slightly below the boiling inception point. The water then enters a region separated from the recirculated stream, where nucleate boiling occurs. The feedwater then joins the recirculating steam.

The effect of the integral preheaters may be more readily understood by referring to Fig. 4.24, which illustrates the temperature-length behavior. The solid lines illustrate behavior in the original design where steam temperature is limited by the temperature approach at the pinch point (P_1). When an integral preheater is added, the secondary fluid on the hot-leg side follows line A, while the secondary fluid in the preheater section follows line

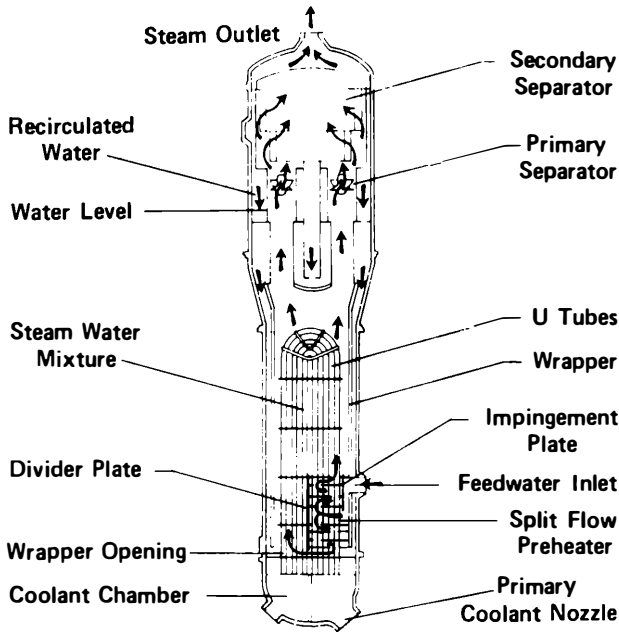


Fig. 4.23 Recirculating U-tube steam generator with integral preheater [from Ref. 223].

B. Since a smaller area per unit length is available for preheating, the fluid in the preheater reaches saturation at a length further from the inlet. The pinch point is at P_2 and thus, the steam temperature is increased. Note that four separate LMTDs are required for estimating the behavior of this design.

The overall resistance to heat transfer is the sum of the resistance of the primary coolant film, the tube wall, fouling, and the boiling film coefficient. If we base the calculations on the outer or steam side area, we have

$$\frac{1}{U} = \frac{1}{h_i} \frac{A_o}{A_i} + \frac{r_o - r_i}{k_w} \frac{A_o}{A_m} + f + \frac{1}{h_o} \quad (4.279)$$

where

h_i = primary coolant film coefficient, Btu/(hr ft² °F)

h_o = secondary side film coefficient

A_o, A_i, A_m = outside, inside, and mean surface areas
= outside and inside radii

k_w = thermal conductivity of tube wall, Btu/(hr ft °F)

f = fouling factor, (°F ft² hr)/Btu.

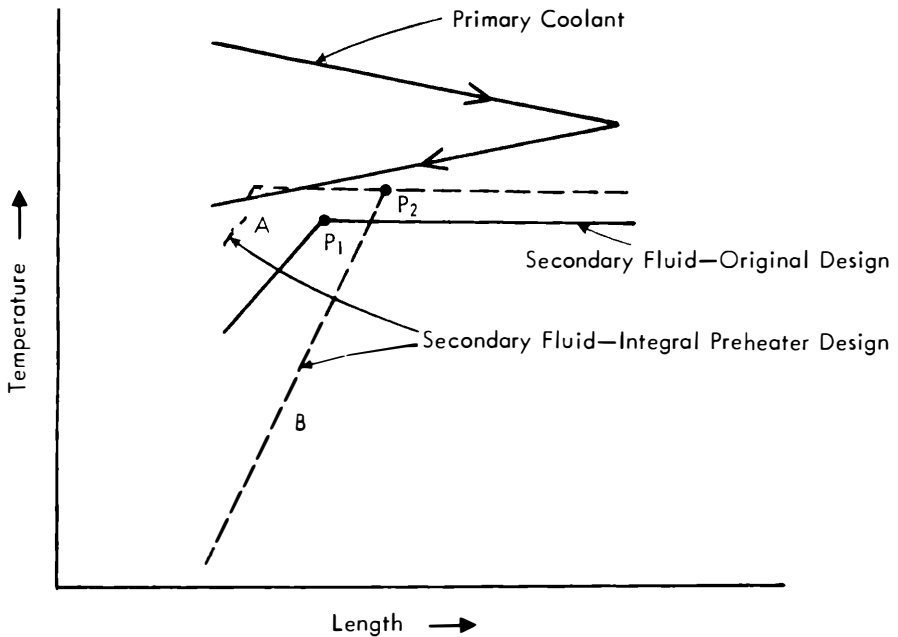


Fig. 4.24 Temperature variation in U-tube steam generator.

The primary-side coefficient can be computed from Eq. (4.154) or another available correlation for a low viscosity fluid flowing turbulently inside a tube. The maximum value of h is usually determined by the highest mass velocity tolerable from a pumping power standpoint. This limit, which has been in the neighborhood of 10 to 15 ft/s, is encountered before reaching velocities at which erosion is a problem.

The correlations available for computing shell-side boiling film coefficients were discussed in Sec. 4.4.1. Both the Rohsenow and Jens-Lottes correlations have been used by designers. Note the possibility that a short section near the secondary-side inlet may not be in local boiling. An appropriate correlation for forced convection single-phase coefficients for flow parallel or across tube bundles should be used (see Sec. 4.3.1) for this portion of the exchanger.

Fouling factors can be considered to represent safety factors that increase the design area of a heat exchanger to compensate for possible scale buildup on heat transfer surfaces. Since the primary coolant is maintained at extremely high purity levels, fouling on the primary side is usually negligible. Secondary system purity has been lower and some fouling can occur. Lewis *et al.*²²² state that an allowance of 0.0003 ($^{\circ}\text{F ft}^2 \text{ hr}$)/Btu is generally accepted as reasonable.

Although it is unlikely that DNB will be encountered in a recirculating unit, it is possible for it to occur (1) in the primary coolant inlet region, where temperature differences and heat fluxes are highest, and (2) at the secondary fluid exit, where steam qualities are highest. The designer compares expected local heat flux with predicted DNB flux for those regions and attempts to maintain an adequate safety margin.

Since DNB is of greater importance in a once-through boiler, reasonable costs can be achieved only if most of the heat is transferred by nucleate boiling. Davies²²⁴ states that it is possible to maintain nucleate boiling up to the position where the quality is in excess of 85%, if low mass flow rates [below 400,000 lb/(hr ft²)] and heat fluxes (under 150,000 Btu/(hr ft²)] are used. At higher qualities, heat transfer can be assumed to be to steam. Baffling is used to provide flow velocities that yield acceptable heat transfer coefficients.

The baffling of the superheating region of a once-through steam generator creates a complex flow path where cross-flow is encountered between baffle plates and something close to parallel flow in the baffle windows. The complex flow path makes exact evaluation of the heat transfer coefficient difficult. In the usual approach,²²⁵ the average heat transfer coefficient, $\bar{h}$, is based on the coefficient for pure cross-flow, h_{id} , and a series of correction factors. That is

$$\bar{h} = h_{id} J_c J_l J_b \quad (4.280)$$

The correction factor for the baffle cut, J_c , includes the effect of window and bundle heat transfer. Its value is 1.0 for an exchanger with no tubes in the window and increases to 1.15 for small baffle cuts. It decreases to 0.65 for large baffle cuts. For a typical well-designed heat exchanger, its value is near 1.0. The term J_l represents the correction factor for baffle leakage effects. It includes both tube-to-baffle and baffle-to-shell leakage with a heavy weight given to the latter. It is a function of the ratio of leakage area to crossflow area and a function of the clearances. A typical value of J_l is in the range of 0.7 to 0.8. Finally, J_b is the correction factor for bundle bypass.

Nahavandi *et al.*²²⁶ considered the optimal design of vertical natural circulation steam generators. They conclude that minimum costs (capital + operating) are achieved by using the smallest tubes permitted by scale formation and maintenance considerations. Increasing tube roughness led to increased costs.

4.6.1.2 Circulation Rate

A natural circulation loop is established in each recirculating steam generator design. A differential head is produced by the difference in density between the water in the downcomer and the two-phase mixture in the tube

bundle. Circulation rate is determined by equating the differential head to the pressure losses of the system.

A crude measure of the adequacy of secondary coolant flow through the tube bundle is the *circulation ratio*, defined as the weight of water circulating in the exchanger per unit weight of steam generated. A small circulation ratio indicates a large fraction of steam in the tube bundle and the possibility of poor heat transfer on the shell side. In addition, a low circulation ratio can lead to flow instability. In the past, units have been designed with circulation ratios such that the steam quality at the exit of the bundle does not exceed 20 to 25%. Measurements of the behavior of a typical large U-tube generator²²⁷ indicate that the circulation ratio decreases from 22 at 11% of full load to 4.4 at full load.

A low circulation ratio can lead to flow oscillations that cause periodic uncovering of a portion of the heat exchanger surface. This "chugging" phenomenon reduces heat transfer efficiency, and flow oscillations of sufficient amplitude can lead to large fluctuations in the water level or steam flow rate.

Experience has shown that flow stability can be achieved by keeping the steam quality in the tube bundle low, or equivalently, by keeping the recirculation ratio high. This necessitates a low pressure drop on the shell side which can be accomplished by increasing the size of the flow paths and, hence, the steam generator vessel. This is undesirable for large units, since it can lead to an uneconomic design.

The circulation rate is not necessarily uniform across the bundle cross section. The nonuniformities arise due to the much higher rate of steam generation along the hot leg and, in older designs, due to the manner in which feedwater is introduced. In many steam generators designed prior to 1972, nearly stagnant regions were found near the tube support plates.

4.6.1.3 Modeling of Steam Generator Behavior

The analysis of steam-generator behavior generally falls into two categories: (1) analysis of secondary side flow paths and (2) analysis of the dynamic response of the steam generator to system transients.

The analysis of secondary side flows is often accomplished using the porous media approach (see Sec. 4.2.2). General-purpose porous media codes, such as COMMIX,⁸¹ may be used or porous media programs especially adapted to heat exchanger design (i.e., FIT-III²²⁸ and ATHOS²²⁹). The available programs appear to be able to predict performance adequately as evidenced by agreement with experimental measurements. The fluid velocities obtained from the porous media analysis can be used to determine whether tubing vibration is a problem (see Sec. 3.2.3).

The analysis of transient response generally uses a one-dimensional model similar to the models used for transient system analysis. Indeed,

application of such reactor system analysis programs as RELAP5/MOD2 to steam generator transient analysis²³⁰ is common. Alternatively, one-dimensional programs similar to reactor system analysis codes may be specially tailored to heat exchanger analysis.²³¹

Appreciably simplified models for analysis of heat exchanger behavior during transients have also been used. Sharon *et al.*²³² devised a model in which the steam generator is nodalized into just three regions: tube bundle, steam drum, and downcomer. The model was capable of predicting all of the major qualitative features and many quantitative features of the transients to which it was compared. Models of this nature have short computing times and are particularly useful for reactor simulators where a real-time response is desired.

4.6.1.4 Operational Problems

U-Tube Steam Generators. A major operating difficulty with U-tube steam generators has been localized corrosion coincident with sludge deposition in low velocity zones on the secondary side. To eliminate such problems, all-volatile chemistry has replaced earlier phosphate secondary water treatment. In addition, designs similar to that shown in Fig. 4.23 incorporate special baffles that significantly increase fluid velocity in the neighborhood of the lower tube sheet. Later designs also use fabrication techniques that eliminate the crevices between the tubes and tube sheet. In Russia, horizontal units with vertical tube support plates have been used in the VVER reactor systems. Experience with VVER steam generators indicates no sludge deposition problems.

Use of all-volatile secondary side chemistry appears to have largely eliminated tube thinning and corrosion-assisted cracking. However, in some units there was an unexpected accumulation of corrosion products in the crevices between the tubes and tube support plates. In a number of instances, the corrosion product accumulation has led to inward denting of the steam generator tubes at these locations. The problem has been most often seen in plants with seawater-cooled condensers and is believed to be due to chloride contamination of the secondary fluid from condenser leakage. Newer plants use improved condenser designs that considerably reduce leakage. In addition, stainless-steel support plates with quatrefoil cutouts are used. The stainless steel reduces corrosion and the quatrefoil cutouts minimize the length of the narrow gap region between the support plate and steam generator tube. Addition of boric acid to the secondary coolant is an additional deterrent to crevice corrosion.

In some European plants, corrosion and denting problems have been minimal despite use of phosphate treatment for the secondary water. These steam generators have used tubes of Incoloy 800. This alloy is more resistant to stress corrosion cracking than the Inconel 600 that is usually used. In

addition, the tubes are spaced by vertical flat bars of stainless steel rather than carbon steel support plates.

In the 1970s an additional problem surfaced, primary water stress corrosion cracking (PWSCC) of Inconel 600²³³. This primary side cracking is observed in highly stressed areas such as the tight bend of the first rows of U-tubes, tube dents at support plates, and the expansion transitions where the tubes enter the tube sheet. The cracking is more likely to be found in the hot-leg side of the tubing since the process is accelerated by high temperature. The problem is considerably alleviated in later designs by using a material less susceptible to PWSCC (Inconel 690), stress relieving the tubing after the manufacture of U-bends and use of explosively expanded tube to tubesheet joints (instead of rolled joints).

It is also possible to treat existing exchangers to minimize the problem. The inner tube rows may be stress relieved by using flexible heaters to heat the tubes during a shutdown period. To reduce problems in the expansion zone adjacent to the tube sheet, shot or rotopeening has been used. This reduces the inner surface tensile stresses so that cracks do not start. However, it appears that peening will not completely stop the growth of cracks that have already started.

It has also been determined that PWSCC is affected by the concentration of lithium in the primary coolant. Chemical shim control using boric acid requires the use of lithium hydroxide to adjust the pH to near neutral levels to keep Zircaloy cladding corrosion at acceptable rates. At the high boric acid concentrations needed at the beginning of life, the pH adjustment may require more than 3 ppm lithium. PWSCC is accelerated at this level. It is thus necessary to determine conditions that balance fuel and heat exchanger corrosion. Good practice appears to require that there be no operation below a pH of 6.9 and that lithium levels be decreased to 2.2 ppm or less as soon as possible. Lithium can then be maintained at that level (to keep primary side corrosion low) until a pH of 7.4 is reached. At that point, the lithium concentration should be decreased as the boron concentration decreases in order to maintain a pH that does not exceed 7.4.

Another operational problem that must be considered is that of steam-generator water hammer.²³⁴ In several abnormal transients (e.g., main feedwater pump trip), it is possible for the feedwater spargers in the steam generator to uncover. Cold auxiliary feedwater is then introduced into the feedwater piping to restore water level and maintain heat transfer. Under some circumstances, this water can form a slug that blocks the pipe cross section and traps steam void upstream. If this happens, the steam void condenses and the void pressure decreases rapidly. The pressure difference created accelerates the water slug through the piping, where it impacts on the first elbow or bend sending a pressure wave through the piping (water hammer). To mitigate this phenomenon, revised designs have been developed. These designs have used a combination of discharging from the top

of the feedwater sparger, keeping feedwater piping short, initiating feedwater flow early during transients which can uncover the sparger,* and limiting the feedwater flow rate until the sparger is refilled with water. Alternatively,²³⁴ means can be provided to furnish feedwater at a rate in excess of the drainage rate whenever the steam generator level drops below the feed ring.

Once-Through Steam Generators. Thermal stresses in heat exchanger tubes during transients and part-load operation give rise to concern in once-through generators. Considerable care is taken to minimize the differential expansion between tubes and shell during full-load operation. In operating plants designed in the 1970s, this is accomplished by causing a stream of slightly superheated steam from the upper part of the tube bundle to flow down an annulus between the tube bundle shell and the vessel over the major part of the vessel length. In addition, the lower portion of the vessel is filled with water preheated (by mixing it with steam) to a temperature close to saturation. The lower portion of the tubes where boiling occurs has a wall temperature close to saturation, while the upper section has a temperature close to that of the primary system. The total expansion of vessel and tubes at full load may be very nearly equalized by proper choice of the upper and lower lengths.

At part loads, the portion of the steam generator in the superheating region increases and the average temperature of the tubes increases. The temperature of the exit steam does not change at part load and, hence, the expansion of vessel and tubes differs. This gives rise to thermal stresses. An additional source of tube-wall thermal stresses is the wall temperature fluctuation seen in the transition boiling region. This is caused by the vertical oscillation of the dryout front at the beginning of the superheating region.

Although fatigue cracks due to varying stresses caused by frequent load changes or transition boiling temperature oscillations are possible, a more likely possibility is the stress-induced breakup or magnetite (iron oxide) layers on the tube surface and subsequent enhancement of corrosion. Failure due to this mechanism is called *corrosion fatigue*.

While tube leakage in once-through generators has been observed, very few units are in operation. Styrkovitch *et al.*²³⁵ believe that the failures are most likely due to corrosion fatigue. However, no detailed description of the situation appears to be available.

Various revised designs, such as undulating or spiral tubes, which would further reduce thermal stresses, have been proposed. However, none have actually been used.

*The sparger is the device that distributes feedwater flow to the steam generator tube bundle.

Note that since all the water entering the tube in a once-through unit is evaporated, the entering water must be very high purity. All of the water fed to these units flows through demineralizers and typically has sodium concentrations of 5 ppb or less.

Another feature of once-through steam generators that must be considered is their low water volume. In the case of feedwater failure, dryout of the unit occurs very rapidly. Emergency feedwater must be provided quickly or the system will not have the capability of removing the reactor heat generation.

Horizontal Steam Generators of VVER Design. In contrast to western plants, the ends of the heat exchanger tubes in the VVER designs are attached to the walls of two vertical pipes. Primary coolant enters through the inlet (hot) headers, passes through the heat exchanger tubing, and exits through the outlet (cold) headers. In a number of VVER steam generators, cracks have been found in the cold header in the ligaments between the tubes. Cracking was observed after 10^4 to 6×10^4 hours of operation.²³⁶

The headers are pearlitic steel containing about 2% nickel, 1% manganese, 0.5% molybdenum, and not more than 0.3% chromium. The cracks originate on the secondary side and begin by intercrystalline corrosion and later propagate by an intergranular mechanism. Damage appears to be confined to steam generators where the tubes were rolled explosively into the header. Corrosion was enhanced by poor water chemistry in some plants.

Revised fabrication procedures have been developed to decrease stress levels and eliminate crevices between the header and tube. Damaged generators are replaced by generators manufactured by the improved techniques. Experience with these revised designs (as of 1993) has been very good.

4.6.2 Pressurizer Heat Transfer

The heat transfer processes occurring in a PWR pressurizer consist of bulk vaporization due to depressurization, bulk condensation due to pressurization, condensation on liquid spray, and condensation on the vessel wall. The simplest approach to pressurizer analysis bypasses all of the foregoing processes and assumes thermodynamic equilibrium between all of the liquid and vapor contained within the unit (single region model). Todreas and Kazimi²³⁷ describe this approach in detail and therefore the description will not be repeated here.

The assumption of complete thermodynamic equilibrium does not lead to satisfactory prediction of rapid transients. Therefore, most current models allow for nonequilibrium between the phases. All of these models couple mass and energy conservation equations with equations of state and appropriate constitutive equations in the manner described in Sec. 3.3.4. Note that the momentum conservation can be ignored since fluid velocities are

low. Because a thorough description of the basis for such models is also provided by Todreas and Kazimi,²³⁷ only the significant features that differentiate some of the models are described here.

In the best estimate, two-fluid model transient analysis codes such as TRAC and RELAP-5, the pressurizer is not treated any differently than any other portion of the system. The pressurizer is modeled as a series of linearly connected control volumes, which are themselves connected to the remainder of the system via junctions with the control volumes representing pressurizer surge and spray lines. Bulk condensation and vaporization rates are usually determined from a mass transfer rate constant and the difference between the actual and saturation temperatures. Spray condensation rates would be determined from the interfacial heat transfer rates.

A number of transient analysis programs have treated the pressurizer with a separate component model. This has generally been the case for programs using a single-fluid model (HEM or dynamic slip). These component models consider a volume that is primarily liquid and one that is primarily vapor. The models have usually allowed for the presence of a superheated vapor or subcooled liquid. However, superheated liquid is not considered and liquid vaporization during a depressurization is determined assuming thermodynamic equilibrium in the lower (primarily liquid) region.^{238,239} The rate at which the vapor produced in the lower region is released to the upper (vapor) region is determined by a bubble rise model^{235,236} (see Sec. 6.5.3.1 for discussion of bubble rise models). Some pressurizer models have included rate equations for both bulk evaporation and condensation due to pressure changes. Baggoura and Martin²⁴⁰ base these rates on the kinetic theory of gases. They calculate the net rate of mass transfer between phases, W , as

$$W = A_i E [M / (2\pi R)]^{1/2} (P_{\text{sat}} T_w^{-1/2} - P T_v^{-1/2}) \quad (4.281)$$

where

A_i = interfacial area

W = rate of mass transfer (mass/time)

M = mole weight of fluid

R = gas constant (J/mol K)

T_v, T_w = vapor and water temperature, respectively (K)

P_{sat}, P = saturation and actual pressures, respectively

E E_c if $P_{\text{sat}} < P$; E_v if $P < P_{\text{sat}}$

E_c, E_v = experimentally based condensation and evaporation coefficients, respectively.

If $P_{\text{sat}} > P$, net vaporization occurs and W is positive. If $P_{\text{sat}} < P$, the net effect is condensation and W is negative. Use of Eq. (4.281) requires experimentally based values of E .

Pressurizer behavior has a very significant effect on most operational transients. The pressurizer is the major factor in setting the system pressure and, in addition, pressurizer inflow and outflow influence the flow and temperature of the circulating fluid.

4.7 COOLING OF STRUCTURE AND SOLID MODERATOR

4.7.1 Temperature Distribution Within Core Structure

In Sec. 1.6, we indicated that the heat distribution in a flat plate due to the absorption of uncollided γ flux follows a distribution of the form

$$q_x''' = q_1''' \exp(-ax) \quad (4.282)$$

where q_x''' = volumetric heat flux at distance x from surface, and q_1''' = volumetric heat flux at the surface adjacent to the source. An equation of this form is often used as an approximate representation of the total heat generation, the effect of buildup factors being approximately accounted for adjustment of constant a . For this flux distribution, the general solution for the temperature distribution, when the heat transferred from each surface of the plate is not the same, is given by

$$T_x - T_c = [q_1''' / (ka^2)] \exp(-dx) - C_1 x + C_2 \quad (4.283)$$

where

$$C_1 = \left\{ \frac{-q_1'''}{ka} \left[\frac{k}{h_1} + \frac{1}{a} + \frac{k \exp(-aL)}{h_2} - \frac{\exp(-aL)}{a} \right] \right\} / \left(\frac{k}{h_1} + \frac{k}{h_2} + L \right)$$

$$C_2 = \frac{kC_1}{h_1} + \frac{q_1'''}{h_1 a} + \frac{q_1'''}{ka^2}$$

and

T_c = temperature of coolant

k = thermal conductivity of plate, Btu/(hr ft °F)

h_2, h_1 = heat transfer coefficients at surfaces $x=L$ and $x=0$, Btu/(hr ft² °F)

L = thickness of plate, ft

q_1''' = volumetric heat flux at surface $x=0$, Btu/(hr ft³).

In a cylindrical shell with a central source, such as a portion of the vessel or thermal shield, the heat generation rate falls more rapidly than in a plate. Since the incident flux varies inversely with radius r , the heat generation rate is then given by

$$q_x'' = (R_1 / r) q_1''' \exp[-a(r - R_i)] \quad (4.284)$$

where R_i is the inside radius. The temperature distribution then becomes

$$T - T_0 = [(R_i q_1) \exp(-aR_i)/(ak)] \{E_1\{-aR_i[1 + (L/R_i)L_1]\} - E_1(-aR_i) - \exp(-aR_i)[1 + (L/R_i)L_m] \ln[1 + (L/R_i)L_1]\} \quad (4.285)$$

where

$E_1(x)$ exponential integral of x

L = wall thickness

$L_1 = r - R_i/L$

L_m thickness at which maximum temperature occurs

T_0 = temperature at $x = 0$.

Equation (4.285) is rarely used for temperature evaluation due to its complexity. Fortunately, for most structural units encountered, the ratio of wall thickness to cylinder diameter is low enough so that a good approximation is obtained by using the solution for a plate. Equation (4.283) can then be used where a simple exponential distribution offers a good approximation of the heat flux distribution. In most instances, a better approximation of heat generation is provided by

$$q_x''' = q_1'''(-a_1x) + q_2''' \exp(-a_2x) + \dots + q_m''' \exp(-a_mx) \quad (4.286)$$

Thus, we can allow for the various energy groups. When the plate is heated from both sides, terms of the form $q_i \exp[-a_i(L-x)]$ can be included; they can be rewritten as

$$q_i[\exp(-a_iL)] \exp(a_ix) = q_i' \exp(-a_i'x) \quad (4.287)$$

where $a_i' = -a_i$. These are compatible with Eq. (4.286) and the subsequent development. By including rapidly decaying terms with negative q_i , we can also closely approximate the effect of buildup factors on heat generation in the region closest to the surface.

For heat generation following Eq. (4.286) in a plate cooled on both sides, temperature distribution is given by

$$T - T_2 = -\frac{1}{k} \left[\frac{q_1''' \exp(-a_1x)}{a_1^2} + \frac{q_2''' \exp(-a_2x)}{a_2^2} + \dots + \frac{q_n''' \exp(-a_nx)}{a_n^2} \right] - C_1x - C_2, \quad (4.288)$$

where

$$C_1 = \left[L + k \left(\frac{1}{h_1} + \frac{1}{h_2} \right) \right]^{-1} \left\{ \frac{q_1'''}{a_1} \left[\exp(-a_1L) \left(\frac{1}{h_2} - \frac{1}{a_1k} \right) + \frac{1}{h_1} + \frac{1}{a_1k} \right] + \frac{q_2'''}{a_2} \left[\exp(-a_2L) \left(\frac{1}{h_2} - \frac{1}{a_2k} \right) + \frac{1}{h_1} + \frac{1}{a_2k} \right] + \dots + \frac{q_n'''}{a_n} \left[\exp(-a_nL) \left(\frac{1}{h_2} - \frac{1}{a_nk} \right) + \frac{1}{h_1} + \frac{1}{a_nk} \right] + (T_2 - T_1) \right\}, \quad (4.289)$$

and

$$C_2 = \frac{C_1 k}{h_1} - \frac{q_1'''}{a_1} \left(\frac{1}{h_1} + \frac{1}{a_1 k} \right) - \frac{q_2'''}{a_2} \left(\frac{1}{h_1} + \frac{1}{a_2 k} \right) - \frac{q_n'''}{a_n} \left(\frac{1}{h_1} + \frac{1}{a_n k} \right) \quad (4.290)$$

where

T_1 and T_2 = coolant temperatures at surfaces $x=0$ and $x=L$,
respectively

L thickness of plate

h_1 heat transfer coefficient at surface $x=0$

h_2 heat transfer coefficient at surface $x=L$.

Since constants C_1 and C_2 contain $1/h$ terms, we observe that the previous expressions cannot be used if one wall is insulated (one h is zero). In that event, we write constants C_1 and C_2 as

$$C_1 = \frac{q_1''' \exp(-a_1 L)}{a_1 k} + \frac{q_2''' \exp(-a_2 L)}{a_2 k} + \frac{q_n''' \exp(-a_n L)}{a_n k} \quad (4.291)$$

$$C_2 = \frac{-q_1''}{hA} - \frac{1}{k} \left(\frac{q_2'''}{a_1^2} + \frac{q_2'''}{a_2^2} + \frac{q_n'''}{a_n^2} \right) \quad (4.292)$$

where

$$q_i'' = \int_0^L q_x''' dL$$

For very thin plates or tubes, it is adequate to use uniform heat generation throughout. In this case, we assume the coolant temperature is the same on both sides of the plate and find

$$T - T_c = \frac{q_1''' x^2}{2k} + \frac{q_1''' x}{k} \left[-\frac{L}{2} + \frac{k}{h_2} + \left(\frac{kL/h_2 + L^2/2}{k/k_1 + k/h_2 + L} \right) k \left(\frac{1}{xh_1} - \frac{1}{Lh_1} - \frac{1}{Lh_2} \right) \right] \quad (4.293)$$

where T_c is the coolant temperature.

Ma²⁴¹ obtained a solution for transient temperature distribution occurring during the heat up of a reactor vessel wall. It is assumed that the vessel is originally at uniform temperature T_0 , is perfectly insulated at its outer radius ($r=r_b$), and is cooled with forced convection h at its inner radius ($r=r_a$). Ma²⁴¹ presents his results in terms of a dimensionless temperature difference, θ , and a dimensionless time, τ , where

$$\theta = ka^2(T - T_0)/q''' \quad (4.294)$$

$$\tau = [(k/\rho c_p)t]/r_a^2, \quad (4.295)$$

where a is the attenuation coefficient and t represents time. Transient temperature distribution is given by

$$\theta(x', \tau) = \sum_{i=1}^{\infty} S_i A_i(\lambda_i x') G_i(\tau) \quad (4.296)$$

where

$$S_i = \pi^2 / 2 \frac{\lambda_i^2 [\lambda_i J_1(\lambda_i) + \beta J_0(\lambda_i)]^2}{\lambda_i [J_1(\lambda_i) + \beta J_0(\lambda_i)]^2 - (\lambda_i^2 + \beta^2) J_1^2(\lambda_i x'_0)} \quad (4.297)$$

$$x' =$$

$$x'_0 = r_a / r_b$$

$$G_i(\tau) = \bar{f}(x') \exp(-\lambda_i^2 \tau) \int_0^{\tau} g(\tau) (\lambda_i^2 \tau) d\tau \quad (4.298)$$

$$\bar{f}(x') = \int_1^{x'_0} x [Y_1(\lambda_i x_0) J_0(\lambda_i x') - J_0(\lambda_i x_0) y_0(\lambda_i x)] q'''(x) dx \quad (4.299)$$

$$\beta = r_a (h/k) = \text{Biot number}$$

$\lambda_i (i=1,2,3)$ = eigenvalues and the positive roots of Eq. (4.300)

$$0 = Y_1(\lambda_i x'_0) [\lambda_i J_1(\lambda_i) + \beta J_0(\lambda_i)] - J_1(\lambda_i x'_0) [\lambda_i Y_1(\lambda_i) + \beta Y_0(\lambda_i)] \quad (4.300)$$

$J_c(\lambda_i x')$ = Bessel function of order c and first kind

$Y_c(\lambda_i x')$ = Bessel function of order c and second kind

$$q''' = q_1''' \exp(-ax) \cdot g(\tau)$$

$g(\tau)$ = function describing the variation of heating rate with time.

4.7.2 Temperature Distribution Within a Solid Moderator

We previously observed that in a solid with internal heat generation, the temperature field must satisfy the partial differential equation

$$\nabla(k \nabla T) = q''' \quad (4.301)$$

where q''' is the volumetric rate of heat generation. Normally, the solid is cooled by the series of parallel channels that contain the fuel elements. At each such channel, we must satisfy the boundary condition

$$k(\partial T / \partial n) = k(T_f - T_w) \quad (4.302)$$

where n = outward normal to the solid boundary, T_f = fluid temperature, and T_w = temperature of the solid boundary. A complete solution of the problem is highly complex since q''' can vary with location, k can be a

function of temperature, and T_f can vary significantly along the channel. Under such conditions, a numerical solution is required. The heat conduction in Eq. (4.301) is written in finite difference form, and an estimate of the heat removed by the coolant is obtained. The flow distribution, coolant axial temperature rises, and pressure drops of the coolant can then be estimated; revised coolant temperatures can then be used to obtain an improved estimate of the solid temperatures. Iteration is continued until the heat flux distributions at the fluid-solid interfaces are compatible with both sets of equations. An example of a computer program for such an analysis is provided by Lee and Gallagher.²⁴²

Computation of the rate of gamma heating of a solid moderator is also quite complex. Gamma heating of graphite-moderated systems has been examined by Hanson and Busselman²⁴³ using both a point-kernal shielding code²⁴⁴ and a transport code.²⁴⁵ The transport code appeared to supply the more reliable results.

For large, graphite-moderated cores, heat generation and coolant conditions can vary quite slowly with distance. A reasonable estimate of the radial temperature distribution in a region can then be obtained by considering a typical channel and the moderator surrounding it. Fend *et al.*²⁴⁶ obtained an approximate analytical solution in circular harmonics for a large, uniformly heated reactor with coolant holes at the vertices of equilateral triangles (Fig. 4.25). Since the region has 30-deg symmetry, only the

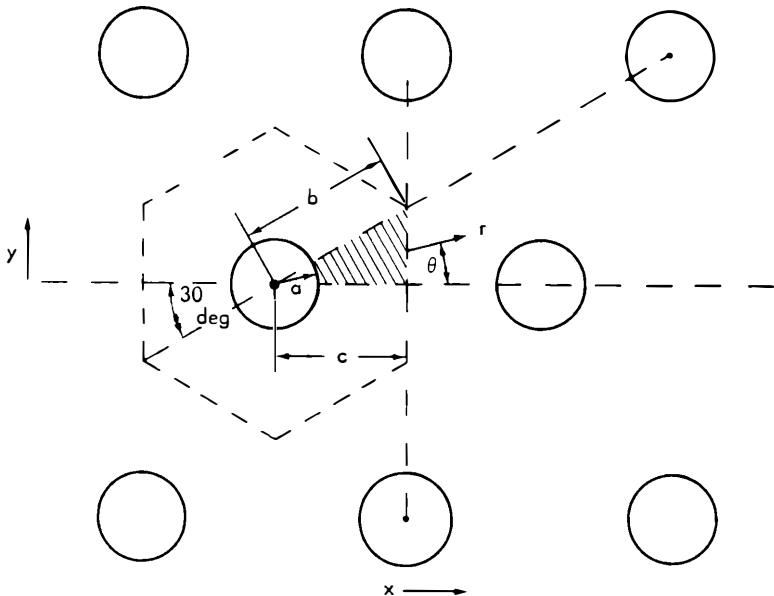


Fig. 4.25 Coolant passages in graphite-moderated reactor.

area bounded by the adiabatic planes $\theta = 0$ deg, $\theta = 30$ deg, and $x = c$ needs to be considered. The temperature at the walls of the coolant channels ($r = a$) is taken as uniform. The final result of this analysis is

$$\frac{T - T_a}{q'' b^2 / k} = \frac{1}{4} [(3\sqrt{3} / \pi) \ln(r/a) - (r/a)^2 + (a/b)^2] \\ - 0.01484(r/b)^6 \cos 6\theta - 0.00021(r/b)^{12} \cos 12\theta \quad (4.303)$$

where T_a = temperature at $r = a$.

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CHAPTER FIVE

Reactor Thermal and Hydraulic Performance

The thermal and hydraulic design of a core depends heavily on the expected performance of the core as indicated by its load follow characteristics, fuel burnup, and core life. This information is used to establish the relative heat flux distribution within the core. The limiting power output of the core design is then determined by the maximum heat removal capability in the hot channel or at the hot spot.

5.1 BASIC THERMAL DESIGN

This section describes conservative design procedures that are based on the use of highly adverse conditions for determining the limiting core power. Core thermal analyses based on this general approach were used for the original design of most PWRs in operation as of 1995. Final core designs and core reload behavior are now generally evaluated using the stochastic DNB methodology described in Sec. 5.4.2 along with the set-point analyses of Sec. 5.6.4. However, the conservative approach described here remains useful for preliminary core design and accident analysis. In addition, this approach forms the basis for understanding the design techniques now being applied.

5.1.1 Thermal Design Limitations and Approaches

Fuel integrity, as well as economy, determines the maximum allowable power output of a UO_2 fuel rod. The major factors considered are:

1. Centerline temperature safely below melting
2. Heat flux below a maximum value allowable by coolant conditions under expected operating situations
3. Burnup and fission gas release limited to avoid excessive internal pressure and cladding creep or embrittlement
4. Heat flux below the limit at which behavior is no longer acceptable during postulated accidents
5. Suitable power density (kW/kg of uranium) for a convenient refueling time and also for a reasonable fuel fabrication cost
6. Rate of power change limited to prevent excessive local stresses.

Each of the first four limitations requires a knowledge of the most adverse heat flux conditions in the core. Section 1.2.4 described the synthesis procedure that determined the maximum heat flux at each axial elevation. By considering the range of conditions possible during load follow operation, the maximum value for F_Q^N was determined as the maximum of the product of the radial peaking factor, $F_{xy}(z)$, and the average axial power $P(z)$. The hot spot factors so estimated constitute an upper envelope below which all expected operating values will lie. The channel which is assumed to contain the maximum heat flux is called the hot channel.* The location in the hot channel to which F_Q^N is assigned is designated the hot spot. Demonstration that the behavior of such a channel is satisfactory provides a conservative demonstration that the behavior of the core is satisfactory. The approach is useful in core analyses, since it can avoid tedious repetition of lengthy calculations. However, it leads to a design that is more conservative than necessary (see Sec. 5.3 for revised approach).

The nuclear hot-channel factors calculated by synthesis procedures are generally multiplied by the quantity F_U^N (where a typical value for F_U^N is 1.05), to allow for uncertainty in the nuclear calculations. In addition, we must account for those deviations from the ideal, which can lead to increases in power level. In the hot-channel concept, all pertinent adverse engineering effects, accounted for by establishing engineering hot-channel subfactors, are combined into the single-flow channel taken to have the highest integrated power output. The hot spot concept compounds all pertinent adverse effects into a single hot spot.

In simplified preliminary calculations, estimated engineering subfactors are often applied simultaneously to a single closed hot channel. The subfactors, either estimated from past experience or extrapolated from model tests, are fabrication tolerance, flow mixing, inlet plenum, and flow redis-

*A single-flow channel in a rod bundle fuel assembly is sometimes called a *subchannel* and is formed by four neighboring rods in a square lattice, or by three neighboring rods in a triangular lattice. The hot channel can be considered one such subchannel.

tribution. The probability of occurrence of various hot-channel factors can be combined in the calculation, but the coupling effects of various subfactors remain. These coupling effects introduce repetition of design penalties. Thus, the simplified analysis can give an overly conservative design. Furthermore, in this simplified approach, interaction of the hot channel with neighboring channels is not considered. The beneficial effects of mixing with neighboring channels is ignored as is the deleterious effect of reduced flow in the hot channel. Most current design approaches attempt to be more realistic. A digital computer code (subchannel analysis program), which more or less fully models core thermal-hydraulic behavior, is generally used to evaluate behavior under steady-state conditions and for those transients where no core damage is to be allowed.

5.1.2 Fuel Rod Design

In selecting a fuel rod design, the independent variables of power density, rod diameter, fuel porosity, and total burnup must be considered. One significant restraint is that center fuel melting be avoided at the peak power expected. This limitation is usually applied at the core hot spot, considering the reactor is operating at a level above its nominal power rating. The increased power assumption allows for the possibility that normal reactor control can allow overpower conditions (e.g., 103 to 110% of nominal power) to exist for brief periods.

The computation of center fuel temperatures was discussed extensively in Chapter 2. We recall that for a given pellet surface temperature, the power required to reach center melting is independent of rod diameter. The temperature difference producing center melting is directly proportional to the linear power rate; i.e., Eq. (2.116). Therefore, for similar coolant conditions, the center melting limitation can be expressed as a limitation on the linear power output (kW/ft) of the fuel rod. The approximate interrelationship of surface heat flux, linear power rate, and power density for a stainless-steel-clad UO_2 fuel rod is shown in Fig. 5.1. If the maximum heat flux is regarded as fixed by the coolant conditions during steady-state or accident situations, rod size can be determined by a compromise between the fuel center temperature (i.e., kW/ft) and power density. The selection process is actually an iterative procedure since coolant conditions can be revised to lead to an improved choice.

Since the 1970s, the requirement that center melting be avoided has generally not been a limiting factor. This is illustrated in Fig. 5.2, which shows limitations on kW/ft versus height imposed by several requirements for a typical core. We see that the LOCA and DNB requirements lead to a kW/ft versus length envelope substantially below the limit set by central fuel melting.

In addition to the limit that fuel centerline temperature be safely below melting (to prevent excessive expansion, instability of the fuel stack, exces-

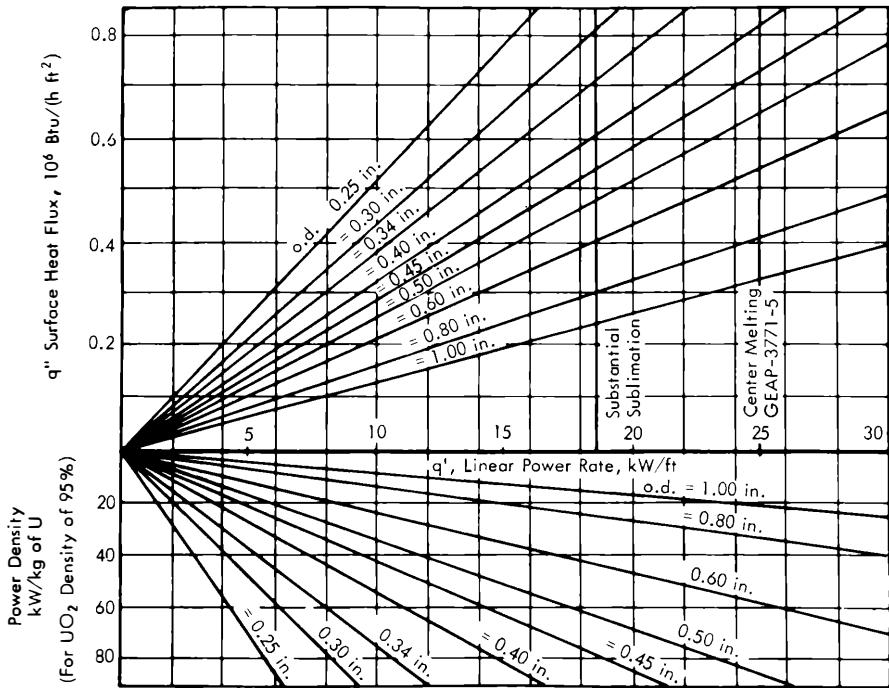


Fig. 5.1 Surface heat flux power density versus linear power for various sized UO_2 rods with stainless steel cladding [from *Nucl. Eng. Des.*, 6, 301 (1967)].

sive release and migration of fission products, and contact of molten fuel with cladding), the following additional design limits are usually imposed on zirconium-alloy-clad UO_2 fuel rods:

1. The temperature at the pellet-cladding interface remains below that at which deleterious fuel-cladding reactions occur ($\sim 675^\circ\text{C}$; see Sec. 2.1.2.1).
2. The maximum cladding strain range (algebraically maximum strain-algebraically minimum strain) is below that at which failure can be expected. Because of irradiation embrittlement and the possibility of further embrittlement due to hydride precipitation, the maximum permissible strain range must be set at a relatively low level.

Early experience¹ indicated that the strain range should not exceed $\sim 1\%$ if failures were to be avoided. However, high-burnup experience with improved pellets, which show little in-pile densification, indicate that this criterion is probably overly conservative.

3. The hydrogen uptake of the cladding at EOL is limited to a range such that excessive embrittlement is avoided—levels above 600-ppm hydrogen are unacceptable while 250 ppm is apparently satisfactory.¹ Typical

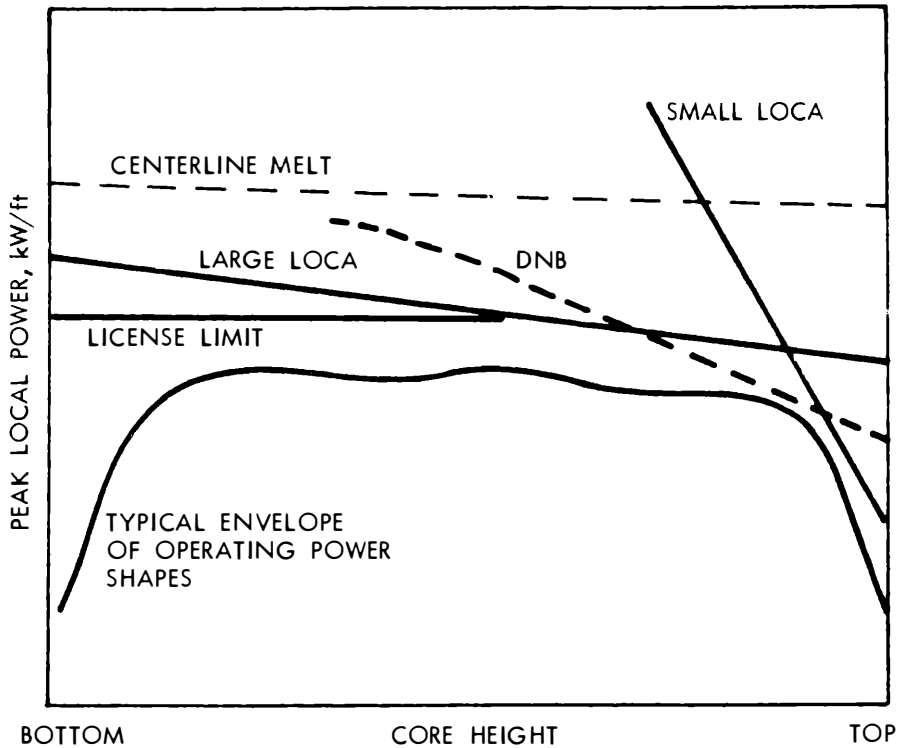


Fig. 5.2 Schematic demonstration of typical kW/ft limits [from Ref. 12].

levels are in the 250- to 350-ppm range for burnups on the order of 40×10^3 MWd/tonne.

4. Maximum EOL cladding corrosion should not exceed a small fraction (e.g., 10%) of the wall thickness—corrosion tends to be autocatalytic since as the oxide layer thickens, the oxide-zirconium interface temperature increases causing the corrosion rate to increase. Improved zirconium alloys, such as Zirlo, show a greatly reduced autocatalytic effect and are preferred for high-burnup designs (see Sec. 2.1.2.1).

5. The internal gas pressure at operating conditions at EOL should not exceed the pressure producing lift-off of the cladding from the fuel pellet. Lift-off can lead to increased fuel gaps, which in turn increase fuel temperatures and further increase the gap. Furthermore, if CHF were briefly exceeded during a transient, cladding ballooning could occur.

6. Cladding stresses during steady state remain within safe limits—the criteria of Sec. II of the American Society of Mechanical Engineers (ASME) Boiler and Pressure Vessel Code are sometimes used to set stress intensity

(algebraically maximum principal stress minus algebraically minimum principal stress). If this were done, the general primary membrane stress intensity would be required to remain below two-thirds of the yield or one-third of the ultimate strength.

7. In conventional PWRs, where EOL strain range is of concern, the cladding should be free standing when subjected to the system design pressure at BOL. This criterion [see Eq. (2.64)] is generally more limiting than the limitation on stress intensity.

8. Cladding stresses during power transients must be below the levels at which stress corrosion failures have been observed—Zircaloy creep reduces stresses induced by rapid fuel expansion against the cladding. However, if the critical contact pressure is too high, stress-corrosion (PCI) failures can be anticipated. As indicated in Sec. 2.5.2, avoidance of PCI failures may require the imposition of plant operating restrictions.

9. The fuel pellet must be stable (exhibit little irradiation-induced densification) and contain no more than a few ppm moisture.

The maximum burnup for a UO_2 fuel rod is primarily determined by strain range, hydriding, and corrosion limitations. The strain range is determined from the sum of the initial inward creep caused by external pressure and the subsequent outward creep caused by internal fission gas pressure and UO_2 swelling under irradiation. Since the porosity of sintered UO_2 can accommodate a part of the fuel swelling, it would seem desirable to use relatively low-density UO_2 pellets for high-burnup designs. However, low-density fuel tends to be unstable during irradiation. Initial pressurization of the fuel rod with helium can greatly reduce initial inward creep and thus significantly reduce the strain range. This pressurization is now (1993) common design practice. While high initial helium pressures are most effective in reducing inward creep, they increase the EOL gas pressure. Selection of the appropriate pressurization level requires the balancing of these two competing factors.

Determination of whether a given fuel rod design is satisfactory first requires estimating the expected overall power lifetime history for the high-power rod (average power level changes at each fuel reshuffling). The response of the fuel rod must then be determined using one of the fuel rod design codes described in Sec. 2.5.4. Where load following operation is anticipated, the most severe power changes expected during a given portion of the lifetime must be included in the power history and maximum cladding stresses must be examined.

The acceptability of fuel rod design during postulated accident situations must be determined after the basic system response has been determined. For the less severe transients, fuel rod codes of the same type as those used for steady-state analysis can be used. Thus, for anticipated loss-of-flow tran-

sients, it would be necessary to establish the fact that neither excessive cladding strain nor cladding temperatures were encountered. For a hypothetical LOCA, a computer code, such as FRAP-T,² would be needed to establish that the ECCS acceptance criteria³ (maximum cladding temperature of 2200°F and 17% cladding oxidation) were not exceeded.

5.1.3 Preliminary Core Design

Establishment of the maximum allowable linear power density, rod diameter, water-to-fuel ratio, and nuclear hot-channel factors serves as a basis of the thermal and hydraulic design. We will illustrate the effect of these parameters by further considering a pressure vessel reactor containing a rod-cluster controlled core.

5.1.3.1 Average Power Output of a Fuel Rod

Since the average power output of a fuel rod strongly influences fuel cost, it is one of the measurements of the effectiveness of a core design. By knowing the maximum linear power density, $q'_{\max}$, and the hot-channel factor for heat flux F_q , we can determine the limiting average power output of a fuel rod:

$$q'_{\text{avg}} = q'_{\max} / F_q \quad (5.1)$$

5.1.3.2 Fuel Rod Array

For a square lattice, pitch s of the rod array is a function of the ratio of moderator to fuel, M/F , and of the cladding thickness, l

$$\left(\frac{M}{F}\right) \left[\frac{\pi}{4} (D_o - 2l - 2g)^2 \right] = p^2 - \frac{\pi}{4} D_o^2 \quad (5.2)$$

where

D_o = rod outside diameter

g = cold gap between fuel and cladding

l = cladding thickness

p = fuel rod pitch (center-to-center distance).

In most stainless-steel-clad PWR cores, p is $\sim 1.3D_o$.

5.1.3.3 Assembly Size and Core Size

The following relationships determine the sizes of assembly and core. First, the minimum fuel rod number is a function of core length L and average heat flux

$$N = (Q)[C_1 / (Lq'_{\text{avg}})] \quad (5.3)$$

where N = minimum number of fuel rods for the limiting q'_{avg} , Q = thermal power output of the core, and C_1 = fraction of power generated in the fuel rod.

Second, the number of fuel assemblies should be determined by considering axisymmetry in the core and a suitable period between refueling. The actual fuel rod number, obtained by selecting the proper combination of the number of assemblies and the number of rods in each assembly, should be close to but greater than the minimum fuel rod number calculated by Eq. (5.3).

Third, the equivalent core diameter, D_{eq} , is a function of the average core power density.

$$C_2 \frac{\pi}{4} L D_{\text{eq}}^2 = \frac{Q}{q'_{\text{avg}}} \quad (5.4)$$

where C_2 is a factor to account for clearance between assemblies and

$$q'_{\text{avg}} = q'_{\text{avg}} / p^2$$

Fourth, to provide reasonable core fabrication costs and neutron economy, the values of D_{eq} and L should satisfy the following relationship:

$$0.9 \leq (L/D_{\text{eq}}) \leq 1.5 \quad (5.5)$$

5.1.3.4 Determining Coolant Requirements

The major limitation on the thermal design of a water-cooled reactor is the necessity to maintain an adequate safety margin between operating conditions and the critical heat flux (DNB). The core design criterion is generally stated in terms of a DNB ratio defined as

$$\text{DNBR} = \frac{\text{DNB heat flux predicted by applicable correlation}}{\text{Reactor local heat flux}} \quad (5.6)$$

The DNB ratio varies along the channel since both local heat flux and fluid enthalpy are varying. For a cosine heat flux, the minimum DNB ratio is found at some portion past the midplane of the core. A typical situation for a base-loaded plant is shown in Fig. 5.3, where reactor and DNB heat flux are plotted versus the ratio of height to total core length (Z/L). We observe that in the region just past midplane, increased fluid enthalpy decreases the predicted DNB flux more rapidly than local flux is decreasing. (See Sec. 4.4.2 for DNB correlations.)

It is, of course, the minimum DNB ratio (at Z^*/L in Fig. 5.3) in the hottest channel that concerns the designer. A typical PWR design criterion is that the minimum $\text{DNBR} > 1.30$ at maximum overpower conditions. The probability of a PWR reaching the specified maximum overpower condition is very small. Even at maximum overpower conditions and normal flow and

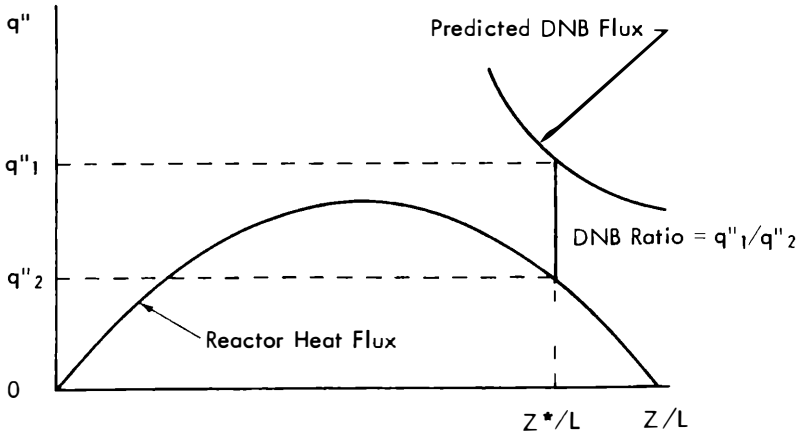


Fig. 5.3 The DNB ratio evaluation [from *Nucl. Eng. De.*, 6, 301 (1967)].

pressure, the number of fuel rods with DNBRs close to 1.30 is again very small.

Note that PWR practice differs from that used in BWR design, where limiting conditions are usually designated in terms of a *critical power ratio* (CPR). The CPR can be defined as the ratio of assembly power at which CHF occurs to the actual assembly power. For any given assembly power, the CPR is lower than the DNB ratio. This is so since the CHF is evaluated at a higher quality in the CPR calculation.

It has been previously observed (see Sec. 4.4.2.3) that exit qualities tend to be higher in a pressure tube reactor of the CANDU type than in the usual pressure vessel type design. Because of this, designers of CANDU reactors assess the margin to CHF by considering both DNB [see Eq. (4.217)] and dryout.⁴ This latter limitation is evaluated using a relationship between exit quality and boiling length [similar to Eq. (4.211)]. The dryout limitation is essentially the same as that imposed on a BWR. Hence, the margin to CHF is most appropriately evaluated by the critical power ratio.

Sufficient coolant flow must be provided to maintain the desired minimum CPR or DNBR. At low qualities, increasing the mass velocity increases CHF; however, increased coolant flow primarily acts to reduce coolant enthalpy rise and, hence, quality. As seen from the CHF correlations of Sec. 4.4.2, a reduced quality substantially increases the predicted critical flux.

Determination of enthalpy rise along the hot channel requires a knowledge of the flow effective in removing heat from the core. In a vessel type PWR, the effective flow rate is the total flow rate minus the bypass flow rate. We then have

$$\Delta H_{\text{reactor}} = Q / W_{\text{total flow}} \quad (5.7)$$

$$\Delta H_{\text{core}} = Q/W_{\text{eff}} \quad (5.8)$$

$$\Delta H_{\text{hot channel}} = F_{\Delta H} \times \Delta H_{\text{core}} \quad (5.9)$$

where Q = total heat output, Btu/hr; W = flow, lb/hr; and $F_{\Delta H}$ = enthalpy rise hot-channel factor. Thus, once bypass flow is determined and a value for $F_{\Delta H}$ has been assigned, we can obtain hot-channel exit conditions. For a preliminary estimate of the enthalpy rise along the hot channel of a base-loaded plant, we assume that the hot channel is closed and the value of $F_{\Delta H}$ is constant along its length. Enthalpy rise at any position along the channel is then obtained by multiplying (ΔH) from Eq. (4.5), or its equivalent for flux distributions other than the cosine, by $F_{\Delta H}$.

5.1.3.5 Hot-Channel Factor Definition

In the original core design approach, which is still used for preliminary design and in establishing hot-channel conditions for one-dimensional accident analysis programs, the total hot-channel factors for heat flux and enthalpy rise include both nuclear effects and engineering uncertainties such as dimensional and flow variations. The effects of engineering uncertainties can be combined and expressed as engineering hot-channel subfactors for heat flux, F_q^E , and for enthalpy rise, $F_{\Delta H}^E$. The total hot-channel factor for heat flux can then be defined as follows:

$$F_q = \frac{\text{Maximum heat flux at hot spot}}{\text{Average heat flux in core}} = F_q^N F_U^N F_q^E \quad (5.10)$$

where F_U^N is uncertainty in the nuclear factor and other quantities have their previous meaning.

The hot-channel factor for enthalpy rise across the core, $F_{\Delta H}$, can be defined as

$$F_{\Delta H} = \frac{\text{Enthalpy rise in hot channel}}{\text{Average enthalpy rise in the core}} \quad (5.11)$$

In terms of its components, we have

$$F_{\Delta H} = F_{\Delta H}^N \cdot F_U^N \cdot F_{\Delta H}^E \quad (5.12)$$

with

$$F_{\Delta H}^N = \text{Hot rod power} / \text{Average rod power} \quad (5.13)$$

Note that the value of F_U^N used in Eq. (5.12) is not necessarily identical to that used in Eq. (5.10).

In a simple, base-loaded core, it is often adequate to obtain $F_{\Delta H}$ from

$$F_{\Delta H} = F_R^N \cdot F_{\Delta H}^E \cdot F_U^N, \quad (5.14)$$

providing F_U^N is large enough to account for the limited variation in F_R^N with lifetime. However, in a load-following plant, there are very substantial variations in flux shapes as the control system responds to load, xenon, and burnup changes. It is then necessary to evaluate $F_{\Delta H}^N$ by determining the maximum ratio of hot rod to average rod power that occurs over all the allowable lifetime power histories. We then have

$$F_{\Delta H}^N = \frac{\max \int_0^L q'(x_0, y_0, z) dz}{(1/N) \sum_{\substack{\text{all} \\ \text{rods}}} \left[\int_0^L q'(x, y, z) dz \right]} \quad (5.15)$$

where N is the number of fuel rods in the core. Typical values for $F_{\Delta H}^N$ have been in the vicinity of 1.55.

Note that the hot-channel factor, F_θ , for the temperature difference between the fuel surface and coolant is usually not significant in basic PWR design since nucleate boiling limits the maximum cladding temperature. Nucleate boiling coefficients are high and even a substantial uncertainty in the heat transfer coefficient would not cause a significant rise in the cladding temperature. However, during post-CHF heat transfer under some hypothetical accident conditions, this hot-channel factor may have to be considered. The value of F_θ would also be important in the design of a naval reactor where nucleate boiling was not allowed.

5.1.3.6 DNBR Evaluation

The maximum value of the integral rod power is used to identify the rod most likely to encounter the minimum DNBR. In a preliminary design, it would be assumed that the hot channel enthalpy rise is determined by the use of Eq. (5.12). The average rod power is then calculated so that it recreates the behavior of the hot rod. It is then common to assume that the rod heat flux follows a cosinusoidal distribution (peak flux at core centerline). The axial hot channel factor is then assigned so that the maximum heat flux corresponds to that predicted through use of F_q . Equation (4.4) or (4.9) then is used to determine heat flux variation along the rod. The heat fluxes and enthalpies along the rod then are used to calculate the DNBR as a function of position and the minimum ratio selected.

Note that this approach is somewhat conservative since the usual operating restrictions on flux distribution lead to peak flux locations that are below the core centerline. Hence, the actual fluid enthalpy in the peak flux region should be lower than that calculated and the true minimum DNBR should be higher than the calculated design value.

In the final core design, we must also consider the limitations that need to be imposed on maximum heat fluxes above the core centerline to main-

tain an adequate DNB margin. Further, we wish to consider the interrelationship between the behavior of the hot channel and those channels surrounding it. For this latter purpose, we use the more sophisticated sub-channel analysis approach that was briefly indicated in Sec. 5.1.1. We consider this approach in more detail in Sec. 5.3.

5.1.4 Hot-Channel Factor Evaluation for Conservative Core Design

5.1.4.1 Nuclear Subfactors

We previously observed (see Sec. 1.5.1) that in modern load-following plants, the rod motion required to follow power demand can cause substantial axial and radial variations in core power distribution. These variations introduce xenon transients, which further complicate the situation. In Sec. 1.5.1, we noted that it was necessary to measure the axial flux imbalance due to such transients by ex-core detectors and to protect against this imbalance. The imbalance may be limited by limiting the axial offset (AO) [see Eq. (1.53)]. Thus, control rod motion is restricted so that the AO remains within a prescribed range. Despite such limitations, there are a large number of possible power histories and control rod maneuvers that must be considered to determine the most adverse power distribution.

We recall (Sec. 1.5.1) that a large number of power histories are studied, and the worst $F_{xy}^N(z)$ and $F_q^N(z)$ and associated AO are determined for each elevation and power history. The data are usually plotted in terms of $F_q^N(z)$ versus AO for each elevation of interest. The data are seen as a cloud of points; this method of data presentation has been referred to as the "fly-speck format." By selecting the maximum $F_q^N(z)$ of the curves at the various elevations, a maximum envelope curve of $F_q^N(z)$ versus z can be constructed. With the appropriate choice of limiting AOs (e.g., zero to some negative value), the maximum $F_q^N(z)$ can be confined to the lower half of the core and the assumptions made in calculating the minimum DNBR are valid. If subsequent calculations indicate that the maximum $F_q^N(z)$ obtained is higher than can be tolerated, the power distribution study can be repeated using a more restrictive AO range. If operating procedures allow values for AO which are significantly above zero, the simplifying assumptions suggested for calculation of the DNBR may not be appropriate. Studies with power peaks in the upper half of the core may then be required.

5.1.4.2 Engineering Subfactors for Preliminary Core Design

Individual effects that must be included in engineering hot-channel factors are usually expressed in terms of engineering subfactors. The most common hot-channel subfactors are:

1. Fuel fabrication tolerance: accounts for flow reduction due to reduced pitch and bowing of the fuel rod

2. Fuel pellet diameter, density, and enrichment: accounts for variation in the quantity of fissionable material in the fuel pellet
3. Lower plenum: accounts for the effect of local flow maldistribution at core inlet
4. Flow redistribution: accounts for reduction in flow in the hot channel due to an increase in pressure drop as a result of nucleate boiling
5. Flow mixing: accounts for flow mixing between parallel and laterally open channels
6. Bypass flow percentage: accounts for flow leakage bypassing the core.

Originally, the overall engineering hot-channel factors, F_q^E and $F_{\Delta H}^E$, were taken as the product of these individual subfactors. As previously indicated, this leads to an overly conservative design. Subfactors that are related to fuel fabrication tolerances (items 1 and 2) are stochastic in nature; hence, the probability that all fabrication effects are simultaneously adverse is low. To combine these effects properly, we must allow for the probability distributions associated with them. The subfactors relating to flow mixing and redistribution are all interrelated and a true independent evaluation of each subfactor is not possible. These factors are accounted for in final design computations by using the subchannel analysis procedures indicated in Sec. 5.3.

5.1.4.3 Statistical Evaluation of Fabrication Tolerance Subfactors

The influence of fuel rod dimensional deviations on thermal design is usually much less in a core of cylindrical rods than in a core of plate-type fuel elements. This is so since the flow area in a plate assembly varies with a single gap, while the flow area of a rod bundle subchannel varies with the average of the four gaps (3 gaps in VVER design). The engineering hot-channel subfactors for fuel element fabrication tolerances have been evaluated for fuel plates by Le Tourneau and Grimble.⁵ Chelemer and Tong⁶ evaluated these subfactors for fuel-rod cores using statistical concepts.

At this point, it is desirable to review a few basic statistical relations. Random variations are distributed in accordance with the normal distribution curve as defined by the following equation:

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\bar{\mu}}{\sigma}\right)^2\right] \quad (5.16)$$

where $g(x)$ is the relative frequency with which value x occurs, $\bar{\mu}$ is the mean or expected value, and σ is the standard deviation. Probability p that x is greater than a value, say a , is given by integrating the above expression from a to ∞ :

$$p(x \geq a) = \int_a^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\bar{\mu}}{\sigma}\right)^2\right] dx \quad (5.17)$$

If substitutions $t_1 = (x - \bar{\mu})/\sigma$ and $t_a = (a - \bar{\mu})/\sigma$ are made, the result is

$$p(x \geq a) = p(t_1 \geq t_a) = \frac{1}{\sqrt{2\pi}} \int_{t_a}^{\infty} \exp(-t_1^2/2) dt_1 \quad (5.18)$$

When nominal conditions exist, the engineering hot-channel factors cause no adverse effect. From Eqs. (5.10) and (5.12), we see that they must then have a value of 1. Since nominal conditions are nothing more than expected conditions, the mean value of any engineering hot-channel factor is 1. Thus, if x in Eq. (5.16) represents the value of the hot-channel factor, $\bar{\mu} = 1$.

We are concerned only with deviations from the nominal that cause adverse effects; e.g., increased enrichment, higher pellet density, etc. Deviations in the other direction are not harmful. Thus, we are concerned only with hot-channel factors in excess of 1. The probability of exceeding any given value of the hot-channel factor is therefore found from Eq. (5.18).

As an example, consider the probability of exceeding a hot-channel factor of $1 + 3\sigma$. Letting $a = 1 + 3\sigma$ gives $t_a = 3$ so that

$$p(x \geq 1 + 3\sigma) = p(t_1 \geq 3) = \frac{1}{\sqrt{2\pi}} \int_3^{\infty} \exp(-t_1^2/2) dt_1 = 0.00135 \quad (5.19)$$

The numerical value is obtained from standard tables.⁷ If a hot-channel factor can be considered a linear function of its governing physical quantities, each of which is normally distributed, then it can be shown⁷ that the hot-channel factor is normally distributed. The value of σ can then be estimated from the relation

$$\sigma^2 \approx (\partial\phi/\partial x_1)^2 \sigma_1^2 + (\partial\phi/\partial x_2)^2 \sigma_2^2 + \dots + (\partial\phi/\partial x_k)^2 \sigma_k^2 \quad (5.20)$$

where the hot-channel factor expression is represented by $\phi(x_1, x_2, \dots, x_k)$: The x terms are the governing physical quantities; their standard deviations are given by σ ; the partial derivatives are evaluated at the mean x values.

To determine whether the hot-channel factor can be considered linear in x terms, the following test suggested by Hald⁷ can be applied:

$$\phi(\bar{\mu}) - \phi(\bar{\mu} - 2.58\sigma) = \phi(\bar{\mu} + 2.58\sigma) - \phi(\bar{\mu}) = 2.58\sigma(d\phi/dx)|_{x=\bar{\mu}} \quad (5.21)$$

where ϕ is a function of variable x with a mean and standard deviation of $\bar{\mu}$ and σ . If the equality is satisfied, the function can be considered linear throughout 99% of the range of the variation in x . This test can be applied for functions of more than one variable by considering one variable at a time, while holding the others constant.

The confidence level of a statistically determined relationship is the fraction of time the relationship is expected to be satisfied. The aforementioned means and standard deviations are obtained from a finite number of samples. To maintain a high confidence level so that the design hot-channel

factors will not be exceeded, the following procedure is adopted. We define m and s as the mean and standard deviation determined from a sample of size n taken from a normal distribution. Confidence parameter k is defined by the following probability equation:

$$p(x < m + ks) = p_k \quad (5.22)$$

where k values are listed in Ref. 8 for various confidence levels γ and resulting probabilities, p_k . For an infinite sample size, the values of m and s will approach $\bar{\mu}$ and σ , respectively, so that

$$p(x < \bar{\mu} + k_\infty \sigma) = p_k \quad (5.23)$$

where k_∞ is the value of k for $n = \infty$. Comparing the above two equations, we get

$$\sigma = (m + ks - \bar{\mu}) / k_\infty \approx ks / k_\infty \quad (5.24)$$

We now use the value of σ , derived from the experimentally observed value of s , to establish design values for the engineering hot-channel factors that have very low probabilities of being exceeded. The engineering hot-channel factor for heat flux F_q^E is proportional to the pellet weight per unit length w' and enrichment e . By assuming that the measurements of w' and e are abundant and that they are independent and normally distributed, we get

$$F_q^E = \left(\frac{w'_{\max \text{ local}}}{w'_{\text{nom}}} \right) \left(\frac{e_{\max \text{ local}}}{e_{\text{nom}}} \right) \quad (5.25)$$

$$\sigma_{F_q^E}^2 = (\sigma_{w'} / \bar{\mu}_{w'})^2 + (\sigma_e / \bar{\mu}_e)^2 \quad (5.26)$$

where σ and $\bar{\mu}$ are the standard deviation and mean, respectively. If a 99.87% probability of not exceeding the design hot-channel factor is required, the value of $(F_q^E)_{\text{des}}$ becomes

$$(F_q^E) = 1 + 3\sigma_{F_q^E} \quad (5.27)$$

The fabrication subfactor of the engineering hot-channel factor for enthalpy rise $F_{\Delta H, \text{fab}}^E$ is defined as

$$F_{\Delta H, \text{fab}}^E = \left(\frac{G_{\text{nom}}}{G_{\text{hot channel}}} \right) \left(\frac{\bar{w}'_{\text{hot channel}}}{\bar{w}'_{\text{nom}}} \right) \left(\frac{\bar{e}_{\text{hot rod}}}{\bar{e}_{\text{noi}}} \right) \quad (5.28)$$

where G is the flow rate in a channel, $\bar{w}'$ is the average pellet weight per length of the fuel rod, and $\bar{e}$ is the average enrichment of the fuel rod.

The parameters in Eq. (5.28) are assumed to be independent and normally distributed. Thus,

$$\sigma_{F_{\Delta H, \text{fab}}^E}^2 = (\sigma_G / \bar{\mu}_G)^2 + (\sigma_{\bar{w}'} / \bar{\mu}_{\bar{w}'})^2 + (\sigma_{\bar{e}} / \bar{\mu}_{\bar{e}})^2 \quad (5.29)$$

A 99.87% probability of not exceeding the design value is again required, and

$$(F_{\Delta H, fab}^E)_{des} = 1 + 3\sigma_{F_{\Delta H, fab}} \quad (5.30)$$

5.1.4.4 Bypass Flow and Core Flow Subfactors

To assemble the fuel assemblies and reactor internals in the vessel, clearances must be provided to account for fabrication tolerances. These clearances also provide passages for the coolant to bypass the core; i.e.,

1. The gap between the baffle and outlet nozzle
2. The gap between fuel assemblies
3. The gap in the control-rod slot or around control-rod thimbles
4. The gap between the baffle and barrel.

The flow bypassed through each of these paths is such that the pressure drop created by flow through the gap equals that set by the main core flow. A reasonable first estimate of the pressure drop set by the main flow can be obtained by ignoring bypass flow. A second iteration generally produces a good estimate of core and bypass flows.

These bypass flows, which can be 4 to 8% of the total flow depending on the size of the design clearances, are not effective in removing heat from the core. Any reduction of these clearances directly increases the effective coolant flow.

The subfactors for core-flow conditions consist of the lower plenum, flow redistribution and flow mixing factors. The flow distribution in the lower plenum is usually evaluated from isothermal scale model tests.¹⁰ The results of such tests are used as one of the inputs to the subchannel analysis used for final core design. As noted earlier, flow redistribution and mixing are evaluated via subchannel analysis procedures. Experience in using subchannel analysis codes for typical reactor core situations allows approximate values of these factors to be determined. Table 5.I lists typical engineering hot-channel factors used in the original design of several PWRs.

5.1.5 Effect of Anticipated Operational Occurrences (AOOs) and Major Accidents on Core Design

Designs of early commercial PWR plants were all based exclusively on normal operating requirements. The behavior of the conservative design so determined was then examined under hypothetical accident conditions to demonstrate that the design met all necessary safety criteria. Generally, the accident analyses did not lead to further restriction on core operation. The more stringent requirements now imposed on core behavior during hypothetical accident situations have changed this situation. Analysis of the hypothetical LOCA now leads to significant restrictions on core operation. Old

TABLE 5.I
Engineering Hot-Channel Factors of Some PWRs

Factors	Reactors		
	Yankee-Rowe	Selni-Sena	Others ^a
Rod diameter, pitch and bowing, pellet density, and enrichment	1.14	1.14	1.08 ^b
Lower plenum subfactor	1.07	1.07	1.03 ^c
Flow redistribution subfactor	1.05	1.05	1.05
Flow mixing subfactor		0.95	0.92 ^d
Total engineering hot-channel factor for enthalpy rise, $F_{\Delta H}^E$	1.28	1.22 ^c	1.08
Total engineering hot-channel factor for heat flux, F_q^E	1.08	1.04 ^c	1.04

^aConnecticut Yankee, San Onofre, Indian Point II, Zorita, Ginna, Turkey Point, Point Beach.

^bRef. 9. ^cRef. 10. ^dRef. 11. ^eRef. 6.

designs had to be reevaluated and, in some cases, backfitting additional safeguards equipment was required. Before setting the design of a new core, both normal and abnormal conditions must be examined.

The previous discussion of fuel rod design (Sec. 5.1.2) pointed out that the hypothetical LOCA imposed limits on power density at a given axial location. These limits, and those imposed by the minimum DNBR and possible license restrictions, are illustrated in Fig. 5.2. The polygon formed by the intersection of these limit lines serves as an upper envelope of allowable power.¹²

The LOCA limitations arise from restrictions placed on the maximum cladding temperature that can be reached. The restrictions are more easily met by providing additional core surface areas. This has led some manufacturers to reduce fuel rod diameter and increase the number of fuel rods per assembly, while holding assembly power and overall size constant. Fuel assemblies with 17×17 arrays are now (1995) more common than those with the previously popular 15×15 array.

In addition to considering core behavior during major accidents, the core behavior during anticipated transients must be examined. An anticipated transient or anticipated operational occurrence (AOO) are those events that are expected to occur one or more times during the plant lifetime. Table 5.II lists the AOO usually considered in PWR analysis.¹³

Since all of these events are considered likely to occur, it is necessary that the reactor core suffer no damage during the transient. This is usually interpreted as meaning that the DNBR remains above the required minimum. From the thermal analyst's point of view, the two transients that are likely to be limiting are the loss-of-flow and control rod drop events.¹³

At full power, the loss of power to all pumps and the flow coastdown that follows generally impose the most limiting conditions. Since there is

TABLE 5.II
Typical Anticipated Operational Occurrences (AOOs)

Control rod withdrawal at power event
Control rod withdrawal during startup event
Moderator/boron dilution event
Idle loop startup/cold water event
Excess load event
Loss-of-load event
Excess heat removal due to feedwater system malfunction
RCS depressurization
Control rod malposition and drop-in event
Loss-of-coolant flow event
Loss-of-ac power event
Asymmetric steam generator transient (ASGT) event
Loss-of-feedwater flow event
Inadvertent operation of emergency core cooling system (ECCS)
Addition of boric acid by chemical and volume control system malfunction

some delay between the time the accident is initiated and the time the control rods enter the core, the lower flow means that the minimum DNBR will be below the steady-state value. It is therefore necessary for the minimum DNBR at the most adverse operating conditions to be sufficiently above the minimum so that the desired minimum is maintained during the transient.

After a control rod drop incident, the reactor protection system does not normally initiate a reactor trip. For boron concentrations that exist later in life, the absence of a turbine runback following the rod drop tends to restore the reactor to nearly full power with a distorted power distribution. The maximum three-dimensional power peaking resulting from this accident must be determined and operating limits adjusted so that (1) the linear heat generation rate is safely below that leading to centerline melt and (2) the minimum allowable DNBR is not reached during the transient.

5.2 ANALYSIS OF CORE PERFORMANCE DURING NORMAL OPERATION

5.2.1 Optimal Core Performance

The production of high-temperature steam is desirable as plant thermal efficiency obviously increases with increasing steam temperature. This is illustrated in Fig. 5.4, where the efficiency of a saturated steam cycle is shown as a function of pressure. The effects of moisture separation and reheat are not included. By increasing the temperature of the primary coolant, higher pressure steam can be generated with a given steam generator

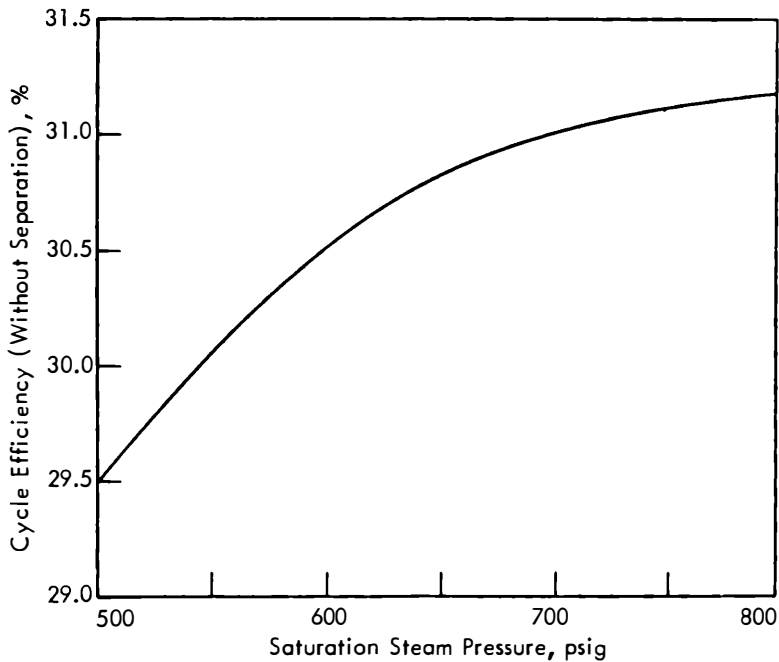


Fig. 5.4 Thermal cycle efficiency as a function of pressure [from *Nucl. Eng. Des.*, 6, 301 (1967)].

area. However, we are limited in this regard since a high coolant enthalpy decreases the CHF and reduces the allowable power generation in the core.

From the discussion of steam generator design in Sec. 4.6.1, it is clear that the total heat transferred is approximately proportional to the product of the area and ΔT_m , the log mean temperature difference between primary coolant and steam. The relationship between primary coolant and secondary steam temperature is graphically illustrated in Fig. 5.5. Thus, higher steam temperatures can be obtained by increasing the heat exchanger area and reducing ΔT_m . This has the disadvantage of significantly increasing plant capital costs. The appropriate selection of primary coolant temperature, primary coolant flow rate, secondary steam pressure, and heat exchanger area leads to the lowest power costs. There is an optimum selection of design variables where benefits due to increased thermal efficiency are just balanced by increased capital and operating costs. An engineering economic study is required for this selection.

Spinks and Groenveld¹⁴ have considered the system changes that would be desirable to accommodate a CANDU pressure tube core with a higher rated power. They desired to accommodate the increased power without

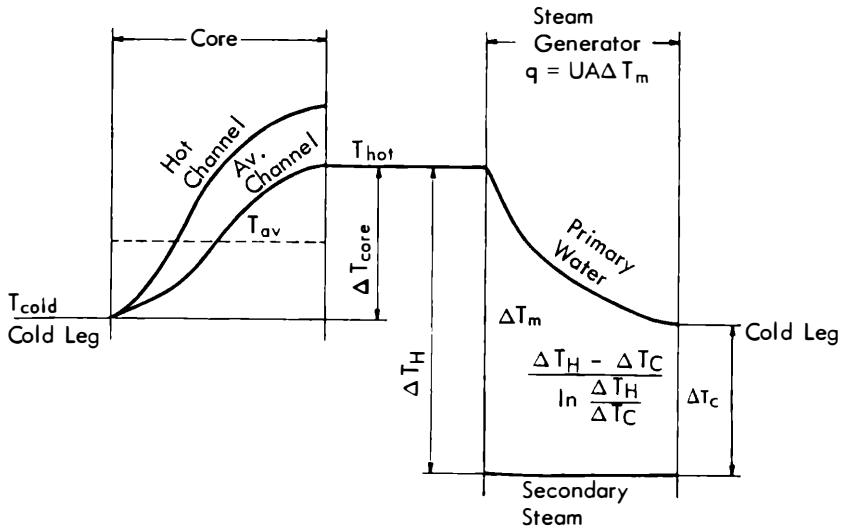


Fig. 5.5 Coolant performance [from *Nucl. Eng. Des.*, 6, 301 (1967)].

increasing the size of the system or increasing the core exit quality. They concluded that this could best be accomplished by reducing the core inlet temperature and the secondary steam temperature. Although the reduced steam temperature produces a lower thermodynamic efficiency, the net power from the plant was still increased. For example, in conjunction with increased system flow (due to decreased bundle resistance) and a decreased core inlet temperature, the increased steam flow, accompanied by a 14°C decrease in steam temperature from 260°C , would allow a thermal power increase of 22% giving an electrical power output increase of 18%. This result appears to be counter to what one would expect intuitively.

With the development of modern optimization theory, the trend has been to consider the economic study to determine thermal design parameters as a mathematical programming problem. In such a problem, we attempt to minimize or maximize a function, called the objective function, subject to a number of constraints. In this case, the objective function would be power costs, and the thermal constraints would be those previously cited; e.g., $\text{DNBR} \geq k$. Additional constraints are imposed by other engineering and physics requirements.

Mathematically, we state our problem as:

minimize

$$C = \phi(y_1, y_2, \dots, y_n) \quad (5.31)$$

subject to constraints

$$\begin{aligned} f_1(y_1, y_2, \dots, y_n) &= 0 \\ f_j(y_1, y_2, \dots, y_n) &\leq 0 \\ f_k(y_1, \dots, y_n) &\geq 0 \end{aligned} \quad (5.32)$$

where $y_1, y_2, \dots, y_n$ are the designer's decision variables, which can include total flow to the core, core-inlet temperature, primary-system operating pressure, secondary-system pressure, fuel-rod diameter, number of fuel rods, etc.

Numerical solutions for large-size optimization problems can now be readily executed on modern computers. The procedures adaptable to problems of the type expected generally fall into two categories.

In the first category, the problem is linearized by writing the objective function and each constraint as a Taylor's series expansion in which only the linear terms are retained. Thus, we would have

$$f_i(y_1, \dots, y_n) \approx A_i + \left(\frac{\partial f_i}{\partial y_1} \right) \Delta y_1 + \left(\frac{\partial f_i}{\partial y_2} \right) \Delta y_2 + \dots + \left(\frac{\partial f_i}{\partial y_n} \right) \Delta y_n \quad (5.33)$$

This linear problem can then be solved by the well-known Simplex procedure. Since linearization is applicable only over a small region, the full step predicted by the Simplex algorithm is not taken; the variables are moved in the predicted direction, but the maximum Δy is limited. The procedure is then repeated for the succession of points indicated until an optimum solution is obtained. An operable example of this class of computer program is POP¹⁵

In the second major category of methods, constrained optimization is converted to an unconstrained problem. This can be done by forming a new objective function, SN , such that if we have m constraint functions,

$$SN = C + \sum_{i=1}^m \delta_i K_i [f_i(y_1, y_2, \dots, y_n)]^2 \quad (5.34)$$

Quantities δ_i are unity for the equality constraints. For inequality constraints, the δ_i are zero if the constraint is met, and unity otherwise. The K_i are positive constants. It can be seen that the greater the violation of the constraint, the higher the objective function. By attempting to minimize this revised function, we minimize constraint violations. Solution of the problem for a series of increasing values of K_i leads to a condition where all constraints are finally satisfied.

To minimize the new unconstrained objective function, SN , a search procedure can be used. An example of a computer program following this

technique is the Optimal Search by Weisman *et al.*¹⁶ Alternatively, the new, unconstrained objective function can be minimized by following its gradient. An example of this technique is Fiacco and McCormick's SUMPT program.¹⁷ The SUMPT program also uses a somewhat different formulation of Eq. (5.34).

Some newer nonlinear programming methods combine linearization and gradient search techniques. In the reduced gradient technique, the problem is linearized and the linear forms of equality constraints are solved to eliminate some variables from the problem. The objective function is then stated in terms of the reduced number of variables. This revised function is minimized along a modification of the gradient that is designed to avoid constraint limits. Since the problem has been linearized, only modest steps along the reduced gradient can be taken before the entire procedure must be repeated. The commercially available GAMS¹⁸ software uses this technique. For extensive review of nonlinear programming, the reader is referred to Reklaitis *et al.*¹⁹

Weisman and Holzman²⁰ applied nonlinear programming techniques in determining both the optimal design and design margin for a PWR. They recognized that many of the design variables and parameters are stochastic rather than deterministic quantities. Therefore, there is always some probability that a design constraint will be violated, even when the nominal values of all quantities indicate a satisfactory design. (In Sec. 5.4, we consider this idea in detail with regard to the DNB constraint.) By using estimated stochastic properties, the probability of violating each design limit was computed. They then formulated their objective function as

$$\min \sum_{i=1}^k C_k + \sum_{i=1}^m p_m C_{fm} \quad (5.35)$$

where

$\sum_{i=1}^k C_k$ = summation of usual capital and operating costs

p_m = probability that the m 'th design constraint will be violated

C_{fm} = cost incurred when failure occurs due to violation of the m 'th design constraint.

Minimization of the foregoing function provides values of major design parameters that lead to low failure probabilities. By determining the ratios of the various design limits to the values computed using the nominal values assigned by the program, a safe and economical design margin is established.

Useful results from any design optimization approach can only be obtained when a realistic and accurate mathematical model of the reactor

plant is available. Unless considerable care is taken in developing and benchmarking such a model, very misleading results can be obtained.

5.2.2 Effect of Core Flow Arrangement on Core Performance

5.2.2.1 *Multipass Versus Single-Pass Cores*

In a multipass core, the core is separated into regions through which the coolant flows successively. The advantages of a multipass core are:

1. High coolant mass velocity increases the DNB margin.
2. High coolant mixing, occurring at core interpasses, reduces $F_{\Delta H}$. However, the benefit of mixing disappears as the radial power distribution becomes flat.
3. Since the coolant of a multipass core is likely to pass through both a high-power region and low-power region, total enthalpy rise in the hot channel probably does not change considerably, even if the high-power and low-power regions interchange in later core life.

The disadvantages are:

1. Mechanical design is complicated and requires more structural material inside the core; this is especially significant in a small core.
2. Pressure drop across the core is greatly increased; this increases the pumping power or reduces the total flow rate.

Vaughan²¹ analyzed the thermal and hydraulic performance of a multipass core. He assumes the total flow rate has been constant and does not consider the additional cost to achieve the higher pressure drop. If the primary pump was not changed, total flow rate would be greatly reduced. Since large PWRs use nearly the largest pumps available, pumping capacity normally cannot be increased without adding additional pumps. This essentially nullifies the advantage of a two-pass core.

5.2.2.2 *Orificing for Closed Assemblies and Pressure Tubes*

An orificed core redistributes the flow so that more passes through the hot channel at the expense of an increased core pressure drop. The increase in the pressure drop mainly depends on the degree of orificing. For example, a PWR core with 40% additional flow passing through the hot assembly requires the other assemblies to be orificed to give the same pressure drop. Thus, an orificed core would have almost twice as much pressure drop as a similar but unorificed core. This high-pressure drop across an orificed core reduces total flow rate to the core, or increases pump cost and pump power. The shift of the power peak during core life greatly reduces the benefit of orificing so that its cost sometimes exceeds its benefits.

5.2.2.3 *Regional Inlet Velocity Control in an Open Core*

When fuel assemblies are open laterally, it is not possible to set the inlet flow to each assembly in accordance with assembly power. Marked assembly-to-assembly differences in inlet flow would lead to large pressure gradients and high lateral flows that could largely negate the effect of orificing. However, it is possible to control flow on a regional basis. If the outermost region were assigned one flow, the adjacent region another, and the innermost region a third, assembly-to-assembly differentials could be kept moderate. Although lateral flows would still occur, they would probably not be large enough to negate all the benefits of orificing.

A three-region design could be beneficial with older in/out refueling schemes, which led to three power regions. However, the checkerboard-like refueling patterns now used in all but the outermost core region would not be helped by three velocity regions. Since the outermost fuel elements do have lower power than the remainder of the core, it is thus possible to orifice the outermost rows to provide somewhat lower inlet flow rates. This design has been employed in operating reactors. Only modest benefits are obtained from this approach because there is appreciable flow redistribution within the core.

5.2.3 **Interaction of Thermal-Hydraulic Design with Other Areas**

The thermal and hydraulic design of a PWR core cannot be established alone. Many parameters have to be determined mutually by nuclear, thermal, mechanical, control, and material engineering considerations. However, the most complicated relationship is the one with nuclear design, which must be carried out iteratively. Before the thermal and hydraulic design can be initiated, the following nuclear information is required:

1. Power distributions during core life, both in steady-state and transient operations. These distributions give peak power that limits reactor power output.
2. Water-to-fuel ratio and fuel rod size for a most economical fuel utilization and suitable reactivity control. This information determines fuel-rod lattice and core size.

After a preliminary thermal and hydraulic design is made, the following information should be given to the nuclear designers for their evaluation:

1. Void distribution, including local boiling void and bulk boiling void. Void distribution can be used to refine the calculation of nuclear power distribution and to evaluate the reactivity loss due to voids in the core.
2. Average fuel temperature at various power levels. These temperatures determine the core power coefficient through the Doppler effect.

5.2.3.1 *Reduction of Power Peaking*

As noted in Sec. 1.2.2.1, current designs aim at cores with low leakage of thermal neutrons and low fast neutron flux at the vessel wall. This has led to fuel assembly arrangements with fresh fuel in the central region of the core. The power peaking this produces is reduced by either the use of burnable poison rods in rod cluster control element thimbles or by fuel rods containing burnable poisons. Since the burnable poison burns up at a different rate than the fissionable material, assembly reactivity may not be monotonic with burnup. An examination of power peaking over core life is generally necessary.

Local peaking within a fuel assembly occurs adjacent to the vacant control element slot or at the fuel element boundary because of the increased water gaps that cause an increased thermal flux. The change from cruciform control rods to rod cluster control appreciably reduced local power peaking because of the reduction in the size of the maximum water gap. If fuel rods containing burnable poison are used, they may be located so as to reduce local flux peaking.

5.2.3.2 *Effect of Fuel Assembly Design*

Some newer fuel assembly designs include axial blankets of about 6 in. of natural uranium dioxide.²² This has the advantageous effect of reducing axial leakage and thus improving uranium utilization. However, since the total power output of such rods is the same as in a single enrichment rod, the maximum power density is increased. Most blanketed fuel assemblies contain flow-mixer grids, which are placed between the usual support grids. The addition of these intermediate mixer grids can more than counteract the adverse effect of the increased linear power on the minimum DNBR.²²

5.2.3.3 *Effect of Boiling Void on Local Power*

The void generated by boiling reduces local moderator density and, thus, reduces the local power level. The ability of a PWR to use its own boiling void as an automatic power peak limiter is one of the most advantageous features of this reactor. This remains true for a chemical-shim-controlled PWR containing boron dissolved in the coolant. Although the void coefficient is reduced by dissolved boron, burnable poisons in the fuel or in burnable poison rods limit the BOL dissolved boron concentration to levels such that a satisfactory negative void coefficient is always maintained.

To achieve design simplicity, the effect of boiling voids on local power distribution has often been ignored. This approach is conservative since the actual power distribution will be less peaked than that obtained assuming no voids.

The effect of coolant voids on the behavior of heavy-water-moderated pressure tube reactors differs considerably from that seen in conventional

reactors. Since nearly all of the neutron moderation is accomplished by the low-pressure D_2O outside the pressure tube, the presence of coolant void causes very little change in reactivity. The coolant density reactivity coefficient in a heavy-water-moderated pressure tube reactor is about one order of magnitude below that of a conventional PWR. In a typical CANDU reactor, the coolant density reactivity coefficient is slightly positive under equilibrium irradiation conditions. However, the overall power coefficient of reactivity remains negative because of the Doppler contribution due to increased fuel temperature.

5.2.3.4 *Effect of Control Rod Motion*

The effect of the control rod insertion required for load-following operation on power distribution was discussed in Sec. 1.5. The skewed flux shapes produced by such rod insertions and subsequent xenon transients must be considered in evaluating the minimum DNBR for any load-following plant. Monitoring of the axial flux distortion produced by rod insertion is required in all plants to demonstrate that the actual flux skewing is within the range that has been shown to produce an acceptable DNBR. To remain within the allowable band, load-following operation must follow a prescribed control rod insertion method such as the "Mode G" or "MSHIM" approach described in Sec. 1.5.

Axial power oscillations due to xenon transients posed a problem (1990) in the Russian-designed 1000-MWe VVER plants. As originally designed, these reactors used a two-year fuel cycle without any burnable poison rods. The resulting positive moderator temperature coefficient at BOL increases the axial power distortion due to xenon transients and makes the reactor difficult to control. Use of shorter control rods, annual refueling, and burnable poison rods should provide a solution.

5.2.3.5 *Effect of Vessel Fluence Reduction Techniques*

While low-leakage core designs substantially reduce the flux at the reactor vessel wall, the reduction so achieved may not be sufficient for some early plants. In some reactor vessels, weld contaminants led to welds that embrittle more rapidly with fast neutron exposure than originally expected. To achieve the fluence reduction needed to keep the vessel nil ductility temperature within acceptable limits, special assemblies may be required adjacent to the weld region. Such assemblies can reduce the wall flux by factors of two to ten.²³ However, since these assemblies generate no significant power, total core power must be reduced slightly to remain within the thermal design limits.

5.2.3.6 *Effect of Increased Power Levels*

It has been found that many PWR plants have the capability for increasing their electrical output above the original design value. The net electrical

output of an existing plant can generally be increased by either²⁴ (1) increasing the core thermal output, (2) increasing steam pressure, or (3) a combination of items 1 and 2. Areas that need to be addressed in such an updating include DNB, limiting linear heat rate, cladding corrosion, steam generator corrosion and steam generator moisture separation.

Increased power and primary coolant temperature will decrease the minimum DNBR and increase the maximum linear heat rate. Improved analysis techniques may be able to compensate for these changes. Both cladding and steam generator corrosion are increased by increased temperatures. If cladding or steam generator corrosion have been problems, such a change should not be considered. Moisture carryover from the steam generator can markedly increase with increased steam flow. It must be determined that the projected increase is within the design capability of the unit.

Most steam turbines have approximately 5% added steam flow capacity at design pressure. Power increases somewhat beyond this level may be possible by modifying the nozzle blocks and the first row of turbine blading.

5.3 SUBCHANNEL ANALYSIS METHODS FOR CORE THERMAL-HYDRAULIC DESIGN

5.3.1 Objectives of Subchannel Analysis Procedures

It was previously observed (Sec. 5.1.1) that a core thermal design based on a single closed hot channel is unnecessarily conservative. Although fabrication subfactors can be combined statistically, the closed hot-channel approach requires the use of the statistically evaluated product of fabrication subfactors and all the remaining hot-channel subfactors to compute maximum conditions. Since these subfactors were evaluated individually, use of the product introduces repetition. For instance, flow starvation at the inlet of a hot channel reduces pressure drop in this channel. It will be repetitive if we apply reduced inlet flow simultaneously with the flow redistribution subfactor to avoid excessive pressure drop in the same hot channel. Similarly, reduced flow at the channel inlet can very well be the correct flow rate for a hot channel with a reduced pitch and fuel rod bowing.

The effect on enthalpy rise can be charged to either of the above reasons, but not to both. A constant mixing subfactor assigned to all reactors with different power distributions is also not proper, because the effect of mixing on enthalpy rise depends on the radial thermal gradient between the channels (i.e., power distribution) as well as flow conditions.

A more realistic evaluation of core performance is accomplished using the subchannel analysis techniques and computer codes described in Sec. 4.2.1. Note that the modeling procedures can be applied to either an entire core or an individual subassembly. A fully satisfactory core analysis requires consideration of both core-wide and hot assembly effects.

In view of the foregoing, we may state the objective for use of subchannel analysis in core design as the evaluation of the minimum DNBR under normal and transient conditions in a manner that considers all of the relevant factors but is not unnecessarily conservative. All subchannel analysis codes are programmed with appropriate CHF correlations (see Sec. 4.4.2) so that the DNBR is automatically determined along with hot channel conditions.

5.3.2 Sequential or Chain Subchannel Procedures

In the sequential or chain procedure, the design process is considered to have two stages. The first stage effectively calculates a hot assembly factor from the core-wide analysis, which accounts for the power distribution among assemblies, inlet flow variation, and mixing between assemblies. The second stage of the calculation effectively calculates a hot-cell factor from an analysis of the hot assembly. Here, effects due to local power distribution, mixing between channels, and the effects of adjacent assemblies are considered.

Chelemer *et al.*²⁵ describe this sequential (or chain) approach in some detail. Assuming a known pressure drop across the fuel assemblies and a given total flow, the core-wide analysis determines the inlet flow to each assembly and the flows to and from the hot assembly as a function of axial position. Originally, the THINC I code²⁶ was used for this link of the chain. More recently, however, the THINC IV²⁷ or COBRA IV²⁸ codes have been used.

The assumption of a uniform pressure across the plenum at core inlet does not allow for variations in inlet velocity which would exist even under isothermal conditions. Such a velocity distribution can be determined by laboratory tests of reactor vessel models. Khan²⁹ describes a computer code that accepts such a velocity distribution as input and then iterates on the inlet pressure distribution until it finds a set of inlet pressures that yield a constant outlet pressure. The pressure distribution so obtained can be used as input data to the first link of the "chain" calculation.

Engineering hot-channel subfactors, based on fabrication tolerances, are also part of the input data. Heat fluxes in the hot channel and at the hot spot are appropriately increased over nominal values. Flow to the hot channel is either appropriately decreased or a reduced flow area is assigned.

In the original THINC CHAIN procedure,²⁵ the hot-assembly analysis (second link of the chain) was performed using THINC II. Flows gained or lost by the hot assembly as a function of axial position were provided as boundary condition to the THINC II program. Since THINC II assumes there is no interchannel pressure gradient, a gain or loss of mass cannot be directly determined for a given subchannel. The effect of flow interchange with the surrounding assemblies is assumed to be felt at the inlet of each length step. The effect of each surrounding assembly is considered inde-

pendently. For flow into the hot assembly, the lateral flow from one side is assumed to mix successively with each row of the subchannels as it flows across the assembly. Thus, fluid entering any subchannel has the enthalpy of the adjacent channel. Increases in axial flow to the subchannels are assigned in a manner that maintains the zero pressure gradient across the hot assembly while maintaining mass balance. The procedure is then repeated for flow from the other surrounding assemblies.

More recent THINC CHAIN procedures use THINC IV³⁰ for both links of the chain. Since THINC IV allows radial pressure gradients, an improved distribution of cross-flow within the hot assembly is possible. The flow to or from the adjacent assemblies can be added to or subtracted from the peripheral subchannels of the hot assembly. The pressure gradients established by the computer program appropriately redistribute the flows within the hot assembly.

A two-link chain procedure may also be devised using the publicly available COBRA codes. COBRA IV²⁸ may be used to solve the core-wide flow distribution problem and the faster COBRA III³¹ or COBRA III C/MIT³² used for the hot assembly flow distribution and enthalpy rise. COBRA III C must be slightly revised to accommodate the flow added or lost by the hot assembly.

COBRA determines the core flow distribution based on a constant pressure drop between upper and lower plenums. Since there is no provision for allowing for a nonuniform pressure in the lower plenum, any flow maldistribution in the lower plenum cannot be input directly. This is sometimes dealt with in the second link of the chain by slightly reducing the calculated flow to the hot assembly.

Note that, with the exception of LOCA analyses, the computer codes commonly used for core thermal-hydraulic analyses are based on single fluid models. This model appears to be adequate for analysis of behavior in the steady state and during most transients.

5.3.3 Single-Pass Computational Methods

The early subchannel analysis computer programs were incapable of considering subchannel regions that varied greatly in size. It was therefore impossible to conduct both a core-wide and detailed hot-assembly examination at the same time. Later computer programs, such as COBRA III C/MIT,³⁵ VIPRE,³³ CETOP,³⁴ and the later versions of FLICA,³⁵ have surmounted this numerical problem. It is now possible to examine the core-wide flow and enthalpy distribution using subchannels that are assembly size or larger and yet simultaneously consider the behavior of some individual coolant subchannels between four adjacent fuel rods (in square lattice).

Moreno *et al.*³⁶ were the first to consider single-pass analysis. They found that a very simplified version of the core could provide a DNB analysis that closely matched the results attainable by more detailed procedures. They recommended the following approach:

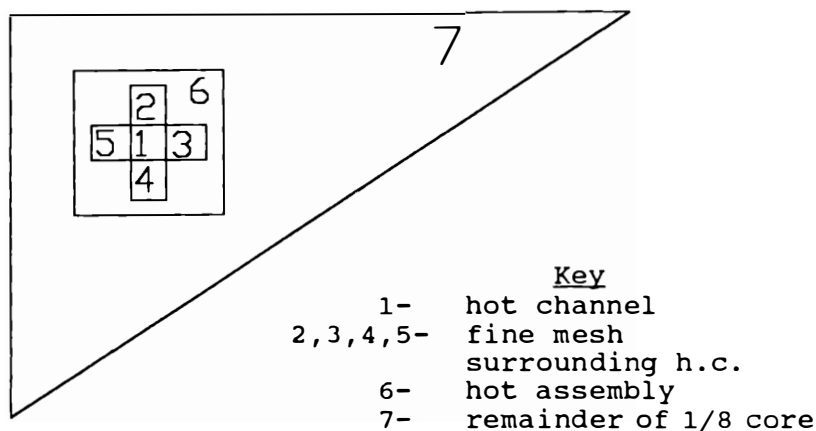
1. Identify the hot channel by virtue of its radial power peaking factor and its placement in the assembly with the highest peaking factor.
2. Surround the hot channel by a fine mesh (each node a subchannel) of the subchannels that directly exchange with it. (Requires 5 fine mesh nodes for one hot channel, 12 fine mesh nodes for two diagonally adjacent hot subchannels.)
3. Lay out at least two coarse mesh nodes representing concentric regions around the hot channel. The interior node usually represents the hot assembly.

This simple scheme recommended by Moreno *et al.*³⁶ for selecting the hot channel may not always be satisfactory. The highest peaking factor may lead to a channel adjacent to an unheated guide tube. While the cold-wall effect may lead to a low DNBR in this channel, an adjacent channel having the largest ratio of energy added to flow area may exhibit the minimum DNBR. It is generally necessary to examine both the subchannel with the highest expected enthalpy rise and the subchannel adjacent to the guide tube having the largest heated perimeter.

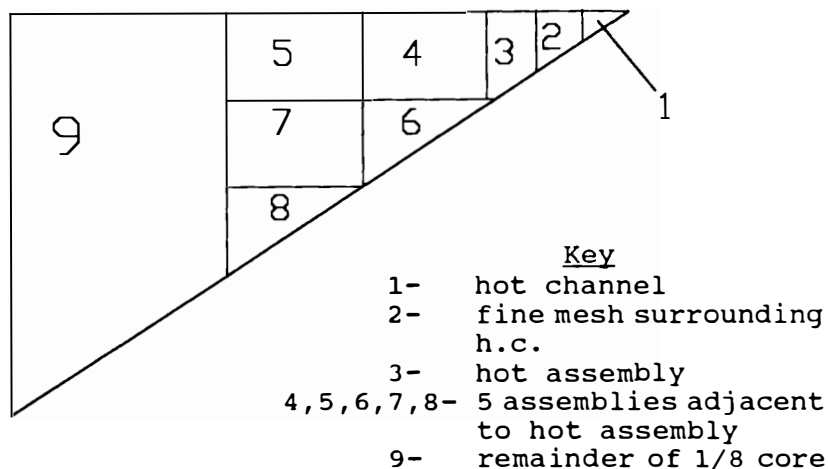
Figure 5.6(a) indicates the arrangement that would typically be used following the one-pass approach of Moreno *et al.*³⁶ By appropriate choice of hot-channel factors and channel equivalent diameter, the simple DNB analysis can generally be brought into fairly close agreement with more exact analyses. Since only seven channels are required, computation time is greatly reduced. This is very useful where repetitive DNB analyses are required.

Since the radial position of the hot channel may change over core life-time, it is sometimes desirable to use a conservative approach and assume the hot channel is at the core center. This eliminates some of the fine mesh flow interchange as well as some of the flow interchange with the entire hot assembly. The resulting higher fluid enthalpy rises provide a more conservative estimate of the DNB ratio.

Figure 5.6(b) shows Chiu's³⁴ recommended arrangement with the hot channel at the core center. In this arrangement, a more detailed model of the remainder of the core is examined. The five assemblies, or parts of assemblies, adjacent to the hot assembly are modeled individually and one additional region models the remainder of the octant. Chiu³⁴ also evaluates the enthalpy transfer from assembly-size and larger regions considering the enthalpy gradient across the channel boundary rather than the lumped enthalpy difference (see Sec. 4.2.1 for discussion of this approach). Chiu's



(a)

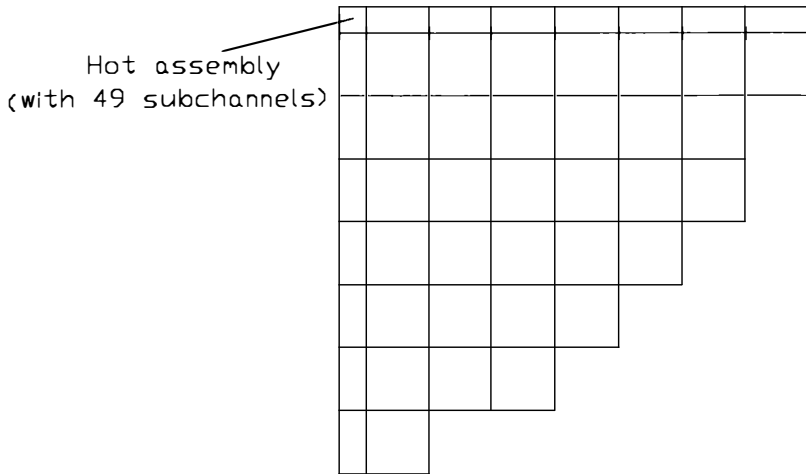


(b)

Fig. 5.6 Some arrangements used for one-pass subchannel core analysis: (a) simple one-pass arrangement and (b) conservative one-pass arrangement.

modeling scheme allows a closer approximation of core behavior than allowed by that of Fig. 5.6(a).

The arrangement of Fig. 5.6(b) allows the examination of the effect of adjacent assemblies on the hot assembly. If two assemblies have nearly the same maximum power, it is possible that the assembly with the slightly lower peak contains the rod with the minimum DNBR. This could occur if



(c)

Fig. 5.6(c) Comprehensive one-pass model for subchannel core analysis.

the average power levels in the adjacent assemblies were such as to lead to a higher enthalpy rise in the channel with the slightly lower peak power. When the highest assembly power is only slightly greater than one or two other assemblies, all of these assemblies should be considered as possible hot assemblies.

The most detailed one-pass approach is that of Raymond *et al.*³⁵ In their use of FLICA-3M a quarter core is modeled on an assembly-by-assembly basis except for the hot assembly. A quarter of the hot assembly is included in the model and it is represented by individual subchannels (49 required). As seen from Fig. 5.6(c), the hot assembly is conservatively placed at the core center. This single-pass model should provide the same accuracy as a two-step chain approach. The choice of modeling approach will depend on the accuracy needed and the number of times the analysis must be performed.

5.3.4 DNB Computations Using Subchannel Analysis Procedures

5.3.4.1 Conservative Approach

The major reason for the development of subchannel analysis procedures is the provision of improved DNB evaluations. For base-loaded plants, detailed digital computer analysis can be carried out simply for the worst power peaking expected (generally that at BOL), and the DNB ratio can be evaluated at the appropriate overpower condition.

In plants designed for load follow, we may proceed in a manner very similar to that used in preliminary core design. The total hot rod power is

again determined using $F_{\Delta H}$ and the desired core overpower condition. However, in this case, $F_{\Delta H}^E$ does not include any allowance for flow redistribution since the hot channel flows are determined by the computer program. Again, the axial power distribution usually is taken as cosinusoidal with the axial hot channel factor chosen so that the maximum linear power agrees with that computed from F_q and the core power. The surrounding rods are taken to have the same axial profile with a rod average power typical of the distribution found in hot assemblies.

Since the position of the hot channel in the core varies over core lifetime and with control rod positioning, a single-reference radial design power distribution is selected for conservative DNB calculation. The assembly power distribution is chosen conservatively to concentrate power in the area of the core which contains the hot channel. This minimizes any benefits due to core flow mixing and redistribution.

In subchannel analysis computations, the fuel fabrication factors are evaluated statistically but plant operating parameters are usually considered as deterministic. To assure conservatism, reactor power, coolant flow, system pressure, and coolant inlet temperature are set at the most adverse conditions (producing lowest DNBR) possible during normal operation. In addition, the nuclear hot-channel factors assigned include an allowance for the uncertainty in their calculation.

An extension of the foregoing procedure can be used to determine the maximum allowable linear power rating at or below the core centerline, which will produce the minimum allowable DNBR. The calculation is simply carried out for successively higher core power levels until the minimum DNBR is obtained.

It is assumed that the local DNBR power limit at elevations below the centerline is the same as that for the centerline. For any power distribution peaked toward the core inlet, the local coolant enthalpy will be lower than that at the core centerline with a cosine flux having the same peak power. Although this power limit is very conservative, it generally does not restrict operations. The limit due to a large LOCA is usually more restrictive in this region (see Fig. 5.2).

The limits imposed by the DNBR on linear power above the core centerline also must be determined. One conservative approach is to continue with the same cosinusoidal power distribution, with peak power at the core centerline, as used above. The calculations then are repeated for a series of increasing core powers, and the DNBRs are obtained as a function of core height and power. The limiting linear power at a given elevation can be conservatively taken as the local linear power under the conditions where the desired DNBR is just reached at the elevation under consideration. This is conservative since in any situation where the peak power is actually above the core centerline, the power distribution is skewed toward the top

of the core. The actual fluid inlet enthalpy then will be lower than that calculated using a cosinusoidal distribution. Therefore, the true DNBR will be above the calculated value.

Demonstration that the core design is safely below the DNB limit under steady-state conditions is not sufficient. This also must be shown to be true for all anticipated transients. Transient versions of the subchannel analysis programs are used for this purpose in conjunction with an appropriate systems analysis code. Core power distribution at the beginning of the transient generally corresponds to that used for steady-state DNBR evaluation.

5.3.4.2 *Realistic DNBR Evaluation*

Chain-type DNB evaluation procedures run too slowly to allow the exhaustive examination of a large number of conditions. Hence, conservative approaches that bounded the operating conditions were used in a limited number of evaluations. The availability of the rapidly running one-pass approach utilizing a small number of channels has made more exhaustive DNBR evaluation feasible. This is particularly useful for load-following plants. Instead of a few conservatively bounding calculations, it is possible to analyze a large number of cases to determine the bounds of safe operation. For a given axial offset, a matrix of conditions, representing the various control rod configurations which can produce that offset and the various burnups during the cycle, are evaluated.³⁷ The power leading to the minimum allowable DNBR and the power leading to the maximum allowable linear power are determined for each condition.* The process is then repeated for a number of different axial offsets. The results, which are then plotted against axial offset, yield a cluster of points as shown in Fig. 5.7 (Note that separate plots are produced for linear heat rate and DNBR.) A bounding curve drawn beneath these data serves as the allowable limit for steady-state operation.

The actual operating power must be sufficiently below the limit curves to allow adequate operational margin. The difference between the operating power and the design limit must be large enough so that the minimum DNBR will not be attained during anticipated transients. As indicated earlier (Sec. 5.1.5), the loss of flow and rod-drop transients are usually the most limiting. These transients must be examined considering the most adverse radial power distribution (lowest steady-state DNBR) at a number of axial offsets. If the transient subchannel analysis program indicates a DNBR below that allowed, then the operational margin must be increased and the allowable operating power decreased until a satisfactory DNBR is attained.

*Modern subchannel analysis programs allow detailed pointwise axial power shapes to be specified in the code input. Thus, the effect of any axial offset can be evaluated.

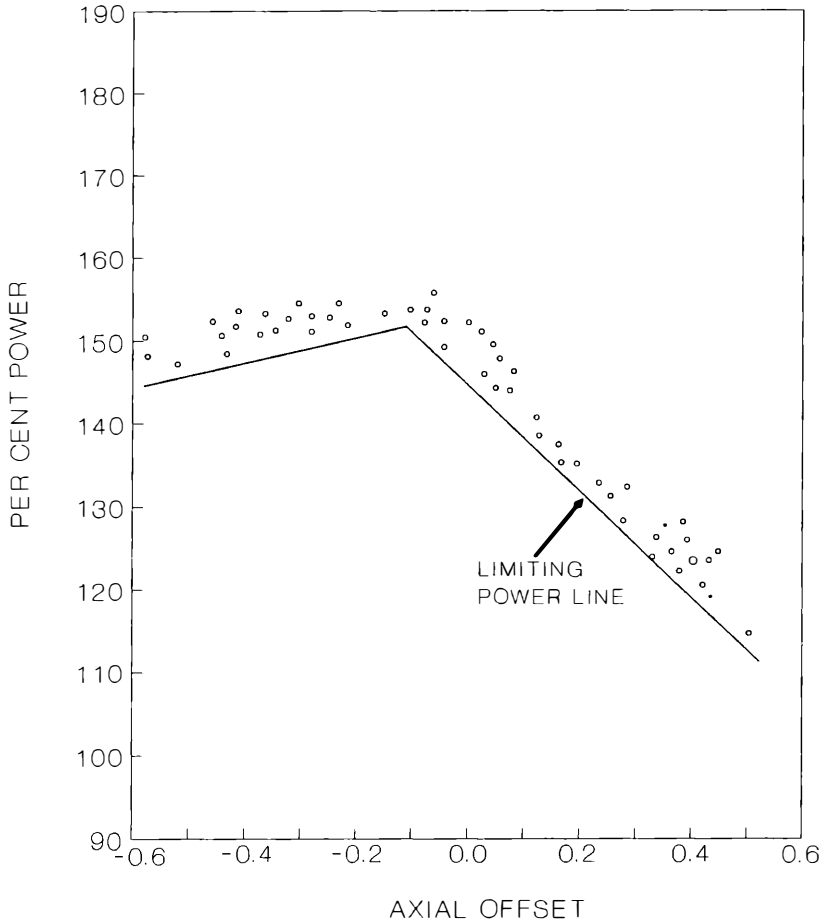


Fig. 5.7 Results of limiting steady-state DNBR calculations.

5.3.4.3 DNBR Evaluation in Mixed Cores

A mixed core may be defined as one containing two different types of fuel assemblies. Such a core may be comprised of different fuel assembly types from the same vendor or fuel assemblies from two different vendors. In both cases, flow redistribution will be affected since the assemblies will almost certainly have different pressure drop characteristics. If the assemblies come from different vendors, the vendor providing the new fuel will not have access to the proprietary DNB correlation used for the fuel originally in the core. A publicly available correlation (e.g., W-3 correlation) must then be used for this fuel.

In one approach to this problem,³⁸ a 3×3 assembly array containing only type A (new) fuel is examined using the chain procedure. The center assembly is assigned the maximum expected power and the minimum DNBR determined. Two additional arrays are then examined. In one array, type A is completely surrounded by B fuel and, in the other, B fuel is on the flats of the center assembly and A on the diagonals. The reduction in DNBR due to the mixed arrays is determined. The process is then repeated with the roles of the A and B assemblies interchanged. In this case, the minimum DNBR is evaluated using a publicly available correlation for the B fuel. On the basis of these runs, an appropriate DNBR penalty is developed for both the A and B type fuel in a mixed core. Safety analyses can then be conducted by treating the mixed core as full core of either fuel type with the appropriate transition core penalty applied to the minimum DNBR.

5.3.4.4 *Combined Thermal-Hydraulic Nuclear Analysis*

Linkage of thermal-hydraulics and neutronic analyses was probably first attempted in the THUNDER program.³⁹ This code linked THINC I to the very detailed neutronic analyses of PDQ-7. The approach was extremely time consuming and of limited use as a design tool. Subsequently the somewhat more rapidly running MEKIN code⁴⁰ was developed. In MEKIN, the core is considered to be made up of neutronically homogeneous regions corresponding to the regions used in the thermal analysis. However, in the neutronic calculations a fine mesh is superimposed on the blocks in the input thermal-hydraulic geometry. This approach was still too cumbersome for general use.

The more recent availability of rapidly running advanced nodal codes and the effective fast group codes (see Sec. 1.3) have made combined thermal-hydraulic nuclear analyses practical. Such analyses are of particular importance in examination of accidents brought about by an anticipated transient followed by a failure to scram. In such an accident scenario, thermal-hydraulic effects are very important in determining the consequences of the accident. The consequences of control rod ejection accidents are also limited by Doppler and thermal-hydraulic effects.

Wu and Weisman⁴¹ linked an effective fast group neutronic calculation to a simple subchannel analysis. Computations of this nature are relatively time consuming since it is necessary to iterate between the nuclear and thermal-hydraulic calculations at each timestep examined. The nuclear computations depend on the void distribution from the thermal-hydraulic calculations, which in turn depend on the power distribution from the nuclear calculations. The process was speeded by using a quasi-steady-state neutronic analysis and requiring only partial convergence of the nuclear calculations prior to returning to the thermal-hydraulic computations. The thermal analysis was accelerated by assuming that no lateral pressure gra-

dient existed at any given core elevation. It was shown that the flow redistribution obtained in this manner differed very little from those obtained by COBRA.

Another example of this general approach is the linkage of the FLICA subchannel analysis program to the CRONOS nuclear code.³⁶ CRONOS is a rapid multidimensional diffusion code, which solves the diffusion equations by a finite element method. At a given time step, feedback effects on macroscopic cross sections due to moderator density and fuel temperature are first computed from the thermal parameters of the previous timestep. The time-dependent diffusion equations are then solved. The power distribution so obtained allows recomputation of the flow and fuel parameters using FLICA.

5.4 STOCHASTIC THERMAL ANALYSIS PROCEDURES

5.4.1 Assessment of Correlation Accuracy

Empirical DNB correlations (see Sec. 4.4.2) are now derived from an experimental base consisting of many rod bundle tests. The raw data consist of axial and radial flux factors, bundle power, inlet mass flux, inlet temperature, pressure, bundle and grid geometry, grid location, and DNB location. These data are input into a subchannel analysis code, which determines the local conditions (mass flux, enthalpy, pressure, quality, void fraction) at the DNB location. These quantities, together with the known DNB heat flux, and proposed functional form of the DNB correlation, are used to evaluate the correlation parameters. The appropriate values of the correlation constants are usually determined by regression analysis. The resulting expression is then compared to the data by (1) substituting the calculated subchannel conditions into the correlation and (2) dividing the calculated DNB flux by the observed flux. This yields the DNB ratio for each of the data points. The collection of DNBR values are then analyzed statistically to determine $\bar{\mu}(R)$, the mean value of the DNB ratio, and s , the standard deviation of the available DNBR values.

Current design (1993) practice requires the minimum allowable DNBR, $R_{\min}$, to be set so that there is a 95% probability that DNB will not occur when $R \geq R_{\min}$. To establish this criterion, we require a probability distribution function that fits the DNB ratio data. It appears⁴² that the major correlations are derived from data that are reciprocal normal. That is, the (measured flux/predicted flux) ratios are normally distributed. Therefore, the tolerance limit on the normally distributed $1/R$ is obtained from

$$\text{limit } (1/R) = \bar{\mu}(1/R) - ks \quad (5.36)$$

where $\bar{\mu}(1/R)$ = mean value of $(1/R)$, s = standard deviation of $(1/R)$ data, and k = confidence parameter [defined by Eq. (5.22) and tabulated in Ref.

7] for 95% probability with 95% confidence level. The minimum allowable DNBR is then

$$R_{\min} = \frac{1}{\bar{\mu}(1/R) - ks} \quad (5.37)$$

For large sample sizes, the value of k approaches 1.645 to provide one-sided 95% probability protection. Values of $R_{\min}$ vary significantly with the correlation. For the W-3 correlation [Eq. (4.206)], $R_{\min} = 1.3$, whereas for the WRB-1 correlation [Eq. (4.216)], $R_{\min} = 1.17$

If a reactor were operating with a DNBR of 1.17, this does not necessarily mean that the power can be increased by 17% before DNB will actually occur. As the power increases, the local enthalpy and quality also increases, thus reducing the predicted critical heat flux in most correlations (e.g., W-3 or WRB-1). Thus the critical power ratio (CPR) will be less than the DNBR. However, this is not true for DNB correlations, such as the EPRI correlation [Eq. (4.215)], which have a denominator of the form $[C + (x_c - x_{in})/q]$ where q is the measured heat flux. Such correlations show DNBRs that are nearly invariant with power level. Thus, requiring a 1.17 minimum DNBR with the WRB-1 correlation [Eq. (4.216)] implies a minimum CPR of 1.07. Hence, the allowance for DNB correlation uncertainty requires only that the power be 7% below the DNB power. However, a DNBR of 1.17 with Eq. (4.215) would imply a $\text{CPR} \approx 1.17$ and therefore the allowance for DNB correlation uncertainty requires the power to be at least 17% below the DNB power. Under the conditions assumed, WRB-1 provides greater operating margins. Comparison of the accuracy of various correlations is therefore best done using CPR statistics.* However, plant safety evaluations are much more conveniently done using DNBR values since these are determined at reactor power while CPR computation requires a search for the critical power level.

5.4.2 Stochastic Design Methods Based on the Total DNBR Ratio

In the conservative design approach discussed in Sec. 5.1, plant operating subchannel analyses for DNB prediction are conducted with parameters and nuclear hot-channel factors set at their most adverse values. The minimum DNBR is then set at the value required to account for correlation uncertainty as described in the preceding section. Analyses based on the most adverse plant operating conditions lead to a very conservative design.

The newer stochastic design methods recognize that it is highly unlikely that all of the plant operating parameters will simultaneously be at their most adverse values. These design methods consider the uncertainties in

*For further discussion of this subject, see M. E. Nissley *et al.*, "Considerations for Comparing CHF Correlations," *Proc. Second Int. Topical Mtg. Nuclear Power Plant Thermal Hydraulics and Operation*, Atomic Energy Society of Japan (1986).

vessel coolant flow, core pressure, effective core flow fraction, nuclear hot-channel factors ($F_Q^N, F_{\Delta H}^N$), engineering hot channel factor ($F_{\Delta H}^E$), core power, and coolant inlet temperature as stochastic quantities. The effect of the uncertainty is combined directly with the DNBR correlation uncertainty to establish a total DNBR uncertainty. The core subchannel analysis to determine DNBR conditions is then conducted with all plant parameters at their nominal values. However, the DNBR so obtained must be above a new higher minimum DNBR. The new minimum DNBR is set to provide an overall 95% probability of no DNB with a 95% confidence level.

Several approaches have been used to relate the variations in plant parameters to DNBR variations. In the simplest of these,⁴³ called the root-sum-square approach, a plant uncertainty factor, Y , is defined by

$$Y = \text{DNBR}(\text{actual}) / \text{DNBR}(\text{nominal}), \quad (5.38)$$

where $\text{DNBR}(\text{nominal}) = \text{DNBR}$ determined with all design parameters at their nominal value, and $\text{DNBR}(\text{actual}) = \text{DNBR}$ determined for actual plant conditions. $\text{DNBR}(\text{actual})$ is a random variable that depends on the uncertainties of the parameters affecting DNBR. If the parameters affecting Y are designated as $x_1, x_2, \dots, x_n$ and Y is expanded about its mean, $\bar{\mu}_Y$, using a Taylor's series

$$Y - \bar{\mu}_Y = \frac{\partial Y}{\partial x_1}(x_1 - \bar{\mu}_1) + \frac{\partial Y}{\partial x_2}(x_2 - \bar{\mu}_2) + \dots + \frac{\partial Y}{\partial x_n}(x_n - \bar{\mu}_n) + \text{higher order terms} \quad (5.39)$$

When the perturbations around the mean value are small, the higher order terms may be neglected and the variance of Y is obtained from

$$\left(\frac{\sigma_Y}{\bar{\mu}_Y}\right)^2 = \left(\frac{\partial Y}{\partial x_1}\right)^2 \left(\frac{\sigma_1}{\bar{\mu}_1}\right)^2 + \left(\frac{\partial Y}{\partial x_2}\right)^2 \left(\frac{\sigma_2}{\bar{\mu}_2}\right)^2 + \dots + \left(\frac{\partial Y}{\partial x_n}\right)^2 \left(\frac{\sigma_n}{\bar{\mu}_n}\right)^2 \quad (5.40)$$

where σ_i^2 is the variance of x_i and $\bar{\mu}_i$ is the mean of x_i . The values of $(\partial Y / \partial x_i)$ are determined by approximating them as $(\Delta Y / \Delta x_i)$ and using a subchannel analysis program for their evaluation. If the mean value and the probability distribution are known for each design parameter, then $(\sigma_Y / \bar{\mu}_Y)$ may be determined for the DNBR.

The central limit theorem states that when a quantity is a function of a number of random variables, the distribution function for the quantity will approach the normal distribution function even though the individual distribution may not be normal. Therefore, Y may be considered to be normally distributed.

The various plant parameters may be expected to vary symmetrically around their nominal value. In many cases the probability density function representing these parameters is unknown but upper and lower bounds for the individual parameters can be established. A common practice is to assume that the parameters are approximately normally distributed and that

the upper and lower bounds represent $\pm 2\sigma$ limits (95% of variations within bounds) with symmetric variation around nominal conditions. If the applicability of the normal distribution is in doubt, a uniform distribution may be conservatively assumed for the parameter. In that case, the standard deviation will be $(a/\sqrt{3})$ instead of $(a/2)$ where a is the magnitude of the upper or lower bound.

To obtain the design limit (minimum allowable DNBR) we must statistically combine the plant parameter uncertainty factor Y with the uncertainty of the DNB correlation. Recall that current DNB correlations follow a reciprocal normal distribution. We will consider the reciprocal of the DNB ratio obtained from the analyses of the experimental data as represented by random variable X ($1/R=X$) following a probability density function $f(X)$. We then can obtain a cumulative probability function $\phi(X)$ from

$$\phi(X) = \int_{-\infty}^X f(X) dX \quad (5.41)$$

For any specified value of X , say A ,

$$\phi(A) = p(X < A) \quad (5.42)$$

where $\phi(A)$ is the value of Eq. (5.41) obtained with A as the upper limit of the integral.

The combined probability that design conditions will not be exceeded is obtained by convoluting the distributions for X and Y . If both X and Y are independent variables with distribution functions ϕ_1 and ϕ_2 , the probability that product XY is less than given value A is numerically approximated by

$$p(XY < A) \leq \sum_i [\phi_1(X_{i+1}) - \phi_1(X_i)] [\phi_2(A/X_i)] \quad (5.43)$$

where the subscript i indicates the i 'th location at which X is evaluated. By assuming a series of values for A , we can determine the distribution function for (XY) . We seek the value of A that will yield $p(XY < A)$ of 0.95. This will provide a 95% nonfailure probability. The value of A satisfying the condition is the minimum allowable DNBR ratio. When this ratio is evaluated using these statistical concepts, it is sometimes called the *stochastic* or *statistical design limit* (SDL).

An alternative stochastic approach is based on the use of a Monte Carlo analysis.^{44,45} In this technique, the effect of uncertainties in the plant parameters and the DNB correlation are considered simultaneously. In a Monte Carlo analysis, an appropriate random number is added to the nominal value of each of the variables representing a plant parameter. The DNBR is then computed for the set of adjusted inputs. This DNBR is adjusted by adding a random number to represent the correlation uncertainty. Alternatively, the DNBR may be multiplied by a random number to reflect correlation uncertainty.

The random numbers required are obtained by first establishing cumulative probability functions [Eq. (5.41)] for each variable of interest. Once these functions are available, a random number between 0 and 1 is generated by a computer subroutine. We then determine the value of the variable which yields a cumulative probability equal to the random number. The variable value obtained is then input to the DNBR calculation. Each of the process variables is determined in the same manner. The random number and DNBR calculations are then repeated a large number of times.

DNB values may be generated using a few-channel, single-pass subchannel analysis procedure for each set of random variables. Alternatively, a response surface that has been fitted to the results of a number of more elaborate subchannel analyses may be used to obtain DNB values. A response surface is a polynomial function fitted to the Monte Carlo output by regression analysis.

The standard deviation of the calculated DNB ratios is found and then used to obtain an estimate of the population variance. The one-sided 95% confidence limit of the population variance, $s^2(95)$, is then obtained using the chi-square distribution and the relationship

$$s^2(95) = \frac{n-1}{\chi_{0.05}^2} s^2 \quad (5.44)$$

where s = sample variance, n = number of Monte-Carlo runs, and $\chi_{0.05}^2$ = chi-square value for $(n-1)$ degrees of freedom and 0.05 probability that population variance exceeds $s^2(95)$.

The stochastic DNBR limit, SDL, would then be set at

$$\text{SDL} = 1 + 1.645s_{95} \quad (5.45)$$

to achieve at 95% probability of safe operation with a 95% confidence level (assuming a normal distribution).

Comparisons^{42,43} have been made of the Monte Carlo approach and the simpler root-sum-square (RSS) methodology. The comparisons indicate that both methods yield essentially the same result.

5.4.3 Assessment of DNB Failure Probability at Operating Conditions

The stochastic design methodology of the previous section may be extended to determine the probable number of rod failures at actual operating conditions. We could confine our attention to the hot channel alone and make the tacit assumption that the nonfailure probability elsewhere is zero. A more reasonable assumption is that a failure is possible in any channel in the core.

At actual operating conditions, the nominal DNBR will be considerably above the SDL value in order to allow for control rod maneuvers and to assure that the SDL will not be exceeded in an anticipated transient. Op-

erating powers are well below the maximum allowable. Therefore, we need to determine the DNB failure probability for the various power levels which actually exist in the core.

The Monte Carlo or RSS approach will yield a cumulative probability function $\phi'(XY)$. For any given channel, the DNB failure probability may be obtained from

$$1 - p(XY < R_j) \leq 1 - \phi'(R_j) \quad (5.46)$$

where R_j = nominal DNBR for channel of interest at operating conditions, and $\phi'(R_j)$ = value of $\phi'(XY)$ evaluated at $XY = R_j$. We divide the fuel rods in the core into m groups representing m different power levels and with n_j rods in each group. The number of rods expected to fail within each group, n_{fj} , would then be

$$n_{fj} = [1 - \phi'(R_j)]n_j \quad (5.47)$$

The total number of rods expected to fail would be obtained by summing n_{fj} over the m fuel rod groups.

The foregoing discussion assumed that the DNB ratio was selected by specifying a hot-channel failure probability and that the effectiveness of this choice was to be assessed. However, the process could be reversed and the core-wide statistical analysis could be used to select a design DNBR. For example, we might specify that the probable number of rods experiencing DNB be less than one.

Other approaches to core-wide statistical analysis have been proposed.⁴⁶⁻⁴⁸ A significant difference between most of these approaches and the method indicated here is the use of the theory of statistics of extremes to specify low probability events.

5.5 COMPUTER-BASED ANALYSIS OF PRESSURE TUBE REACTOR CORES

The problems facing the designer of a pressure tube reactor (e.g., of the CANDU type) differ somewhat in character from those encountered in the design of the vessel-type PWR. The CANDU reactor core can be considered a network of pipes between constant pressure inlet and outlet headers. The designer's problem is the determination of the flow and enthalpy distribution in the various core channels given the power distribution, pump and heat exchanger characteristics, and pressure drop characteristics of the flow paths.

The usual approach to this problem is based on one-dimensional, single fluid modeling of the reactor circuit. The NUCIRC program⁴⁹ considers the reactor circuit to consist of one pump, one boiler, one reactor header and one reactor outlet header, and a core containing up to 95 channels. When used in the "circuit" mode, the system boundary conditions (reactor inlet

header temperature, header-to-header pressure drop, and core flow) are determined for a given reactor outlet header pressure, channel power distribution, and steam side conditions in the boiler. The flow and average outlet quality are determined by using an appropriate weighting of the 95 representative core channels. Note that an iterative procedure is required since initially neither the total flow or individual channel flows are known. The first portion of the design effort is devoted to determining the appropriate size of feeder piping and inlet orifices to obtain a satisfactory core flow distribution. This distribution is usually based on a constant enthalpy rise across each channel to assure that channel flows are matched to channel powers. Note that because of the on-line refueling of pressure tube type cores, the radial power distribution remains relatively fixed during reactor operation.

Once the appropriate feeder and orifice sizes are established, the final values of reactor inlet temperature, header-to-header pressure drop, and core flow are determined. The NUCIRC program⁴⁹ may then be used in a single-channel mode. Channel conditions may then be evaluated for the high-power channels. The margin to DNB and dryout is computed by NUCIRC and the limiting channel determined. As previously noted (Sec. 4.4.2.3) the lower operating pressure (~1400 psi) and higher exit qualities in CANDU designs require dryout to be considered. Hot-channel qualities can be as high as 25% under some conditions.

In contrast to vessel-type cores, no subchannel analysis of the hot pressure tube is used. Both DNB [see Eq. (4.217)] and dryout are evaluated for bundle average conditions. The CHF correlations used are based on tests of 28- or 37-rod clusters (usual number of fuel rods in pressure tube). The test clusters are designed so that the radial power distribution of the electrically heated rods simulates the fine flux dip across a reactor fuel bundle. The results of the tests, in terms of bundle average heat flux, may then be used directly for design purposes.

In establishing the appropriate margin to CHF, allowances must be made for the effect of bundle misalignment and pressure tube creep.⁵⁰ Misalignment of the fuel bundles during refueling will cause increased pressure drop and therefore a decreased flow. An appropriate penalty (of the order of 5%) is applied for this flow decrease. While the misalignment may also increase turbulence and thus lead to some improvement in CHF, no credit for this enhancement is now (1993) claimed.

The Zircaloy pressure tubes creep outward under reactor pressure (outward creep of the order of 1% every nine years).⁴³ Since CANDU pressure tubes are horizontal, the fuel bundles will lie at the bottom of the pressure tube and there will be additional clearance near the top of the pressure tube. The additional flow through the open area means slightly less flow elsewhere and a reduced margin to CHF. A subchannel analysis of a tube with this flow bypass is used to determine the CHF penalty caused by this.

After the misalignment and pressure tube creep penalties are applied, additional margin to CHF is provided to allow for the reduced CHF values seen during anticipated transients.

In addition to outward creep, the CANDU pressure tubes exhibit irradiation-induced axial growth. This growth is greatest in the tubes in the central portion of the reactor where the neutron flux is the highest. To assure that the total tube length remains within the range of the automatic refueling machine, the tubes in the central portion of the reactor have an initial length slightly below that of tubes in outer regions.

5.6 REACTOR PROTECTION AND CORE MONITORING

5.6.1 Safety Criteria

The basis for the design of the reactor protection system is contained in the U.S. Code of Federal Regulations, "General Design Criteria for Nuclear Construction Permits,"⁵¹ which states, "The reactor core shall be designed to function throughout its design lifetime without exceeding acceptable fuel damage limits which have been stipulated and justified. The core design, together with reliable process decay and heat removal systems shall provide for this capability under all expected conditions of normal operation with appropriate margins for uncertainties and for transient situations which can be anticipated, including the effects of loss of power recirculation in pumps, tripping of the turbine generator set, isolation of the reactor from the primary heat sink, and loss of all off-site power." This criterion is generally interpreted as meaning that failures due to exceeding the CHF or an excessive linear heat production must be avoided during any reasonably anticipated situation.

Following these guidelines, the revised Code of Federal Regulations⁵² defines "safety limits" and "safety settings." "Safety limits are limits upon important process variables which are found to be necessary to reasonably protect the integrity of each of the physical barriers which guard against the uncontrolled release of radioactivity." The "maximum safety settings are settings for automatic protective devices related to variables on which limits have been placed. A maximum safety setting shall be so chosen that automatic protective action will correct the most severe abnormal situation anticipated before a safety limit is exceeded." Thus, the safety limit on reactor power would be the power level at which operation is deemed to become unsafe, while the maximum safety setting would be the power level at which a trip is initiated. The maximum safety settings must take into account measurement and instrument uncertainties associated with process variables.

In PWR thermal design, the safety limits are derived from the nuclear steam supply system (NSSS) overpressurization limit and the Safe Accept-

able Fuel Design Limits (SAFDL). For the thermal designer, two SAFDLs are of importance. The first of these is the linear heat generation rate (LHGR) limit, which corresponds to the onset of fuel centerline melting. The LHGR depends on core power, axial power distribution, radial power distribution, azimuthal tilt magnitude, and fuel rod design.

The second SAFDL is that of DNBR. This limit requires that the DNBR not go below 1.0. The DNBR value will be affected by core power, axial power distribution, radial power distribution, coolant inlet temperature and mass flow, and primary coolant pressures.

When the SAFDL and overpressurization limits are expressed in terms of limits on process variables, they become the safety limits. These limits, as tabulated in plant technical specifications, are combinations of selected variables that will prevent SAFDL violation and overpressurization.

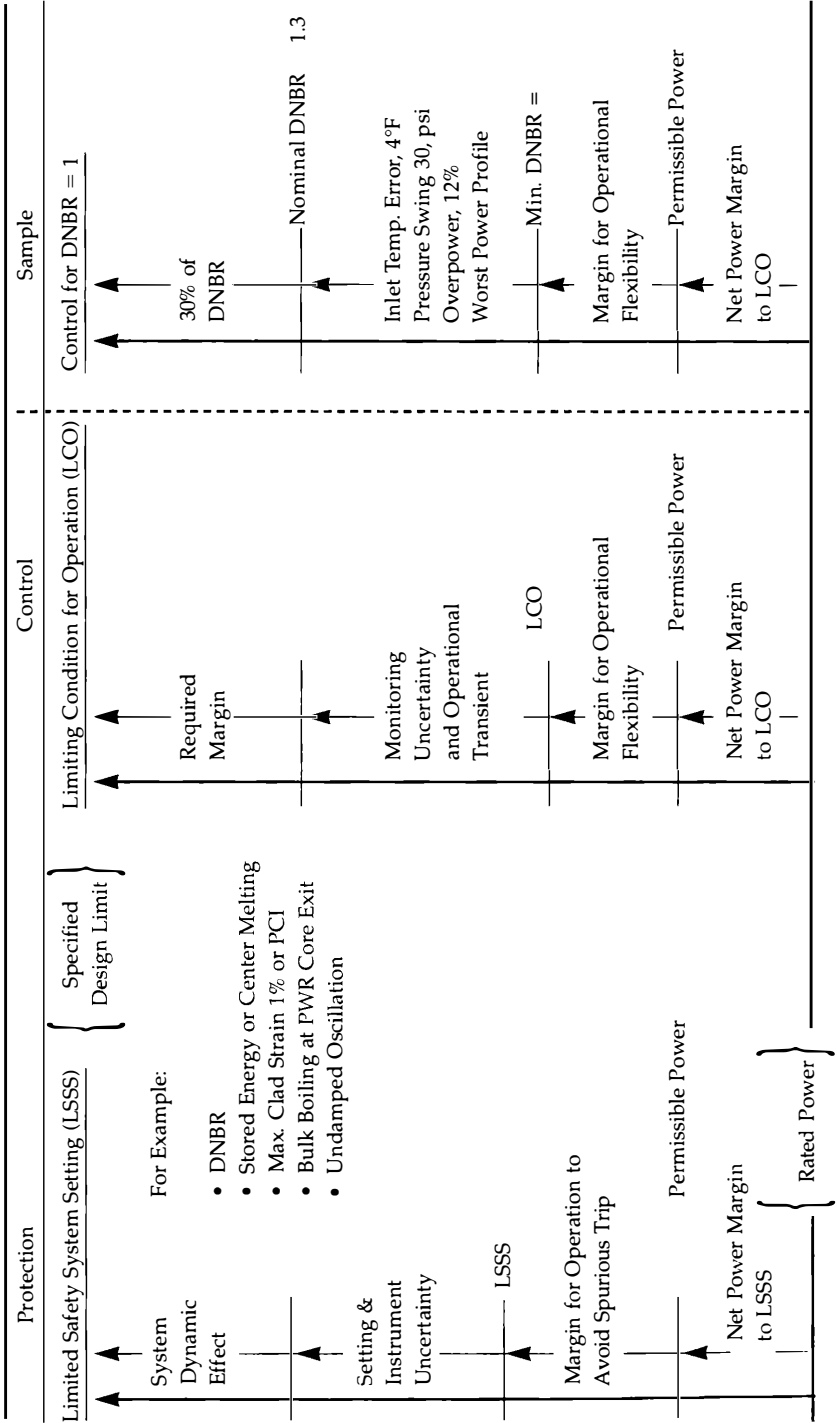
Since safety acceptance criteria require the absence of core damage during normal operation and any anticipated operational occurrence (AOO), the SAFDL may not be violated under these conditions. The Limiting Safety System Settings (LSSS) are determined so that reactor protection system (RPS) action (reactor trip) will prevent pertinent plant parameters reaching values which would cause a SAFDL to be exceeded. As indicated in Table 5.III, the LSSS must allow for uncertainties in the setting and instrument reading as well as transient or dynamic effects.

Only a limited number of parameters are monitored by the RPS (i.e., primary and feedwater flow, coolant temperature, pressure, core flux levels). In some cases, the reactor protection system cannot monitor the variable that is most important during the transient. The approach to a SAFDL may then not be sensed. In such cases, we use analyses of system behavior during the accident to determine the preaccident conditions that must be maintained to assure acceptable consequences. The operating envelope thus prescribed is termed the Limiting Conditions for Operation (LCO). Usually, the LCO includes steady-state core power, pressure, minimum coolant flow rate, radial peaking factors, control rod positions, and axial offset (AO). As seen in Table 5.III, the LCO must also allow enough margin for any correlation uncertainties in the calculated parameters (required margin) as well as monitoring uncertainties and transient effects.

The LCO generally limit preaccident DNBR, reactor shutdown margin, ejected rod worth, and LHGR. As noted previously, the limit on the maximum cladding temperature that may be reached during a LOCA leads to a limit on the steady-state LHGR. This restriction is one of the most significant imposed on normal operation. Section 5.3 discussed the operating limitation imposed on the axial offset (AO) by DNBR requirements. The LOCA LHGR places an additional restriction on AO. The allowable envelope is set by the most restrictive of the two requirements (see Fig. 1.37).

As seen in Table 5.III, the LCO operating margin is obtained by summing the allowances required for correlation uncertainties (indicated as required

TABLE 5.III
Thermal Margins for Reactor Power and Safety



margin in Table 5.III), monitoring or instrument uncertainty, operating flexibility, and the allowance for anticipated operating occurrences (AOOs). The required overpower margin (ROPM) for a given transient is determined from an analysis of the accident. The required margin may be related to the margin consumed during the event. Most AOO are DNB limited. The required margin⁵³ may then be defined as

$$\text{ROPM} = \left[\frac{(\text{CPR})_N}{(\text{CPR})_{\text{mi}}} - 1.0 \right] 100 \quad (5.48)$$

where $(\text{CPR})_N$ = critical power ratio at nominal operating conditions, $(\text{CPR})_{\text{min}}$ = minimum value of critical power ratio during transient, and ROPM = operating margin in % of design power.

Although the DNBR would normally be calculated in the transient analysis, the CPR corresponding to this can be obtained by a knowledge of the relationship of CPR to DNBR for the particular correlation used.

5.6.2 Core Monitoring Instrumentation

The ex-core ionization chambers that measure axial offset are an important part of any core monitoring system. We previously noted (Sec. 1.5.1) that this measurement is often obtained from full-length ionization chambers that are split into upper and lower segments. Current designs often split these units into four axial segments. Electronic comparison of the signals from upper and lower segments allows the offset to be read directly. The AO signal provides an effective means for the operator to monitor and control the axial core power distribution.

Generally the four ex-core ionization chambers (see Sec. 1.5.1) are positioned around the core so that one is outside each core quadrant. The azimuthal power tilt may be determined from these readings. The *quadrant power tilt* is often defined as the ratio of the reading of one ionization chamber to the average of all four segments at the same elevation. This ratio is normally required to be close to unity (e.g., ~ 1.02) since any at-power asymmetry can cause radial xenon transients to follow load reductions. Such transients would magnify the steady-state asymmetry.

The information provided by the ex-core detectors is supplemented by that obtained from the in-core instrumentation. Such in-core instrumentation was first used to demonstrate reactor power capability. Early reactor designs were generally very conservative and actual operating experience was helpful in demonstrating that increased power output could be achieved safely. A typical example is the Yankee-Rowe plant where in-core instrumentation was valuable in justifying the increase in plant output from 392 to 600 MWth.

In-core instrumentation has generally consisted of flux measuring devices plus thermocouples placed at the centers of the exit nozzles of a num-

ber of fuel assemblies. Mixing vanes are often placed at fuel assembly outlets to assure that thermocouple measurements reflect true bundle-average outlet temperatures.

In-core flux measuring devices have taken a variety of forms. They have included:

1. *Flux wires*: flexible wires inserted into the core at a number of radial positions and irradiated for a preset time. Wires are then removed from the core and counted. Wire activation is proportional to flux and, hence, axial flux variation is determined by axial variation in wire activity. These units have generally been replaced by item 2.
2. *Movable in-core detectors*: detectors connected to flexible cables that can move detectors axially in fixed radial positions. By making a traverse of the core, axial variation in flux at a given x - y location is determined. Detectors can be kept out of the core when not in use and, hence, radiation damage is limited.
3. *Fixed in-core detectors*: at given x - y locations, several fixed detectors are located at a series of axial positions. Detectors provide a continuous picture of core-flux distribution.

Movable in-core detectors are miniature fission chambers with enriched uranium, in the form of U_3O_8 , as fissionable material.⁵⁴ The detectors may be used by themselves or in conjunction with fixed in-core detectors. In the latter role, they merely serve as periodic calibrating devices for the fixed in-core detectors. When used independently they are likely to be inserted at monthly intervals when the reactor is operating. The detectors are inserted into retractable guide thimbles, which remain in the core except during refueling. The thimbles extend out of the bottom of the reactor vessel through conduits. Since only four to six detectors are used, a number of traverses are required to examine the approximately 30% of the fuel assemblies containing flux thimbles.

Most fixed in-core flux detectors now (1994) in use are self-powered rhodium detectors.⁵⁴ Rhodium-103 has a 150-barn cross section for thermal neutrons. The rhodium-104 capture product emits energetic beta particles. Some beta particles escape from the emitter and produce a positive charge that can be detected. The insulated detector and lead wire is contained in an inconel sheath. The individual detectors are grouped into an assembly of four to seven detectors, which are located at different axial locations. In some designs, the detectors are grouped around a hollow central tube in which a movable detector may be inserted.

Fixed in-core detectors are used primarily for the determination of the radial flux distribution with the ex-core detectors indicating the axial flux distribution. Instead of using fixed neutron-flux detectors, some core detector monitoring systems⁵⁵ use a number of thermocouples placed at fuel assembly exits to provide information on radial power distribution. Use of

coolant exit temperatures in this manner implies that assembly average exit conditions are not expected to be above the boiling point. Statistical analysis procedures are used to distinguish between signal changes due to actual reactor condition changes and thermocouple malfunctions.⁵⁵

A complete picture of core conditions requires a number of measurements in addition to flux levels or core exit temperatures. This usually includes core power level, control rod positions, core inlet temperature, core flow, and system pressure. Control rod positions are obtained from control rod position indicators associated with the rod drive system. Plant instrumentation directly provides system pressure and core inlet temperature. Core flow may be estimated from reactor coolant pump speed measurements. Core power is usually determined via the heat balance method. This heat balance is determined from the magnitude of the feedwater flow to the steam generators and the difference in enthalpy between the inlet feedwater and outlet steam.

Some plants have had problems with the accuracy of core power measurements due to degradation of the feedwater flow measuring device. This problem can be avoided by basing the flow measurement on a Kalman filter estimation.⁵⁶ A Kalman filter is a statistical technique that combines several measurements and their uncertainties so as to obtain a more reliable estimate of the quantity being measured. The power estimates that can be used for this purpose are condenser power plus turbine power based on first stage pressure, heat balance power, neutron flux power from ex-core detectors, and neutron flux power from in-core detectors. An on-line computer can perform the required analyses and provide a reliable power estimate, which will not be unduly biased by difficulties with any one signal.

In modern practice, all of the information describing core behavior is fed to one or more on-line computers that synthesize these data. The computer system determines how close plant conditions are to those specified by LCO and LSS requirements. The system may be used solely for plant surveillance or it may be integrated into the core protection system. These uses are discussed in greater detail in Sec. 5.6.4.

5.6.3 Core Protection

We may think of the reactor protection system as a control system that, in routine operation, remains an observer acting only if the reactor system reaches the limit of permissible operation (limiting safety setting, LSS). Most reactor trips are based on a single monitored parameter and a fixed set point. In this category, the following are typical of PWR abnormal operating conditions that usually cause a reactor trip^{36,37}:

1. *High neutron flux level*: high flux level indicates excessive power generation in the reactor; e.g., inadvertent rod withdrawal while operating at

power. Coincident high flux level signals from two power range channels cause a trip.

2. *High reactor startup rate*: reactor startup rate indication, computed by the startup channels, gives the operator information on the rate of power change. High startup rate signals from source range instrumentation actuate an alarm and a rod-stop circuit. High startup rate signals from intermediate range channels, if the startup rate continues to increase, initiate a trip signal. The power range channels provide a further backup if flux exceeds a preselected value on two of the four power range detectors.

3. *Loss-of-reactor-coolant flow*: trip is required on loss of flow in one or more reactor coolant loops, or on loss of power to one or more reactor coolant pumps. The need to trip on loss of flow in one loop is a function of the power level at the time of the accident and the number of loops in the plant. This protection is necessary to prevent excessive fuel temperatures resulting from this accident.

4. *Manual trip*: trip initiated by the reactor operator.

5. *High water level in the pressurizer*: high level in the pressurizer initiates trip to prevent filling the pressurizer and discharging water through the safety valves.

6. *Initiation of safety injection operation*: this trip provides a backup for low-pressure trip.⁶ Indication of safety injection operation is provided by several independent measurements.

7. *Turbine-generator trip*: above ~10% power level, a turbine trip initiates a scram to prevent activation of the steam-generator safety valve.

8. *Low steam generator water level*: redundant instrumentation is supplied to protect against loss of steam generator inventory. Should this still occur, reactor trip is indicated.

9. *Feedwater-steam flow mismatch*: alternative protection against loss of steam-generator inventory. Mismatch signal generally must be correlated with a slightly low level in the steam generator.

10. *High system pressure*: coincident signals from two channels initiate a trip if the system pressure exceeds the set point. The trip circuit is an additional backup for high power trips.

11. *Malpositioning of control rods*: an axial flux difference as monitored by in-core or out-of-core measurement (design varies with vendor) results in turbine cutback. This procedure protects against heat flux significantly outside acceptable limits.

Dropping a control rod assembly due to deenergizing a drive mechanism also results in a reduction of turbine load. This is required since the value

of F_q is increased when a dropped rod assembly distorts the normal flux pattern.

12. *Turbine overspeed*: turbine trip originates if turbine speed exceeds a preset level. Loss of load leads to reactor trip.

In addition to the foregoing single monitored parameter, fixed set-point trips, each plant has additional thermal protection trips. These trips, which have characteristics that vary with the reactor vendor, fall into two classes. The first class is that of a single monitored parameter with a variable set point.⁵³ An example of this trip is the high thermal power level trip as indicated by the temperature difference across the core. Coincident high ΔT measurements from two channels cause a trip. This circuit would provide a backup for the flux level trip.

The set point for this trip would normally depend on the measured average core temperature and measured axial offset. By incorporating these variables, the trip protects against local fuel rod power exceeding allowable limits even though the total core power is within acceptable limits.

Another trip category⁵³ is that of "a calculated monitored parameter with a fixed set point." Minimum DNBR trips are in this category. The allowable minimum DNBR is fixed and the approach to this set point is continuously computed from monitored plant variables. In a typical analog-controlled plant, such a trip might be calculated from reactor power, coolant inlet temperature and pressurizer pressure. Figure 5.8 shows the safety limits established for pressure, temperature, and power at 100% flow in a typical three-loop design. They assume conservative values of the hot-channel factors. The limits are based on DNBR margin and no boiling at the vessel exit. The maximum safety setting would be derived from Fig. 5.8 by allowing for instrument and measurement uncertainties. In later load-following reactors, this approach to the minimum DNBR would also include a term representing the effect of the measured axial offset. This is required as the control rod motion needed to adjust power level can change the axial power distribution (see Sec. 1.5.1). Earlier base-loaded reactors operated continuously at essentially full power with control rods remaining in the nearly fully withdrawn positions and producing a nearly constant axial power shape.

Figure 5.9 shows the relationship between the key thermal protection trips in the protection system just described and the allowable reactor operating range at nominal flow. Figure 5.9 also shows the limitation imposed by the steam generator safety valve.

The VVER pressurized water reactors designed in the former Soviet Union utilize core protection systems that differ significantly from those of U.S. or West European designs. The VVER protection system appears to be based on naval reactor practice where great importance is placed on maintaining some reactor power. In contrast to the previously described systems

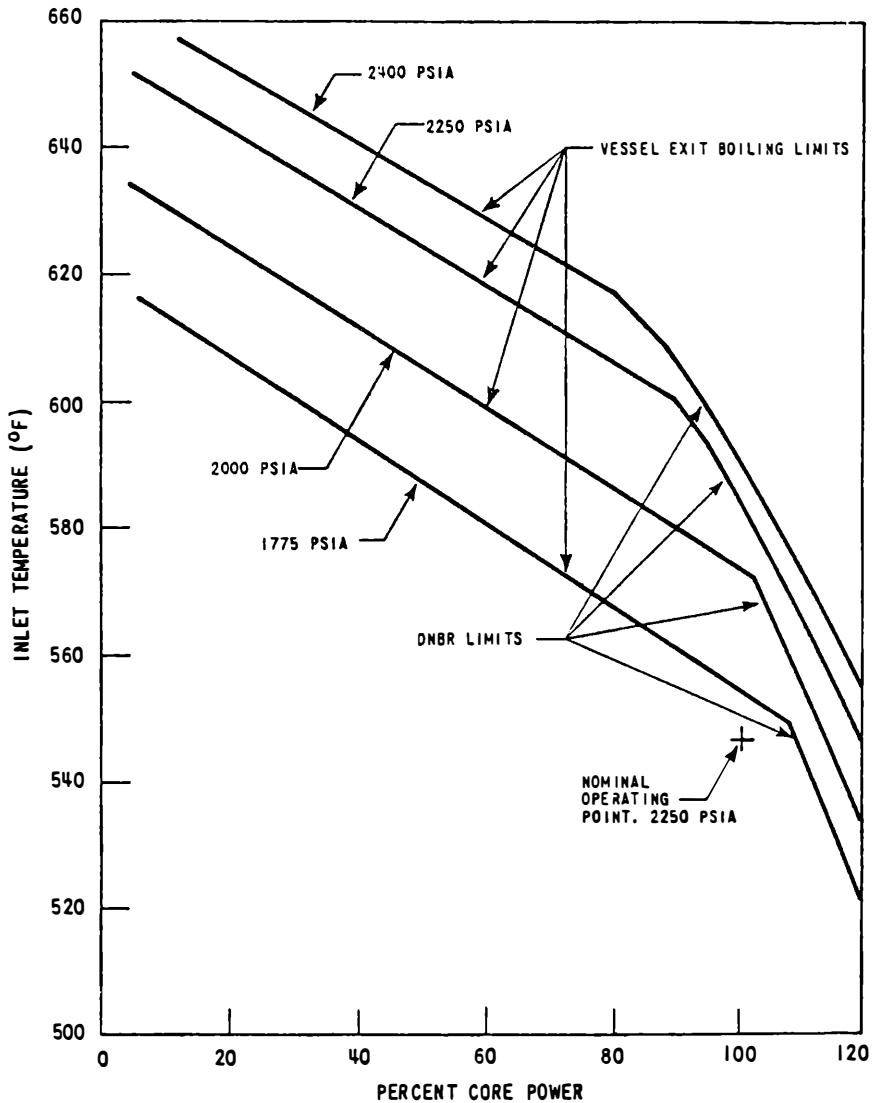


Fig. 5.8 Safety limits for typical three-loop plant. (Based on data from Ref. 53.)

in which most off-normal conditions result in a scram, the original VVER system allows four stages of response. These are full scram, fast rod insertion, slow rod insertion, and rod stop. For example, the loss of one reactor coolant pump would not lead to a reactor scram but to a decrease in reactor power by using a fast rod insertion. The VVER design leads to failure mode possibilities that are not present in PWRs following western designs.

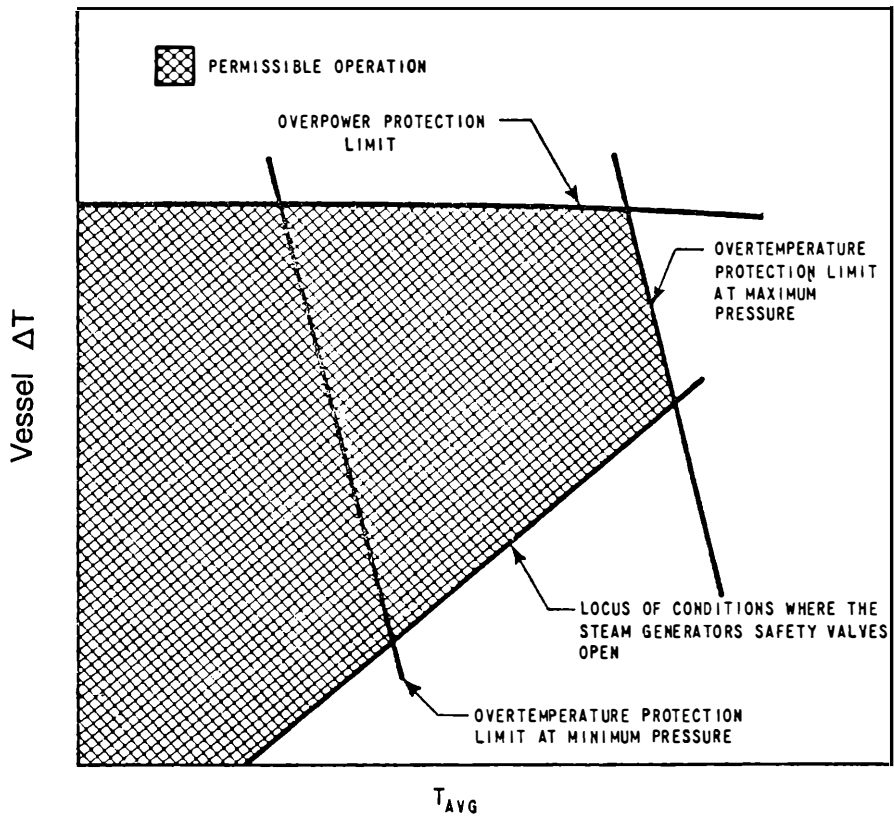


Fig. 5.9 Region of permissible operation as defined by core protection system. (Based on data from Ref. 53.)

5.6.4 Set-Point Analysis (Conventional PWRs)

The determination of the limiting conditions of operation (LCO) or the limiting safety setting (LSS) for parameters having a fixed set point and based on a single monitored variable is fairly simple. In general, the anticipated transients that involve the set point of interest are modeled and a set-point setting (analysis set point) that yields satisfactory results determined. The source and magnitude of the various uncertainty components affecting the monitored variable must then be ascertained. The actual set point is then obtained by biasing the analysis set point by an amount that accounts for the uncertainty and which allows for operating maneuverability.

The determination of LCO or LSS conditions required for thermal protection is more involved. These set points involve several system parameters and require that the monitored process variables be correlated to the reactor parameters affecting the design limit. This necessitates defining the

design limits in terms of monitored parameters and developing a process that will provide coefficients and/or correlations which conservatively relate the monitoring quantities to the design limit. The design processes used for establishing thermal protection set points vary appreciably with reactor vendor and whether digital or analog control is used. The following sections describe some of the approaches used.

5.6.4.1 *Analog Control Based on Thermal Overpower and Overtemperature Trips*

The thermal overpower and overtemperature trips are based on the DNBR and vessel exit boiling limit curves of Fig. 5.8, calculated by analog circuits. The overpower trip equation is given by⁵³

$$\Delta T_{\text{set point}} = C_1 - C_2 T_A - C_3 (T_A - T_R) - f(\Delta I) \quad (5.49)$$

where

$\Delta T_{\text{set point}}$ = overpower ΔT set point (percent of full power ΔT)

C_1 = a preset, manually adjustable bias (percent of full-power ΔT)

C_2 = a constant that compensates for piping and thermal time delays (percent of full-power $\Delta T/^\circ\text{F}$)

C_3 = a constant that accounts for the effects of coolant density and heat capacity on the relationship between ΔT and thermal power (percent of full-power $\Delta T/^\circ\text{F}$)

T_R = average reference reactor coolant temperature at full power ($^\circ\text{F}$)

T_A = average reactor coolant temperature ($^\circ\text{F}$)

$f(\Delta I)$ = a function of the neutron flux difference between upper and lower ion chamber sections (percent of full-power ΔT).

In indicating how the constants for Eq. (5.49) are derived, we assume, for the sake of simplicity, that the $f(\Delta I)$ term can be ignored. With this assumption, the set-point equation may be based on the limits of Fig. 5.8 replotted in terms of vessel temperature difference (ΔT) and the average reactor coolant temperature⁵³ [Fig. 5.10(a)]. The overpower temperature limit equation is then based on the intersection of the DNB limit lines and lines for each pressure representing a prescribed overpower limit (e.g., 118% nominal power). The intersection of the overpower lines and the DNB limit lines are shown by the circles in Fig. 5.10(a). The dashed line drawn through these circles represents the overpower trip line. From this line the constants C_1 and C_3 of Eq. (5.49) can be determined.

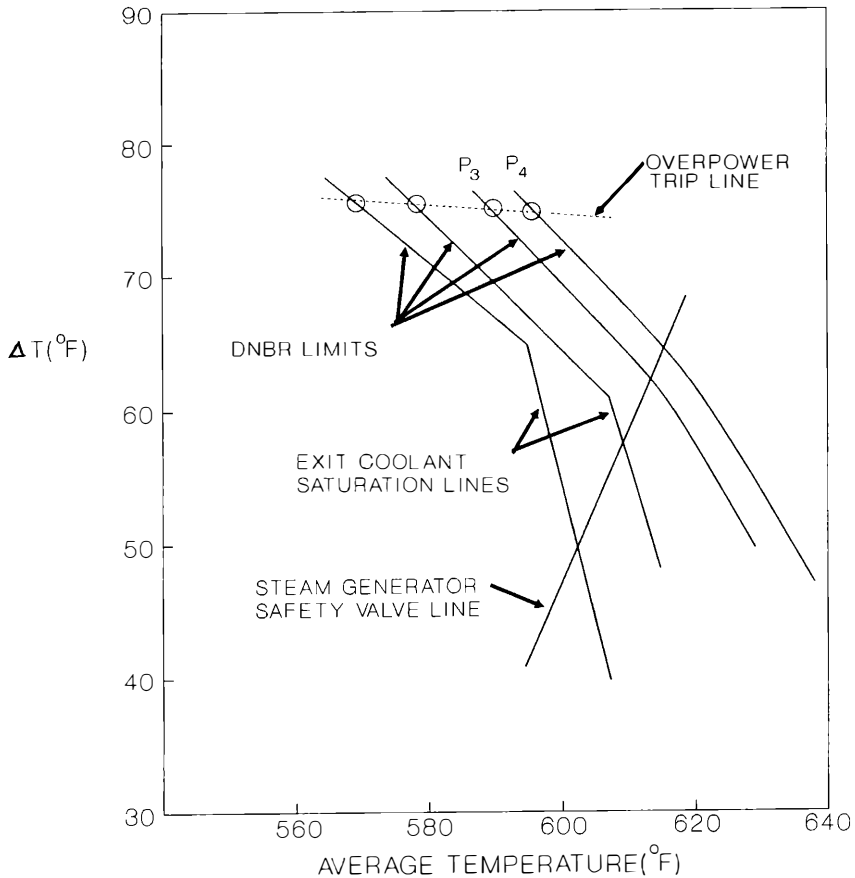


Fig. 5.10(a) Core limits used to determine overpower trip. (Based on data from Ref. 53.)

The region that must be protected by the overtemperature trip (or minimum DNBR trip) is bounded by the protection provided by the overtemperature trip and the steam generator safety valve. This region is bounded by points A, B, C, and D of Fig. 5.10(b) where⁵³:

Point A: intersection of allowable overpower line and DNBR limit at pressure of high-pressure trip

Point B: intersection of allowable overpower line and DNBR limit line at pressure of low-pressure trip

Point C: intersection of steam generator safety valve line and DNBR limit at pressure of high-pressure trip

Point D: intersection of steam generator safety valve line and DNBR limit at pressure of low-pressure trip.

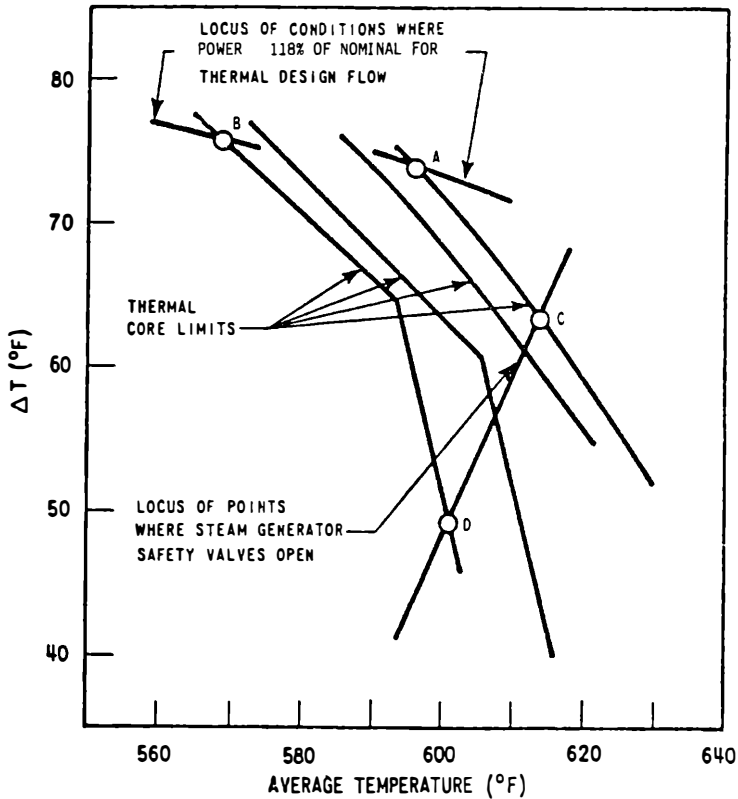


Fig. 5.10(b) Core thermal limits used to determine overtemperature (minimum DNB) trip. (Based on data from Ref. 53.)

These points are used to determine the constants in the overtemperature (DNBR) protection equation. The set point is continuously calculated by the analog circuitry according to the following equation⁵³:

$$\Delta T_{\text{set point}} = C_4 - C_5(T_A - T_R) + C_6(P - P_R) - f(\Delta I) \quad (5.50)$$

where

$\Delta T_{\text{set point}}$ = overtemperature ΔT set point (percent of full-power ΔT)

C_4 a preset, manually adjustable bias (percent of full-power ΔT)

C_5 = a constant based on the effect of temperature on the design limits (percent of full-power ΔT /°F)

C_6 a constant based on the effect of pressure on the design limits (percent of full-power ΔT /psi)

T_A = average reactor coolant temperature (°F)

T_R = average reference reactor coolant temperature at full power (°F)

P = pressurizer pressure (psia)

P_R = reference reactor coolant system pressure (psia)

$f(\Delta I)$ = a function of the neutron flux difference between upper and lower long ion chambers (percent of full-power ΔT).

If we again ignore $f(\Delta I)$, the equations for four separate overtemperature protection limit equations are determined from points A, B, C, D of Fig. 5.10(b) by requiring:

1. Slope parallel to AC, passing through A at P_4 and B at P_1
2. Slope parallel to AC, passing through A at P_4 and D at P_1
3. Slope parallel to BD, passing through B at P_1 and A at P_4
4. Slope parallel to BD, passing through B at P_1 and C at P_4 .

The four equations meeting the above criteria are determined and each is tested to ensure that all thermal limits are covered. The equation that provides the maximum operating margin while covering all limits is the one selected.

The foregoing would normally be carried out initially for an axially symmetric flux. The entire analysis would then be repeated for a number of skewed flux conditions in order to evaluate $f(\Delta I)$.

Note that the steady-state DNBR value used in establishing the limiting conditions must be set high enough to ensure that the true minimum allowable DNBR will not be exceeded during an anticipated transient. The analysis of the rod drop and loss-of-flow transients must therefore be performed prior to the set-point analysis.

LCOs required with the foregoing protection system are those governing control rod insertion. For constant axial offset control, the LCOs set power-dependent control rod insertion limits and the allowable axial offset. Rod insertion limits are set so as to assure an adequate shutdown margin, to assure adequate margin to meet $F_{\Delta H}$ requirements, and to limit the consequences of a rod ejection accident.

5.6.4.2 *Analog Control Based on Axial Offset, Minimum DNBR, and Variable Overpower Trips*

In contrast to the approach described in the previous section where the effect of nonsymmetric axial power distributions (effect of axial offset) is included with other thermal limits, this approach provides a reactor trip upon unacceptable axial flux distribution. This provides direct protection against unacceptable LHGRs.

The axial offset trip is based on the parameters P_L and S , which may be defined as⁵³:

$$P_L = 100 \left(\frac{\text{LHGR producing centerline melt}}{\text{Peak LHGR in core}} \right) \quad (5.51)$$

$$S = AS_E + B \quad (5.52)$$

where A = shape annealing factor, S_E = difference between upper and lower ex-core detector signals (% of total power), and B = base term. The value of P_L , which depends on core power, rod positions, burnup, and xenon distribution, is determined at a given power level for a large number of conditions. Each condition examined corresponds to an allowable operating condition and a particular (S, P_L) pair. All of the points determined are plotted on a graph whose coordinates are S and P_L . A bounding curve, which is below all of the calculated points, is then drawn. This bounding curve is then further reduced to allow for uncertainties in both power and axial offset measurement and by the increase in power peaking resulting from a control rod drop. The resulting curve must be consistent with the variable overpower trip. The entire process is then repeated for several different power levels. The S values for which $P_L = 100$ are then used to construct a trip function in terms of S and power level.

The DNB thermal margin limit (minimum DNBR) trip function has the form⁵³

$$P_{\text{trip}} = aQ_{\text{DNB}} + bT_c + c \quad (5.53)$$

where

T_c = reactor coolant inlet temperature

a, b, c = constants

$Q_{\text{DNB}} = (Q_A)(Q_R)$

Q_A = function of symmetric offset (function of S_E)

Q_R = power level function dependent on control rod position.

The Q_A function is based on the determination of the parameter P_D , which is defined as the power level, in percent of rated power, at which DNB would occur if all other parameters were kept constant. The calculation is carried out in a manner similar to that used to determine P_L . Each calculation now corresponds to an ordered pair (S, P_D) for many allowable operating conditions for a given control bank insertion. For each rod insertion examined, adverse conditions of flow and pressure are assumed and the DNB conditions are evaluated by an appropriate subchannel code. The procedure is repeated for other control bank insertions. An enveloping S versus P_D curve is then drawn for each control bank insertion examined.

The constants A , B , and C of Eq. (5.53) are generated from a reference set of DNB thermal limits at an assumed value of S_E and given rod bank insertion. The limit lines are generated at the maximum overpower that would occur when the reactor is nominally at full power.

The Q_A function is determined from the P_D limit line corresponding to the conditions used to set A , B , and C . The points along this line are divided into the maximum overpower margin, which occurs at the peak of the curve. The Q_R function is then obtained using the P_D versus S_E envelopes generated for a number of different control bank insertion limits at various power levels below full power. The functions are determined by trial and error and by requiring that the previously determined values of A , B , and C remain valid. The Q_R function thus corrects for the effect of control rod insertion and the increased radial peaking which this causes.

The computation used to determine P_L may also be used to evaluate the LCO set by the limiting LHGR required by LOCA analyses. One could define a P'_L based on the ratio of the LOCA limiting LHGR to the peak LHGR in the core. The P'_L versus S curve for this limit would be obtained in exactly the same manner as used for the P_L versus S curve. After appropriate allowance power and axial offset uncertainties, the final curve would provide the desired operating limit.

The operation limits on S_E derived above would also have to satisfy the requirement that the increase in LHGR that occurs in a dropped rod accident, plus appropriate uncertainties and margin allowance, would not violate the fuel melting limit (SAFDL). The more restrictive of the LOCA or drop limits would govern the allowable value of S specified by the LCO.

5.6.4.3 *Digital Control and Surveillance Set Points*

The newer use of a digital control and surveillance system has drastically altered the manner in which thermal set points are determined in reactor cores to which this system has been applied. A typical digital core protection calculator conducts on-line monitoring of control rod positions, core power level, axial flux distribution as determined by ex-core detectors, system pressure, and core inlet temperature.^{56,57} The radial and axial peaking factors are inferred from the control rod position measurements and axial offset from the ex-core detectors. The radial peaking factors may be based on a reference set of startup peaking factor measurements. The accuracy of the peaking factors being used can be periodically checked against movable flux wire measurements and appropriate adjustments made. The peaking factors and power level are used to determine the actual LHGR. After appropriate application of uncertainties, this is compared to the overpower trip limit (based on fuel melt or LOCA imposed limit). An LHGR exceeding the LSS actuates a trip.

In addition to comparing the linear power with the LSS, a digital control system will also calculate the DNBR, compare it with the design limit, and actuate a reactor trip if required. A surveillance display is also usually provided. The DNBR computation is based on the estimated axial and radial peaking factors, and measured inlet temperature, pressure, and flow. Since a close estimate of the actual peaking factors is used, the digital system

removes the conservatism added by the use of upper envelope peaking factors required in analog protection systems.

Values of the DNB ratio are computed on-line using a simplified single-pass subchannel analysis. In the simplest arrangement used, the hot channel and core are represented by only four channels. In this approach, the location of the limiting assembly is preselected off line and conservative values are assigned to such parameters as relative hot assembly to adjacent assembly power distribution and core inlet flow.

The computed DNBR value is compared to a DNB limit, which is determined by adding margin and transient effects to a statistical combination of uncertainties.⁵⁶ These uncertainties are (1) modeling and DNB correlation uncertainties, (2) state parameter uncertainties (uncertainties in measured parameters), and (3) system parameter uncertainties (unmeasured parameters such as fuel rod dimensions).

In reactor systems containing stationary in-core detectors, a separate digital computer system is provided to determine the margin to operating limits (LCO). The system is similar to the core protection system just described except that the core power distribution is synthesized from the in-core detector signals (instead of ex-core detector signals) and radial peaking factors based on rod positions. Both the linear heat generation rate and DNBR are computed on-line and provided in a surveillance display.

The DNBR computations are, as before, based on a single-pass subchannel analysis. In some systems, up to 19 channels have been used to model the core. In these systems, the location of the hot assembly is not preselected, but is instead determined by the computer.⁵⁶ Based on the core power distribution, the quadrant with the highest total power is selected. Within this quadrant, pairs of neighboring assemblies are compared by defining the quantity $(\Delta \text{DNBR})_{ij}$ for assemblies i and j as:

$$\Delta \text{DNBR}_{ij} = a(\%P_{\text{HOT}}) + b(D_p) + c(\%M) + d(D_m) \quad (5.54)$$

where

$$\%P_{\text{HOT}} = \frac{(P_1)_i - (P_1)_j}{\left[\frac{(P_1)_i + (P_1)_j}{2} \right]} \times 100 \quad (5.55)$$

$$D_p = \frac{(P_1)_i - (P_1)_j}{(P_1)_i} - \frac{(P_1)_j - (P_1)_i}{(P_1)_j} \quad (5.56)$$

$$\%M = \left\{ \frac{(M)_i - (M)_j}{\left[\frac{(M)_i + (M)_j}{2} \right]} \right\} \times 100, \quad (5.57)$$

$$D_M = \frac{(M_1)_i - (M)_j}{(M_1)_i} - \frac{(M_1)_j - (M)_j}{(M_1)_j} \quad (5.58)$$

and

$(P)_i$ = average radial peaking factor of five assemblies adjacent to and including assembly i

$(P_1)_i$ = radial peaking factor for assembly i

$(M)_i$ = average inlet flow rate of five assemblies adjacent to and including assembly i

$(M_1)_i$ = inlet flow rate for assembly i

a, b, c, d constants.

If $\Delta \text{DNBR} > 0$, assembly j is limiting. If $\Delta \text{DNBR} < 0$, i is the limiting assembly. The process is repeated by comparing each assembly to the previously limiting unit. The constants a , b , c , and d are determined by means of a large number of off-line full-core analyses. The operating margin to the LCO for DNB is significantly improved by the on-line approach since the conservative assumptions required concerning peaking factors are largely eliminated.

A separate digitally controlled surveillance system may also be provided in reactors that utilize core exit thermocouples to monitor the radial power distribution. In one such approach,⁵⁸ a very rapid advanced nodal program⁵⁹ provides a 3-D simulation of the core every 15 min or when plant conditions change. The measured coolant exit temperatures and axial flux distribution are used by the nodal program to update a reference core power distribution. (The reference core power distribution is obtained from movable in-core detector system during a monthly flux map.) The 3-D flux distribution from the nodal code provides a set of peaking factors, which are used in conjunction with the on-line monitoring of system parameters (flow, pressure rod positions, inlet temperature, axial flux, assembly exit temperature) to provide on-line computations of the DNBR and LHGR, which are compared to LCO. Note that the use of an advanced nodal code for determining the core power distributions results in more operating margin than one based on the use of preselected coefficients dependent on rod position.⁵⁸

If a digitally controlled thermal protection or surveillance system must be taken off line, then the plant is restricted to a lower power level, which is consistent with conservative estimates of the most adverse operating conditions.

5.6.5 Thermal Protection in Pressure Tube Reactors

Although an overpower protection is provided by a trip initiated by ex-core ion chambers, the primary thermal protection in CANDU pressure

tube reactors is provided by the regional overpower protection (ROP) system.⁴ The system, which uses self-powered in-core neutron flux detectors, consists of two sets of detectors. One set (vertically placed) activates the control rod shutdown system. The second set (horizontally placed) usually actuates a poison (gadolinium nitrate) injection shutdown system. A typical core layout of the detectors actuating the control rods would contain 34 detectors in 16 vertical detector assemblies. The detectors for a given shutdown system are grouped into three logic channels. If detectors in any two of the three logic channels reach their set point, the reactor is tripped.

In the usual design, a margin of about 20% separates the maximum channel power (7.3 MW in CANDU 600s) at which trip probability is required to be low from the dryout power (~ 8.7 MW) at which trip probability must be very high.¹⁴ By setting the ROP detectors so that they trip at 10% above the normal maximum flux, an operating margin of 10% above the set point is obtained.

The regional nature of the ROP trip takes into account the effect of control rod insertion on peak power. However, since the trip is based only in power measurements, it is necessary to assume the most adverse values of assembly flow, inlet temperature, pressure, and pressure tube dimensions in calculating CHF power.

5.6.6 Alternative DNB Design Limit

As indicated previously, current (1994) practice sets one of the safe acceptable design limits (SAFDL) as a DNBR of 1.0. However, there is evidence that brief (15-sec) post-DNB operation would not cause fuel damage up to a maximum temperature of about 870 K.⁶⁰ Somewhat similar observations are reported by Snoek,⁶¹ who operated simulated fuel elements at power levels that were about 10% above the CHF. Snoek observed that no dry patch spread completely around any element and that the sheath temperature increase was modest (less than 200°C). He concluded that no fuel damage would result from such operation.

If brief post-DNB operation is allowed during anticipated transients, a power margin gain of the order of 5 to 12% is possible. Replacing the DNB limit by a mechanistic fuel failure criterion is allowed by the U.S. Regulatory Commission as an option.⁶² However, in view of the limited gains, the difficulty in justifying this approach has deterred use of a revised criterion by the designers and operators of vessel-type PWRs. Use of a fuel-failure-based criterion has received more favorable consideration from designers of pressure tube reactors.

At CANDU reactor conditions, the CHF power corresponds to what has been described as slow dryout.⁴ There is no immediate excursion into film boiling but a gradual rise in temperature as power is increased. The initial CHF condition is sometimes called the *onset of intermittent dryout* (OID),

while the film boiling condition has been described as *onset of dry sheath* (ODS). The ODS power may be up to 5% above the OID power. Acceptance of thermal limits based on ODS power would therefore allow greater operating power levels. If the ODS limit were adopted, the allowable bundle power would be computed solely from a dryout correlation based on the boiling length hypothesis, and correlations based on OID conditions [e.g., Eq. (4.217)] would be abandoned.

5.7 ADVANCED PWRs

5.7.1 Design Objectives and Specific Goals

Advanced PWR designs fall into two categories: (1) reactor core designs directed toward improving fuel utilization and conversion and (2) reactor core and system designs directed toward improvements in reliability and safety. In the former category we find tight lattice cores producing high conversion ratios (see Sec. 1.2.2.3) as well as more conventional cores designed to limit the use of chemical shim control. However, most current interest is directed toward the advanced plants having broad system changes leading to simpler and more reliable units. The specific objectives of such designs are⁶³ as follows:

1. Provide a plant with greatly simplified systems, equipment, operation and maintenance.
2. Provide increased reliability by reducing the number of and simplifying plant systems.
3. Provide a higher degree of public safety and licensing confidence.
4. Reduce cost of nuclear power so that it is fully competitive with non-nuclear sources.
5. Provide a short construction schedule.
6. Minimize environmental effects.
7. Provide for increased plant lifetime.

To develop plants that meet the desired objective, Tong⁶⁴ has suggested a set of basic principles that should be applied to the design and operation of such plants. He suggests the following:

- Priority should be given to the safety significant (or the risk-reduction) items in the implementation of reactor safety improvement.
- Feedback of operating experience, including the requirements of simplicity, design margin, and ease of operation and maintenance, should be incorporated into the design of new reactors.
- Probabilistic risk assessments (PRAs) should not be limited to only the worst individual case envelope but should also include various sets of

probable multiple failures with high risks, even if the individual effects of each single failure are of a minor nature.

- Standardization should be maintained in the functions, performance, and basic specifications of the systems and components affecting safety.
- A “living” PRA program should be used in safety evaluations.

Tong believes that a “living” PRA is needed because to be useful the PRA must be updated to reflect actual plant conditions. For example, should a component or a system not function properly, the original failure probabilities and the risk of the plant need to be reevaluated. To maintain the recalculated plant risk below the tolerable value in the new safety evaluation, it may be necessary to implement a temporary safety improvement, such as operating with a special caution, activating a backup safety system, or reducing the operating power. Alternately, long-term changes may be needed, such as an equipment or system modification or replacement. The PRA must be amenable to the repeated adjustments and uses for supporting timely decision-making. Safety improvements could be confidently identified by using such a “living” PRA.*

To meet the general design objectives of simpler and more reliable plants, a set of specific design goals has been developed.⁶⁵ The major design goals are tabulated in Table 5.IV. Features of some of the plant concepts designed to achieve these goals have been indicated in Sec. 1.1.3.

5.7.2 Core Modifications in Advanced Design Reactors

Tight lattice cores that produce high conversion ratios and use plutonium as the fissile material remain of interest in countries where fuel reprocessing is available. However, the behavior of such high-pressure-drop cores during accidents, such as a large LOCA, remains (1995) a concern. The passive safety advanced designs are largely based on fuel assemblies similar to the advanced fuel assembly designs for presently (1995) operating reactors (see Sec. 2.1.2.3). Nevertheless, the advanced reactor design goals lead to some differences in core design.

Table 5.V indicates the typical progression of reactor designs from the late 1950s to the large plant designs of the mid-1970s, which are typical of many currently (1995) operating reactors. The general increase in total thermal power density is readily observed. In contrast to this, it will be seen that the advanced reactor design (AP-600) reverses this trend. Significantly lower peak heat fluxes, linear power density, core power density, and fuel power densities are used. The lower peak linear power and heat flux mean that there is considerably more margin between operating conditions and the DNBR and fuel melting design limits. The probability of an abnormal

*See Sec. 6.8.8 for a description of probabilistic risk assessment (PRA).

TABLE 5.IV
Design Goals for Passive Advanced LWR Exhibiting Improved Reliability and Safety*

General Design Requirements	
Safety system concept	Simplified safety system concepts: primarily passive systems; safety grade, ac power generators shall not be required
Plant design life	60 years
Design philosophy	Simple, rugged, high design margin, based on proven technology
Plant siting envelope	Must be acceptable for most available sites; 0.3g; shutdown earthquake
Safety and Investment Protection	
Accident resistance	Design features that limit the likelihood and effect of initiating events, such as: Fuel thermal margin $\geq 15\%$ Adequate time to respond to plant upset conditions through features such as reduced core power density and increased coolant inventory Use of best available materials
Core damage prevention	
Core damage frequency	$< 10^{-5}$ per reactor year to be demonstrated by PRA
LOCA protection	No core uncover for up to a 6-in. beak
Station blackout coping time for core cooling	8 hours
Mitigation	
Severe accident frequency and consequence	Demonstrate by PRA that the cumulative frequency $< 10^{-5}$ per reactor year for sequences resulting in > 25 rem at 0.5 miles from reactor

*From Ref. 65.

continued

event resulting in fuel damage is therefore greatly reduced. This change responds to the design goals of increased thermal margin and adequate time to respond to plant upset conditions.

The lower core power density also aids in meeting the goal of a 60-year plant life. One of the factors limiting plant lifetime is the fast neutron embrittlement of the reactor vessel (see Sec. 5.2.3.5). Fast neutron (energy > 1 MeV) irradiation appreciably raises the nil ductility temperature (temperature below which a vessel failure would be brittle rather than ductile). Current (1995) U.S. regulations (Code of Federal Regulations 10 CFR 50.61) require that the nil ductility temperature (NDT) be above 270°F for plates and axial welds and above 300°F for any circumferential welds in the core beltline. The plate NDT limit is exceeded after about a fast fluence of about 8×10^{18} n/cm² (Ref. 66). Currently operating (1995) reactor vessels reach this limit in about 25 to 35 years. Since the increase in NDT varies approx-

TABLE 5.IV (continued)

Containment	Large, rugged containment building with design pressure based on licensing design basis pipe break
Radiological source term	More realistic source term for licensing
Hydrogen generation	Licensing design basis hydrogen concentration less than 13% in containment for 75% active clad oxidation.
Emergency planning zone	Technical basis for reduction of zone.
Additional passive plant requirements	No core uncover for up to 72 hours without operator action for licensing design basis events and station blackout conditions No soluble boron required Maintain containment integrity for at least 72 hours in the event of a severe accident without the use of portable equipment.
Licensability	
Regulatory stabilization	Stable regulatory basis for ALWR, including: Meet NRC regulatory requirements NRC safety issues resolved Optimize regulatory requirements for ALWR
Severe accidents	Meet NRC severe accident policy statement; treat severe accidents separately from licensing design basis
Performance	
Design availability	87%
Refueling interval	24-month capability
Unplanned automatic scrams	<1/year
Maneuvering	Daily load follow
Load rejection	Loss of load without reactor trip

imately with the fluence to the 0.27 power,⁶⁶ increasing lifetime from 35 to 60 years requires a flux reduction by about a factor of 7.5. The required fast neutron flux reduction is readily accomplished in designs such as the SIR where there is a substantial distance between the core and the vessel wall. The required flux reduction presents more difficulty in designs such as the AP-600 where the vessel wall remains fairly close to the core.

In designs like the AP-600, the lower power density used will lead to some reduction of the fast neutron flux at the vessel wall. The flux reduction will be aided by the very-low-leakage refueling schemes expected to be used with such cores. An additional reduction in vessel flux is achieved by the use of a reflector, consisting of zirconium oxide pellets in zirconium alloy cladding, in the region between the core baffle and the core barrel. The reflector also helps to flatten the power distribution in the core. A further vessel-wall flux reduction is obtained by providing additional space

TABLE 5.V
Thermal Parameters of Typical Power Core Designs

Year of Design	58	64	62	64	66	67	68	74	92
Reactor	Yankee Rowe	Conn. Yankee	Selni	San Onofre	IPP-II	Salem No. 1	TVA Sequoyah	South Texas	AP 600
Thermal power, MWt	392	600	825	1346	2758	3338	3411	3800	1993
Coolant inlet temperature, °F	496	506	512	553	543	544	547	560	530
Avg. Reactor ΔT , °F	30	41.6	56.5	44.2	53	64.7	63.0	66.8	65
Avg. heat flux, 10^3 Btu/hr ft ²	86	128	130	143	175	186	190	185	143
Max. linear power, kW/ft	6.5	9.9	12.4	15.0	18.5	12.4	12.6	13.3	10.7
Core Power Density, kW/liter	58	89	64	72	85	102	103.8	97	78.8
Fuel power density, kW/kg of uranium	19	28.7	21	23.4	31.5	37.7	38.5	41.5	28.9

between the core and vessel wall. Alternatively, axially zoned burnable poison rods can be placed in peripheral assemblies so as to reduce preferentially the peak vessel flux. The combination of all of these effects can lead to the required 60-year lifetime.

In addition to the reduction in core power density, the core inlet temperature has been reduced. This is in accordance with the desire to keep the core exit temperature of advanced PWRs below 600°F (Ref. 65). The reduced primary coolant temperature leads to a less corrosive environment, which is particularly important in reducing steam generator tube failures. The AP-600 design thus trades a slightly reduced thermal efficiency for increased component reliability.

Elimination of boric acid chemical shim control requires the increased use of fuel rods containing burnable poison. Additional compensation for the reduction in reactivity with core burnup can be accomplished using water displacement rods. Some control rod spiders can be used for the gradual removal of control elements containing zirconium oxide pellets from the core. As the ZrO_2 is removed from the core, water fills the vacant slot. This increases the neutron moderation and increases core reactivity. To provide a significant reactivity change, one control spider could control displacement elements in four fuel assemblies. The water displacement rods would be removed from the core near the end of the cycle when the burnable poison has been mostly depleted. Such a scheme can help reach the desired extension in the period between refueling. Elimination of chemical shim control will also lead to a more positive moderator density coefficient and hence to a smaller power increase during some transients.

The thermal analysis approaches described in this chapter are expected to be generally adequate for design of those advanced reactor systems (e.g., AP-600) having a configuration not greatly different from present (1995) operating reactors. Revised techniques will be needed for designs (e.g., PIUS) having configurations that vary substantially from those of present PWRs.

5.7.3 Primary System Component Modifications

The component modifications discussed in this section are those applicable to the evolutionary technology reactors, such as the AP-600, which retain the external loop design characteristic of currently (1995) operating plants. Most of the system components for these designs are generally similar to those of earlier operating plants.

A significant variation from current practice occurs in the sizing of the pressurizer. As will be seen in Chapter 6, several of the transients considered to be anticipated operating occurrences result in pressure increases that cause power-operated relief valves (PORV) and safety valves to open. In most of the evolutionary plant designs, appreciable larger pressurizer

volumes are used (approximately 33% increase in volume in some designs⁶⁷). The increased volume results in substantially reduced pressure surges during transients. This can lead to the elimination of PORVs and a greatly reduced likelihood that safety valves will open. The probability that a sticking valve will produce a small LOCA is therefore also reduced.

Most of the system designs in this category use improved reactor coolant pumps. In some cases, centrifugal pumps with improved seals are to be used. Alternatively, the mechanical seal pumps are replaced by canned motor pumps. Because seals are eliminated, this removes the possibility of a small LOCA following a station blackout (see Sec. 1.1.1.4). Because canned motor pumps generally have a fairly low inertia, a heavy metal disk (e.g., clad uranium) would be attached to the rotor. This supplies the increased inertia needed during a loss-of-flow accident.

Both currently operating (1995) reactors and the advanced designs use large accumulators to inject water during the blowdown period of a large LOCA. In some advanced designs a fluidic flow control device is used to control the flow from the accumulator.⁶⁷ When the accumulator is full, water flows through a standpipe and bypasses the control device. When the accumulator level drops below the height of the standpipe, the exiting fluid proceeds through the control device. The liquid then enters through a side connection that sets up a vortex, which restricts the flow. This allows a high initial flow followed by a prolonged period of low flow. This is particularly useful in a small LOCA.

Whereas the AP-600 uses vertical steam generators of a design similar to the improved models used for replacement of steam generators in current (1995) plants, horizontal units are used in other designs. As noted in Sec. 1.1.1.3, the horizontal units are believed to have reduced sludge build-up on tube plates and greater resistance to seismic events. In both vertical and horizontal units, an increased secondary inventory is provided to slow transient response and provide additional margin to trip set points.⁶⁸

Note that, in addition to the changes in the major components described in this section, there are a number of changes and additions to the safety system in advanced designs. These typically would include passive containment cooling, a passive residual heat removal system, an automatic depressurization system, and a passive containment spray system.

5.8 FUTURE DESIGN DIRECTIONS

It is obvious that design procedures will continue to become more sophisticated as the knowledge and experience base expands. Analyses may be expected to become increasingly realistic and to include additional applications of statistical methods. The increased analytical realism will almost certainly be supported by expanded use of digital control and monitoring systems.

Emphasis on increased reliability will probably be the major focus of future design efforts. This will no doubt result in increased design margins, which enhance operational flexibility and further reduce the challenges to the safety systems.

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CHAPTER SIX

Safety Analysis

The importance of reactor safety analysis has led to a major effort in this area with the consequent development of very extensive subject material. Thorough coverage of this material is not possible in a single chapter and therefore only an overview will be presented. Attention will be focused on the thermal-hydraulic aspects of off-normal behavior.

Since the major portion of the thermal-hydraulic design effort is devoted to those events which must be analyzed for plant licensing, these accidents are emphasized. The behavior during more severe accidents, in which the original core configuration is not maintained, remains uncertain. Predictions are highly dependent on the assumptions made. However, the general problems relating to these more severe accidents, and means for reducing the frequency of such accidents, are briefly examined.

6.1 SAFETY APPROACHES

6.1.1 Safety Philosophy

To prevent radioactivity releases to the environment, the nuclear industry has adopted the defense-in-depth approach. This approach consists of (1) careful design, construction, and operation of nuclear reactors and related facilities so that malfunctions that can cause major accidents are unlikely; (2) incorporation of systems within the plant that prevent such malfunctions as may occur from becoming major accidents; and (3) incorporation of plant features (e.g., vapor container, emergency cooling) that limit the consequences of major accidents.

To determine the effectiveness of the second and third of the aforementioned lines of defense, a series of analyses is conducted. A set of design-basis accidents is postulated and the accident consequences are modeled using a computer code that contains the necessary reactor and system models.

Each design-basis accident is determined by a specified failure, or limited set of failures, and conservatively specified initial conditions. With the exception of the limited initial set of failures, all control and safety systems are assumed to operate as designed. Since the system behavior follows a prescribed path, the analyses are described as "deterministic." A reactor system design is licensable if these analyses show that the established design criteria are met. Most of the present (1995) safety analysis efforts are devoted to these studies.

In contrast to the deterministic analyses required for plant licensing, probabilistic analyses may be used to determine the likelihood of an accident that results in core damage or release of radioactivity to the environment. Failure of each system is assumed to have some probability. These techniques, generally described as probabilistic risk assessment (PRA), are receiving increasing attention (see Sec. 6.8.8). Future licensing regulations may require that such analyses show that the probability of a significant radioactivity release would be below some specified low limit.

6.1.2 Safety Criteria

In setting the safety limits and maximum safety settings described in Sec. 5.5.1, it is assumed that the transient causing a departure from normal reactor conditions is such that proper action of the protection system will prevent any core damage. While this is possible for the most probable events, it is not possible to prevent some core damage in less likely, but more severe, accidents.

The basic principle of safety system design is that the potential magnitude of injury or damage should be considerably less for accidents that are likely than for those with a low probability of occurrence. It is desirable to define sets of departures from normal operation that differ from set to set in the probability of their occurrence.¹

Category I includes conditions that occur frequently or regularly in the course of normal operation, refueling, or maintenance; e.g., isolated rupture of a fuel element, load changes, approach to criticality, etc. It is required that the plant be designed such that the full range of these conditions can be handled without shutting the plant down manually or automatically.

Category II comprises incidents of moderate frequency. This set includes all faults that are not expected during normal plant operation, but reasonably can be expected during the life of a particular plant. This category

consists of the anticipated operating occurrences (AOOs) delineated in Chapter 5 (see Table 5.II). Plant design must ensure that no such fault can cause others that are more serious or cause fuel damage in excess of that which is normal in a rapid reactor shutdown. As indicated in Sec. 5.6.1, the events in this set correspond to those for which maximum safety settings prevent safety limits from being reached. This is consistent with the requirements of the U.S. Code of Federal Regulations. Since both category I and II events are expected, they are usually called "transients" rather than "accidents."

A third category consists of infrequent events. Any one of these events might occur during the lifetime of a given plant. Such accidents as a minor loss of primary coolant, minor secondary steam break, or inadvertent boric-acid dilution fall into this category. For such conditions, it is desired that the fault create no necessity for interruption or restriction of public use of the areas in the plant vicinity to which the public normally has access.

The fourth category consists of what are sometimes termed "limiting faults." These are accidents which are not expected during the life of a given plant but whose consequences include the potential for serious public damage or injury. Thus, despite their low probability, safeguards must be provided. In this category, the thermal designer would have to consider a major rupture of the primary or secondary system, ejection of the maximum worth control rod, or the consequences of movement of the fuel or structure. For these drastic conditions, the possibility of significant plant damage is accepted, but it is required that in the event of such a fault, conservative calculations of the off-site radioactivity dosage indicate that the limitations of the Code of Federal Regulations,² Sec. 10CFR100 would not be exceeded. This requirement is stipulated for major accidents in Ref. 3.

The major design-basis accident situations that the thermal designer considered for the safeguards analysis of most PWRs now operating (1995) were (1) loss-of-flow accident (LOFA) due to pump power loss, (2) reactivity insertion, (3) rod drop accident, (4) steam-line break accident due to a rupture of the secondary system, and (5) loss-of-coolant accident (LOCA) due to a rupture of the primary system, which can include the rupture of a control rod housing and subsequent control assembly ejection. The first three of these accidents are considered AOOs and therefore they are category II situations. A small size steam-line break is considered an infrequent event and would be treated as a category III event. A LOCA due to a small size piping rupture or small valve failure would also be considered an infrequent event and placed in the third category. A LOCA due to a large size break or a break with a rod ejection is clearly a limiting fault and a category IV incident. Other limiting faults include a large secondary system break, steam generator tube rupture, and reactor coolant shaft seizure or free wheeling pump rotor.

In addition to the foregoing, the thermal design is now required to consider the various AOOs in conjunction with the assumption that the reactor fails to trip. These incidents are referred to as anticipated transients without scram (ATWS). Although such an event appears highly unlikely, the current regulatory approach considers these as if they were category III events.

An additional accident category that now also receives consideration is that consisting of events which can lead to severe accidents (a severe accident is one in which a substantial portion of the core is heavily damaged). Such accidents may result from a category III or IV* event followed by (1) failure of another significant component, or (2) failure of a protective system, or (3) operator error that negates or terminates the action of the protective systems. A brittle failure of the reactor vessel is another, although highly unlikely, cause of a severe accident. While no definitive standards exist for the limits of such events, systems analyses are directed toward the development of operational procedures that will minimize the consequences of the accident. These studies are also directed toward the establishment of accident "signatures" that will allow the operator to diagnose the problem rapidly.

6.2 ACCIDENT EVALUATION APPROACHES

6.2.1 Thermal-Hydraulics Approaches

As indicated in Sec. 6.1.1, the analyses of all PWR transients and accidents are based on the use of an appropriate digital computer program that represents the entire reactor system. This would include the primary system, secondary system, control and protection systems, and auxiliary and safety systems. Where required by the accident, appropriate containment conditions are also specified. The primary system is represented by a series of linearly interconnected volumes. The time variation in system behavior is then determined by solution of the energy, mass, and momentum conservation equations for the reactor coolant in each of the volumes. The primary system is acted on by external events imposed as program input and interacts with the models representing secondary, control and auxiliary systems. The plant features, which must be incorporated for a full systems analysis, are tabulated in Table 6.I.

Essentially, all of the currently used accident analysis programs model the primary coolant system using the finite difference approach described in Sec. 4.1.2.4. The earliest computer codes for primary systems analysis were of the node and branch type. However, while the recent versions of

*The accident categories of this section differ from the official U.S. NRC classification. In the NRC system, classes 1 through 7 deal with radioactivity releases and fuel accidents, class 8 includes all design-basis accidents, and severe accidents are class 9.

TABLE 6.1
Plant Features Required by Systems Analysis Programs*

Reactor protection system—trip set points and delays
Feedwater flow control program (pressure and level)
Engineered safety features—actions and set points
Steam dump and bypass system—flow control program
Main steam line and feedwater isolation systems—set points and delays
Primary and secondary safety and relief valves—set points and flows
Boron injection system—concentration and flow
Pressurizer protection system—trip, set points, and delays

*Table based on material presented by M. Kennedy, G. Peeler, and P. Abramson in "PWR Systems Transient Analyses" as part of *Guidebook to Light Water Reactor Safety Analysis*, P. Abramson, ed., Hemisphere Publishing Company, Washington, DC (1986).

the FLASH code⁴ remain node and branch, the other major analysis programs (e.g., RELAP,⁵ RETRAN,⁶ TRAC,⁷ CATHARE⁸) all use the control volume approach (see Sec. 4.1.2.4). Two-phase flow modeling in the earlier programs used the single-fluid approach. Later programs have tended to use the dynamic slip or two-fluid model (see Sec. 3.3.4).

Since the systems analysis codes are usually very long running, most attempt to limit computer time by using a relatively simple model of the reactor core. In some cases, the core will be modeled by only one channel representing average core conditions. In other cases, an average channel and the hot channel will be used for representing core behavior. When this is done, the output of the systems analysis program may be used as input to a subchannel analysis code. The subchannel analysis provides a more detailed and accurate evaluation of the approach to limiting conditions. Exceptions to the requirement for sequential use of a subchannel analysis code are systems analyses conducted with those few systems programs that contain a subchannel code as one of their segments (namely, COBRA TRAC program⁹).

Computer programs designed specifically for licensing analysis of AOOs are often considerably simpler than those designed for the full range of possible accidents. A marked reduction in running time can be achieved by decoupling the transient momentum equation and only solving the continuity and energy conservation equations simultaneously. A quasi steady-state version of the momentum equation is then used to determine the flows for the next timestep. This approach is satisfactory for most AOOs since very rapid flow and pressure variations are not encountered. The IRT code¹⁰ is an example of a generally available program of this type. A rapid and reasonably accurate program of this nature can also be very useful in conducting parametric analyses.

Codes that deal with category III or IV events may be subdivided into evaluation models and best estimate models. The evaluation models are

those programs or program versions that (1) are used for the analyses required by regulatory bodies and (2) make conservative assumptions with respect to all the physical phenomena involved. A best estimate model is one that is designed to provide the most realistic estimate of all of the phenomena involved and has usually been adjusted to provide agreement with experiments. While best estimate codes have been most frequently used as an aid in understanding system behavior, they can be used as a basis for regulatory submittals with a proper allowance for the uncertainty in their predictions and (1) conservative modeling of the reactor protection system and (2) conservative selection of input parameters.

The computer programs normally used for analysis of accidents in categories II through IV assume that a coolable reactor core geometry is maintained. Separate analysis programs (e.g., MAAP,¹¹ MELCOR¹²) are usually required for the analysis of severe accidents resulting in a degraded core. However, the degraded core analysis can be linked to a systems analysis code to provide an integrated view of the entire accident.¹³

6.2.2 Neutronic Modeling

The effects of reactivity changes must be considered in a number of transients and accidents. Since most system codes model the reactor core in a very simplified way, such codes cannot address the spatial changes in reactivity. A simple point kinetics model is therefore usually used. Such a model¹⁴ solves the equations

$$\begin{aligned}\frac{dN}{dt} &= \left(\frac{\delta k - \bar{\beta}}{l^*} \right) N + \frac{1}{l^*} \sum_{d=1}^7 \bar{\beta}_d C_d \\ \frac{dC_d}{dt} &= \lambda_d (N - C_d)\end{aligned}\quad (6.1)$$

where

N = neutron density

$\bar{\beta}_d$ = effective neutron fraction, delayed neutron group d

$$\bar{\beta} = \sum_{d=1}^7 \bar{\beta}_d$$

C_d = precursor concentration, delayed group d

l^* = prompt neutron lifetime

λ_d = decay constant, delayed neutron group d ,

and where δk is defined by

$$\delta k = \delta k^0 + \delta k(t) + \delta k_{TC} + \delta k_p + \delta k_{exp} + \delta k_{Doppler} \quad (6.2)$$

The reactivity forcing functions are δk^0 , an initial step insertion, and $\delta k(t)$ a time-varying reactivity insertion. The feedback components are due to water temperature change (δk_{TC}), water density change (δk_p), fuel expansion (δk_{exp}), and fuel temperature change ($\delta k_{Doppler}$). The effect of fuel temperature variation is designated $\delta k_{Doppler}$ since this reactivity change is due to the Doppler broadening of the ^{238}U neutron absorption resonance peaks with increasing fuel temperature.

In using the point kinetics model, the total reactor power is assumed proportional to neutron density, N . The heat generation rate is assumed to be a separable function of space and time, with the spatial relationship preassigned on the basis of steady-state values. The Doppler effect in Eq. (6.2) is also generally based on preaccident conditions. Since this ignores any local flux peaking, the effect would be conservatively underestimated.¹⁵

Local power is computed by multiplying the local power at steady state by the ratio of the current value of N to the steady-state value of N . Once a scram has occurred, the heat release is determined from the stored energy released by the fuel (see Sec. 2.6) plus the decay heat and delayed neutron fission power (see Sec. 1.5.2).

Since the core average channel is represented in any systems analysis, it is possible to use a one-dimensional (axial) kinetics model instead of point kinetics.¹⁶ If this is done, the flux is again factored into an amplitude and shape function. The rapidly varying amplitude function is determined from the overall core reactivity [Eq. (6.1)]. The more slowly varying shape function is based on the solution of a modified 1-D neutron diffusion equation, which accounts for the effect of local reactivity (including void, Doppler, and control rod effects) and local delayed neutron precursor concentrations. It is thus possible to avoid a preassigned spatial relationship in the axial direction.

More accurate modeling of reactivity effects is possible when nuclear effects are modeled in conjunction with a subchannel analysis program. Such an analysis allows the spatial power shifts that occur during an accident to be considered in the thermal-hydraulic modeling. As noted in Sec. 5.3.4.4, the availability of rapid modern nodal and coarse mesh diffusion ($1\frac{1}{2}$ or 2 group) codes makes this feasible.

The most sophisticated approach to combined nuclear and thermal modeling is one in which the subchannel computations are linked to a space-time kinetics program, which directly solves the neutron kinetic equations for each node or mesh volume. The discrete form of the diffusion equations for the i 'th interior mesh volume may be written for the fast and thermal neutron groups as¹⁷

$$\begin{aligned} \frac{1}{v_g} \frac{d}{dt} \phi_{gi} = & \sum_j D'_{gj} \phi_{gj} - \sum_{gi}^R \phi_{gi} \\ & + \sum_{g'=1}^2 \left(\chi_g \frac{1-\beta_i}{k_{eff}} v \sum_{g'i}^f + \sum_{g' \rightarrow gi}^S \right) \phi_{g'i} + \sum_{l=1}^6 \chi_{gl} \lambda_l C_{li} \quad , \quad g=1,2 \end{aligned} \quad (6.3)$$

and

$$\frac{d}{dt} C_{li} = -\lambda_l C_{li} + \frac{\beta_{li}}{k_{\text{eff}}} \sum_{g'=1}^2 v \Sigma_{g'i}^f \phi_{g'i} \quad l=1, \quad 6 \quad (6.4)$$

where

ϕ_{gi} , C_{li} = volume-averaged group flux and precursor density, respectively

j = center mesh and its six neighbors

D'_{gj} = diffusion coupling coefficient

k_{eff} = steady-state multiplication factor

v_g = neutron velocity in group g (assumed to be constant)

χ_g = fraction of prompt neutrons born in group g

χ_{gl} = fraction of delayed neutrons born into energy group g from precursor group l

λ_l = decay constant for precursor of delayed neutrons in group l

β_i = fraction of all neutrons which are delayed

β_{li} = fraction of all neutrons which are delayed and obtained from precursor group l

$\Sigma^R, \Sigma^f, \Sigma^s$ = macroscopic absorption, fission and scattering cross sections, respectively

ν = number of neutrons produced per fission.

After the flux distribution has been determined for a given timestep, the thermal source strength in a given volume is then obtained from

$$q''' = G' \sum_{g=1}^2 (\phi_{gi} \Sigma_{gi}^f) \quad (6.5)$$

where q''' = thermal source (energy/time vol) and G' = energy/fission.

It is more usual to use a nodal representation of the spatial flux than the coarse mesh approach illustrated by Eqs. (6.3) and (6.4). The transient analog of the steady-state nodal equation [Eq. (1.36)] may be written

$$\begin{aligned} \frac{1}{\Delta x} [J_{Cx+}^K - J_{Cx-}^K] + \frac{1}{\Delta y} [J_{Cy+}^K - J_{Cy-}^K] + \frac{1}{\Delta z} [J_{Cz+}^K - J_{Cz-}^K] \\ + \sum_G^{R,K} \bar{\Phi}_G^K + \frac{1}{v} \frac{d\bar{\Phi}_G^K}{dt} = \bar{Q}_G^K \end{aligned} \quad (6.6)$$

where v = neutron velocity and other symbols have the same meaning as in Eq. (1.36). The average neutron source in the node, $\bar{Q}_G^K$, would be computed using the last two terms of Eq. (6.3). The AROTTA program¹⁸ is an example of this approach.

A computational procedure that is intermediate between the point kinetics approach and the solution of the neutron kinetic equations for each control volume is the modified adiabatic approach* previously noted in Sec. 5.3.4.4. In the original adiabatic method,¹⁹ it is assumed that the neutron flux distribution may be determined by the product of an amplitude function and a shape function. The amplitude function is proportional to the total core power and is determined by the point kinetics equation. Since most transients are relatively slow compared to neutron lifetime, the shape function is then determined by solution of the steady state diffusion or nodal equation. The modified approach of Wu and Weisman²⁰ allows for the fact that the delayed neutron production rate is not proportional to the current power level. The concentration changes in the delayed neutron precursors are followed and the shape function is determined using Eq. (6.3) for the fast group with $d\phi_{gi}/dt$ set to zero. The $1\frac{1}{2}$ group approximation (see Sec. 1.3.3) is used and the thermal flux is determined directly from the fast neutron flux [see Eq. (1.44)]. The simultaneous solution of the diffusion equations for the various control volumes yields the relative neutron fluxes and the eigenvalue (k_{eff}). At the given timestep, the k_{eff} value so determined is used in the point kinetics equation to determine the reactor power level (amplitude function) for the next timestep. The modified adiabatic method allows for longer timesteps than are possible with the direct solution of the kinetic equations for each node. Hence, this approach is considerably faster.

When the spatial changes in power are considered, the neutronic calculations are closely linked to a subchannel analysis program in a single code package. The power distribution determined by the neutronics code is used in the subchannel analysis code, which determines revised flow, fluid temperature, and void distributions. It is also necessary to include fuel temperature calculations as part of the subchannel analysis program so that the Doppler effect can be considered in evaluating fuel cross sections. The revised fluid and fuel conditions are used to compute revised nuclear constants which are used in the next step of the nuclear computations. Iteration between the subchannel analyses and neutronics program at a given timestep may be required.

In most cases, the neutronics-subchannel analysis program is not now (1994) directly linked to the systems analysis code. In the tandem or cyclical approach, the systems analysis program is first run to provide forcing functions of time-dependent pressure, inlet flows and inlet temperature. These are then provided the combined neutronic-subchannel analysis program. The output of the latter code provides a time-dependent core power, which is used as a forcing function for the systems analysis program. Dias *et al.*²¹

*Other reactor space-time kinetics approximations include the quasi-static and improved quasi-static methods [see K. Ott and R. Neuhold, *Nuclear Reactor Dynamics*, American Nuclear Society, La Grange Park, IL (1985)].

describe the cyclical use of the RETRAN systems codes with the ARROTTA/VIPRE subchannel-neutronics program. They found that four to five cycles (ARROTTA/VIPRE run followed by a RETRAN run) generally led to a converged solution. The coordinated cyclical use of a systems program followed by a coupled neutronics-subchannel analysis program is clearly a precursor to codes that fully integrate all three portions of the computation.

6.2.3 Computer Configurations

Traditionally, the long-running systems codes designed for licensing applications were executed on a mainframe machine. With the increased computing power available at workstations, many of the systems analysis codes can be satisfactorily executed at such a station.

In addition to computer programs designed for licensing purposes, there are a number of system analyses programs designed primarily as learning tools. One category of these teaching programs is designed for execution on a microcomputer (personal computer). These programs keep execution and memory requirements low by using a small number of control volumes and simplified modeling. The PRISM²² and PC-TRAN²³ programs are examples of this approach. These programs attempt to provide a generally reasonable representation of system behavior during transients but do not seek high accuracy.

A related set of computer programs are those designed for the full-scale reactor simulators used for operator training and engineering studies. These programs are similar in many respects to the teaching programs but have the additional requirement that they must operate in real time and provide outputs that may be translated into the signals driving control room instrumentation. In addition, these programs must allow for operator interaction. They tend to use a more detailed modeling than the simple teaching codes and are often executed on a minicomputer. Modified versions of these programs can also be economically used for a variety of engineering studies (i.e., effect of operator actions, effect of control system or component variations). The TREAT code²⁴ is an example of such a program.

6.3 ANTICIPATED OPERATING OCCURRENCES

Anticipated operating occurrences (AOOs), or anticipated transients, were considered in Chapter 5 as part of the discussion of the reactor protection system. Since these are category II events, the design criteria of Sec. 6.1.2 require that these transients result in no plant damage. The core monitoring and protective system is designed to see that this is accomplished. This section considers the procedures used for analysis of reactor and system behavior during these transients.

The transients that are generally examined (see Table 5.II for a list of typical AOOs) may be categorized into five groups; reduction in reactor coolant flow, undercooling transients, overcooling transients, change in reactor coolant inventory events, and reactivity and power distribution anomalies. Those of most significance to the thermal designer (limiting transients) are in the undercooling and power distribution anomaly category. Since AOO transients are relatively mild, computer codes based on a single fluid model are satisfactory for these analyses.

Evaluation of AOO events is normally based on operation at full power. However, consideration must be given to the possibility of some system impairment (e.g., plugging of a number of steam generator tubes). It is also desirable to examine these events under shutdown conditions where some safety-related systems may be out of service.

6.3.1 Loss-of-Flow Accident (LOFA)*

As noted previously, the loss of flow occurrence is generally the most limiting of all the anticipated operating occurrences. Primary coolant flow may be reduced by single or multiple reactor coolant pump trips. Since the simultaneous loss of all pumps is the most severe, this accident is the one which is of concern.

The first step in the analyses is the determination of the variation of the flow versus time. One approach to this problem is that used by Fuls²⁵ in the FLOT computer program. For a given pump, Fuls approximates the pump characteristics by

$$H' / \omega^2 = (C_1)(F / \omega)(|F / \omega|) + C_2 \quad (6.7)$$

where H' = pump head (ft); F = volumetric flow through pump (ft^3/s); ω = pump speed (rad/s); and C_1, C_2 = constants for given pump. The absolute value of flow divided by speed provides an increasing negative pressure drop in case of reverse flow through the pump. To utilize Eq. (6.7), we must know the pump speed during the transient. This is obtained by applying Newton's second law to the pump to give

$$\Sigma T = \frac{d\omega}{dt} \left(\frac{I}{g_c} \right) \quad (6.8)$$

where I = moment of inertia of the rotating parts (lb ft^2) and ΣT = net torque rotating part (lb ft).

The major torques on the pump are the windage and bearing torques and the hydraulic torque on the impeller that arises as it imparts head to

*Although loss of flow is best described as a transient, the LOFA designation is in common use.

the fluid. For any given pump, the windage and bearing torques, T_{wb} , can usually be described by

$$T_{wb} = C_3(\omega/\omega_s)^2 \quad (6.9)$$

where C_3 = constant and ω_s = normal pump speed (rad/s).

If the pump were 100% efficient, the hydraulic torque would be obtained by dividing the energy imparted to the fluid by impeller speed. In any real pump, actual hydraulic torque is greater than we would calculate in this manner due to inefficiency in the transfer of energy to the fluid. These inefficiencies are taken as proportional to the square of the relative velocity between the impeller and the fluid. Thus, total hydraulic torque, T_h , is given by

$$T_h = \frac{\rho F H'}{\omega} \left(\frac{g}{g_c} \right) + C_4(\omega r - u)^2 \quad (6.10)$$

where C_4 = constant for a given pump; r = effective impeller radius (ft); and u = fluid velocity (ft/s). Equation (6.8) then becomes

$$\frac{d\omega}{dt} \left(\frac{I}{g_c} \right) = -C_3(\omega/\omega_s)^2 - \frac{\rho F H'}{\omega} \left(\frac{g}{g_c} \right) - C_4(\omega r - u)^2 \quad (6.11)$$

The flow in a given loop is determined by rewriting Eq. (3.31) to allow for the pump head. If there is only one pump per loop, for the j 'th loop we write

$$\begin{aligned} & \left(\frac{1}{g_c} \right) \frac{d}{dt} \left[\left(\frac{W' L'}{A'} \right)_R + \left(\frac{W' L'}{A'} \right)_L + \left(\frac{W' L'}{A'} \right)_{SG} \right] \\ &= - \left\{ \left[\frac{C'}{\rho} \left(\frac{W'}{A'} \right)^{n_1} \right]_R + \left[\frac{C'}{\rho} \left(\frac{W'}{A'} \right)^2 \right]_L + \left[\frac{C'}{\rho} \left(\frac{W'}{A'} \right)^{n_2} \right]_{SG} \right\} + \rho H' \end{aligned} \quad (6.12)$$

where

L' = axial length of component (ft)

W' = mass flow through component (lb/s)

C' = pressure loss coefficients for individual components

A' = cross-sectional area of component (ft²)

and the subscripts are

R = reactor vessel

SG = steam generator

L = loop.

Exponents n_1 and n_2 allow a pressure-loss relationship other than one dependent on the square of the fluid velocity. By using separate equations for each loop, transients in which only some of the pumps are lost can be

considered. A set of continuity relationships must be written to relate the flows in the various components.

A finite difference method can now be used to solve the problem. Consider the simplest situation in which all pumps are lost simultaneously; we can consider the entire system as if it were a single loop. Steady-state values for flow and pump speed are used to determine the rate at which the pump speed changes [Eq. (6.11)]. A short time interval is assumed and the reduced head obtained from the pump is estimated from Eq. (6.7). The pump head is then used to obtain the change in flow over the time interval, Eq. (6.12). If desired, an average value of the original and reduced flow can then be used to reestimate the changes in pump speed, pump head, and loop flow. If the changes over the interval are within the desired limits, the calculation proceeds to the next time period using the reduced flow, speed, and head to estimate the new changes in conditions.

Most general-purpose systems analysis programs can be used to analyze a LOFA. Essentially, all of these programs make use of homologous curves to obtain pump characteristics. Thus, Eq. (6.7) would be replaced by an equation or table representing the homologous head-pump speed relationship [equivalent of Fig. 3.9(a)]. Similarly, Eq. (6.10) would be replaced by the appropriate homologous hydraulic torque-pump speed relationship [equivalent of Fig. 3.9(b)]. The hydraulic torque, together with the pump inertia, I , is then used in Eq. (6.8) to obtain the change in pump speed with time. The systems program would then calculate loop flow during a given time interval by the simultaneous solution of the momentum, energy, and mass balance equations for each of the control volumes in the loop. The pump analysis would provide the momentum balance for the control volume representing the pump.

If the transient were to lead to two-phase flow at the pump suction, the pump head would fall below the single phase value. Revised homologous curves would then be required (see Sec. 3.3.6).

Flow coastdown information obtained from this analysis normally would be used subsequently in a subchannel analysis of the thermal behavior of the reactor fuel and coolant. Time variation in flow and reactor power would be the input to a computer program such as COBRA (Ref. 14), which analyzes transient core behavior (see Sec. 4.2.1.4). As previously observed, the analysis must show that there is adequate margin to DNB throughout the accident. Additional margin can be gained by using a combined subchannel neutronics analyses. Since higher coolant enthalpies and void fractions are produced in the hottest sections of the core, the additional moderator feedback reduces the reactivity, and hence power, in these regions. The reduced power increases the DNB ratio. A number of computer programs that accomplish this are available²⁶⁻²⁹ and are used for design purposes.³⁰

6.3.2 Undercooling Transients

Undercooling transients are those in which there is a decrease in the rate of heat removal by the secondary system. Typical transients³¹ of this nature are loss of load (turbine trip), increase in feedwater temperature, and loss of feedwater flow. In each of these cases, the degradation of secondary side heat removal capability leads to a rise in the secondary system pressure and temperature.

6.3.2.1 *Loss-of-Load Transient*

The loss-of-load transient is, perhaps, the most likely of these events. In this transient, the sudden reduction in steam flow causes a rapid reduction in heat removal from the primary coolant. Heat addition in the reactor core initially remains constant and exceeds the rate of heat removal. This results in rapid heatup of the coolant and a pressurizer insurge: a flow of water from the loop into the pressurizer. Increased coolant temperature progresses around the loop; when it reaches the core, it decreases reactivity and, hence, power. The reduction in power can be to a level below that of the rate of heat removal. This can lead to a pressurizer outsurge. Pressurizer heaters and sprays are used to maintain system pressure within the control band.

Since steam is no longer removed from the steam generators after a turbine trip, the continued transfer of heat from the primary system causes the secondary system pressure to rise. To prevent actuation of the steam generator safety valves, a controlled release of steam to the condenser (steam dump) is generally initiated whenever the load loss occurs at above about 10% of full power.

The licensing analyses of this transient must demonstrate that (1) the relief and safety valves keep the reactor coolant system pressure below the design pressure and that (2) the DNB design limit is not reached.

Analysis of these transients requires an accurate modeling of the pressurizer. Early pressurizer models were devised by Margolis and Redfield³² and Baron.³³ More recent pressurizer models have been discussed in Sec. 4.6.2. The successful models all consider two regions (one primarily liquid, one primarily vapor) which are not in thermodynamic equilibrium. Non-equilibrium models with varying assumptions have proven capable of providing agreement with experimental data.

The control system is generally designed so that a step loss of load in excess of 50% provides a direct reactor trip signal. If the direct trip fails, a reactor trip should be initiated by either high pressure or high level in the pressurizer. This sequence of events is no longer an AOO. Here it would be necessary to show that the design pressure is not exceeded and that fuel damage is very limited or does not occur.

Tong³⁴ has considered the situation where the direct scram fails and the secondary scram occurs shortly after DNB has occurred. Tong³⁴ conducted a parametric study of fuel rod behavior using a simplified model in which the thermal resistances and capacitances of the UO_2 pellet and metallic cladding were respectively lumped (see Sec. 2.6.1.2). If we define

$$q'_2 = \theta / R_2 \quad (6.13)$$

we can rewrite Eq. (2.221) as

$$q'_n = C_1 C_2 R_1 R_2 q'_2 + (C_1 R_2 + C_2 R_2 + C_1 R_1) q'_2 + q'_2 \quad (6.14)$$

Hence, the fuel rod transfer function is

$$\frac{q'_2(s)}{q'_n(s)} = \frac{1}{C_1 C_2 R_1 R_2 s^2 + (C_1 R_2 + C_2 R_2 + C_1 R_1) s + 1} \quad (6.15)$$

The effective time constant of a UO_2 fuel rod is different for various heat transfer mechanisms. The heat transfer coefficient to water, before DNB, is usually very high and cladding temperature is approximately equal to the water temperature. Thus, only the pellet capacitance is considered in a power transient. The resulting time constant is $C_1 R_1$, which is ~ 8 s for the Yankee-Rowe fuel rod. In case of film boiling, the heat transfer coefficient at the rod surface decreases sharply causing a rapid rise in the cladding temperature. The time constant for transient heat transfer between the pellet and cladding is $R_1 C_1 C_2 / (C_1 + C_2)$, which is ~ 2.5 s for the Yankee-Rowe fuel rod. Tong³⁴ used this general approach for a parametric study of fuel rod transient behavior. He showed that maximum cladding temperature ($T_{2,\max}$) is a function of τ_1 , steady-state power, $q'_n(0)$, and the post CHF R_2 where $\tau_1 = t_1 / r_1^2$; r_1 = pellet radius; and t_1 = (time to scram – time to reach DNB).

The results of Tong's analysis are shown in Fig. 6.1. The estimates of this figure must be considered as conservative upper bounds since no account was taken of the reduction in peak power prior to scram due to the effect of local void production.

6.3.2.2 Loss-of-Feedwater Transient

After a loss of feedwater, the reactor would normally trip and the auxiliary feedwater flow initiated. The accident analysis must show that the auxiliary feedwater system can remove the decay heat and stored energy. This will prevent either overpressurization of the primary system or excessive primary coolant loss if the pressurizer valves open.

If the primary trip on loss of feedwater flow does not occur, a secondary trip on pressurizer level or pressure should take place. Under these conditions, it is necessary to show, with a subchannel analysis of the reactor core, that the minimum allowable DNB ratio is not reached. As noted pre-

viously, a subchannel analysis that is combined with a neutronic analysis provides additional margin since the moderator feedback will reduce the peak power. Conservative values of the moderator feedback (high boron concentration) should be used.

6.3.3 Reactivity and Power Distribution Anomalies

Transients in this category include uncontrolled control assembly bank withdrawal for low or full power, control assembly misalignment or drop, startup of inactive loop, and chemical control system malfunction, which results in a boron dilution.

It was noted in Sec. 5.1.5 that the rod drop accident tends to be one of the transients that limits reactor power. A dropped rod results in a negative reactivity insertion and, if the rod worth is high enough, a negative flux rate reactor trip. However, for low-worth rods, the reactor may not be

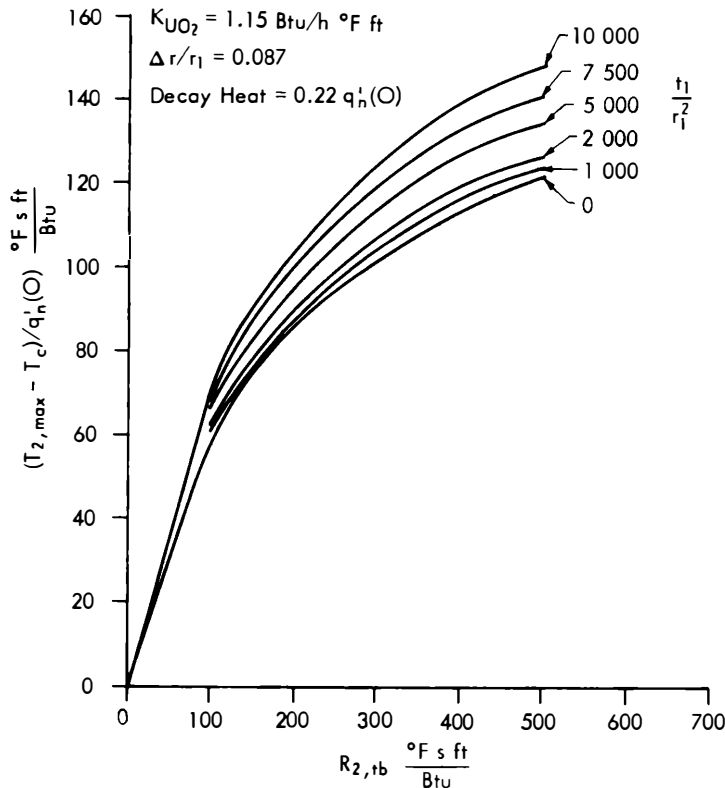


Fig. 6.1 Thermal transient parameters of a fuel rod [from *Nucl. Sci. Eng.*, 9, 306 (1959)].

tripped and the original power level can be reestablished by control bank motion or reactivity feedback. Operation with the dropped rod then results in a significant increase in reactor power peaking. The effect on the minimum DNB ratio must therefore be determined for the limiting power distributions. While a combined subchannel-neutronic analysis provides the most accurate analysis of the situation, the power distribution is often determined separately ignoring thermal-hydraulic feedback. The conservative power distribution so determined would then be used in a subchannel analysis of the reactor core.

6.3.4 Overcooling Transients

Overcooling transients include those caused by³¹ an increase in feedwater flow, decrease in feedwater temperature, excessive load, and an inadvertent opening of a steam generator relief, safety, or dump valve. Since these events cause a decrease in the primary coolant temperature, both core reactivity and power increase. The transient analysis must show that the minimum DNB ratio is not reached.

In addition to an increase in power level, the transient also gives rise to an initial decrease in pressure and pressurizer level. A reactor scram may be initiated by high flux levels, high thermal power, low pressurizer load, or low pressurizer pressure. A conservative analysis of this transient is usually accomplished using the maximum moderator reactivity feedback and the minimum fuel temperature feedbacks. The systems analysis must be followed by a subchannel analysis to determine the minimum DNB ratio.

6.3.5 Changes in Reactor Coolant Inventory

The AOOs that lead to an increase in reactor coolant inventory include the inadvertent operation of the emergency core cooling system (ECCS) and malfunction of the chemical and volume control system.³¹ Since ECCS operation means injection of borated water, core power is reduced. This means that the DNB ratio increases during the accident and hence it is not limiting.

If the chemical and volume control system fails and water at reactor conditions is injected, the pressurizer level rises with little change in power. The transient is usually examined to show that the operator has time to end the accident before the pressurizer is filled.

Decreases in system inventory may be due to inadvertent valve opening or minor leaks. For very minor leaks, the makeup system may be able to make up the loss and there may be no reactor scram. For an inadvertent valve opening, the charging system probably cannot keep pace. The let-down system would be isolated and the event probably terminated by low pressurizer level or pressure. If an open safety or relief valve fails to close on reduced pressure, the event then becomes a small loss of coolant accident.

6.4 ACCIDENTS DUE TO SECONDARY SYSTEM RUPTURE (STEAM-LINE BREAK)

Accidents involving the rupture of a secondary system line may also be considered overcooling transients. As such, the increase in reactivity that this overcooling creates must be modeled. The most serious of these accidents is a main steam-line break based on the assumption of an instantaneous guillotine rupture. Since such an accident is not expected during the life of a given plant, it is a category IV event.

When a steam-line break occurs, there is a sudden increase in steam flow from the steam generator. This brings about a rapid cooling of the secondary system, which in turn causes the primary system to cool and depressurize. A reactor trip is generated by a low steam generator pressure or level, low primary system pressure, or a low calculated DNB ratio. The main steam isolation valves close and limit the blowdown to just one steam generator. Low pressure also actuates borated water injection by the safety injection system.

During the initial phase of the accident, it is necessary to determine whether the minimum DNB ratio is reached. However, most of the concern in this accident is directed toward a return to power after the reactor trip. This is a possibility since conservatism requires the assumption that the most reactive control rod is stuck in a fully withdrawn position. Whether a return to power will occur depends on the rapidity with which the safety injection system is actuated. If the reactor returns to power with the most reactive rod out of the core, there will be a high power peak and hence the possibility that some fuel rods go through DNB.

This transient has often been studied using a point kinetics reactivity model.¹⁴ Such a model may not be entirely satisfactory without some corrections. Core inlet temperatures are asymmetric and reactivity feedback is larger than would be obtained using a uniform inlet temperature. A conservative allowance for this is sometimes obtained by taking the core inlet temperature as that of the cold sectors of the core.

The system analysis must be followed by a subchannel analysis of the core to determine the extent of the DNB problem. This analysis requires a knowledge of the power distribution with one stuck rod. In the past, the flux distribution was often based on static calculations. This is reasonable since the reactivity variation initiated by the steam-line break is relatively slow. The THUNDER code²⁴ was one of the early programs that combined fine-mesh diffusion neutronics and subchannel analysis for steady-state conditions. More satisfactory results are obtained from the newer programs which combine nodal-code neutronics with transient subchannel analysis (e.g., FLICA-CRONOS combination³⁰). The most sophisticated analyses directly link the combined neutronic-subchannel analysis code with the systems analysis program. The previously noted (see Sec. 6.2.2) linkage of the

ARROTA/VIPRE subchannel package with the RETRAN systems analysis code was developed for analysis of the steam-break accident.²¹

Detailed studies of the steam-line break analysis by Kennedy *et al.*³⁵ indicate problems in obtaining a satisfactory evaluation of this accident. Their study indicates that the core bypass flow to the reactor vessel head should not be modeled if conservative results are desired. In addition, asymmetric thermal hydraulics is important in the determination of reactivity feedback. Proper modeling of the mixing of the coolant returned to the core is needed to establish realistic operating conditions. (Recall that Sec. 3.2.2 points out that, in the case of asymmetric steam generator behavior, there is little mixing in the lower plenum but significant mixing in the upper plenum of the reactor vessel.)

6.5 LARGE-BREAK LOSS-OF-COOLANT ACCIDENT (LBLOCA)

6.5.1 System Behavior

The most serious accident generally considered in the safeguards analysis is the hypothetical LOCA occasioned by the double-ended rupture of a main coolant pipe. If such an unlikely accident were to occur, the course of expected events would be:

1. *Subcooled blowdown*: as described in Sec. 3.3.5, pressure waves are propagated through the system at sonic velocities. The system is rapidly depressurized until the vapor pressure of the coolant is reached. This period can be on the order of 5 to 50 ms long.

2. *Saturated blowdown*: after system pressure falls to the level of coolant vapor pressure, steam voids are formed. Sonic velocity is drastically reduced, pressure waves are damped out, and choked flow occurs at the break. The length of this period depends on the location and size of the break.

During this period, the fuel rods are cooled by fluid in the reactor core. Initially, high forced convection or nucleate boiling heat transfer coefficients prevail. As fluid enthalpy increases, the boiling crisis can be reached, leading to a marked reduction in core cooling. Cladding temperature rise is reduced as the time between blowdown initiation and boiling crisis is increased.

To extend the period during which the core is well cooled, an ECCS is provided. When the primary system pressure falls to a predetermined level, a large quantity of borated coolant water is injected from high-pressure accumulators. In most PWR designs, bottom flooding is used and ECCS water enters the reactor vessel through the downcomer. (See Fig. 6.2 for the ECCS systems arrangement.)

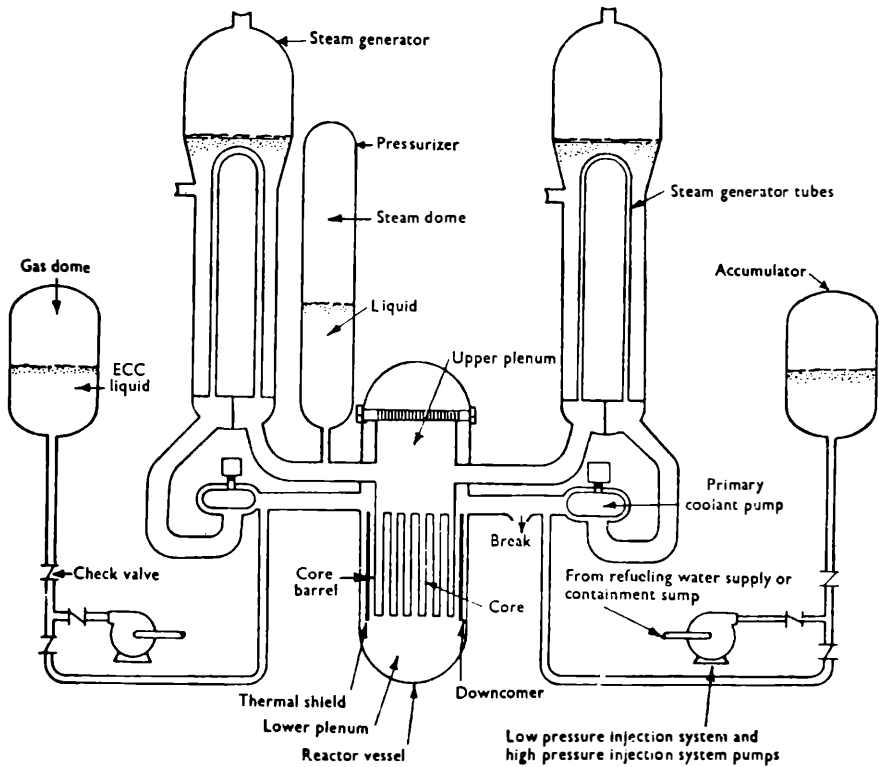


Fig. 6.2 Multiple-loop PWR system during reflood.

3. *Dry period:* after the accumulator inventory is exhausted, continued blowdown leads to a condition where the liquid, or rather the liquid-steam froth, falls below the top of the core. When the froth level is above the bottom of the core, the dry portion is cooled by steam generated in the lower region. When the froth level drops below the bottom of the core, the only core cooling is by radiation to the surrounding structure and by convection and radiation to the small amount of steam generated by froth contact with the vessel wall.

During this period, cladding temperatures are expected to rise to a level at which the zirconium-steam reaction proceeds at an appreciable rate (see Sec. 2.6.2.2). The heat produced by this exothermic reaction must be considered in assessing the maximum temperature rise.

4. *Vessel reflood:* after the accumulator injection has stopped and system pressure has been further reduced, high flow pumps of the ECCS system add water to the reactor vessel to recover the core. Since most designs call for bottom reflooding, substantially improved cooling does not result until

the froth level reaches the bottom of the core. The steam produced from quenching the lower portion of the fuel rods flows upward and cools the upper portion of the core. This steam cooling generally limits the maximum temperature reached by the cladding. The water from ECCS pumps gradually refills the core; the transient is terminated when the vessel is refilled. Provision is made for the subsequent addition of water at a low rate to replace that lost through boiloff.

Behavior of the hypothetical accident depends to some extent on whether a break has been assumed in the hot-leg or cold-leg piping. If a break in the cold-leg piping has been assumed, steam and liquid will be flowing up through the downcomer to the cold-leg break during the blowdown portion of the accident. The upflow of steam can be so rapid that the entry of ECCS water to the downcomer is restricted. This downcomer flooding phenomenon, discussed in Sec. 3.5.4, would not be encountered in a hypothetical hot-leg break accident. Because of this phenomenon, some conservative system models assumed that no ECCS water can penetrate to the core during the blowdown portion of a cold-leg break accident.

It is most common to add ECCS water through lines connected to the cold legs of the main piping. In the cold-leg line where a break is postulated, there is a high velocity flow of fluid that is largely steam. It has been postulated that this steam flow could prevent the entrance of ECCS water to the main piping line. However, experimental tests of injection tees under stimulated accident conditions do not appear to support this conclusion. Analysis of the test data indicates the water does enter the primary piping, that it is effective in condensing the exiting steam, and that the average pressure drop across the run of the injection tee can be estimated from a one-dimensional momentum balance.

Some reactor designs reduce delays in ECCS coolant penetration of the core by using vent valves and direct injection of cooling water to the downcomer. When a steam-water mixture is flowing toward the cold leg, pressure in the core is higher than at the cold leg. Check valves located in the upper region of the core barrel then open to vent steam and water directly from the area above the core to the upper region of the downcomer. The steam-water mixture then exits through the broken cold leg. Vent valves nearly equalize the pressures in the core and downcomer. Emergency cooling water, injected into the downcomer below the entrance of the main coolant lines, thus is able to enter the core.

Another approach has been to inject the water from high pressure accumulators directly to the upper head of the reactor vessel. Water in the upper head is discharged above the fuel assemblies through hollow support columns and control rod guide tubes. Injection water penetrates during the blowdown period since the flooding phenomenon in the downcomer is avoided. However, there is still a brief period following the quenching of a major portion of the core, where the steam generation rate is so high that

entry of water from the upper head is prevented. At the conclusion of upper head injection, the core is reflooded by water entering from the main piping lines.

Other modified schemes proposed for ECCS injection include features such as direct injection of water to the lower plenum, use of accumulators that are not pressurized until an abnormally low pressure is sensed, and installation of check valves in the cold-leg nozzles.

6.5.2 Regulatory Requirements

Somewhat simplistic views of LOCA were held when the design of the first light-water power reactors began. At that point, it was assumed that a LOCA would result in some degree of core meltdown. The fission products released from the molten fuel would be retained by the vapor container. It appeared that public safety would be adequately protected. While this may be correct, further study of the accident consequences led to several questions. The possibility that a release of molten fuel into the water in the lower portion of the vessel or into the vapor container sump could lead to a sudden generation of steam (steam explosion) was raised. A high-efficiency steam explosion in the sump could conceivably lead to a pressure pulse that could rupture the vapor container. In addition, molten fuel could melt through the reactor vessel, drop to the floor below, and melt some distance into the lower structure. It was considered that this could also lead to a release of radioactivity to the environment.

In view of the possible problems involved in a partial core meltdown, it was decided that reliable emergency core cooling sufficient to prevent any fuel melting would be provided. The ECCS previously described was designed and sized to prevent any melting of fuel element cladding. Further study of this LOCA then led to questions on the conservatism of the no-clad melting criterion. It was argued that very destructive failures of highly oxidized cladding could occur at temperatures below melting. If fuel pellets were released from rods undergoing brittle failure, local geometry might be ill defined and adequate cooling of the core could not be guaranteed. This has led to the philosophy that fuel conditions must be limited to preserve the original core geometry with at most moderate and calculable changes.

As a result of extensive study and public hearings, the U.S. regulatory agencies set up a series of criteria that all light-water-cooled reactors must meet.³⁶ The major requirements of these criteria, which assume Zircaloy-clad elements containing UO_2 pellets, are:

1. The maximum calculated fuel element cladding temperature is not to exceed 2200°F.
2. Total oxidation of the cladding is not to exceed 0.17 times the total cladding thickness.

3. The total amount of hydrogen generated by chemical reaction of the cladding is not to exceed 0.01 times the hypothetical amount generated if all the cladding around the fuel reacted.
4. Calculated changes in core geometry must be such that the core is still amenable to cooling.
5. After the initial successful operation of the ECCS, appropriate long-term cooling must be provided to keep the core at an acceptably low temperature.

The purpose of the first two criteria is to ensure that the Zircaloy cladding remains sufficiently intact to retain the UO_2 fuel pellets. Severe oxidation at high temperature can render the cladding brittle; destructive damage could result from rod bursting. By limiting the maximum temperature and degree of oxidation, sufficient ductility to avoid brittle failure is assured.

The third criterion assures that hydrogen will not be generated in amounts that could lead to explosive concentrations. The final criteria simply restate what has always been generally accepted.

The regulations concerning hydrogen releases were subsequently augmented in a 1985 addition to the Code of Federal Regulations (10CFR50.44). This later regulation requires that the vapor container be provided with hydrogen control devices (e.g., catalytic recombiners, spark-plug-type igniters) and means for assuring a mixed atmosphere. It must also be shown, prior to the hydrogen control device being effective, that either no combustion will occur or that the containment can withstand such combustion.

The consequences of the radioactivity release to the containment due to a LBLOCA must also be evaluated. This evaluation is usually based on the very conservative assumption that all noble gases, 10% of all halogens, and 1% of all solid fission products are released from the fuel.¹ By use of the expected vapor container leak rate and appropriate atmospheric diffusion models, the radiological dose is determined as a function of distance from the vapor container. U.S. regulations² required that the dose not exceed 25 rem to the whole body or 300 rem to the thyroid after exposure for either (1) 2 hr at the exclusion area boundary or (2) 30 days at the low population zone boundary.

6.5.3 Key LOCA Phenomena

6.5.3.1 Blowdown and Dry Periods

Although the largest break flow area is based on a guillotine rupture of the primary system piping, the full break area would not be available for blowdown flow. The pipe segments are expected to be connected to some extent after the rupture. Some reactor vendors have therefore based their double-ended break analysis on a reduced flow area (e.g., 40% of pipe cross-sectional area).

Early LOCA analyses considered only the guillotine break. However, it is now recognized that safety systems based solely on the largest break size may not be satisfactory for accidents initiated at some smaller break sizes. A range of break sizes must therefore be examined. Accidents are generally considered in the LBLOCA category if the break size examined is above 0.5 ft^2

The initial phase of the accident is largely determined by the critical flow through the break (see Sec. 3.3.5 for critical flow prediction). For example, in a double-ended guillotine break of the cold-leg piping, the initial flow through the break will generally exceed the flow in the intact cold legs.³⁷ This leads to a flow reversal at the core inlet. The resulting flow stagnation inside the core produces CHF conditions and a cladding temperature increase. This situation is shortlived (2 to 3 s) as the system pressure moves to saturation and the break-flow drops. The flow from the intact cold legs is then greater than the break flow and the core flow is restored. This usually initiates a partial or core-wide quenching. Subsequent flashing in the reactor vessel and degradation of pump performance can lead to flow oscillations and a second clad heat up. A quench may then occur due to flow reversal at the core outlet.

In a fast blowdown with flow reversal, the CHF occurs at a high local void fraction (e.g., $\alpha \geq 0.85$) and is often estimated using Hsu and Beckner's approach.³⁸ Heat transfer correlations useful throughout the blowdown and dry periods are indicated in Table 6.II.

During portions of the blowdown and reflood periods, large volumes, such as the core, will only be partly full of liquid. Hence, when the node and branch approach is used, it is inappropriate to use the homogeneous assumption for nodes representing the reactor vessel and pressurizer. Consequently, a steam phase and a steam-water mixture were both assumed to exist within these nodes. Steam bubbles were assumed to be created by bulk flashing of the liquid and then to rise. A mass balance on the steam trapped below the surface of the liquid was used to compute the height of the two-phase system. Initially, a constant bubble rise velocity of 2 ft/s relative to the container was assumed.³² The time, t_B , required for a bubble to leave the froth in a given volume was

$$t_B = z_f / u_B \quad (6.16)$$

where u_B is bubble velocity (2 ft/s) and z_f is height of froth. Redfield and Murphy⁴⁹ report that better agreement is obtained with experimental data when z_f is replaced by z_L where z_L is the height of liquid only in the froth layer. The value of u_B remained 2 ft/s .

Somewhat more sophisticated bubble rise models have been developed subsequently. In the approach developed by Wilson *et al.*⁵⁰ the motion of the vapor relative to the froth-vapor interface is given in terms of a superficial steam velocity (volumetric flow of steam divided by the cross-

TABLE 6.II
Blowdown Heat Transfer

Mode of Heat Transfer	Transition Between Modes	Range of Interest	Sensitivity to Cladding Temperature	Suggested Correlations	Limitations in Range of Applicability	Remarks
Single-phase convection		P 2000 to 22000 psi G (0 to 2.5×10^6 lb/ft ² h)	Insignificant	Dittus-Boelter ³⁹ for Re > 2000 Rohsenow-Choi ⁴⁰ for Re < 2000		
	Incipience of boiling	P 2000 to 700 psi G (0 to 2.5×10^6 lb/ft ² h)	Insignificant	Bergles ⁴¹	P 15 to 2000 psi	If computer time is of concern, simplified transition criterion of $h_{NB} > h_{FC}$ is satisfactory
Nucleate boiling		P 500 to 2000 psi G (0 to 4×10^6 lb/ft ² h) χ_0 to 0.95	Insignificant	Chen ⁴²	P 14 to 1000 psi G (0.04 to 3×10^6 lb/ft ² h) $\chi < 0.7$	Gungor and Winterton ⁴⁴ [Eq. (4.174)] acceptable alternative
	Transient CHF	P 400 to 1700 psi G (0 to 2.5×10^6 lb/ft ² h)	Significant	Hsu and Beckner ³⁸ for high flow Modified pool boiling [Eq. (4.228)] for low flow	P 700 to 1500 psi G (0.01 to 0.5×10^6 lb/ft ² h) G < $.05 \times 10^6$ lb/ft ² h	EPRI correlation ⁴⁷ or Table 4.IV Look-up table suitable alternative

TABLE 6.II (continued)

Mode of Heat Transfer	Transition Between Modes	Range of Interest	Sensitivity to Cladding Temperature	Suggested Correlations	Limitations in Range of Applicability	Remarks
Post-CHF transition		P 200 to 1500 psi G (0 to 3) $\times 10^6$ lb/ft ² h	Significant	Tong and Young ⁴³ for high flow. Exponential decay between T_{CHF} and min. film boiling temp.	P 500 to 1400 psi G (0.5 to 3) $\times 10^6$ lb/ ft ² h χ 0.15 to 1.10 Low flow or low χ	
Film boiling		P 20 to 1600 psi G (0 to 3) $\times 10^6$ lb/ft ² h $\chi > 0.15$	Significant	Bishop <i>et al.</i> ⁴⁵ if T_w < Leidenfrost temp. and pressure high Modified Condie- Bengston [Eq. (4.245)] if $T_w >$ Leidenfrost temp. or P low	P 500 to 3190 psia G (0.9 to 2.5) $\times 10^6$ lb/ft ² h G < 3.8 $\times 10^6$ lb/ft ² h P ≥ 60 psia	
Steam cooling		P 20 to 1600 psi G (0 to 3) $\times 10^6$ lb/ft ² h		Modified Dittus- Boelter [Eq. (4.155)] for high flow Webb-Chen ⁴⁶ [Eq. (4.250)] for intermediate flow Natural convection equation for low flow.	G > 0.25 $\times 10^6$ lb/ft ² h P ≥ 60 psia	Bishop <i>et al.</i> ⁴⁸ suitable alternative if $P >$ 1000 psi Webb-Chen equation automatically extends to steam convection as quality $\rightarrow 1.0$

sectional area of the vessel). The RETRAN version of this model computes the superficial velocity, of the rising steam as

$$u_{bl} = (\bar{\alpha})^{1/c_2} C_w \quad (6.17)$$

where $\bar{\alpha}$ = average mixture void fraction

$$C_w = \frac{C_3}{\left[C_1 \left(\frac{\rho_g}{\rho_l - \rho_g} \right)^{0.32} \left(\frac{\sqrt{\frac{\sigma}{\rho_l - \rho_g}}}{De} \right)^{0.19} \right]^{\frac{1}{C_2}}}$$

$$C_3 = \left[g \sqrt{\frac{\sigma}{\rho_l - \rho_g}} \right]^{0.5}$$

$$\left. \begin{array}{l} C_1 = 0.136 \\ C_2 = 1.78 \end{array} \right\} \quad \text{when } u_{bl} < 5.5 C_3$$

$$\left. \begin{array}{l} C_1 = 0.75 \\ C_2 = 0.78 \end{array} \right\} \quad \text{when } u_{bl} > 5.5 C_3$$

In the later approaches¹⁶ to blowdown modeling, it is recognized that in a decompression of a large volume, the partial density of steam bubbles should be lowest at the bottom of the vessel. A linear density variation is usually assumed where

$$\rho_m = m(z/z_m) + b \quad (6.18)$$

and ρ_m = mixture density at height z (mass steam/vol); z, z_m = height under consideration and height of mixture, respectively (L); and b, m = model parameters. The parameters b and m are not independent and are related by

$$\left. \begin{array}{l} m = 2C^1[(M_m/V_m) - \rho_l] \\ b = (1 - C^1)(M_m/V_m) + C^1\rho_l \end{array} \right\} \quad \text{for } 0 < \bar{\alpha} < 0.5$$

$$\left. \begin{array}{l} m = 2C^1(\rho_g - M_m/V_m) \\ b = (1 + C^1)(M_m/V_m) - C^1\rho_g \end{array} \right\} \quad \text{for } 0.5 < \bar{\alpha} < 1.0 \quad (6.19)$$

where

C^1 = model parameter adjusted to provide agreement with data
($C^1 < 1$)

M_m = total mass of vapor-liquid mixture in volume

V_m = vapor-liquid mixture volume = Az_m

A = cross-sectional area of volume

z_m = height of vapor-liquid mixture.

The height of the froth (vapor-liquid mixture) in the volume may be determined from initial conditions and mass and volume balances on the froth. From the mass balance, we obtain

$$\frac{dM_m}{dt} = u_{bl} \rho_g A + \sum_{in} w_j - \sum_{out} w_j \quad (6.20)$$

$$\frac{dz_m}{dt} = -u_{bl} + \dot{m} z_m \rho_l (1 - \bar{\alpha}) v_{fg} + \frac{1}{A} \left(\sum_{in} w_j / \rho_j - \sum_{out} w_j / \rho_j \right) \quad (6.21)$$

where

$\bar{\alpha}$ = mean void fraction of mixture = $\rho_l - (M_m / V_m) / (\rho_l - \rho_g)$

ρ_g, ρ_l = vapor and liquid density, respectively (M/L^3)

v_{fg} = change in specific volume on evaporation (L^3/M)

$\dot{m}$ = vapor generation rate per unit mass of liquid [$M/(Mt)$]

u_{bl} = effective bubble rise velocity [L/T] determined from Eq. (6.17) at $\alpha(z_m)$

w_j = flow rates to or from j 'th connecting volume (M/t)

ρ_j = density of fluid flowing to or from j 'th connecting volume.

Note that the density of fluid leaving the froth volume would depend on the elevation of the exit line.

When system modeling is based on the control volume approach, the core is usually modeled as a set of vertically stacked control volumes. The change in density and void fraction with elevation can then be directly modeled and a bubble rise model is no longer needed for the core. However, in the large, nearly stagnant volume contained within the pressurizer, a single fluid model does not provide an adequate representation of component behavior. It is usual to model the pressurizer separately by considering it as a single node with a primarily vapor region and a primarily liquid region.

The increased fuel and cladding temperatures during the dry or steam cooling period may be determined using the models described in Sec. 2.6.1. The heat source term must include the heat released by the exothermic zirconium-steam reaction. Reaction rates limited by oxygen diffusion across the oxide may be determined from Eq. (2.251). However, at the end of the dry period, where steam flow is low (flow becomes laminar), the rate determined by hydrogen diffusion [Eq. (2.253)] must also be computed. The governing reaction rate is the lowest of the two rates.

6.5.3.2 Refill and Reflood Phases

When the system pressure has decreased sufficiently, injection of the cold water from the low pressure safety injection (LPSI) system begins. As ob-

served in Sec. 6.5.1, the penetration of this water may be delayed by flooding in the downcomer. The models of Sec. 3.5.4 may be used to determine the boundaries (countercurrent flow limit, CCFL) of the flooding region.

The initial penetration of the cold injection water can cause substantial oscillations in the cold-leg flow due to the formation of liquid slugs in the cold leg. As previously noted in Sec. 3.4.5, these oscillations appear to be generated by stratified-slug flow pattern instability. A two-fluid model incorporating an appropriate stratified-slug transition criterion and interfacial transfer rates which vary with the flow pattern can predict such oscillations.⁵¹ It is also possible to treat this problem using a single fluid model systems code containing a special component model for the ECCS injection region. Rothe⁵² successfully modeled this behavior by an on-off condensation rate. When the liquid plug is upstream of the injection region no condensation is assumed. With a downstream plug, the condensation rate is that required to achieve thermodynamic equilibrium.

The LPSI flow to the core may also be limited by what is called "steam binding." This is the generation of a back pressure in the steam generator by evaporation of droplets that were entrained in the core upper plenum, cold legs, and steam generator plenum. The evaporation of the droplets produces a void expansion and back pressure, which reduces ECCS flow to the core.

Once the downcomer flooding and steam binding have ended and the vessel pressure is near atmospheric, core reflooding becomes more effective. Core cooling by steam from the lower plenum is followed by quenching. The portions of the core which are above the quench front will be in the inverted annular flow regime for high reflooding rates or in the dispersed droplet regime for low flooding rates. Both of these exhibit poor rates of heat transfer but they do provide some cladding cooling prior to quenching. As noted in Sec. 4.4.4.3, some investigators divide the inverted annular flow (IAF), into smooth IAF, rough-wavy IAF, agitated, and post-agitated IAF. The heat transfer correlations appropriate to these regimes and the highly dispersed regime⁵³ are shown in Table 6.III. Note that in Table 6.III the wall to vapor heat transfer is obtained by multiplying the Webb-Chen result (h_{w-C}) by a factor of 1.2. This factor is an approximation of the heat transfer enhancement provided by the grid spacers in the rod bundle.

If the heat transfer computations are coupled to a two-fluid systems code, interfacial drag and heat transfer rates are also required. The interfacial transfer correlations proposed by Nelson and Unal⁵³ for use in the TRAC code are shown in Tables 6.IV and 6.V. Note that the interfacial heat transfer relationships of Table 6.V consider two possibilities. When the liquid temperature is above saturation, interfacial heat transfer is taken as governed by flashing. When the liquid temperature is below saturation, interfacial heat transfer is based on evaporation.

TABLE 6.III
Wall-to-Fluid Heat Transfer During Reflood*

Regime **	Wall to Liquid	Wall to Vapor
Smooth inverted annular flow	$h_{\text{conv}_1} + h_{\text{rad}}$ where h_{conv_1} = Denham correlation h (Eq. 4.261) $h_{\text{rad}} = (1 - \alpha)q_{\text{rad}} / (T_w - T_{\text{sat}})$	Unused
Rough-wavy inverted annular flow	$h_{\text{conv}_2} + h_{\text{rad}}$ where $h_{\text{conv}_2} = h$ from Eq. (4.262)	$1.2h_{\text{we}}$ where $h_{\text{we}} = h$ from Webb-Chen correlation (Eq. 4.250)
Agitated and post-agitated inverted annular flow	$h_{\text{conv}_3} = h$ from Eq. (4.263)	$1.2 h_{\text{we}}$
Highly dispersed flow	Unused	$1.2 h_{\text{we}}$

*Based on proposals of Nelson and Unal (*Nucl. Eng. Des.*, **136**, 277 (1992)).

**See Fig. 4.21 for regime definitions and transition criteria.

Nelson and Unal⁵³ determine the axial location of the various film boiling regimes in terms of the distance above the CHF location, z_{CHF} . The film boiling regime boundary locations are correlated in terms of capillary number, Ca , as indicated in Fig. 4.21. The axial location of the critical heat flux in the mostly liquid region of the hot channel must be determined from an appropriate CHF correlation. Since reflooding velocities are low, a modified pool boiling CHF correlation is often used for this purpose [see Eqs. (4.228) and (4.229)].

Substantially improved cooling of the cladding begins at the upper boundary of the transition boiling region. This is usually considered the rewetting point. In the Nelson and Unal⁵³ approach, the length of the transition boiling region is determined by rearranging Eq. (4.240) with q'' corresponding to the minimum film boiling heat flux, q''_{fb} . We then have

$$z_{tb} - z_{\text{CHF}} = \ln(q''_{\text{CHF}} / q''_{fb}) / B \quad (6.22)$$

where

$$B = K Ca^{-0.5} \text{ for } \alpha < 0.8$$

$$K = 16 \text{ (1/m) if } Re_g < 2000$$

$$K = 10 \text{ (1/m) if } Re_g > 2000$$

$$Ca = (V_l \mu_l / \sigma_{\text{CHF}})$$

$$Re_g = (\alpha p_g D_e V_g / \mu_g)_{\text{CHF}}$$

$$\sigma_{\text{CHF}} = \text{surface tension at CHF location.}$$

TABLE 6.IV
Interfacial Drag Correlations For Reflood Phase at LOCA*

Flow Pattern	Characteristic Dimension or Number	Void Fraction	Drag Coefficient or Friction Factor
Bubbly or dispersed bubble (nucleate boiling)	$d_b = 2[\sigma/g\Delta\rho]^{1/2}$ $Y = 0.01 \left[\frac{\sigma De}{0.01197G^2/2\rho_l} \right]^{1/2}$ $\epsilon' = 0.01Y$	$\alpha_{fr} = \alpha - \alpha_{w}$ $\text{if } T_l \leq T_{sat} + 5K$ $\alpha_{w} = \frac{\pi Y}{6De}$ $\text{if } T_{sat} - 5K < T_l < T_{sat}$ $\alpha_{w} = \frac{\pi}{6} \frac{Y}{De} [0.2(T_{sat} - T_l)]$ $\text{if } T_{sat} \leq T_l \quad \alpha_{w} = 0$	$C_{i,} = \frac{\rho_l \alpha_{fr}}{d_b} \left[\frac{1 + 17.671(1 - \alpha_{fr})^{1.3}}{18.76(1 - \alpha_{fr})^{1.5}} \right]$ $\times \frac{(C_1 V_g - C_0 V_l)^2}{(V_g - V_l)^2}$ $C_{i,w} = \frac{2\rho_g}{De} [1.14 - 2 \log_{10}(\epsilon'/De)]^{-}$ Total momentum weighted drag coefficient given by $C_{i,sh} = \frac{0.00175}{\alpha_{fr}} C_{i,}$ $C_{i,sh} = \frac{C_{i,w} V_l^2 + 0.00175 C_{i,fr} (V_g - V_l)^2}{(\alpha_w + \alpha_{fr})(V_g - V_l)^2}$
Smooth inverted annular flow	$R_g = \frac{(De - D_{core})\alpha\rho_g V_g}{2\mu_g}$		$C_{i,si} = 2\rho_g f_{sl}(1 - \alpha)^{1/2} / De$ laminar $f_{sl} = 16/Re$ turbulent $f_{sl} = 0.079Re^{-0.25}$

TABLE 6.IV (continued)

Flow Pattern	Characteristic Dimension or Number	Void Fraction	Drag Coefficient or Friction Factor
Rough-wavy inverted annular flow	$\epsilon' = 80d_d$ $d_d = 1.838 \left[\frac{\sigma}{g(\rho_l - \rho_g)} \right]^{1/2} N_\mu^{1/3}$ $N_\mu = [\mu_g / (\rho_g \sigma)]^{1/2} \left[\frac{\sigma}{g(\rho_l - \rho_g)} \right]^{-}$		$C_{i,rv} = 2\rho_g f_{i,rv}^{-1/2} / De$ if $\frac{\epsilon'}{De} < 1$, $f_{i,rv} = \left[1.14 - 2\log_{10} \left(\frac{\epsilon'}{De} \right) \right]^{-}$ if $\epsilon' / De > 1$, $f_{i,rv} = 0.769$
Dispersed droplet flow	$Re_d = \frac{\rho_l d_d (V_g - V_d)(1 - \alpha_d)^{2.5}}{\mu_g}$ $V_d = V_g - 2.462 \left[\frac{(\rho_l - \rho_g) g d_d}{2\rho_g} \right]^{1/2}$ $\delta_f = \frac{0.0025\rho_l V_g^2}{[g\rho_l - 0.75\rho_g V_g^2 / De]}$	$\alpha_d = 1 - \alpha_f - \alpha$ $\alpha_f = p_c \delta_f / A$ p_c = unheated perimeter A = total flow area	$C_{i,dl} = \frac{C_{i,dd} (V_g - V_d)^2 + C_{if} V_g^2}{[V_g - \alpha_d V_d / (1 - \alpha)]^2}$ where $C_{i,dd} = 0.75\alpha_d \rho_g C_d / d_d$ $C_d = 24(1 + 0.1Re_d^{0.75}) / Re_d$ $C_{i,if} = 2\rho_g f_{i,if} / De$ $i_{if} = 0.0051(1 - 75\alpha_f)$

*Based on correlations of Nelson and Unal [Nucl. Eng. Des., 136, 277 (1992)].

Symbol Definitions for Tables 6.IV and 6.V

- d_b = bubble diameter
 d_d = droplet diameter
 D_{core} = diameter of vapor core
 T_l = liquid temperature
 T_g = temperature of vapor
 V_g, V_l = gas and liquid velocities
 a_i = interfacial area unit vol (1/m)
 α = overall vapor fraction
 α_d = fraction of vol. occupied by droplets
 $\alpha_{f,r}$ = fraction of volume occupied by liquid film
 δ_f = thickness of liquid film on unheated surfaces
 C_i = interfacial drag coefficient
 C_o = momentum weighting coefficients
 h_l = heat transfer coefficient between liquid and interface
 h_i = heat transfer coefficient between vapor and interface

TABLE 6.V
Interfacial Area and Heat Transfer Correlations for Reflood Phase of LOCA*

Flow Pattern	Interfacial Area (area/vol)	Liquid to Interface Heat Transfer (w/m ² °C)	Vapor to Interface Heat Transfer (w/m ² °C)
Bubbly, dispersed bubble	$a_{tb} = 6\alpha/d_b$ d_b = bubble diameter	$T_l \leq T_{sat}$ $Nu_b = Nu_1 + Nu_2$ $Nu_1 = 0.185Re^{0.7} Pr_l^{0.5}$ $Nu_2 = 2 + (0.4\sqrt{Re} + 0.06Re^{2/3})Pr_l^{0.4}(\mu_w/\mu_x)$ $T_l > T_{sat}$ $h_l = [(3.714) \times 10^{-6} \rho_v H_{fg}^2]/T_{sat}^{1.5} = h^*$	$h_v = 1000$
Annular, annular mist	$i_m = a_{tb} + a_{if}$ $a_{td} = \frac{\alpha}{1 - \alpha_d} [6\alpha_d/d_d]$ $a_{if} = \frac{4}{De} \sqrt{\frac{\alpha}{1 - \alpha_d}}$	$T_l > T_{sat}$ $h_l = h^*$ $T_l \leq T_{sat}$ h_l computed as in bubbly flow when $T_l \leq T_{sat}$	$h_v = 1000$

TABLE 6.V (continued)

Flow Pattern	Interfacial Area (area/vol)	Liquid to Interface Heat Transfer ($w/m^2 \text{ } ^\circ\text{C}$)	Vapor to Interface Heat Transfer ($w/m^2 \text{ } ^\circ\text{C}$)
Inverted annular flow (IAF)	$T_l \geq T_{sat}$ $a_{ii} = 4D_{core}/De^2$ $D_{core} = De\sqrt{1-\alpha}$ $a_{i,fr} = 6\alpha_{fr}/d_b$ $a_{total} = a_{i,fr} + a_{ii}$ $T_l \leq T_{sat}$ $a_{total} = a_{ii}$	$T_l > T_{sat}$ $h_l = h^*$ $T_l < T_{sat}$ h_l computed as in bubbly flow when $T_l \leq T_{sat}$	$h_v = 3000$ $h_v = 1000$
Dispersed droplet flow	$i = \frac{6\alpha_d}{d_d} + \frac{6\alpha_{fr}}{De}$	$T_l > T_{sat}$ $h_l = h^*$ $T_l \leq T_{sat}$ h_l computed as in bubbly flow when $T_l \leq T_{sat}$	$h_v = 0.126 \exp(-30P/P_{cr}) [V_g \rho_g (1-\alpha_d)/\alpha]^{0.55}$ $\times \frac{k_g Pr_g^{0.33}}{\mu_g^{0.33} [2\sigma/(g\Delta p)]^{0.725}}$ $\times \left[\frac{(\rho_g \sigma \sqrt{\sigma/(g\Delta p)})^{1/2}}{\mu_g} \right]^{0.4833}$

*Based on correlations of Nelson and Unal [Nucl. Eng. Des. 136, 277 (1992)].

See symbol definitions at the bottom of Table 6.IV.

Heat transfer rate across interface per unit volume of mixture $= a_i (T_g - T_l) / \left(\frac{1}{h_l} + \frac{1}{h_v} \right) (W/m^3)$.

Many investigators have not been willing to relate the onset of transition boiling to hydrodynamic conditions but have preferred to base this point on the so-called rewetting temperature—the temperature at which the liquid can rewet the hot cladding. In Sec. 4.4.1 it was noted that Weisman and Kao⁵⁴ recommend that this be taken as simply $(T_{\text{CHF}} + 100^\circ\text{C})$ where T_{CHF} is the wall temperature at the CHF point. However, Yeh⁵⁵ argues that there is a hydrodynamic effect and that the hot rod can be wetted more easily as the liquid velocity increases. He proposes the empirical correlation

$$T_{\text{rw}} - T_{\text{sat}} = (76.4 - 0.449V_l + 0.1454V_l^2)/(1 + 6.192V_l) \quad (6.23)$$

where

$T_{\text{rw}}, T_{\text{sat}}$ = rewet and saturation temperature, respectively ($^\circ\text{C}$)

$V_l = [V_{\text{in}} - (\dot{m}_v / \rho_l A)] / (1 - \alpha_{\text{rw}})$ = liquid velocity at rewet location (cm/s)

$\dot{m}_v$ = steam flow rate at rewet front (mass/s)

V_{in} = inlet liquid velocity (cm/s)

A = flow area (cm^2)

ρ_l = liquid density (mass/ cm^3)

α_{rw} = void fraction at rewet location.

Both the estimation of the liquid velocity at the rewet location and the critical heat flux require a computation of the void fraction in the mostly liquid region. The Yeh and Hochreiter void-fraction correlation⁵⁶ described by Eq. (3.98) is specifically designed for reflood conditions.

There is a very sharp cladding temperature gradient in the vicinity of the rewet location. This is caused by the change from a very low film-boiling heat transfer to high nucleate boiling heat transfer over the short transition boiling region. The axial heat conduction through the cladding is the major mechanism by which the cladding at the edge of the film boiling region is brought to the rewet temperature. Because of this axial conduction, the cladding temperature begins to drop significantly at a temperature somewhat above the rewet temperature. The temperature at which this drop begins is usually referred to as the *quench temperature*.

Computer programs utilizing generalized thermal-hydraulic models must couple these models to cladding models which allow for axial conduction if the quench behavior is to be adequately predicted. The sharp temperature change in the vicinity of the rewetting point requires very short axial nodes be used in order to model the axial conduction properly. This is impractical in many computer codes and therefore a separate quench front velocity calculation is often carried out.

The simplest modeling of quenching front motion during reflooding uses an empirical equation, based on reflooding experiments, to compute the velocity of the front. This approach has been used by Zemlianoukhin *et al.*⁵⁷

for rewetting of VVER fuel. For 9.15-mm rods with a 0.65-mm zirconium alloy cladding, they propose

$$u_q = 0.37G^{0.7}(1 - T_w/1200)[1 + 0.07(\Delta T_{\text{sub}})^{1/2}] \quad (6.24)$$

where

u_q = quench front velocity (cm/s)

G = mass flux [kg/(m² s)]

T_w = wall temperature 250 mm above quench front (°C)

ΔT_{sub} = local liquid subcooling at quench front (°C).

This approach is coupled with a simple correlation of the heat transfer coefficient as a function of distance from the quench front.

Use of experimentally observed quench front velocities may not be desirable since electrically heated rods filled with high conductivity filler quench more slowly than fuel rods. An alternative approach is to compute the quench front velocity based on analysis of the cladding conduction problem. For a thin cladding exposed to low quench front motion, the behavior may be approximated by a one-dimensional model. It may be shown that the quenching rate is constant with time.⁵⁸ This allows the use of the Lagrangian approach with the coordinate x being based on the distance from the quench front. The basic differential equation for the cladding temperature, T , then becomes

$$\frac{d^2T}{dx^2} + \frac{\rho_c c_p u_q}{k} \frac{dT}{dx} = \frac{h}{k\delta} (T - T_b) \quad (6.25)$$

where

ρ_c = density of cladding

c_p = specific heat of cladding

k = thermal conductivity of cladding

δ = cladding thickness

T_b = temperature of coolant (sink)

u_q = velocity of quench front.

To solve this differential equation, we simplify the problem by ignoring the transition boiling heat transfer. It is assumed that h is constant at h_b in the region below the rewet temperature and h is zero above the rewet temperature. Equation (6.25) thus has one solution for $T > T_{rw}$ and $x > 0$ and one for $T < T_{rw}$ and $x < 0$. By obtaining the two solutions and equating the two temperature gradients at $T = T_{rw}$ we find

$$u_q = \frac{1}{\rho_c c_p} \left(\frac{hk}{\delta} \right) \frac{(T_{rw} - T_b)}{(T_f - T_b)^{1/2} (T_f - T_{rw})^{1/2}}, \quad (6.26)$$

where T_f = initial temperature in film boiling region.

At higher quench rates, the temperature gradients through the cladding (y direction) must be considered. Duffey and Porthouse⁵⁸ show that the two-dimensional solution may be approximated by

$$u_q \approx \frac{2h}{\pi \rho_c c_p} \frac{(T_{rw} - T_b)}{(T_f - T_b)} \quad (6.27)$$

Note that at high quench rates the quench front velocity is independent of the thermal conductivity and thickness of the cladding. Duffey and Porthouse⁵⁸ indicate that the boundary between the solutions of Eqs. (6.26) and (6.27) occurs at $(h\delta/k) \approx \pi^2/4$.

Although axial conduction effects in the fuel are small, this effect may be considered via the exact solution of the quench front propagation velocity problem provided by Yeh and Hochreiter.^{56,59} These authors also note that a typical quench process consists of three stages. In the first stage, encountered at the lowest and coolest level of the core, the cladding temperature is below the rewet temperature and the quench front velocity is identical to the inlet liquid velocity. In the second stage, at slightly higher elevations, the cladding temperature is above the rewet temperature but there are no voids present in the coolant. The quench velocity is then calculated from the applicable quench rate equation using appropriate boiling correlation (e.g., Jens-Lottes) for the wet side h . In the third stage, there are appreciable voids in the coolant and quench rates are lower. Here, the quench velocity is again calculated from the applicable quench rate equation but the wet-side heat transfer coefficient should be obtained by multiplying the boiling heat transfer coefficient by $(1 - \alpha)$.

In the dispersed droplet region above the quench front, the rate of heat transfer will depend significantly on the quantity of liquid carried along with the steam. If a two-fluid model is used as the basis for the computation, the conservation and constitutive equations will provide an adequate description of fluid conditions. If a simple reflooding model is used, then an estimate of the droplets carried over with the steam is required. This may be obtained by use of an entrainment correlation developed for reflood conditions [see Eq. (3.243)]. Alternatively, the total mass effluent rate, above the quench front, $\dot{m}_0$, may be calculated from a mass balance on the mostly liquid region.

$$\dot{m}_0 = \dot{m}_{in} - A(\rho_l - \rho_g) \left[-\frac{d}{dt} \int_0^{L_q} \alpha(z) dz \right] \quad (6.28)$$

where

A = flow area

$\dot{m}_{in}$ = inlet mass flow rate (mass/time)

$\alpha(z)$ = void fraction at elevation z

L_q = quench front elevation.

Sun and Duffey⁶⁰ suggest that Eq. (6.28) may be roughly approximated by

$$\dot{m}_0 = \dot{m}_{in} - \rho_l A u_q \quad (6.29)$$

where u_q is the velocity of the quench front. If it is assumed that all liquid in the fluid stream above the quench front will be entrained, then the droplet mass flow rate, $\dot{m}_d$, is approximated by

$$\dot{m}_d = \dot{m}_0 - \left[\int_0^{L_q} q(z) dz - \dot{m}_{in} c_p (T_{sat} - T_{in}) \right] / h_{fg} \quad (6.30)$$

where $q(z)$ = total heat input per unit length per unit time (energy/Lt) providing a saturated mixture of liquid and vapor is present at the quench front and $\rho_l \gg \rho_g$.

The time variation of the hot rod cladding temperature is obtained by coupling the hydrodynamic and wall-fluid heat transfer to a fuel rod conduction model. The heat transfer calculations are, however, complicated once the cladding temperature reaches the level where the internal gas pressure causes cladding ballooning and bursting. Early computations of the effect of the channel blockage due to ballooned and burst rods were based solely on the effect of the reduced steam flow through such channels. Allowance for the increased turbulence in the blocked region provided some amelioration of the increased temperature caused by the lower flow.⁶¹ More recent work⁶² recognizes that the reduced size of the passage causes droplet atomization. This, together with flow acceleration and turbulence enhancement, increases heat transfer. Experimental data⁶² show that at area blockages of about 60% the improvement in heat transfer due to droplet atomization dominates. In an artificial area blockage of 90%, reduction in mass flow was sufficient to counteract the atomization effect and a slight increase in peak temperature was observed. However, experimental measurements of fuel rod burst strains indicate that the actual circumferential burst strains are about 50%.⁶³ With unidirectional flow through the bundle during the reflood phase, this limits the maximum cooling channel blockage to about 70%. At this blockage level, the peak cladding temperature would not be expected to exceed that calculated for an unblocked channel.

6.5.4 Fluid-Structure Interactions

6.5.4.1 Forces on Internal Structures

In the subcooled and early saturated blowdown portions of the accident, the designer is concerned with the effect of pressure pulses (waterhammer) on the core, reactor vessel structure, and loop components. This is of particular concern in modeling core-barrel behavior. If the accelerations caused by the pressure-pulse loads are sufficiently large, they could cause deformation of fuel elements and control rods. Such damage must be prevented if the reactor is to be safely shut down.

To obtain the forces acting around the system, a detailed hydraulic analysis is required. By solving the wave equations (Sec. 3.3.5) algebraically, Fabric⁶⁴ developed a relatively rapid method for computing system behavior during subcooled blowdown. The solution procedure requires the assumption of a constant sonic velocity, and, therefore, cannot be used once portions of the system fall below saturation pressure. Since significant pressure waves can still exist, it may be desirable to extend the analysis into the two-phase region. Fabric⁶⁴ has done this using the method of characteristics that was outlined in Sec. 4.1.2. A similar procedure was followed by Goulding,⁶⁵ but Nahavandi,⁶⁶ on the other hand, chose to represent the system by a node-and-branch model (Sec. 4.1.2.4) and to integrate the resulting differential equations numerically.

All of these early approaches computed the forces on the structural members assuming that the structural members were rigid. This is not the case and appreciably lower forces are computed when proper allowance is made for structure flexibility.⁶⁷ In the case of the core barrel, the structure moves in the direction of the load. This reduces the pressure on one side while increasing the pressure on the other side. The peak load is thus significantly lower than computed assuming a rigid barrel.

The method of characteristics can be extended to include fluid-structural interactions. Takeuchi⁶⁸ accomplishes this by writing the mass and momentum conservation equations using a variable flow area, A . For a one-dimensional system these conservation equations may then be written as

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial z}(GA) = 0 \quad (6.31)$$

and

$$\frac{\partial}{\partial t}(GA) + \frac{\partial}{\partial z}(uGA) + A \frac{\partial P}{\partial z} = -\rho FA \quad (6.32)$$

where u = fluid velocity (length/time); G = mass velocity (mass/area time); and F = sum of friction and gravitational head losses per unit length. The characteristic equations then become

$$\frac{dG}{dt} + \frac{1}{a} \frac{dP}{dt} = u \frac{d\rho}{dt} - F \pm a\rho \frac{\dot{A}}{A} \quad (6.33)$$

with $dz/dt = u \pm a$, $\dot{A} = dA/dt$, and a = sonic velocity. The hydraulic force on a component is a function of the pressures determined by solution of the hydraulic equations. The flow areas are a function of the structural deformation, which is determined by the equation of structural matrix. The terms $a\rho \dot{A}/A$ provides the coupling between the hydraulic and structural systems.

A fully adequate representation of the system requires a two-dimensional representation of the system (axial and circumferential dimensions). In a one-dimensional system, a given point, say, point P , would receive information from points upstream and downstream (x direction). The characteristic equations [Eq. (6.33)] may then be approximated by two finite difference equations. In a two-dimensional system, a given point receives information from four adjacent points. Two additional finite difference equations representing the characteristic equations in the y direction must be added.

The use of Cartesian coordinates restricts a sound wave to travel along the Cartesian directions. To account for the increased distance the sound wave must travel, the sonic velocity must be increased by a factor of $\sqrt{2}$ for a two-dimensional problem ($\sqrt{3}$ for three dimensions).⁶⁸

Most currently used (1993) computer programs have used two-dimensional finite difference models of the fluid behavior. A typical example of such a code is SOLA-FLX,⁶⁹ which represents the fluid between the core barrel and vessel as an unwrapped slab. The thickness of the flow zone is governed by the motion of the barrel which is obtained from the fluid pressures applied to the barrel. The inlet and outlet nozzles are represented by point sources and sinks, respectively. The barrel shell is modeled by a 3-D finite difference form of the standard shell equations. To avoid further complications, the codes linking fluid and structural effects have all been restricted to single phase behavior. This is reasonable since the largest forces occur at the initiation of the accident where the system is still single phase. In a typical large LOCA, the pressure differentials across the barrel become negligible in less than 50 ms.

If a pipe break were to occur at a steam generator exit line, significant forces may be exerted on the divider plate separating the flow streams in the lower head. An analysis of the effect of this loading on the divider plate is desirable.⁶⁷

The pressure forces on components are sensitive to the time period over which the pressure at the break is assumed to go to atmospheric (break time). The shorter the assumed break time, the larger the forces. For the sake of conservatism, break times of a few milliseconds are usually assumed.

6.5.4.2 Blowdown Forces on Primary System Piping

The decompression and compression waves moving at sonic speed through a piping system after a pipe ruptures produce substantial forces on the piping system. In a piping segment bounded by an elbow or bend at either end (bounded segment), the pressure at one end of the segment exceeds the pressure at the other end, thus producing a net longitudinal force. In

the segment that has ruptured, there is also a net thrust created by the discharging fluid.

The net force, F_w , on a piping segment, is obtained from a force momentum balance. If we ignore body forces, we have

$$F_w = - \int \int P dA + \int \int V(\rho V dA) + \frac{\partial}{\partial t} \int \int \rho V dv \quad (6.34)$$

where V = fluid velocity and v = volume.

In a bounded segment, the sum of the first two terms of Eq. (6.34) is zero and the net time-dependent force (wave force) is due solely to the last term on the right-hand side of Eq. (6.34). For a pipe of constant area, A , we then have⁷⁰

$$F_w = \frac{A}{g_c} \frac{\partial}{\partial t} \int_{z_l}^{z_f} \rho(z,t) V(z,t) dz = \frac{A}{g_c} \frac{\partial}{\partial t} \int_{z_l}^{z_f} \frac{W(z,t)}{A} dz = \frac{L}{g_c} \frac{dW(t)}{dt} \quad (6.35)$$

where $W = \rho V A$ and L = segment length.

In the segment containing the break (open segment), the pressure and momentum force sum is not zero. The time-dependent force on an open segment of constant flow area is then

$$F_w = AP(L,t) + \frac{A}{g_c} \rho(L,t) V^2(L,t) + \frac{L}{g_c} \frac{dW(t)}{dt} \quad (6.36)$$

(blowdown force) (wave force)

Note that, even if the blowdown rate is constant, there will still be a force on an open segment.

The blowdown and wave forces produced by a large break would be quite substantial. The pipe support system must be designed to accommodate these forces. In addition, the jet of fluid discharging from the break would produce an impingement force on any object in its path. If the entire jet is intercepted by a single component, the thrust on that component is equal to the blowdown force on the open pipe segment from which the discharge originated.⁷¹ The jet issuing from the broken pipe expands to area A_∞ at some distance from the break. Moody⁷¹ estimates that A_∞ is approximated by

$$A_\infty = \frac{W^2}{g_c F_w \rho_\infty} \quad (6.37)$$

where ρ_∞ = homogeneous density of discharge fluid mixture at containment conditions. An object in the path of the jet having interception area A_I , where $A_I < A_\infty$, would be subject to an impingement force of $(F_w A_I)/A_\infty$.

Computation of the wave and blowdown forces requires a correct evaluation of the time variation of the fluid pressure, velocity and density in the pipe segments of interest. These quantities may be determined from a

one-dimensional systems model providing the timesteps chosen are sufficiently small so that the pressure pulses may be followed [see Eq. (4.74)] during the early blowdown period. A post-processor may be added to such a program to automatically make these force calculations.⁷⁰

6.5.5 Evaluation Models for LBLOCA Analysis

6.5.5.1 *Structure of Evaluation Models*

Any of the techniques capable of following the waterhammer analysis into the two-phase region must provide a detailed representation of the reactor system. Therefore, such computations of system behavior, covering more than a few hundred milliseconds of real time, become very expensive. However, we have noted that in about 50 ms, the pressure pulses are generally damped to levels that are no longer of concern. The designer is then concerned with determining the rate at which core temperatures are changing. To accomplish this, he needs an estimate of the rate at which the system is drained and the time histories of the pressures, flows, and froth level within the core. It was initially concluded that an approximate representation of the system would be adequate for the saturated blowdown phase. Margolis and Redfield³² and Redfield and Murphy⁴⁹ used this approach in the early FLASH programs.

In the initial FLASH programs, the system was represented by a limited number of volumes, as shown in Fig. 6.3. Each volume was considered a node, and conservation of mass and energy equations were written for each node. Connections between volumes were considered branches; a one-dimensional momentum equation based on homogeneous flow was written for each branch.

A bubble rise model allowed a steam phase and a steam-water mixture region to exist in each of the large nodes. The rate of system depressurization and liquid level in the core is determined by the critical flow out of the break (indicated as "leak" in Fig. 6.3). In an early version of the program the Moody model,⁷² or this model adjusted by an empirical multiplier, was used for critical flow computations.

A continued study of system behavior led to the conclusion that the limited number of nodes of the early FLASH codes was adequate for small break areas, but was insufficient for large double-ended breaks. Newer versions of the FLASH code,⁷³ capable of considering a large number of control volumes, are now available. The improved codes also contain improved modeling of system components and improved numerical procedures.

A number of proprietary codes (e.g., SATAN, CRAFT, MODFLASH), either based on FLASH or using similar branch and node techniques, have been developed. The most important of the FLASH derivatives is RELAP (Ref. 74) which was developed for the U.S. Nuclear Regulatory Commission and is in the public domain. The RELAP code has been extensively com-

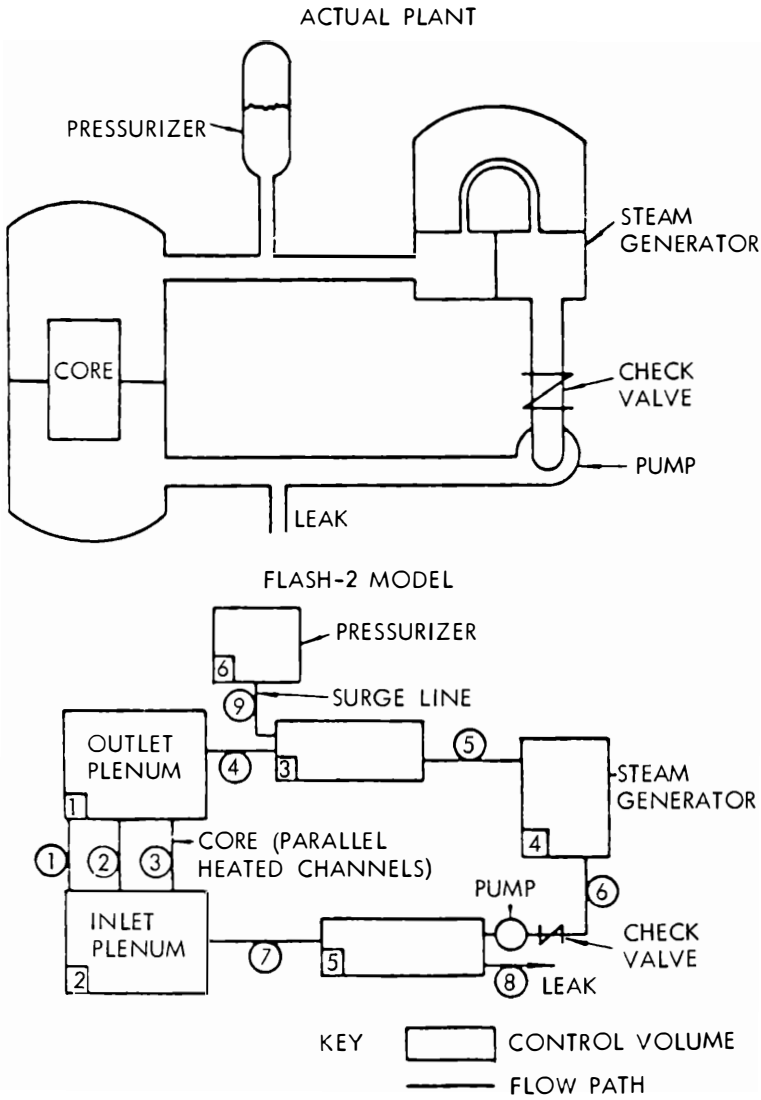


Fig. 6.3 The FLASH representation of a single-loop reactor plant [from *Nucl. Appl.*, 6, 127 (1969)].

pared to system blowdown data and has been modified to obtain agreement with the available data.

Essential system features that are modeled by RELAP are shown in Fig. 6.2. The fluid system is modeled by fluid volumes (nodes) and by fluid junctions (flow paths or branches) between the volumes. The RELAP-4 computer program⁷⁴ uses a one-dimensional modeling approach in treating the

reactor system transient response to postulated accidents such as coolant loop rupture, circulation pump failure, power excursions, etc. The RELAP-4 code solves an integral form of the fluid conservation and state equations for each user-defined volume and computes a continuing record of system conditions. The user can also enter the fluid conditions as special boundary conditions.

The major parts of the RELAP-4 program are (1) models of fluid behavior, (2) heat transfer, and (3) reactor kinetics. To obtain fluid behavior, standard conservation equations, given in differential form for fluid mass, energy, and flow, are used to derive differential stream-tube fluid equations. These are integrated over mathematically defined "control-volumes" to obtain mass, energy, and flow values. Thermodynamic properties are defined by state-property relations in terms of specific internal energy and density. The resulting set of simultaneous equations is linearized and advanced for a small time increment by a fully implicit numerical technique. The quantities, known within any volume after the mass and energy equations are integrated over a timestep, are total water mass, total air mass, and the combined internal energy of both water and air (air considered when vapor container pressure is determined). From these variables, specific thermodynamic states of water and air are determined.

A tabular representation of pump behavior forms part of the RELAP input. The data table is derived from two sets of homologous curves representing single phase and high-void behavior. An empirical weighting factor, which is a function of void fraction, allows interpolation between the two data sets for any set of pump conditions.

Heat transfer to and from the fluid in given control volumes is described by a heat conductor model, where the conductors can be described as rods, pipes, cylindrical vessels, and plates. The heat addition rate to the fluid of a volume is calculated as the product of surface heat flux and heat transfer area at the conductor surface in contact with the fluid.

When a portion of the core is uncovered, the heat generated in that region must include any additional heat that can be supplied from the exothermic zirconium-water reaction. When sufficient steam is available, the rate of this reaction is conservatively assumed to follow Baker and Just's parabolic rate law⁷⁵ (see Sec. 2.6.2). When less steam than would be consumed by a reaction following the Baker-Just equation is available, a steam-limiting rate law is assumed in which all available steam reacts. The RELAP-4 code does not account for the consumption of steam and the evolution of hydrogen during the metal-water reaction in the mass and momentum equations. The RELAP-4 code conservatively assumes that the inside surface of the cladding continues to react after rupture occurs. The reaction within the cladding is limited to that portion of the rod(s) modeled within the heat slab where the rupture occurs. It is assumed that an oxide layer is not present on the interior of the cladding at the time of rupture.

The model delivers the heat of the interior reaction directly to the exterior of the cladding.

The RELAP-4 code uses a point reactor kinetics model to calculate power, while reactor kinetics equations are solved using a method similar to that employed in the IREKIN program.⁷⁶ Contributions to the reactivity include a time-dependent (scram) reactivity and such feedback effects as fuel temperature, water density, and water temperature. In a LBLOCA, feedback effects are not significant because a scram occurs quickly. The total power in a given region is the sum of direct fission and radioactive decay power.

Figure 6.4 illustrates a typical nodalization used for the evaluation of a LBLOCA with RELAP-4. Note that a single loop, of appropriate size, used to model all of the intact loops. The total number of nodes used is substantially greater than those used in the early FLASH representation illustrated in Fig. 6.3. In particular, note that the core is now modeled by two series of axially stacked nodes. One of the linearly interconnected set of nodes represents core average behavior while the second set represents the hot channel. In addition, the core downcomer is explicitly modeled.

A systems code, such as FLASH or RELAP, requires a great deal of computer time. At the end of the blowdown period, flow rates are quite low and a number of approximations can be made. In particular, momentum storage terms are negligible, thus allowing a simplified (quasisteady state) momentum equation to be used. Much longer timesteps can then be taken. In addition, a drastically reduced number of control volumes, or nodes, are satisfactory for representing the system. The longer timesteps and smaller number of nodes allow a much more rapid determination of system behavior. The WRELFLOOD code⁷⁷ is an example of this approach. Most reactor vendors use simplified codes of this nature to analyze system behavior during reflood.

The need to keep the running time of system behavior codes reasonable limits the number of nodes or control volumes that can be placed within the core. Under some circumstances, results of the systems code may not provide a sufficiently detailed view of core behavior. When this is the case, significant results of the systems calculations, core-inlet flow, and enthalpy as a function of time, are used as input to a more detailed calculation of core temperatures. Such calculational procedures were described by Walters *et al.*,⁷⁸ Iyer,⁷⁹ and Cybulskis *et al.*⁸⁰

The last step in analyzing core behavior can be the employment of a transient fuel element code, such as FRAP-T (Ref. 81) (see Sec. 2.6.2). The fuel element code takes its input from a core heat transfer code or a system code such as RELAP. In the latter case, the fuel rod code replaces the core heat transfer code. Alternatively, the core heat transfer code can contain sufficient fuel rod modeling so that a separate fuel rod code is not needed to meet regulatory requirements. Fuel rod modeling in such core heat transfer codes generally has been much less complete than in FRAP-T.

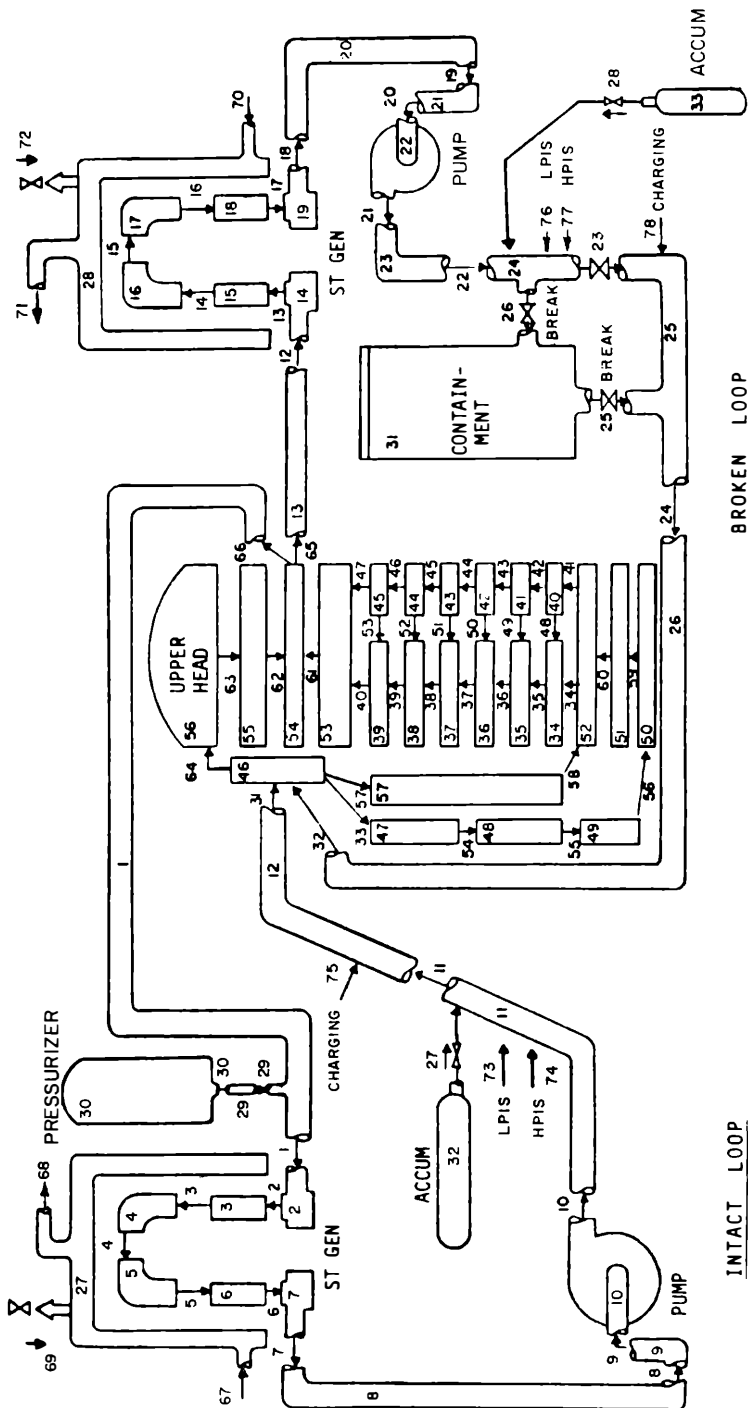


Fig. 6.4 Typical system nodalization diagram for a RELAP-4 blowdown evaluation.

The WRAP-EM system⁸² is a publicly available program which links GAPCON, RELAP-4, a refill-reflood model and FRAP-T in a single package. The steady-state fuel design code GAPCON provides initial fuel rod conditions to the RELAP module. RELAP then computes system behavior during the blowdown phase. The core hydraulic conditions during blowdown are transferred to FRAP-T to be used in hot fuel pin analysis. Once the blowdown concludes, system behavior is determined by the refill-reflood model (FLOOD) with FRAP computing fuel temperatures.

The FLOOD code module⁸³ in WRAP allows for the fact that, when water first contacts the core, the steam produced increases core pressure and pushes the water back. Vaporization then diminishes, pressure falls, and reflooding rate increases sharply. The process continues in a cyclic manner as the core is recovered. In the FLOOD code, these system dynamics are coupled to detailed core cooling calculations.

The ATHLET code system,⁸⁴ developed in Germany, consists of the DRUFAN module for blowdown and the FLUT module for refill and reflood. DRUFAN is a 1-D, single fluid, drift flux model using the node and branch approach. FLUT is a 1-D, two-fluid model which divides the reactor system into a series of control volumes. A modified version of the FLUT module has been developed for application to tight-lattice cores.

6.5.5.2 Conservatisms Included in Evaluation Models

In the United States, the Code of Federal Regulations (10CFR50, Appendix K) imposes specific restrictions on the models that may be used during a LOCA evaluation. These are based on models having a sophistication similar to the WRAP-EM package.⁸² The major conservative assumptions mandated by these regulations are:

1. *Stored energy:* Initial steady-state temperatures to be determined using those correlations yielding the highest cladding temperature or optimally the highest stored energy in the fuel.
2. *Decay heat:* Heat generation rates from radioactive decay are 1.2 times the 1971 ANS Standard. (This is the equivalent of the 1973 ANS standard curve shown in Fig. 1.38.)
3. *Metal-water reaction:* Rates based on the conservative (high rates) Baker-Just model. If cladding ruptures, both inner and outer surfaces are assumed to react.
4. *Discharge from break:* Critical flow is based on the conservative Moody model⁷² multiplied by discharge coefficients varying from 0.6 to 1.0. The discharge coefficient to be used is that which results in the highest cladding temperature. [The Moody model determines critical flow based on the assumption of thermodynamic equilibrium and that the

slip ratio (S) is $(\rho_g / \rho_l)^{1/3}$. If the Moody critical flow rates are multiplied by 0.6, the resulting product is close to HEM predictions.]

5. *ECCS bypass*: During most of the blowdown period for a cold-leg break, the ECCS injection water must be assumed to be ineffective in refilling the system and not to contribute to the water inventory. The end of the bypass period is to be taken as the end of upward flow in the downcomer or at an upward steam velocity below that at which droplet entrainment occurs.
6. *No return to nucleate or transition boiling*: Once CHF has occurred in the blowdown period, no return to nucleate or transition boiling is allowed during blowdown. This is required even if subsequent calculations indicate the heat flux is below the critical value or the wall temperature drops below that at which transition boiling occurs. Return to nucleate boiling must be postponed until the reflood period.
7. *Film boiling correlations*: Conservative correlations (tending to underpredict data) are required for the film boiling region.
8. *Single failure*: The calculations must assume that one of the ECCS components fails. The failure leading to the most damage must be that considered. (This usually means that one of the two ECCS trains is taken as inoperable.)

Use of the foregoing assumptions leads to peak cladding temperatures in excess of those which would be expected based on experiments such as LOFT. The Loss-of-Fluid Test (LOFT) facility⁸⁵ was a small reactor designed to model a four-loop PWR by retaining the same volume to power ratio on a 1:50 volume scale. The small reactor core was 1.7 m high and contained nine square and four triangular assemblies of standard diameter PWR rods. Both large- and small-break LOCA experiments were conducted and the results compared to computer program predictions.

6.5.6 Best Estimate Computer Models for LBLOCA Analysis

The best estimate models were originally developed to obtain model predictions in agreement with available experimental data. The degree of agreement between such predictions and the variety of experimental data available is usually taken as an indication of the degree to which the actual physical phenomena are understood. Best estimate models also allow the degree of conservatism present in the evaluation models to be quantified.

Best estimate computer models, such as TRAC,⁷ RELAP-5,⁵ CATHARE,⁸ and COBRA-TRAC⁹ differ from the evaluation models by (1) modeling the system as a series of control volumes instead of a set of nodes and connecting branches, (2) using a two-fluid model instead of a single fluid model, (3) using the same program throughout the blowdown and reflood

instead of a separate program for the refill and reflood portions of the accident, and (4) using best estimate correlations for the various physical phenomena encountered instead of conservative or bounding correlations.

In a LOCA evaluation using a one-dimensional best estimate code, the division of the primary system into control volumes is generally essentially the same as that shown for the RELAP-4 noding indicated in Fig. 6.4. However, the pressurizer would usually be represented by a series of vertically stacked volumes instead of a single node. No special pressurizer component model would be used and the interphase-transfer process in the pressurizer would be determined by the basic two-fluid model incorporated within the program.

In a program such as COBRA-TRAC with three dimensional capability, the primary system, with the exception of the core, is represented in a one-dimensional manner. The 1-D system representation is appropriately linked to three dimensional representation of the reactor core.

The core behavior computed by a best estimate program differs considerably from that obtained with the standard evaluation model assumptions. Since the evaluation model does not allow a return to nucleate boiling and all ECCS water injected during the core bypass period is presumed lost, an evaluation model will generally show a continuous increase in cladding temperature until the steam cooling from the recovered portions of the core finally reduces the temperature. A best estimate model computation for a cold-leg LBLOCA will generally show three peaks. The cladding temperature begins to increase during blowdown after the CHF is reached. The first peak (blowdown peak) occurs when the lower heat flux allows a return to nucleate boiling. The temperature again increases when the fuel is uncovered. At the end of ECC bypass, the liquid ECCS inventory held outside the reactor can enter the vessel and this causes a relatively small temporary decrease in cladding temperature (first reflood peak). The cladding temperature then rises again and does not fall again until the steam from the recovered portion leads to another reflood peak shortly before final quench. For the usual large plant (as of 1994) the final quench occurs 100 and 150 s after initiation of the accident.

It is now possible to use a best estimate program as the basis for a license evaluation of a LOCA. The evaluation would still have to be carried out assuming the most damaging simultaneous failure. However, use of a best estimate approach eliminates the requirements for use of 120% of the 1971 ANS standard decay heat, full ECCS bypass during blowdown, no return to nucleate boiling, and zirconium-steam reaction rate from the Baker-Just equation. Break flow rates would not have to be based on the Moody critical flow equation.⁷² Evaluation of accident behavior using the critical flow rate at several assumed break locations would be needed. The discharge from a break near a reactor vessel nozzle would not be at thermodynamic equi-

librium and critical flow rates appreciably above the HEM values would be obtained for this location.

The 1988 revised rule by the U.S. NRC on ECCS acceptance criteria provides that best estimate methods may be used provided the license submission quantifies the uncertainty of the estimates and includes this uncertainty when the calculated results are compared to the mandated acceptance limits. In addition to quantifying the uncertainties in the results, it is necessary to demonstrate that the best estimate code shows the capability for scale up from test facility information to full-scale plant application and that the code is applicable to the scenario for which it is to be used. Boyack *et al.*⁸⁶ have developed what they call "*Code Scaling, Applicability and Uncertainty*" (CSAU)¹¹ evaluation methodology to answer these questions. The applicability of the computer code is determined by (1) comparing documented code capabilities with the analysis requirements and (2) evaluation of the correlations and closure relationship used in the code against their applicable data base. The scaling ability and accuracy of the program are evaluated by comparing the results of computer code predictions with experimental data from a number of LOCA or separate effects tests. The ability to scale is seen by the examination of tests on models of varying size. By varying model parameters, the variation needed to gain agreement with variable experiments is determined.

The effect of the uncertainty in code modeling of parameters on key acceptance criteria is evaluated by determining the effect of model parameter changes on code results (e.g., the peak cladding temperature). The phenomena that have the greatest effect on the peak temperature are fuel stored energy, core heat transfer, pump performance, critical break flow, ECCS bypass flow, and steam binding (see Sec. 6.5.3.2).

In addition to the modeling uncertainties, uncertainties arise due to uncertainty in the knowledge of reactor state and operating conditions at the beginning of the transient. To combine the individual modeling and state parameter uncertainties, a Monte Carlo sampling approach was used. All of the parameter and state uncertainties can be conservatively represented by a uniform distribution over the expected range. By using slightly less than 200 computer runs, for a full-scale plant, an acceptable response surface can be generated.⁸⁶ This in turn provided a statement of uncertainty in terms of a mean peak clad temperature and the safety margin necessary to ensure that the actual temperature would be below the limit at least 95% of the time. To this margin one must add any separate bias for factors whose uncertainty is not evaluated but whose effect was determined by a bounding calculation.

Boyack *et al.*⁸⁶ have applied CSAU methodology to a cold-leg LBLOCA in a four-loop plant using the TRAC-PFI/MODI program. They found that the total safety margin (including biases) required for a 95% probability

limit was about 200°C for the first reflood peak and about 455°C for the second reflood peak. However, the mean peak cladding temperature declined from about 630°C during blowdown to about 400°C at the second reflood peak. They found that for a typical LBLOCA the maximum cladding temperature (mean value plus uncertainty) is only about 910°C, which is well below the 1200°C regulatory limit.

6.5.7 Initial Conditions for Conventional PWRs

The core power distribution along the hot channel prior to initiation of LOCA will, of course, significantly influence the maximum cladding temperatures calculated by the accident analysis. Since the cost of a LOCA analysis is high, it is expensive to repeat the calculation for all the possible power distributions. To avoid the necessity for repetitive calculations, it was common practice to conduct the system analysis for base loaded plants using a very conservative power distribution along the hot channel. One way this can be done is to assume that the power distribution along the hot channel corresponds to the upper envelope of $F_q(z)$ determined by the power distribution studies previously described. If the hot fuel rod can be shown to meet the regulatory requirements under these conditions, it is clear that the design is satisfactory.

Current regulatory requirements make it difficult to show compliance using a bounding power distribution. Further, load follow requirements on many plants lead to significant rod motion with axial skewing of the flux (significant axial offset, AO). Current practice is to assume a particular axial offset along with the most adverse radial power distribution which can prevail with that AO. The peak cladding temperature, fuel oxidation, hydrogen generation, and cladding distortion that result if the accident begins at full power are then computed for the limiting LBLOCA. If any computed quantity is above the allowable limit, additional computer runs are made to determine the power at which the limits are met. The computation is then repeated for a series of axial offsets. The limiting powers are then used to determine an axial offset versus allowable power curve (AO tent) similar to that shown in Fig. 1.37

6.5.8 Long-Term Recovery Following LBLOCA

The assumption that the core will remain recovered once initial recovery has been completed may not hold under some conditions. Shortly after a large cold-leg break, the loop seals in the reactor coolant pump suction piping are likely to refill and clear in an oscillatory fashion. However, when the loop seal clearing takes place at a low power, it is possible to depress the loop seal quasistatically. The upper portion of the core could then be uncovered for a significant time period.

This problem is not of concern in those plants containing internal vent valves, which provide a large path for core steam to reach the cold-leg break. For plants without vent valves, the lower the elevation of the loop seals the greater the allowable depression and hence the greater concern.

The behavior⁸⁷ may be understood by reference to Fig. 6.5. The steam generated in the core can flow to the break through three paths. For simplicity, we shall ignore the flow through the upper head bypass (path 1) and assume steady-state pressure drops apply. In path 2, steam flows through the heat exchanger and up the loop seal. The pressure drop through this path is approximated by

$$\Delta P_2 = \rho_f (1 - \bar{\alpha}_{LS}) (\Delta z_{LS}) g \quad (6.38)$$

where $\bar{\alpha}_{LS}$ = average void fraction on upflow side of seal and Δz_{LS} = elevation change, cold leg to loop seal inside bend (see Fig. 6.5). In path 3, flow is downward from the upper plenum, through the core and up through the downcomer. Hence, the pressure drop is approximated by

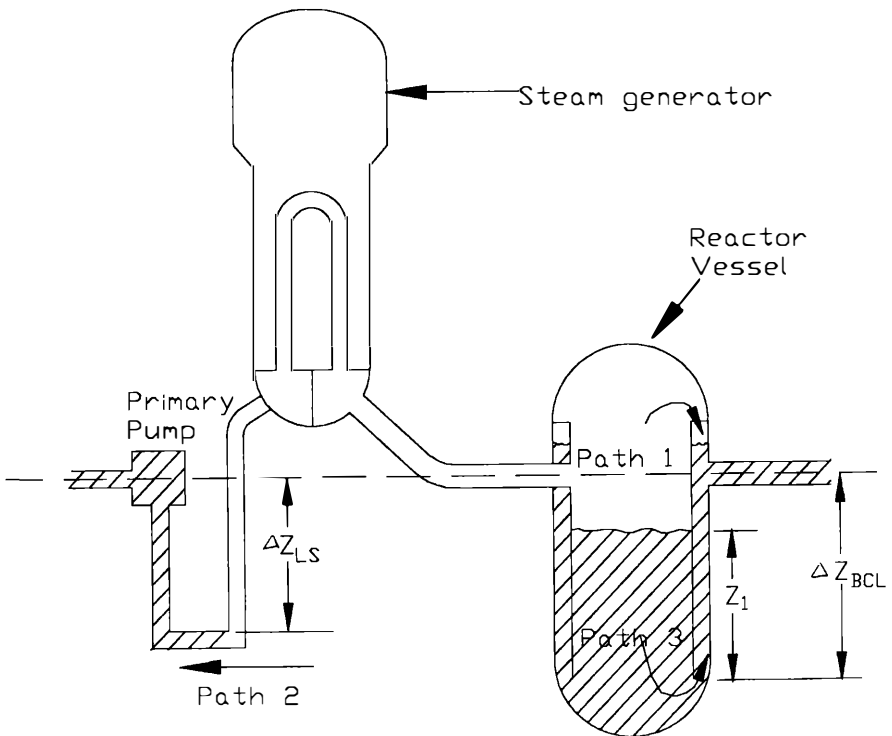


Fig. 6.5 Loop seal behavior during the post-reflood period.

$$\Delta P_3 = \rho_f (\Delta z_{BCL} - z_1)g \quad (6.39)$$

where Δz_{BCL} = elevation change, core bottom to cold leg (see Fig. 6.5) and z_1 = collapsed liquid level in core. By equating Eqs. (6.38) and (6.39), we have

$$z_1 = \Delta z_{BCL} - (1 - \bar{\alpha}_{LS})\Delta z_{LS} \quad (6.40)$$

The actual level in the core will be $(z_1 / \bar{\alpha}_c)$ where $\bar{\alpha}_c$ is the average void fraction in the core. The collapsed liquid height in the core, z_1 , will be a minimum when $(\bar{\alpha}_{LS})$ is zero.

Based on test in a 1/48 volumetrically scaled, full height and pressure facility, Kukita *et al.*⁸⁷ conclude that a quasistatic core level depression can take place if the break size is large enough to allow termination of continuous single or two-phase natural circulation and if the core steaming rate is almost balanced with the steam condensation rate in the primary system. As the core power decreased, the core level depression is reduced and finally disappears.

Computations of long-term core recovery behavior⁸⁸ using RELAP-5 indicated an oscillating behavior of the core level. This oscillation was not observed experimentally.

The fact that a portion of the core uncovers after initially reflooding does not necessarily mean that unacceptably high fuel temperatures result. The uncover appears to occur late in the transient when decay heat generation is relatively low. Further, the steam produced by the covered portion of the core is available to cool the fuel elements.

6.5.9 LBLOCA in CANDU Reactors

The most severe LBLOCA in a CANDU reactor corresponds to the guillotine break of an outlet or inlet header. The pressure tubes connected to the broken header void quickly (in ~ 1 s). The rapid voiding, coupled with a positive coolant reactivity coefficient, leads to a significant power pulse prior to reactor shutdown by rod motion. This high initial power and reduced cooling leads to rapid increase in cladding temperature. Once the power is reduced, the rapid steam flow out of the break reduces the cladding temperature. In an outlet header break, the initial peak in cladding temperature is in the neighborhood of 1200°C (Ref. 89). This occurs at about 7 s after initiation of the accident.

As the steam flow out of the break diminishes with lower system pressures, the cooling of the fuel is reduced. Cladding temperature again rises. A second cladding temperature peak of $\sim 900^\circ\text{C}$ is reached in 30 to 35 s (Ref. 89). This corresponds to the time at which ECC begins to reach the fuel channels. A final quench occurs at about 55 s after accident initiation.

The emergency core cooling system providing protection against the consequences of a LOCA is generally similar to that used in a vessel-type re-

actor. A gas pressurized accumulator provides coolant during the early stages of the accident. Pump supplied medium and low pressure injection systems provide emergency core cooling during the latter phases of the accident.

Banerjee and Hancox⁹⁰ describe the methods for analyzing a LOCA in a CANDU-type PWR. The blowdown phase of the accident is analyzed using an explicit finite difference technique embodied in the RODFLOW code. The RODFLOW code uses the control volume approach rather than a node and branch model. Modeling based on the assumptions of thermodynamic equilibrium and homogeneous flow was shown to fit the results of blowdown experiments.

The assumption of homogeneous flow is inappropriate for modeling the reflooding of a horizontal pressure tube. A control volume contains a specified axial length of the pressure tube and the rod bundle contained within it. During reflood, conditions are not uniform within such a control volume. The injected water first runs along the bottom of the channel rewetting the lower elements. The steam produced rises toward the top of the channel and flows along gaining substantial superheat (up to several hundred degrees). Both water and steam streams cool the bundle, but the elements in the steam see a lower heat transfer coefficient and stay at a higher temperature. As injection continues, the water levels rise and elements are progressively rewet starting from the end farthest from the break.

To model the complex rewetting process, Banerjee and Hancox⁹⁰ considered the difference in temperatures between phases. However, they assumed that different velocities were not required since the two streams tend to be homogenized at the heated section exit and entrance by channel hardware. Therefore, they wrote a single momentum equation with $V_{\text{gas}} = V_{\text{liquid}}$, but with separate energy and mass balance equations for each phase. The mass balance equations are solved using an implicit finite difference procedure. The liquid level is followed and the rates of heat transfer to the steam and liquid portions are adjusted according to this level and the previous thermal history of the fuel in the two regions.

Later analyses have used the FIREBIRD III or SOPHT codes.⁸⁹ Both of these programs use similar one-dimensional single-fluid models. The conservation equations are expressed in finite difference form and solved by an implicit solution procedure. The assumption of equal fluid velocities used by RODFLOW is removed and vapor and liquid velocities are computed via drift flux or slip correlations. In both codes, the entire pressure tube is modeled by a set of serially connected segments. However, a separate hot rod calculation allows a calculation of peak cladding temperatures. In FIREBIRD III, a flow pattern map allows a determination of when stratified flow occurs. A reduced coolant heat transfer coefficient may then be applied in the hot rod calculation. SOPHT does not contain a flow map but the user can choose sets of correlations to evaluate the heat transfer

coefficients under given options. The SOPHT code also contains a reasonably detailed model of plant components and control systems so that it may be used for accidents other than a LOCA.

To examine the power pulse that follows a LBLOCA in a CANDU reactor, the early FIREBIRD versions used a point kinetics model. Later versions are coupled to a 3-D kinetics program. In addition, a multiple channel model of the reactor core is used in order to obtain the effect of spatial distribution of core void rates on core reactivity.

CATHENA,⁸⁹ a more recent best estimate model code, resembles FIREBIRD in many respects but uses a one-dimensional two-fluid model that considers thermodynamic nonequilibrium. The conservation equations are solved using a one-step semi-implicit staggered-grid finite difference approach.

An additional concern in CANDU reactors is the behavior of the pressure tube during a LOCA. When flow stratification occurs in a fuel channel, both the exposed upper fuel rods and adjacent portion of the pressure tube heat up. This could lead to a significant temperature gradient around the tube with unacceptably high strains. Computation with the AMPTRACT code⁹¹ indicates that the emptying of the channel (in about 1 s) leads to a relatively flat temperature distribution around most of the tube circumferences. It is expected that the pressure tube would strain radially into contact with the calandria tube. The stored energy would then be transferred to the moderator and pressure tube integrity would be preserved.

6.6 SMALL-BREAK LOSS-OF-COOLANT ACCIDENT (SBLOCA)

The SBLOCA is generally considered to be a LOCA initiated by a primary system rupture with a break flow area of less than 0.5 ft². Such an accident can be produced by the rupture of an instrumentation line, rupture of a steam generator tube, small size break in the main piping of the primary system, a stuck open relief valve, or the rupture of a control rod housing. The latter accident is a special case since it would be accompanied by ejection of a control rod.

In the SBLOCA accident, the depressurization rate is much slower and the duration of the event much longer than in a large LOCA. The consequences of the accident are primarily mitigated by the high-pressure safety injection (HPSI) pumps, which are usually actuated when pressures have dropped to 1500 to 1600 psia range. Since the gas-pressure-actuated accumulators [sometimes called safety injection tanks (SIT)] are usually actuated in the 400- to 600-psi range, their action only affects accidents with break sizes near the upper end of the small-break spectrum. With smaller break sizes, the system pressure does not fall to the level where SIT flow will

occur. The low-pressure safety injection (LPSI) system is not actuated in any SBLOCA. The specific behavior seen in a small-break LOCA is highly dependent on the break size.

6.6.1 Breaks Actuating Accumulator Flow

Breaks ranging in area from about 0.1 to 0.5 ft² are generally sufficiently large so that the primary system depressurizes rapidly.⁹² The depressurization behavior is shown in Fig. 6.6(a). Pressure quickly drops to point A, which corresponds to the saturation pressure for the water in the upper plenum of the reactor vessel. After the liquid-vapor interface falls below the break height, point B, the rate of energy removal at the break increases. Depressurization ceases at point C where accumulator flow is initiated.

The HPSI pumps have little influence on behavior for this break size range as their rate of injection is considerably below the rate of discharge for the break. (This is similar to what is seen in an LBLOCA). Heat removal is primarily by means of natural circulation flow through the steam generators. Forced circulation is not available since the primary coolant pumps are generally tripped shortly after the accident begins. The pump trip is believed to reduce the rate of coolant loss from the system.

Uncovering of a portion of the core is likely during this class of small break. However, the core is recovered before a substantial cladding temperature rise can occur. Breaks in this size class are therefore not limiting.

6.6.2 Medium-Size Small Breaks

Medium-size small breaks (0.02 to 0.1 ft²) tend to show a depressurization behavior similar to that shown in Fig. 6.6(b). As for the larger SBLOCA, pressure quickly drops to ~1050 psi. When the break is uncovered (point D), rapid depressurization lasts briefly. The pressure reaches a slightly

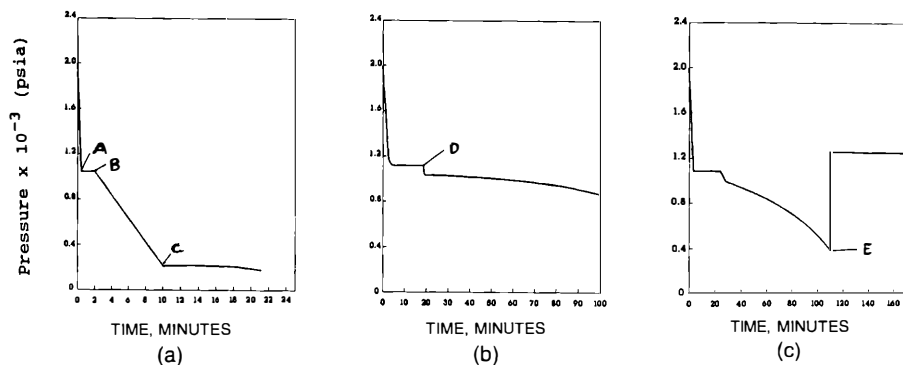


Fig. 6.6 Typical pressure decay curves for SBLOCA.

reduced level where the rate of heat removal by the steam generator and flow out the break equals the rate of energy production. The system pressure continues to drop slowly as a larger fraction of the heat produced is removed via the break flow.

As the pressure slowly decreases, the break flow diminishes. At some point, the break flow is below the HPSI flow and primary system water inventory increases.

A portion of the core will generally be uncovered during accidents in this category. Since the transient is very slow, the uncovered period may be relatively long. The potential for core damage during a SBLOCA is therefore greatest for breaks in this size range.

As HPSI flow is not capable of initially exceeding the break flow, the single phase natural circulation at the beginning of the transient is succeeded by two-phase natural circulation. The vapor produced in the core is condensed in the steam generator and then drains back to the core. When the system subsequently refills after HPSI flow exceeds break flow, the steam generator will again refill. Single-phase natural convection then returns.

During the refill process, water accumulates in the piping U-bends (loop seals) between the pumps and U-tube steam generators. At some point, the pressure on the cold-leg side of the U-tube is sufficient to clear the liquid from the seal and force a portion of the water into the reactor vessel. This additional core inventory generally determines the point at which the core heat up is terminated.

6.6.3 Very Small Size Breaks

With break areas of less than about 0.02 ft^2 , the flow from the HPSI pumps exceeds the loss through the break. The reactor core is never uncovered and the core is cooled by single-phase natural circulation through the heat exchanger.

The primary system depressurizes very slowly as shown in Fig. 6.6(c). When the entire primary system is completely refilled with water (point E), pressure rises sharply. Since the pressure remains high for a long period and little energy is removed via the break flow, the initial inventory of the heat exchangers cannot provide all the cooling required. In the absence of main feedwater flow, auxiliary feedwater is needed to maintain core cooling.

Since the core remains covered with liquid at all times, no significant cladding temperature rise occurs. However, the long period at a relatively high pressure with low-temperature HPSI water flowing in the downcomer raises the possibility of *pressurized thermal shock* (PTS). A PTS event refers to a transient causing a very rapid cooling (thermal shock) of the reactor

vessel wall while the vessel is maintained at high pressure. The concern is that the cooling will bring a substantial portion of the vessel to a temperature where a brittle fracture may occur.

As indicated in Sec. 1.2.2.1, exposure of the reactor vessel wall to a fast neutron flux results in embrittlement of the vessel. The degree of embrittlement is generally measured by the so-called *nil-ductility temperature* (NDT) of the material. At the NDT, the yield stress and the ultimate stress are coincident. At temperatures equal to or below the NDT, a preexisting flaw will propagate and lead to a brittle failure if the material is subjected to a stress above the lower fracture propagation stress. This stress, which is of the order of 6500 psi for reactor vessel steel, is generally below the actual stress level during a considerable portion of a very small break accident. It is therefore necessary to show that, in such transients, a substantial portion of the vessel remains sufficiently above the NDT so that a brittle fracture cannot propagate through the vessel. The degree to which the vessel wall will be cooled depends on the temperature of the fluid in contact with the vessel. If there is complete thermal stratification in the cold-leg downcomer, the wall will be subjected to the temperature of the cold HPSI water. If there is significant mixing in the downcomer, the most highly embrittled portion of the vessel (adjacent to reactor centerline) will be subjected to a considerably higher temperature.

Experimental measurement⁹³ in downcomer geometries under conditions simulating the injection of HPSI water have shown that there is appreciable mixing in the downcomer. Comparison of the experimental results with computations by the multidimensional porous-media code COMMIX-IB (see Sec. 4.2.2) indicate good agreement.⁹³ However, the large number of nodes required by such a simulation and the long duration of the accident make evaluation of the entire transient by COMMIX, or similar codes, quite expensive.

Several empirical mixing models have been developed to allow reasonable prediction of downcomer behavior without resorting to lengthy best estimate codes. Based on COMMIX calculations, a very simple model has been proposed by Chexal *et al.*⁹⁴ As shown in Fig. 6.7, the downcomer is divided into four regions. The fluid velocities and temperatures within each of the regions are computed as follows:

Region I:

$T_I = T_{MM}$ = mixed mean temperature obtained from 1-D transient analysis code for given time

$V_I = \bar{V}$ = average velocity in downcomer (from 1-D transient code).

Region II:

V_{II} = velocity in plume assuming HPSI flow falls in plume with parabolic profile

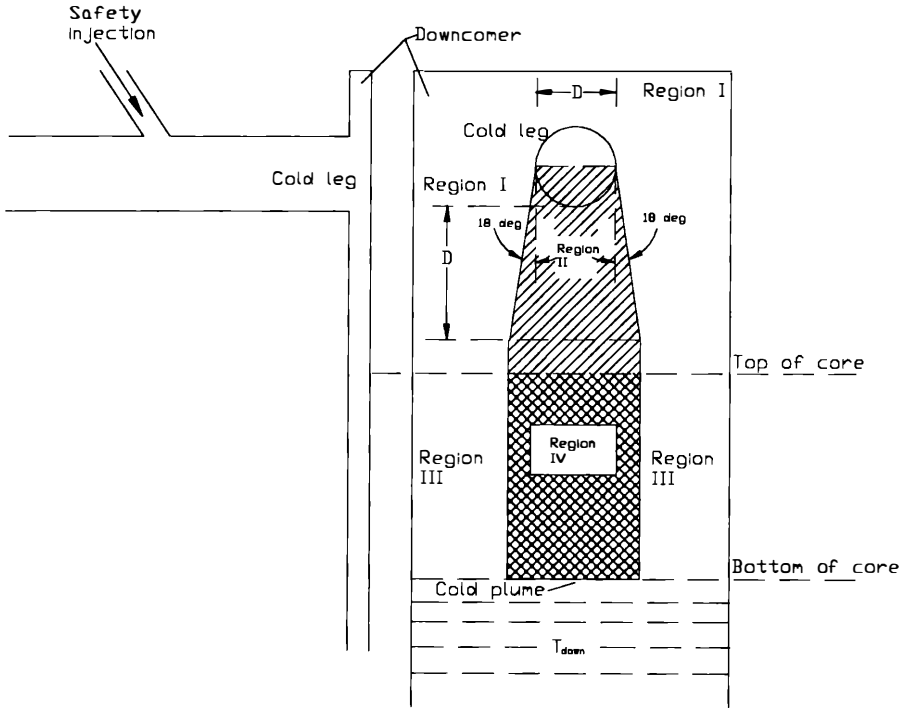


Fig. 6.7 Reactor vessel downcomer mixing plume (from Ref. 94).

$$T_{II} = T_{MM} + (T_L - T_{HPSI}) \left[\frac{0.01}{60} \left(\frac{V_L / V_{HPSI}}{\Delta \rho / \rho_{HPSI}} \right) - 0.07 \right] \quad (6.41)$$

where

T_L = cold leg temperature upstream of HPSI nozzle

V_L = cold leg velocity upstream of injection nozzle

V_{HPSI} fluid velocity in HPSI line

ρ_{HPSI} density of fluid in HPSI line

ρ_L density of fluid in cold leg upstream of injection point.

Region III:

$$V_{III} = V_I \quad (6.42)$$

$$T_{III} = (T_I + T_{down}) / 2 ,$$

where T_{down} = downcomer lower plenum temperature obtained from 1-D transient code for given time.

Region IV:

$$V_{\text{IV}} = V_{\text{II}} \quad (6.43)$$

$$T_{\text{IV}} = (T_{\text{II}} + T_{\text{down}}) / 2.0$$

Chexal *et al.*⁹⁴ find that the foregoing model provides a conservative lower bound (temperatures about the same as or below COMMIX predictions) for the downcomer fluid conditions.

Calculation of vessel wall temperature in the embrittled region requires a computation of the convective heat transfer coefficients in regions III and IV. Chexal *et al.*⁹⁴ recommend calculating the Reynolds number in these regions based on the velocities and temperatures computed via Eqs. (6.39) and (6.40). If $\text{Re} > 10^4$, forced convection dominates and the Dittus-Boelter equation is used to obtain h . Free convection is believed to dominate for $\text{Re} < 10^4$, and h is computed from a composite correlation considering heat transfer due to a net downward flow with a buoyant upward flow along the wall. They suggest

$$\begin{aligned} \text{Nu} &= \text{Nu}_0 \left(\frac{\text{Nu}}{\text{Nu}_0} \right) \\ &= \frac{\text{Nu}}{\text{Nu}_0} \left\{ \text{Re} \, \text{Pr} \left(\frac{C_F}{2} \right) / \left[12.7 \left(\frac{C_F}{2} \right)^{1/2} (\text{Pr}^{2/3} - 1) + 1.07 \right] \right\} \end{aligned} \quad (6.44)$$

where

$$C_F = 1 / [3.64 \log \text{Re} - 3.28]^2 \quad (6.45)$$

$$\frac{\text{Nu}}{\text{Nu}_0} = [1 + 4500 \, \text{Gr} / (\text{Re}^{21/8} \, \text{Pr}^{1/2})]^{0.31} \quad (6.46)$$

$$\text{and } \text{Gr} = \left[\left(\frac{\rho_b - \rho_w}{\rho_b} \right) g D_{\text{gap}}^3 / \nu^2 \right] \left[\frac{1 + \beta (T_w - T_b)}{2} \right] \quad (6.47)$$

and

β = volume expansion coefficient

ν = kinematic viscosity at T_b

T_b, T_w = bulk and wall temperature, respectively

ρ_b, ρ_w = fluid densities at T_b and T_w , respectively.

The heat transfer calculations must be coupled with a transient conduction calculation to evaluate the wall response.

Based on a number of studies of PTS events, it has been concluded that brittle failures will not arise providing the NDT has not risen excessively. The U.S. NRC has therefore established fracture toughness requirements for PTS events (Code of Federal Regulations 10CFR50.61). These require that, RT , a reference temperature, which equals the nil-ductility temperature plus margin, remains below a specified value. For plates, forgings, and axial welds this temperature is 270°F; for circumferential welds, it is 300°F. The regulations require that RT (in °F) be calculated from the lower value of

$$RT = I + M + [-10 + 470 \text{ Cu} + 350 \text{ CuNi}] \left(\frac{\phi t}{10^{19}} \right)^{0.1} \quad (6.48)$$

or

$$RT = I + M + 283 \left(\frac{\phi t}{10^{19}} \right)^{0.194} \quad (6.49)$$

where

I = initial nil-ductility temperature of unirradiated material (if none measured, value between 0 and -56°F specified depending on weld material)

M = required margin [in Eq. (6.48) 48°F if measured value of I used, 59°F otherwise; in Eq. (6.49) 0°F if measured value, 34°F otherwise]

Cu, Ni weight % copper and nickel in material, respectively

(ϕt) fast ($E > 1.0 \text{ MeV}$) neutron fluence (n/cm^2) at cladding-base metal interface in location receiving highest fast neutron flux.

Operators of the Russian VVER plants have recognized that many of their vessels will not have adequate fracture toughness at the end of life if untreated. A number of these vessels have been subjected to high-temperature annealing in place to remove the fast neutron radiation damage.⁹⁵

6.6.4 Break Location Effects

The depressurization rates shown in Fig. 6.5, together with the foregoing descriptions, specifically apply to a small rupture in the main piping or rupture of an instrumentation line. Slightly different behavior may be expected for failures at other locations. One of the other situations that needs to be considered is a SBLOCA due to a stuck-open power-operated relief valve (PORV). Since these valves are connected to the steam space in the pressurizer, break flow rates for early phases of the accident will be determined by the critical flow of steam through the valve throat. After initial depressurization, the steam generated in the core can cause flooding in the pressurizer surge line. This would prevent the liquid in the pressurizer

from draining into the core and lead pressurizer instruments to indicate the pressurizer is nearly full. Reactor operators may then erroneously conclude that the entire primary system is full.⁹² Such an erroneous conclusion led the operators to turn off the ECCS injection during the severe accident at Three Mile Island (see Sec. 6.8).

The rupture of one or more steam generator tubes is a significant possibility as tube failures in steam generators have been fairly frequent.⁹⁶ In this accident, the secondary system is made radioactive by the primary-to-secondary side leakage. It is thus desirable to depressurize the primary system using the PORV to terminate this leakage. The depressurization must be accompanied by adequate ECCS flow to assure that the core does not become uncovered.

A third small-break location of importance is the rupture of one of the control rod housings on the upper head of the reactor vessel. Such a rupture would probably be accompanied by ejection of a control rod from the core. This situation is therefore usually referred to as the *rod ejection accident*. Although no such accident has ever occurred, cracks attributable to primary water stress corrosion cracking have been found in some control rod drive penetrations.⁹⁵

This accident differs from the other relatively large size SBLOCA in that it is accompanied by a significant reactivity transient which is terminated by a reactor trip. The increased power reached by the reactor prior to shutdown means that DNB will be reached earlier than in the same size break without a rod ejection. Analysis of this accident must show that the maximum cladding temperature attained is at an acceptable level.

6.6.5 SBLOCA Analysis

The U.S. Regulatory requirements for acceptance of a SBLOCA licensing analyses, as imposed in 10CFR50, Appendix K, are the same as those imposed on LBLOCA analyses (see Sec. 6.5.2.1). In addition, the conservatisms required of an LBLOCA evaluation model (see Sec. 6.5.5.2) are also required for an SBLOCA.⁹⁷

The evaluation model conservatism mandating no return to nucleate boiling during blowdown is usually less burdensome in SBLOCA analysis than for LBLOCA evaluation. The blowdown period is generally considered as just the initial rapid depressurization to the saturation pressure. In SBLOCA analysis, the critical heat flux is usually not encountered until the slow depressurization that follows. When the heat flux subsequently drops sufficiently so that nucleate boiling actually returns, credit can be taken for this. The initial cladding temperature rise is therefore mild. Core uncover occurs later in the accident than in a LBLOCA. Since the decay power is considerably reduced by that time, the cladding temperature reached during the uncovered period is generally significantly below that seen in a large LOCA.

When the reflux condensation mode is encountered in the steam generator, countercurrent flooding limitations (CCFL) must be considered. Tests in the ROSA-IV facility⁹⁸ have shown that the primary coolant inventory distribution is controlled by CCFL in the hot leg riser of the steam generator as long as CCFL is not reached at the U-tube inlet. It was found that the flooding conditions at the U-tube inlet could be correlated by an equation similar to Eq. (3.261) but with different coefficients. That is,

$$(j_g^*)^{1/2} + (j_f^*)^{1/2} = 0.88 \quad (6.50)$$

with (j_g^*) and (j_f^*) being defined by Eqs. (3.259) and (3.260) using the tube diameter as the characteristic length.

All of the major 1-D computer codes designed for LBLOCA analysis (RELAP-4, RELAP-5, TRAC, CATHARE) can also be used for SBLOCA evaluation. In addition, some of the computer programs primarily designed for non-LOCA studies (e.g., RETRAN⁹⁹) are also able to analyze SBLOCA events.

Analysis of the rod ejection accident requires that the systems code be capable of a neutronic analysis. As indicated in Sec. 6.2.2, a point kinetics model with conservative weighting coefficients is usually employed for this purpose. However, a one-dimensional (axial) kinetics approach may also be used.¹⁶ Ejection of the maximum worth rod is always assumed.

Three-dimensional models¹⁰⁰ of core thermal-hydraulics and kinetics have been used to check the conservatism of the point kinetics and 1-D models used in licensing evaluation. The input of the systems code with a point kinetics or 1-D model is used to provide the time variation in core flow and pressure. Such analyses¹⁰⁰ indicate that the single rod ejection from a given power may be characterized by the rod worth as long as the peak fuel enthalpy is the major concern.

Any rupture of the primary system poses the problem of radioactivity releases to the containment. The primary system coolant is always slightly radioactive due to the presence of the short-lived N^{16} from irradiation of the coolant itself. More important is the longer lived radioactivity due to (1) irradiation of corrosion products and tramp uranium and (2) leakage of uranium oxide and fission products from pinhole defects in fuel elements. The iodine resulting from irradiation of uranium in the coolant or direct leakage of fission products is of greatest concern. It is postulated that, as a result of a sudden change in the primary system pressure, a significant amount of iodine may be released from the pinhole defects. This phenomenon is called an iodine spike.

The U.S. Nuclear Regulatory Commission requires that the radiological consequence of possible iodine releases be determined. The NRC guidelines⁹⁷ require two analysis categories to be examined: (1) consequence of iodine release assuming an initial iodine concentration in the coolant of 60

$\mu\text{Ci/g}$ and (2) an iodine spike originating at the time of the accident in which the iodine release rate increases by a factor of 500 over the preaccident release rate. Studies⁹⁶ of a number of actual steam generator ruptures have shown actual releases are much lower than called for by these guidelines.

6.6.6 SBLOCA in CANDU Reactors

The depressurization initiated by a SBLOCA automatically initiates a reactor trip and the emergency coolant injection system (ECIS). No significant cladding temperature rise occurs for any small-size break so long as the emergency coolant system functions.⁸⁹ CANDU licensing requirements mandate that the SBLOCA accident be considered in conjunction with failure of the ECIS.⁸⁹ These conditions correspond to a severe accident and will be examined in Sec. 6.8.

6.7 ANTICIPATED TRANSIENTS WITHOUT SCRAM

6.7.1 Regulatory Requirements

Regulatory requirements mandate an analysis of the likelihood and consequences of various anticipated transients (category II conditions) under the hypothetical assumption that no reactor trip occurs. These occurrences are generally referred to as anticipated transients without scram (ATWS).

The range of ATWS events considered includes all anticipated operating occurrences (see Table 5.II) followed by a failure to scram. For a PWR, the most severe ATWS situations are usually taken as (1) loss of flow due to loss of offsite power, (2) pressurizer safety valve stuck open, (3) rod withdrawal at power, (4) loss of feedwater flow, (5) a reactor coolant pump coastdown, and (6) maximum steam load increase. Criteria established¹⁰¹ for these hypothetical accidents require that:

1. Any power excursion be limited (turned around) by coolant and fuel reactivity feedback.
2. Radiological consequences remain within the limits set by the Code of Federal Regulations.
3. System pressures remain below the peak pressure limit set in accordance with the ASME Boiler and Pressure Vessel Code for emergency conditions, ~ 3250 psi for current (1994) plants.
4. Fuel will survive peak internal and external pressures and will have an enthalpy of < 280 cal/g.
5. Fuel will either not experience DNB or, if it does, the maximum cladding temperature must remain below 2200°F .
6. Containment pressure must not exceed design pressure.

Standard PWR designs are capable of meeting these criteria.¹⁰² To make certain that the probability of an ATWS event is low, the U.S. Code of Federal Regulations (10CFR50.62) requires that reactors be provided with two completely independent electrical circuits, including the originating signal, which can shut the reactor down. In addition, each reactor is required to have circuitry, which will trip the turbine and initiate auxiliary (or emergency) feedwater system under conditions indicative of an ATWS. This last requirement is imposed as the ATWS event originating with a loss of feedwater is considered the most severe ATWS transient.

6.7.2 Characteristics of ATWS Events

All of the postulated ATWS events are self-limiting due to reactivity feedback. Events producing an increase in reactor power (e.g., rod withdrawal) produce power transients that are limited by the reactivity reduction due to void formation and fuel temperature rise. These events are terminated by operator-initiated injection of borated water. The power transient produced may heat the coolant sufficiently to overpressurize the primary system. The overpressurization is usually not severe.

Events involving a decrease in the heat removal capability of the system tend to be more serious than those producing power transients. In these events, the primary coolant heats up rapidly causing (1) a significant increase in the pressure of the primary system and (2) a decrease in reactor power due to moderator and fuel reactivity feedback. The increase in pressure is limited by the discharge of the pressure relief valves. However, after all the steam is discharged from the pressurizer, the rate of liquid discharge may be insufficient to prevent a further rise in system pressure.¹⁰³ The event will finally be terminated by manual initiation of borated water injection.

6.7.3 Loss-of-Feedwater ATWS

Primary system overpressurization is most severe in the ATWS event caused by loss of normal feedwater.¹⁰¹ This event is mitigated by the automatic initiation of auxiliary feedwater and turbine trip on loss of normal feedwater. In order that spurious actuation of this safety circuit be avoided, the safety signal is usually delayed. The delay between the receipt of the loss of flow signal and safety circuit actuation increases with decreasing power level. At full power, the turbine trip signal may be delayed by about 30 seconds.¹⁰⁴

Typical behavior of the system pressure after a loss of feedwater ATWS from 100% full power is shown [moderator temperature coefficient -0.02 ($\$/^{\circ}\text{C}$)] in Fig. 6.8.¹⁰⁴ Initially, system pressure changes little as the steam generator inventory is sufficient for removal of the heat generated. However, at about 30 to 35 s into the transient, the pressure begins to rise and

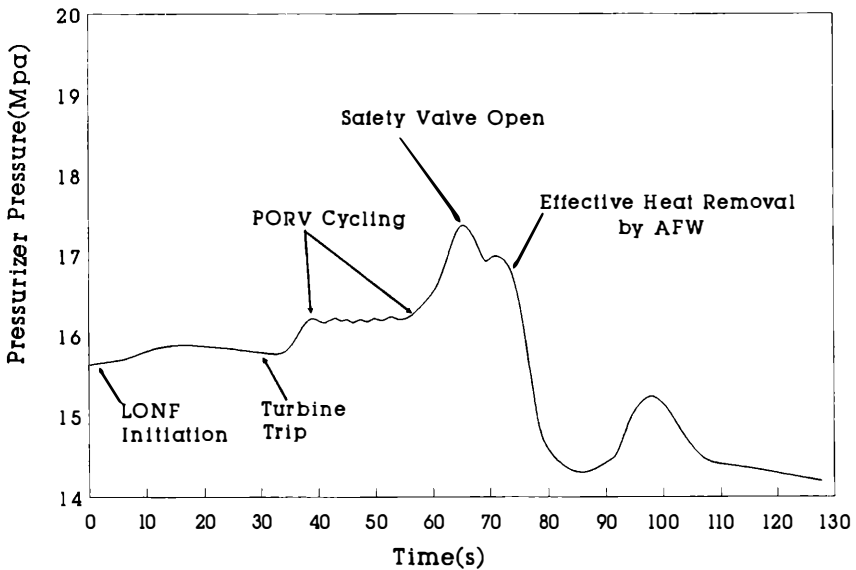


Fig. 6.8 Typical system pressure behavior following ATWS initiated by loss of normal feedwater (LONF) [from Ref. 104].

shortly thereafter the PORV begins to cycle. The power generation rate is high enough to increase the coolant temperature and cause a further rise in system pressure. Reactivity feedback subsequently causes a reduction in power so that a peak pressure is reached at about 65 s. At the reduced power, the auxiliary feedwater and steam dump remove all the power generated and the system pressure drops sharply.

The peak pressure reached is sharply dependent on the moderator temperature coefficient. With an insufficiently negative moderator coefficient,* the primary system pressure safety limit can be exceeded. Analysis of this accident must therefore be based on the BOL conditions where high boron concentrations produce the least favorable moderator reactivity coefficient.

The primary system inventory may be sufficiently depleted by the discharge during the pressure transient that the pressurizer empties on final cooldown. Additional coolant would then be added to the system by the HPSI pumps.

*The moderator coefficient is the change in reactivity per unit change in moderator temperature [see Eq. (6.2)].

6.7.4 ATWS Analysis Procedures

The major systems analysis programs (RELAP, TRAC, etc.), originally developed for LBLOCA analysis, may all be used for analysis of ATWS events. In addition, those system analyses codes, such as RETRAN,⁹⁹ developed primarily for transient analysis may also be used. All of the foregoing programs solve the mass, momentum, and energy conservation equations, simultaneously. In addition, they contain a point kinetics or 1-D reactor kinetics model.

Since momentum effects have little influence on ATWS events, simultaneous solution of all conservation equations is not required. In many programs used by reactor vendors (e.g., CADDs¹⁰⁵) only the continuity and energy conservation equations are solved simultaneously. Momentum conservation is imposed as a boundary condition. In addition, thermodynamic equilibrium is assumed everywhere but in the pressurizer where a separate component model is used. This is the approach taken by the publicly available IRT program¹⁰ and others developed for analysis of anticipated operating occurrences. Linkage of this fluid modeling approach with a point kinetics neutronic model provides a rapidly running ATWS analysis program.

Reactor core models combining 3-D thermal hydraulics and neutronics are available (see Sec. 6.2.2) but are not now (1994) routinely incorporated in ATWS licensing evaluations. However, they are used for evaluation of the adequacy of the simpler neutronic models used for licensing evaluation.

Determination of the radiological consequences of an ATWS event requires a knowledge of the fuel failures produced by the accident. Although cladding temperature increases due to ATWS occurrences are limited, fuel failures may be produced by pellet cladding interaction (PCI). Such failures may arise because of (1) sudden collapse of the cladding onto the pellet due to the increased primary system pressure or (2) hard pellet-to-cladding contact due to the fuel temperature rise caused by a rapid power increase (e.g., in rod withdrawal event). In both cases, appreciably increased cladding stresses result. The magnitude of the stresses expected may be determined by using the power and system pressure changes as part of input of an appropriate fuel element design program (see Sec. 2.5.4.1). The program would have to examine the ATWS-generated conditions for a number of previous power-time histories typical of the fuel elements in the reactor core. The stress and burnup data can then be provided to a fuel failure estimation program (see Sec. 2.5.4.2) to determine the additional failures as a result of the accident.

The estimated number of fuel failures allows a calculation of the additional radioactivity contained by the primary system due to fission product release. This quantity, together with the knowledge of the total coolant released by the relief and safety valves, allows determination of the radioactivity release. The radioactivity will only be released to the vapor con-

tainer if the valve discharge is sufficient to overpressure the rupture disk on the reactor coolant collection tank to which the valves discharge.

6.8 SEVERE ACCIDENTS

6.8.1 Overall Perspective

A severe nuclear reactor accident is an event, brought about by multiple failures, which results in changes to the reactor core configuration and significant releases of radioactivity outside the primary system. In worst case scenarios, the reactor core becomes molten and the reactor containment is breached.

Prior to the 1970s, safety studies and analysis of nuclear power plants were essentially limited to the design basis accidents discussed in the previous sections of this chapter. Events beyond these accidents were considered “incredible.” That is, these events were thought to be of such low probability that it was unnecessary to show that these consequences did not harm the public. This viewpoint was altered by the 1979 accident at Three Mile Island (TMI). Concern over such events was increased by the 1986 accident at Chernobyl in the Ukraine.

The TMI accident¹⁰⁶ resulted from a loss of feedwater to the steam generators, which in turn led to overpressurization of the primary system. The overpressure was relieved by opening of a relief valve (PORV), which subsequently failed to close when the pressure decreased. This produced a SBLOCA, which activated the ECCS. However, the operators misinterpreted the measurement showing the pressurizer was full and assumed the entire primary system was full. They therefore shut off the emergency coolant (the high level in the pressurizer was apparently caused by flooding due to countercurrent flow in the line leading to the pressurizer; see Sec. 6.6.4). The operator action led to severe overheating of the core and partial melting. The accident progression was finally limited when the operators realized their error and the high-pressure injection flow was resumed some 200 min into the accident.

Although TMI demonstrated that severe accidents were not incredible, it also demonstrated that it is possible to recool a severely damaged core. Figure 6.9 shows the end-state of the cooled core based on postaccident examination of core-bore samples. Since there were no significant releases of radioactivity outside the containment, TMI also illustrated that in the presence of containment, and with appropriate accident management, there need be no radioactivity releases beyond the site boundary.

The Chernobyl accident¹⁰⁷ was far more serious since it caused major radioactivity releases to the environment. In this graphite-moderated, water-cooled reactor, the accident was initiated by a sudden reactivity insertion. The reactivity insertion was exacerbated by the positive void reactivity of the reactor. The slowly moving control rods were not effective in ter-

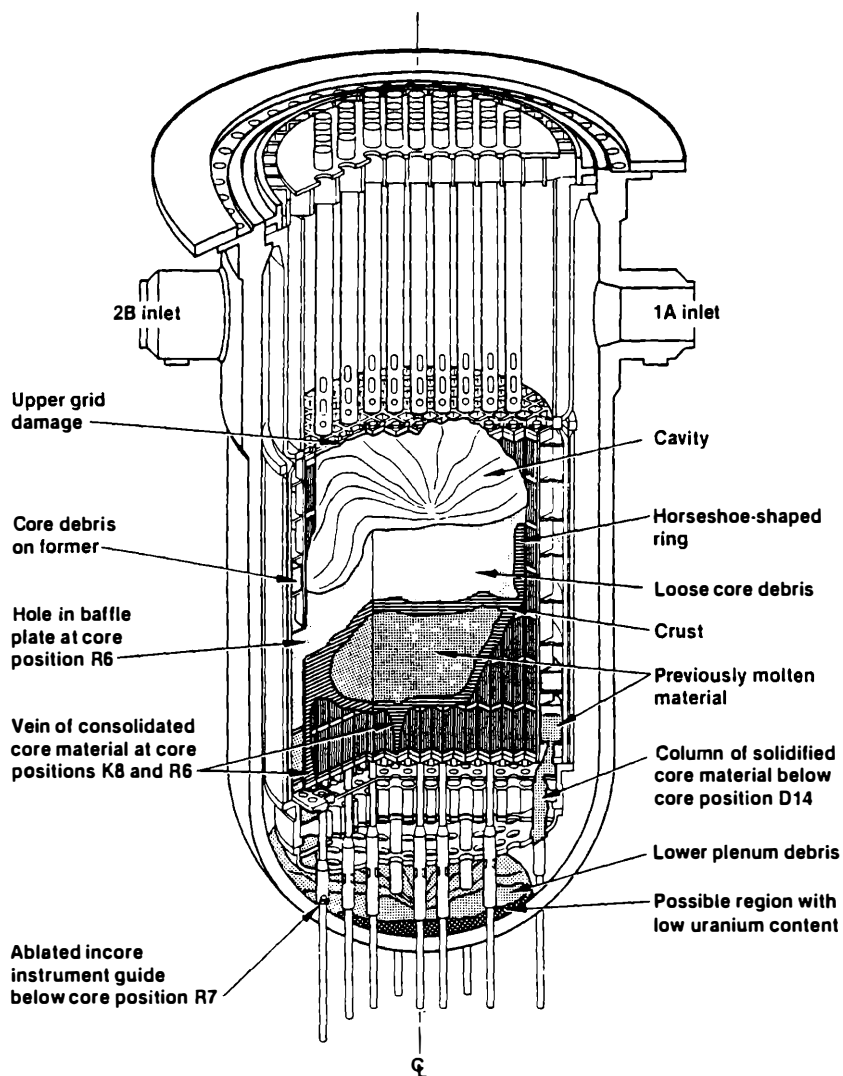


Fig. 6.9 TMI-2 end-state configuration [from Ref. 106].

minating the excursion and gross fuel and cladding melting, with accompanying fuel-coolant interaction (steam explosion), occurred. This severely damaged the reactor. Since no vapor container was provided, the resulting radioactivity release was widely dispersed. The designs of western light water reactors provide containment and limit reactivity insertions to a magnitude far below that seen at Chernobyl. Hence, the Chernobyl accident is

not directly relevant to PWR safety. However, the event again demonstrated that severe accidents cannot be considered incredible.

Concerns over the possibility of another severe accident led to several significant actions affecting currently operating plants. The first of these was a major improvement in reactor operator training to reduce the possibility of human error exacerbating an accident. The second action was the development of severe accident management plans so that reactor operators had clear guidelines to follow in dealing with extraordinary situations. In addition, the U.S. NRC required all plant owners to conduct a probabilistic safety assessment to determine the likelihood of severe accidents at each individual plant and the possible consequences of these events. It was expected that the assessments would be examined to see what steps, if any, could be taken to significantly reduce risk.

Severe accidents are now considered in the design of all new plants. All new plant designs now incorporate features that are expected to reduce both the likelihood and consequences of a severe accident significantly.

It is obviously extremely difficult to conduct experiments under conditions that truly model reactor core behavior after the onset of melting. In view of this, it should be recognized that all descriptions and modeling of post-melt behavior are subject to a high degree of uncertainty. The severe accident discussions in the sections which follow must be regarded with this uncertainty in view.

6.8.2 Severe Accident Causes

All severe accidents result from events that lead to situations where adequate core cooling cannot be maintained. When all reactor types are considered, two broad classes of accidents that produce inadequate cooling may be distinguished; namely, reactivity insertion accidents and core uncover accidents.¹⁰⁸ The Chernobyl accident was in the former class and TMI in the latter class. As indicated previously, PWR design is such that reactivity insertions sufficient to cause a severe accident are virtually impossible. Accidents in this class do not contribute significantly to PWR severe accident risk.

Severe accidents due to core uncover may be internally or externally generated. Externally generated accidents include those generated by seismic events (earthquakes), flooding, or fires. A sufficiently strong earthquake could cause a LOCA while simultaneously disabling emergency core cooling systems and/or reactor control systems. Flooding or fire can cause loss of load coupled with loss of feedwater and ECCS systems.

For many plants the dominant internal initiating event is a system-caused LOCA. A severe accident results when a LOCA is coupled to the failure of either the ECCS or long term decay heat removal system. Since small- or medium-size LOCAs are far more likely than a large LOCA, the

smaller LOCAs are much more of a contributor to severe accident risk in most plants.

In addition to severe accidents beginning with a loss of coolant due to piping failure, a LOCA originating from a brittle failure of the reactor vessel is conceivable. Such an event could arise during low-temperature operation with the primary system "water-solid." This low-temperature overpressurization (LTOP) accident is made unlikely by administrative procedures and available overpressure mitigation systems.¹⁰⁹ A second possible source for a vessel failure is a pressurized thermal shock event following an SBLOCA. (See Sec. 6.6.3 for actions taken to make this event unlikely.) Core melt following vessel fracture is considered to be almost certain.

Loss of all offsite power coupled with failure of emergency power (station blackout) is another significant contributor to risk for many plants. Anticipated transients coupled with loss of load and failure of accident mitigating systems are also a possible source of severe accidents. However, accident risk studies show that this source makes only a small contribution to the probability of a severe accident in most plants.

Probabalistic safety analyses of a number of conventional PWRs show that the dominant contributors to severe core damage frequency vary significantly from plant to plant.¹¹⁰ While LOCA initiated events are the most likely cause at most plants, events initiated by anticipated transients, loss of offsite power and earthquakes are the most likely causes at some individual plants.

Probabalistic risk analyses of the CANDU-6 system¹¹¹ conclude that, as with vessel type units, reactivity insertion accidents make an insignificant contribution to the risk of a severe accident. The major severe accident initiator was loss of service water because this affects the moderator, calandria vault, secondary heat transport, and emergency coolant injection heat exchangers. A LOCA initiated event is the next most frequent cause. This is followed by station blackout events. Several other causes including ATWS events, loss of steam generator inventory, and loss of long-term cooling make up the remaining 4% of initiating events.

6.8.3 Severe Accident Sequences in Conventional PWRs

6.8.3.1 Degraded Core

After core uncover, the fuel elements heat up rapidly. In the absence of adequate cooling, reaction of the Zircaloy cladding with steam leads to cladding embrittlement, and fuel rod failure with some fuel geometry degradation. Upon further heating, melting of the cladding in the hot regions of the core allows the fuel pellets to be released from their normal positions. An unstructured debris bed may then form in the region of cladding melt. Table 6.VI indicates the fuel rod surface temperatures at which the various levels of fuel damage occur.

TABLE 6.VI
Temperature at Which Various Levels of Fuel Damage Occur*

Temperature of Fuel Rod Surface (°C)	Damage Description
700–750	Ag-In-Cd alloy in control rods melt
800	Fuel rod swelling and bursting
900	Zr-steam reaction begins; fuel rod temperature rises more rapidly
1300–1500	Visible liquid formation due to (a) Inconel-Zircaloy eutectic formation at support grids and (b) UO_2 -Zr reaction on exposed internal surfaces
Above 1500	Vaporization of volatile fission products
1850–1950	Zircaloy melting plus Zr + UO_2 reaction
2400–2650	ZrO_2 and UO_2 - ZrO_2 mixture melt

*Based on data from J. H. Gittus, J. Mathews, and P. Potter, "Safety Aspects of Fuel Behavior During Faults and Accidents in PWR's and in Liquid Metal Sodium Cooled Reactors," *J. Nucl. Mater.*, **166**, 132 (1989).

If accident management procedures allow adequate emergency core coolant flow to be restored at this point, the accident progression may be halted. Prevention of further damage requires that the debris bed consisting of fuel pellets and oxidized and refrozen cladding be satisfactorily cooled.

In the absence of adequate ECCS flow, core damage will progress. Fuel melting will occur with relocation and refreezing in a lower portion of the core. If adequate ECCS cooling is begun before fuel melting has become too extensive, it is possible to halt core degradation at this point. The core would then contain a debris bed above a coherent molten zone. Cooling by convection at the outer surfaces of the molten zone would then produce a frozen crust, which would gradually propagate into the melt. The damaged fuel is thus contained within the reactor vessel. The situation corresponds to that observed in the TMI accident.

6.8.3.2 Molten Core at High Pressure

If core cooling is not achieved in a timely manner, heating will result in a larger molten core region. This situation is most dangerous when it occurs at a high primary system pressure. It was thought that such a condition could be produced by the station blackout accident.

At high pressures, the volumetric heat capacity of steam is a significant fraction of that of water. This leads to energy redistribution due to natural circulation loops established between the reactor vessel and steam generators. The circulating gases which are heated by the core to temperatures above 725°C, can heat loop components to failure.¹⁰⁸ The loop failure would then depressurize the system. Current (1993) calculations indicate that primary loop failure and system depressurization would occur prior to significant core meltdown. However, uncertainties in these calculations have

led to some continued concern over the behavior of a molten core at high pressure.

If core melting were to continue at high pressure, a sequence of melting-freezing steps would cause the gradual movement of a core wide melting-crusting front in the downward direction. Eventually the molten core material, generally referred to as *corium*, would reach the lower plenum of the reactor vessel. If water remains in the lower plenum, an energetic fuel coolant interaction (FCI) is possible. This would cause failure of the vessel and dispersal of the corium in the vapor container.¹⁰⁸

If the reactor vessel does not fail because of FCI, it will fail subsequently due to melting or creep rupture of the lower head. The rapid corium expulsion would be followed by a high velocity blowdown of the vapor in the primary system. Such a blowdown would disperse the corium throughout the containment.

Corium dispersal through the containment gases as a result of either a blowdown or FCI could produce a rapid heating of the containment gases. This phenomenon, called *direct containment heating*, could lead to pressures and temperatures which produce containment failure.

Although dispersal of corium at high pressures would produce very serious consequences, the likelihood that this would happen is small. If the primary system does not fail prior to core melt, there is adequate time for the reactor operators to depressurize the system. Essentially all current PWRs of Western design can be reliably depressurized by operation of the PORVs.

More recent calculations have cast doubt on the latter parts of the station-blackout scenario. Heames and Smith¹¹² showed that when allowance is made for failure of the seal on the primary system pumps (seals require high-pressure coolant, which is not available after loss of power), the system slowly depressurizes. Vessel failure then occurs much later and at a pressure below 3 bar. High-pressure melt ejection and direct containment heating would then not occur.

6.8.3.3 Molten Core at Low Pressure

The most likely outcome of a very severe accident is the production of a molten core at low pressure. Hence, this is the major contributor to population risk following a severe accident. Redistribution of decay heat due to natural circulation is not significant. Upper internals remain relatively cool. There is no danger to the containment boundary until after complete melting has occurred. This would require about 15 to 30 min in the absence of all core cooling.¹⁰⁸

The potential for severe damage arises when the corium slumps into the lower plenum. The behavior which will be observed is uncertain. One possibility is a very energetic FCI, which fails the reactor vessel and could also lead to containment breach via missile production. A more likely outcome

is a very limited FCI in which some steam and hydrogen are produced. The melt would then reagglomerate on the lower head of the vessel. If adequate cooling of the vessel head is established, the corium can be retained within the reactor vessel. Operator action to keep the cavity below the reactor vessel flooded is likely to be successful in this regard.

If lower head cooling is not established, melt through of the lower vessel head would occur. If water is present in the cavity below the head, the potential for a FCI exists. The fuel reaching the bottom of the cavity will interact with the concrete there. This interaction produces water vapor and non-condensable gases (CO_2 and H_2). These gases could lead to an over-pressurization of the vapor container. Of particular concern is the high pressure due to a possible detonation of an explosive hydrogen-air mixture. The required presence of hydrogen recombiners or igniters in U.S. reactor systems (see Sec. 6.5.2) should be able to keep hydrogen concentrations below the level at which detonation can occur.

6.8.4 Severe Accident Sequences in CANDU Reactors

6.8.4.1 *Degraded Core*

In contrast to vessel-type PWRs, LOCAs are not the originators of the most severe accidents for CANDU pressure-tube reactors. The cooling available from the heavy water moderator in the calandria limits accident consequences even when no emergency core coolant injection is available.¹¹³ However, a LOCA coupled with failure of emergency coolant injection will lead to a severely degraded core.

Without adequate convective cooling in a fuel channel, fuel temperatures rise rapidly. The hot fuel then transfers some of the heat generated to the horizontal pressure tube via radiation and conduction. The hot pressure tube then deforms by sagging if the primary system pressure is low or by ballooning if the pressure is high. The deformed pressure tube contacts the calandria tube thus establishing a radial heat loss path from the fuel channel to the moderator.

The exact sequence of events depends on stored energy content of the fuel when steam cooling begins. The most threatening sequence is that posed by "early heat up" caused by rapid voiding of the channel at the start of the accident.¹¹³ Fuel temperatures and system pressure increase rapidly and cause the pressure tube to balloon into contact with the calandria tube. The exterior elements then level off in temperature due to heat transfer to the moderator. However, the channel distortion caused by the ballooning allows a significant portion of the steam flow to bypass the fuel bundle. The interior elements are inadequately cooled and their temperatures continue to rise. Eventually these elements become so hot that they sag into contact with the outer elements. This establishes a heat flow path to the moderator.

Channel integrity could be challenged if the heat flux to the calandria tube were sufficient to cause film boiling on the calandria tube surfaces and then interrupt heat flow. Computations indicate that this does not occur for any realistic fuel slumping conditions.¹¹³ Further, the pressure tube strains produced by the circumferential temperature gradient around the tube are not sufficiently high to cause rupture.¹¹⁴ Similar conclusions are arrived at for pressure tube behavior with "late heat up."

6.8.4.2 Molten Core

The severe accident initiating events that can lead to molten fuel include total loss of service water, station blackout, simultaneous loss of all feed-water supply systems and emergency water supply, and loss of long-term emergency core cooling. All of these events lead to the dominant core melt category that has been designated as *late core disassembly* (LCD).¹¹⁵

The LCD accident scenario resulting from loss of all service water is considered typical of this accident category.¹¹⁵ Service water loss is coupled with a station blackout, which in turn produces a small LOCA. The resulting pressure tube contact with the calandria tubes allows substantial heat flow to the moderator. Since service water is unavailable for moderator cooling, the moderator temperature rises to boiling. The calandria liquid then boils away, uncovering successive rows of reactor channels. The uncovered pressure tubes heat up rapidly to failure and then dump fuel fragments to the bottom of the calandria. An FCI is possible at this juncture. The fuel debris remains cool for about an additional 3 hr until the calandria water boils away.

Once the calandria liquid is boiled away, the shield water surrounding the calandria is boiled off. When the shield water level is below the debris bed, heat transfer is insufficient to prevent fuel melting. At the end of the shield boil-off, the molten corium relocates to the concrete shield tank around the calandria vault. Since the calandria vault contains a considerable quantity of water, a further FCI is likely here. The reaction of the molten corium with the concrete produces noncondensable gases, which, in concert with the steam generated, overpressurize the prestressed concrete vapor container, causing some cracking.

The vault floor is eventually breached and the mixture of fuel material and concrete decomposition products falls to the basement of the containment building. The very large quantity of water in the basement (from the unflashed and condensed liquid of the primary system, calandria, calandria vault, and containment douse) is believed to quench and disperses the debris. Heat transfer by steaming and conduction through the floor maintains the debris in place.

The progression of an LCD event is quite slow. Containment overpressure occurs about 25 hr after initiation of the accident. There is ample time

for operator intervention to restore cooling and retain the molten material within the calandria. This scenario, designated *partial melt* (PM) results in little, if any, radioactivity release outside the site boundary.

6.8.5 Important Physical Phenomena in Severe Accidents

6.8.5.1 Debris Bed Cooling

If the unmolten debris or rubble bed within the core cannot be adequately cooled, then the resumption of core cooling will not be able to prevent subsequent core melting. Two debris bed geometries are possible. In the first of these, the debris bed is unblocked at the bottom and coolant flow from below is possible. Such a situation may occur if cooling is resumed before any significant fuel melting has occurred. In the second geometry, the bottom of the bed is blocked by molten corium.

With upward flow from the bottom of the debris bed, cooling is possible only if the pressure head due to liquid in the downcomer can force enough fluid through the bed to remove the heat generated in the bed. A conservative lower bound of the heat removal rate may be obtained by assuming satisfactory cooling so long as the exit quality remains below 1.0. If saturated liquid enters at the bottom of the bed, the variation of quality, $x(z)$, with distance along the bed is determined by¹¹⁶

$$x(z) = \frac{1}{GH_{fg}} \int_0^{z_l} (1 - \epsilon) Q_v(z) dz \quad (6.51)$$

where z_l = debris bed height; Q_v = volumetric rate of energy generation (energy/ L^3t); and ϵ = bed porosity (bed volume - particle volume)/bed volume.

The two-phase pressure drop through the bed can be estimated (for low pressures) from Naik and Dhir's¹¹⁷ extension of the Kozeny-Carmen equation for single-phase pressure drop through particle beds. We then have

$$-\left(\frac{dP}{dz}\right) = a \left[\frac{\nu_v x}{\alpha} + \frac{\nu_l (1-x)}{(1-\alpha)} \right] G + b \left[\frac{x^2}{\rho_g \alpha^2} + \frac{(1-x)^2}{\rho_l (1-\alpha)^2} \right] G^2 + \rho_g g + \rho_l (1-\alpha) g \quad (6.52)$$

where

ν_g, ν_l = kinematic viscosities of vapor and liquid, respectively
 μ/ρ

ρ_g, ρ_l = vapor and liquid densities

a $[150(1-\epsilon)^2]/(\epsilon^3 d^2)$

b $[1.75(1-\epsilon)]/(\epsilon^3 d)$

d debris bed particle diameter

α = void fraction = x^m

$$m = 0.359 + 626/G$$

x = quality

G mass flux (kg/m² hr).

Along with Eq. (6.51), Eq. (6.52) may be integrated over the bed height for any assumed G . The correct value of G is that which yields a pressure drop equal to the downcomer head ($z_l \rho_l g$). If the quality at the bed exit is less than 1.0 with this G , adequate cooling is available. For realistic conditions, Dhir¹¹⁶ estimates that 1% of full power can be removed by this process if a heat sink is available.

The more limiting situation occurs when the debris bed is blocked by a coherent mass of molten corium or by the lower head of the reactor vessel. The latter geometry may arise when a jet or gravity driven drain of molten corium flows downward from the primary melt region and is quenched in the lower plenum. For such beds, liquid coolant can only enter at the top. A counterflow situation develops with downward flowing liquid and upward flowing vapor. Cooling can be maintained only if the critical heat flux (dryout) is avoided.

It is generally agreed that the critical head flux is determined by the limiting countercurrent flow process in the upper layer of the debris bed. The results have been expressed either as a flooding limit or in terms of the frictional coupling between phases. Similar expressions result although there is considerable variation in numerical predictions.

The frictional coupling approach is conceptually similar to that approach taken for cooling of beds with an unobstructed bottom. The interfacial friction force due to countercurrent flow is set equal to the density head difference between the rising and falling fluid columns. At some power the vapor flow is high enough to reduce the downward flow of liquid sufficiently so that all of it evaporates before it reaches the bottom of the bed (dryout condition).

The more sophisticated of these approaches includes the effect of capillary forces within the bed. This effect can be satisfactorily approximated by adding the capillary head to the hydrostatic head.^{118,119} For deep beds, which are the most difficult to cool, capillary effects can be ignored. Lipinski¹¹⁹ presents a closed-form solution for the dryout heat flux, q_c'' (W/m²), for deep beds with an impervious bottom boundary

$$q_c'' = \left(\frac{q_i^4}{4q_L^2} + q_L^2 \right)^{1/2} - \frac{q_i^2}{2q_L} \quad (6.53)$$

where

$$q_L = \frac{(\rho_l - \rho_g)gd^2\epsilon^3 H_{fg}}{150(1 - \epsilon)^2(v_g^{1/4} + v_l^{1/4})^4} \quad (6.54)$$

$$q_t = H_{fg} \left[\frac{\rho_g \rho_l (\rho_l - \rho_g) g d \epsilon^3}{1.75(1 - \epsilon)(\rho_g^{1/6} + \rho_l^{1/6})^6} \right]^{1/2} \quad (6.55)$$

and d = particle diameter (m); ν_g, ν_l = kinematic viscosity of vapor and liquid (m^2/s); ρ_g, ρ_l = density of vapor and liquid respectively (kg/m^3); and ϵ = bed porosity. The quantities q_l and q_t refer to the dryout fluxes at the limits of all laminar and all turbulent flow, respectively. If the debris bed is formed in a manner which leads to particle diameter being a function of elevation, then Eq. (6.53) should be applied to each level in the bed.

The flooding limit approach for estimation of dryout flux, q_c'' , of coarse beds is described by Theofanous.¹⁰⁸ After expressing the flooding limit in terms of heat flux, the relationship for water near atmospheric pressure is

$$q_c'' = 2.21 H_{fg} \left(\frac{\epsilon^3}{1 - \epsilon} \right)^{1/2} \left[\frac{d}{F} \right]^{1/2} \left(\frac{\rho_g}{\rho_l} \right)^{3/8} \quad (6.56)$$

where q_c'' is in (W/cm^2) and

F = ratio of particle surface area to that of an equal volume sphere

d = particle diameter (cm)

H_{fg} = heat of vaporization (kJ/kg),

and other symbols have their previous meaning.

The heat fluxes predicted by Eq. (6.56) are substantially lower than those predicted by Eq. (6.53). Theofanous¹⁰⁸ notes that there is a group of experimental observations on bed dryout that agrees with Eq. (6.53) while another group of observations agrees with Eq. (6.56). The use of the more conservative approach of Eq. (6.56) would appear sensible in view of the disagreement.

6.8.5.2 Core Melt Progression and Cooling

Figure 6.10 shows a schematic representation of the core melting process. Table 6.VII summarizes the general sequence of events leading to core melting and eventual vessel failure. The unperturbed sequence shown assumes no attempt to recool the core. This is highly unlikely since in all postulated accident sequences there is a minimum of several hours available for effective operator action to forestall vessel failure.

Dallman and Duffey¹²⁰ point out that under some conditions it may not be possible to prevent the molten core mass from increasing in size until all the fuel is molten. They argue that a solid crust will form on the surface of the molten region. All heat generated within the molten region must be transferred across the crust by conduction. For any given decay heat generation rate and crust thickness, there is a maximum diameter molten mass that is coolable. When the total heat flux is above that which can be trans-

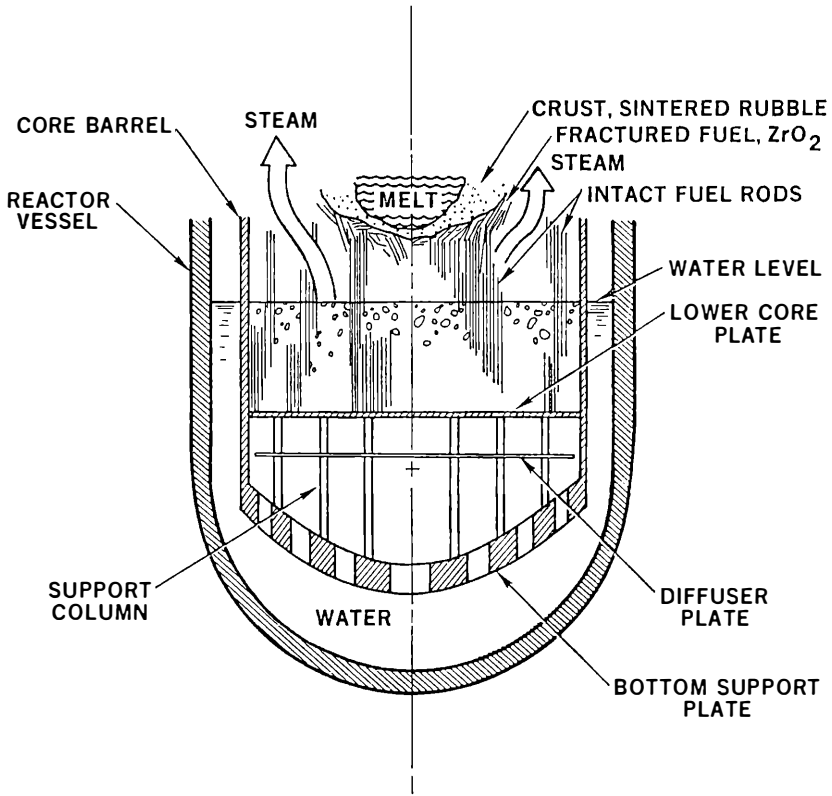


Fig. 6.10 Schematic representation of core melting (courtesy of Sandia National Laboratory).

ferred by conduction across the crust, the mass is above the coolable limit. The crust would then melt and the molten region expand. With a crust thermal conductivity of 6 (W/m K) , and a melt heat generation of 3 MW/m^3 , the maximum coolable melt radius is approximately 0.4 m . If the heat production rate is reduced to $1.5 \text{ (MW/m}^3)$, the coolable radius expands to about 0.6 m . The calculations are conservative in that they do not account for any heat transfer to the vapor above the hemispherical mass.

6.8.5.3 Fuel-Coolant Interactions (FCI)

A fuel-coolant interaction (FCI) refers to the rapid generation of steam and hydrogen due to the mixing of water and molten corium. This interaction can take place within the reactor vessel if (1) molten corium slumps or pours into a pool of water in the lower head or (2) a rapid reactivity transient causes the sudden ejection of molten fuel from core fuel elements. An

TABLE 6.VII
Core Damage Sequence

Flashoff and/or boiloff	Melt grows radially
Fuel heatup begins	Radial structures attacked
Zircaloy oxidation begins	Below-core structures uncovered
Clad balloons, bursts, spalls	Core support structures fail
"Early" debris forms	Core slumps downward
Binary, ternary (U-Zr-O) melts form	Partial quench, fragmentation
"Stop and go" downward relocation	Remelt
Large-scale blockage	Vessel breach
Coherent melt forms	

ex-core interaction is possible if the lower head of the reactor fails and corium contacts a water pool in the reactor cavity.

The vapor production in an FCI leads to a significant increase in pressure. The intensity of the thermal interaction is governed by the degree of mixing of the corium and water. Under appropriate conditions, a portion of the corium can be finely dispersed in the coolant. The large interfacial area then allows an intense thermal interaction with pressure generation on an acoustic timescale. The resulting detonation-like phenomenon is referred to as a *steam explosion*.

A steam explosion resulting from transient induced fuel rod failures is believed to be a major contributor to the severe accident at Chernobyl. However, reactivity transients of the magnitude required to produce sudden fuel rod failures appear to be virtually impossible in a PWR.* Concern over steam explosions is therefore based on core behavior after a loss of cooling leads to melting of the uncovered core region.

An energetic FCI is usually considered to have three phases: fuel-coolant mixing, triggering by a local pressurization event, and propagation. The multiphase mixtures existing prior to the propagating phase are referred to as *premixtures*. Premixing transients that escape triggering are called benign FCIs.

The magnitude of an energetic FCI is largely determined by the behavior of the premixture. A number of procedures for the prediction of FCI events have therefore been based on the detailed modeling of premixture dispersion and interfacial transport phenomena (e.g., Refs. 121 and 122). Unfortunately, there is (1994) incomplete knowledge of the details of the fuel break-up and heat transfer phenomena which must be described.

An alternative approach to the problem is to determine an upper limit on the fuel-coolant mixing possible and then use this result in a simple

*See J. Kubowski *et al.*, "An Evaluation of the Control Rod Ejection Accident in Pressurized Water Reactors of VVER-440 Type," *Kernenergie*, 33, 11, 417 (1990).

thermodynamic calculation of the pressure produced. Corradini and Moses¹²³ examined a number of models of the mixing process and concluded that the molten material participating in the reaction increased with the mixing length and water depth. They concluded that for substantial water depths (e.g., 3 m) and mixing lengths at the upper limit of those expected (e.g., 100 mm), the total fuel mass mixed is limited to <7% of the core mass.

With an upper limit on the mass of fuel which is premixed, the maximum energy available for a detonation can be estimated. In the early thermodynamic models,¹²⁴ the effect of the explosive energy release was treated in a manner similar to that used for chemical detonations. However, experimental data show that chemical explosions generate much larger pressure pulses ($\sim 10^4$ MPa) over shorter timescales (~ 0.1 ms) than energetic FCIs (typically 10 MPa over 1 ms).¹²⁵

Hall¹²⁵ has proposed a thermodynamic model based on the assumption that the compression of the fluid surrounding the fuel-coolant premixture is adiabatic and reversible. In addition, it is assumed that, at the end state, there is thermal equilibrium between the fuel and coolant of the original mixture. By taking the system as isolated, we may write

$$W_g + \Delta E = 0 \quad (6.57)$$

where W_g = work done as a result of interactions and ΔE = change in internal energy of the corium coolant mixture. Since internal energy is only a function of state, one may use the simplest possible process for linking the end and initial states. The ΔE for the essentially incompressible fuel is determined by the product of its effective heat capacity, c_p , and the temperature change. The coolant is assumed to follow a constant volume path to the end-state pressure and then an isobaric expansion to the final volume. The value of ΔE is then given by

$$\Delta E = m_f c_f \left\{ T_{se} - T_{fi} + \left(\frac{z'}{1-z'} \right) \left(T_{se} - T_{ci} + \frac{xH_{fg}}{c_c} \right) \right\} - P_e V_{gi} \left\{ 1 - \left(\frac{P_i}{P_e} \right)^{1/\gamma} \right\} \quad \text{for } 0 \leq x \leq 1 \quad (6.58a)$$

$$\Delta E = m_f c_f \left\{ T_{ce} - T_{fi} + \left(\frac{z'}{1-z'} \right) \left(T_{se} - T_{ci} + \frac{c_v}{c_c} [T_{ce} - T_{se}] + \frac{H_{fg}}{c_c} + \frac{P_e v_c}{c_c} - \frac{RT_{se}}{(MW)c_c} \right) \right\}, \quad \text{for } x \geq 1 \text{ (superheated vapor)} \quad (6.58b)$$

where

c_l, c_v = specific heat at constant volume for coolant in liquid and vapor states, respectively

c_f = effective specific heat of corium (effective value should include variation of specific heat with temperature and include latent heat of fusion)

$$z' = m_c c_c / (m_f c_f + m_c c_c)$$

m_c, m_f = mass of coolant and fuel, respectively, in the premixture

MW = molecular weight of coolant

T_{fi} = initial temperature of molten fuel

T_{ci}, T_{ce} = initial and final temperatures, respectively, of fuel

T_{se} = coolant saturation pressure at end-state pressure

P_i, P_e = initial and final (end-state) pressure, respectively

V_{gi} = initial volume of cover gas

v_c = specific volume of coolant at initial conditions

x = quality of coolant at end state

R = ideal gas constant

γ = ratio of specific heat at constant pressure to specific heat at constant volume for coolant vapor.

The unknown values of x , T_{ce} , and T_{se} may be expressed in terms of the final pressure, P_e . The final quality is approximated, providing the volume of the vapor is much larger than the liquid volume, by

$$x = \frac{MWP_e}{RT_{se}} \left\{ v_c + \frac{V_{gi}}{m_f} \frac{c_c}{c_f} \left(\frac{1-z}{z} \right) \left(1 - \left[\frac{P_i}{P_e} \right]^{1/\gamma} \right) \right\} \quad (6.59)$$

From the Clausius-Clapeyron equation

$$T_{se} = \left\{ \frac{1}{T_b} - \frac{R}{(MW)H_{fg}} \ln \left(\frac{P_e}{P_b} \right) \right\} \quad (6.60)$$

where T_b is the saturation temperature at the base pressure, P_b . If $x \leq 1$, then $T_{ce} = T_{se}$. For $x \geq 1$,

$$T_{ce} = T_{se} + xH_{fg}/c_v \quad (6.61)$$

Final closure is obtained by relating the final pressure P_e to the work done, W_g . If the cover gas is ideal and water compressibility can be ignored,

$$-\Delta E = W_g = \frac{P_i V_{gi}}{\gamma - 1} \left\{ \left(\frac{P_e}{P_i} \right)^{(\gamma-1)/\gamma} - 1 \right\} \quad (6.62)$$

An iterative numerical solution of the foregoing equations provides an estimate of the pressure produced by the postulated FCI. Since kinetic energy effects have not been included, the pressure estimate would theoretically be a qualified lower bound. However, the pressures computed by this approach appear to be in reasonable agreement with data from small-scale tests.¹²⁵

6.8.5.4 Lower Head Coolability

The ability to cool molten corium improves once the molten mass has fallen and is held by the lower head of the reactor vessel. The molten mass is no longer hemispherical but the molten material spreads out over the lower vessel head surface.

If, as in the TMI accident, the corium drains into the lower head without an energetic FCI, the head can be cooled by inflow of water through the insulation.¹²¹ Accumulation of water around the lower portions of the reactor vessel is to be expected during a LOCA in a number of containment configurations. For configurations where the water accumulation does not normally occur, means for flooding this area could be provided.

The molten corium in the lower head would circulate by natural convection causing a portion of the heat generated to be transferred upward to the steam in the vessel and the remainder downward through the vessel head. Based on a modification of earlier correlations, Henry and Fauske¹²⁶ suggest that the downward heat flux, q''_d , can be obtained from

$$q''_d = 0.54 \frac{k\Delta T}{R} \text{Ra}^{0.18} \left(\frac{L}{R}\right)^{0.26} \quad (6.63)$$

and the upward heat flux, q''_u , from

$$q''_u = 0.36 \frac{k\Delta T}{R} \text{Ra}^{0.23} \quad (6.64)$$

where

R = outer radius of reactor vessel head

L = distance between top of molten material and center point of inner surface of vessel head

k = thermal conductivity of corium

ΔT = (corium temperature) – (water saturation temperature)

Ra = Raleigh No = $g\beta q''' R^5 / (\alpha' \nu k)$

α' = thermal diffusivity = $k / \rho c_p$

q''' = volumetric heating rate in corium

ν = kinematic viscosity of corium

β = coefficient of volume expansion.

The total heat generation in the bed is equated to the sum of the heat transferred to the upper and lower surfaces. The resulting equation is rearranged to yield

$$\Delta T = \frac{RV_m q'''}{k} \left[0.36 A_u \text{Ra}^{0.23} + 0.54 \text{Ra}^{0.18} A_d \left(\frac{L}{R}\right)^{0.26} \right]^{-1} \quad (6.65)$$

where A_d, A_u = heat transfer areas at lower and upper surfaces of corium, respectively; and V_m = volume of corium melt.

The value of ΔT obtained may then be substituted into Eq. (6.63) to obtain the heat flux at the lower head surface. Table 6.VIII lists estimated properties for molten corium needed for this calculation.

The vessel head will be coolable if the value of q_d'' is below the critical heat flux and if sufficient fluid can flow through the insulation passages to make up for that evaporated. The boiling heat transfer would take place in the narrow passage between the reflective insulation and the reactor vessel. The critical heat flux for a segment of such a passage has been predicted by assuming a circulation flow determined by equating the driving head of liquid to the pressure drop in the passage segment.¹²⁶ The heat removal rate is then correlated by

$$q'' = \frac{s}{L'} H_{fg} \left[\frac{xg\rho_l L' \sin\theta}{1 + fL'/2s} \right]^{1/2} \quad (6.66)$$

where

f = friction factor

s = width of gap between vessel and insulation

L' = insulation gap length

θ = angle of inclination of surface

x = exit quality.

The upper limit on the heat flux is determined by setting x in Eq. (6.66) equal to unity.

The total rate of energy removal through the lower head is also limited by the total flow of water into the gap region. The water entering the gap flows through the submerged insulation gaps and steam exits through the insulation gaps above the liquid surface. A conservative estimate of this flow rate, m_w , is obtained from¹²⁶ a computation of the total natural circulation flow. Upon equating driving head and pressure drop, we obtain

$$m_w = C \left[\frac{2(\rho_l - \rho_g)gh}{\frac{1}{\rho_l L_l^2} + \frac{1}{\rho_g L_g^2}} \right]^{1/2} \quad (6.67)$$

TABLE 6.VIII
Estimated Properties of Molten Corium*

$k = 3 \text{ W/(m K)}$	$\alpha' = 7 \times 10^{-7} \text{ (m}^2/\text{s)}$
$\nu = 6 \times 10^{-7} \text{ (m}^2/\text{s)}$	$\beta = 10^{-4} \text{ K}^{-1}$

*Data from Henry and Fauske, *Nucl. Eng. Des.*, **139**, 31 (1993).

where

c = empirical constant (estimated as 10^{-4} from test data)

h = water height

L_l = length of gap submerged in water

L_g = gap length for steam venting.

Henry and Fauske¹²⁶ conclude that, for typical head and insulation geometry, the head cooling available will be able to remove the decay heat generated.*

6.8.5.5 Fission Product Release and Transport

The evaluation of the radiological consequences of a severe accident requires a knowledge of the quantity and chemical state of the fission products released during the accident. The fission product release mechanisms may be categorized as:

1. *Gap release*: release upon cladding failure of the fission products in the interconnected void volume of the fuel rod
2. *Meltdown release*: escape of fission products as fuel becomes molten
3. *Vaporization release*: release resulting from sparging of molten core by gases from concrete decomposition
4. *Oxidation release*: release from finely divided corium droplets that are formed as the result of any steam explosion.

A portion of the fission products released will deposit on the internal surfaces of the primary loop. A portion of those fission products reaching the vapor container may be deposited on container and equipment surfaces or removed by the vapor container spray. Once the release to the vapor container atmosphere is determined, the radioactivity released to the environment can be estimated. The product of (1) an index measuring the radiological consequences of the release (e.g., potential fatalities) and (2) the probability of the series of events producing the release, is a measure of the relative risk of the particular accident sequence.

The complex subject of fission product release and transport is considered beyond the scope of this text. It is mentioned here only to alert the reader to this problem area.

*Confirmatory experimental research in the CYBL facility at Sandia National Laboratory [P. V. Chu *et al.*, "Large Scale Testing of In-Vessel Debris Cooling Through External Flooding of Reactor Pressure Vessel in the CYBL Facility," paper presented at 21st Water Reactor Safety Meeting, Washington, DC (1994)] indicates that external flooding should be capable of providing the necessary cooling.

6.8.5.6 Direct Containment Heating (DCH)

It was previously observed that if the lower head of the reactor vessel failed at high pressure, the corium it contains is likely to be dispersed widely in the containment volume. The dispersed core debris would transfer an appreciable fraction of its energy directly to the vapor in the containment. Computations for a typical PWR reactor indicate that adiabatic transfer of all the energy from dispersal of 50% of the core would result in a vapor container pressure of about 4.5 bar and 100% dispersal would result in 12 bar (Ref. 127). The pressure due to the unlikely 100% release would clearly fail the vapor container.

The DCH scenario can be avoided by providing an accident management system that depressurizes the primary system to a safe level prior to core melt. It is therefore desirable to be able to predict the pressure below which dispersal rates would be tolerable. Early estimates of the dispersal pressure were based solely on Ku , the Kutateladze number, where

$$Ku = \rho_g U_g^2 / [\sigma g (\rho_l - \rho_g)]^{0.5} \quad (6.68)$$

σ = surface tension, and U_g = gas velocity in cavity. This dimensionless number is a well-known criterion for entrainment phenomena. However, the small scale experiments of Tutu *et al.*¹²⁸ indicated that the fraction of the melt that would be retained in the cavity to which it was released was also dependent on the hole size and the driver gas density. The data were used to develop a preliminary criterion¹²⁸ for the low-pressure cutoff for debris dispersal. For typical PWR melt conditions, low-pressure cutoff predictions are in the vicinity of 20 to 30 bar.

6.8.5.7 Corium-Concrete Interaction (CCI)

When molten corium contacts concrete, the concrete is eroded by thermal decomposition. That is, the heat transferred to the concrete releases chemically bound water and carbon dioxide, vaporizes water, and melts the refractory oxides remaining (CaO , SiO_2). The H_2O and CO_2 released from the concrete are reduced to H_2 and CO as they pass through the corium pool. In the presence of igniters, the CO and H_2 released from the pool will form CO_2 and water vapor. The heat released from the combustion of hydrogen will of course increase the containment pressure. The water vapor may be condensed if containment cooling is available. However, the CO_2 is non-condensable and a long-term (several day) pressure buildup leading to vapor container failure is possible. Note that major CO_2 releases are only possible if limestone was used in the coarse aggregate of the concrete.

The CCI phenomenon can be divided into two phases.¹²⁹ In the first (high-temperature) phase, the molten corium is above the melting range of individual oxidic and metallic layers. This transient phase is determined by the stored energy in the melt. After a relatively short time (approximately

1 hr), the corium temperature falls below the solidification temperature of one or more of the melt constituents. A portion of the energy loss from the pool is taken up by the corium solidification. In the second phase, the rate of heat loss decreases. The melt reaches a quasi steady state in which heat loss balances energy gain due to decay power and exothermic chemical reaction.

During the initial transient period, the rate of heat transfer from the melt to the concrete is largely determined by the heat transfer coefficient between the corium melt and the concrete. A conservative (possible overestimate) of the rate of concrete penetration is obtained if it is assumed that the molten concrete slag dissolves in the corium. The gases released by the corium-concrete interaction considerably complicate the heat transfer at the interface. Bradley and Suo-Anilla¹²⁹ found that the Kutateladze correlation for bubble driven convection gave reasonable penetration values. This heat transfer coefficient, h_c , is therefore estimated from:

$$\frac{h_c \lambda}{k} = \begin{cases} 1.5 \times 10^{-3} Ku_1^{2/3} & \text{for } \frac{u_g \mu}{\sigma} < 4.3 \times 10^{-4} \\ 3.0 \times 10^{-5} Ku_1^{2/3} \left(\frac{u_g \mu}{\sigma} \right) & \text{for } \frac{u_g \mu}{\sigma} > 4.3 \times 10^{-4} \end{cases} \quad (6.69a)$$

where

λ = Taylor wavelength = $\sqrt{\sigma/[g(\rho_l - \rho_g)]}$

k = thermal conductivity of corium

Ku_1 modified Kutateladze number = $P u_g \text{Pr} / g \mu$

P = system pressure

Pr = Prandtl number = $C_p \mu / k$

u_g = superficial velocity of gases released from melt

μ = viscosity of corium.

Heat transfer from the upper surface of the corium would be by radiation and natural convection to the containment atmosphere. If we assume that the corium is well mixed by the gases from concrete decomposition, a heat balance gives

$$V \rho_c (c_p) \frac{dT_c}{dt} = q''' V - (T_c - T_a) A (h_{\text{rad}} + h_{co}) - A U (T_c - T_m) \quad (6.70)$$

where

q''' volumetric heat generation rate in corium (energy/mass t)

A corium-concrete contact area

(c_p) = specific heat of corium

h_{co} = natural convection heat transfer coefficient to containment atmosphere

- h_{rad} = radiation heat transfer coefficient $= q''_{\text{rad}} / (T_c - T_a)$
 T_a, T_c = temperature of atmosphere and corium, respectively
 T_m = melting point of concrete
 V = volume of corium
 U = overall heat transfer coefficient between corium and concrete
 (equal to h_c if no slag assumed).

A numerical solution of the finite difference approximation of Eq. (6.70) allows the corium temperature to be estimated.

During the quasi-steady-state phase of CCI, a concrete slag would clearly be present between the solidified corium and the remaining concrete. Heat transfer rates to the slag would also be expected to be predicted by the Kutateladze equations [Eqs. (6.69a) and (6.69b)]. However, because of the uncertainty in slag properties, some investigators¹²⁹ suggest that Eqs. (6.69a) and (6.69b) be modified so as to yield conservative heat transfer rates.

To determine the rate of concrete penetration from a given heat transfer rate to the concrete, q''_{con} , requires an estimate of the decomposition enthalpy of concrete. Theofanous¹⁰⁸ estimates this to be approximately 2700 kJ/kg concrete. If a constant velocity of the molten concrete boundary is assumed, the penetration rate, v , is then obtained from

$$q''_{\text{con}} = v \rho_c H_d + q''_c \quad (6.71)$$

where

$$q''_{\text{con}} = UA(T_c - T_m)$$

H_d = decomposition enthalpy change (energy/mass)

ρ_c = density of concrete

q''_c = conduction heat transfer rate to solid concrete slab (energy/area time)

v = penetration rate (length/time).

The value of q''_c is obtained by considering the problem from the standpoint of a moving heat source in an infinite solid. The basic differential equation is then¹³⁰

$$\frac{d^2 T}{d\xi^2} + v \rho_c c_p \frac{dT}{d\xi} = 0 \quad (6.72)$$

where $\xi = x - vt$; c_p = specific heat of concrete; and x = distance from original concrete boundary.

Solution of Eq. (6.72) and application of the boundary conditions that $T = T_m$ at $\xi = 0$ and $T = T_i$ at $\xi \rightarrow \infty$ yields

$$T = (T_m - T_i) \exp\left(\frac{-v\xi}{\alpha}\right) + T_i, \quad (6.73)$$

where $\alpha = k/\rho c_p$. The heat flux transferred to the solid is obtained at $\xi = 0$ from

$$q_c'' = -k \frac{dT}{d\xi} \Big|_{\xi=0} = \frac{kv}{\alpha} (T_m - T_i) \quad (6.74)$$

By combining Eqs. (6.71) and (6.74) with the rate of heat transfer from the corium, we obtain the penetration rate, v , as

$$v = \frac{UA(T_c - T_m)}{\rho H_d + \rho c_p (T_m - T_i)} \quad (6.75)$$

With the assumption that q_{con}'' corresponded to the heat generated in a 50-cm-thick layer of corium generating 0.5 W/cm³, Theofanous¹⁰⁸ estimated the value of v would be 6.5 cm/hr.

In a number of plant designs, the reactor cavity is separated from the sump water by only a modest thickness of concrete shielding. Upon melt-through of this concrete, the corium would be brought into contact with the sump water. The vapor container pressure would be significantly increased by the steam produced by this contact. Farmer *et al.*¹³¹ note that the rate of heat transfer will be determined by radiation and gas-sparging enhanced film boiling. They attributed the enhanced heat transfer primarily to an enhanced contact area. Based on small-scale molten lead-freon tests¹³² they correlate this enhanced area as

$$A_{\text{enhanced}}/A_{\text{flat}} = 1 + 4.5 u_g/u_t \quad (6.76)$$

where u_g = superficial velocity of concrete decomposition gases; u_t = bubble rise velocity = $1.4\{[g(\rho_l - \rho_g)\sigma]/\rho_l^2\}^{1/4}$; and σ = surface tension.

The total heat transfer coefficient is effectively the sum of the conduction and radiation heat transfer coefficient across the vapor film (entrainment rate very low at expected condition and entrainment heat transfer negligible). The value of q_c'' , the conduction heat transfer across the film, is determined from

$$q_c'' = q_{\text{FB}}'' (A_{\text{enhanced}}/A_{\text{flat}}) \quad (6.77)$$

where q_c'' = conduction component = enhanced film boiling heat flux based on nominal (flat) area; and q_{FB}'' = minimum film boiling heat flux from Berner correlation [Eq. (4.239)].

Farmer *et al.*¹³¹ suggest that the radiant heat transfer across the film be calculated in the usual manner except that A_{enhanced} be used instead of the flat contact area. Farmer *et al.*¹³¹ conclude that radiation is the predominant mode of heat transfer across the film under actual accident conditions.

6.8.5.8 Hydrogen Generation

Hydrogen generation to the zirconium-steam reaction is well understood as long as the core retains something close to its original geometry

(see Sec. 2.6.2.2). Additional hydrogen production is, however, to be expected from steam interaction with core rubble or molten corium. If core cooling is resumed, before a large fraction of the core has become molten, it is likely that essentially all of the remaining Zircaloy will be oxidized. This is borne out by the postaccident examination of the TMI core, which showed no metallic zirconium in the high-temperature regions.¹³²

Estimates of the hydrogen production from molten corium pouring or slumping into a pool of water in the lower head are less certain. The available calculations¹³³ suggest that additional oxidation of this core material will add a few percent to the hydrogen production. However, no firm upper limit on the hydrogen production can be supported on the basis of the available (1994) models.¹³²

A major concern in a severe accident sequence is the possibility that a hydrogen detonation will occur. The resulting pressure pulse could lead to the failure of the primary containment. This concern has led to the U.S. NRC requirement that all vapor containers be equipped with sufficient hydrogen igniters or recombiners to insure that a detonation will not occur. Possible detonations are of greatest concern in the small containments associated with vapor suppression devices (e.g., ice condenser) since these containments will have the highest hydrogen concentrations.

In order for a hydrogen detonation to occur, a combustible mixture must be present. The solid line of the concentration diagram of Fig. 6.11 shows the limit of such combustible mixtures.¹³⁴ Mixtures to the left of the solid line are combustible. The shaded and crosshatched regions of Fig. 6.11 indicate those mixtures where a detonation is possible. However, a mixture can detonate only if it is present in a compartment or volume whose smallest width is greater than the so-called "detonation cell width" of the mixture. The mixtures in the shaded region of Fig. 6.11 have detonation cell widths less than 3 m (Ref. 134). Such mixtures would be able to support a detonation within the vapor container. Although there are compartments in a typical PWR which have widths larger than 3 m, they are not believed to have sufficient obstacles to create the flame acceleration required by a detonation.¹³⁴ The region between the solid line and shaded region is the region where ignition systems can operate. The modest band between the detonation and flammability limits clearly indicates the necessity of taking steps to avoid local peaks in the hydrogen concentration (e.g., wide distribution of hydrogen igniters, means for gas circulation). Additional protection can be provided by use of catalytic hydrogen recombiners which can operate below the ignition limit.

6.8.6 Computer Programs for Analysis of Severe Accidents

To attempt an overall assessment of the consequences of severe accidents going beyond core melt, a number of computer programs (MAAP,¹¹

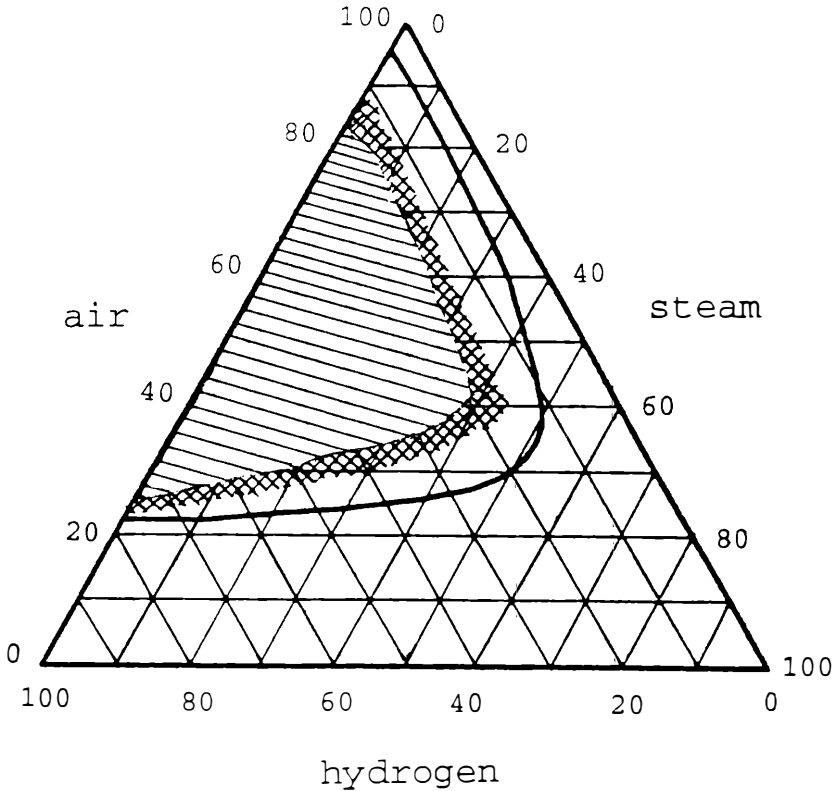


Fig. 6.11 Combustion and detonation limits in hydrogen-steam air mixtures [from U. Behrens *et al.*, *Nucl. Eng. Des.*, **130**, 43 (1991)].

MARCH,¹³⁵ MELCOR,¹² MELPROG^{136,137}) have been written. Each of these programs contains mechanistic models for the spreading of the molten region, relocation of molten corium, and failure of structure components. Some of these programs (e.g., MELCOR, MAAP) are stand-alone codes, which contain a relatively simple model of the entire reactor system. Others (e.g., MELPROG) model the core alone and are linked to a systems code when used (e.g., MELPROG/TRAC¹¹²). Computer programs have also been written to examine specific aspects of the accident (e.g., CORCON^{138,139} for molten core concrete interactions).

At present (1994), the mechanistic core behavior models used are based on very limited data and are highly tentative. Because of the great difficulty involved in carrying out relevant large-scale experiments, it may never be possible to fully verify the mechanistic models proposed. Nevertheless, the

computer programs are valuable for scoping calculations and as a means for determining problem areas.

6.8.7 Severe Accident Management

All severe accident studies indicate points where appropriate and timely operator intervention can substantially ameliorate the course of the accident. The most obvious intervention is system depressurization during a severe accident at high pressure. Whenever high core outlet temperatures are seen in conjunction with high system pressures, the system should be depressurized to avoid high-pressure melt ejection from the reactor vessel. Nearly all PWRs have reliable depressurization systems which can be manually and/or automatically actuated. Connection of the power operated relief valves to the emergency battery power supply will ensure that these valves can be operated even during a station blackout.

Current U.S. NRC regulations require that additional auxiliary electric power generators be available for connection if the diesel generators fail to start during a power outage. The connection of such generators eliminates the most severe consequences of a station blackout.

If the system is at low pressure, the unavailability of emergency core coolant can be circumvented, at least in part, by connecting the system to an outside water supply. Similarly, failure of the water supply to the shutdown cooling system, which provides long-term heat removal, can be remedied by connection to an external water supply.

Section 6.8.5.2 noted that it appears that molten corium on the lower head of the reactor vessel can be cooled if water has flooded the reactor cavity. If such water would not normally be present, operator action can be taken to assure that the cavity is flooded.

Modern operating practice includes (1) the preparation of detailed plans for coping with severe accidents and (2) making certain that reactor operators can recognize the various situations possible and know what actions should be taken.

6.8.8 Probabilistic Risk Assessment

A probabilistic risk assessment (PRA) is a procedure that (1) identifies and delineates the combination of events that, if they occur, will lead to an undesired event or events; (2) estimates the occurrence frequency for each event combination; and (3) estimates the consequences of the undesired events.¹⁴⁰ When applied to a nuclear reactor, the undesirable event is usually taken as core melting (with the undesirable consequences being the quantity of radioactivity released to the environment).

The application of PRA methodology to nuclear power plants behavior was introduced by Rasmussen, Levine, and coworkers.¹⁴¹ This pioneering study assessed the risk to the public from accidents occurring in light-

water-cooled reactor systems. Public risk was defined in terms of a curve of events per year versus fatalities. The number of yearly events was based on the assumption that 100 reactors were operating in the United States. Similar curves of events versus fatalities were drawn for the other man-caused disasters, e.g., airplane crashes. The curves for reactor-caused fatalities and, hence, risk to the public from reactor accidents, were orders of magnitude below any of the other man-caused disasters. The curve for reactor-caused fatalities was approximately two orders of magnitude below the curve for fatal injuries caused by airplane crashes to persons standing on the ground.

After the TMI accident, which was economically disastrous but not injurious to public health, the contributions that PRA studies could make to reactor safety were recognized. PRA studies of existing plants are now conducted to (1) determine areas that are significant contributors to the probability of a severe accident and (2) evaluate the effect of proposed design or operating changes on severe accident probability and consequences. A PRA assessment of the ability of new plant designs to reduce the probability of severe accidents is one of the ways such designs are now evaluated. Since thermal-hydraulic design changes are likely to be partially evaluated through a PRA study, an acquaintance with general approach is required.

A PRA is made up of sets of *event trees* and *fault trees*. The process begins with an event tree defined by an initial failure within the plant. The failure may be internally generated (e.g., pipe rupture) or externally generated (e.g., earthquake). The tree then considers the events which follow as determined by the operation or failure of the various safety systems. A separate event tree is required for each potential initial failure.

Figure 6.12 illustrates a greatly simplified event tree for a sequence initiated by a small LOCA (far left of diagram). At each subsequent significant event, a failure probability, p_i , must be assigned. The probability of each sequence of events is the product of the probabilities of the events in the direct chain.

Each failure probability needed by the event tree is determined from a fault tree. A fault tree begins with the failure of the system and then determines, using mathematical and engineering logic, the ways in which the system failure can occur. The likelihood of the failure is then estimated from the failure rates for the components making up the system and the likelihood of operator or maintenance errors. The probability estimation is complicated by the need to consider "common cause failures," which are simultaneous failures caused by a single event (e.g., cable tray fire). A full description of the approach used for event and fault tree generation is provided by Fullwood and Hall.¹⁴²

In the so-called level 1 PRA, the final states of the PRA analysis are safe shutdown of the reactor or various levels of core damage. In a level 2 PRA, the final states are various categories of radioactivity releases. Since level 2

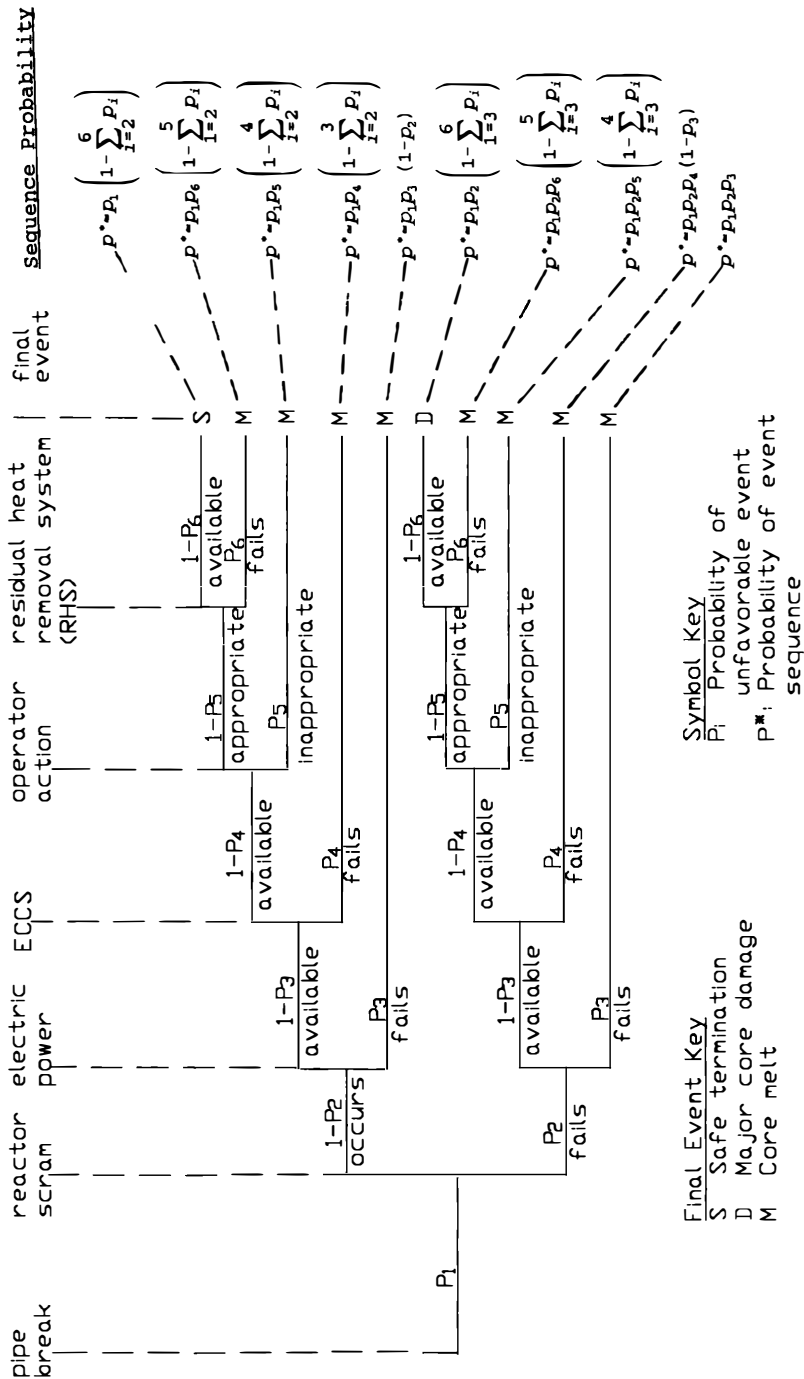


Fig. 6.12 Simplified event tree for a large-break LOCA.

analyses involve the uncertain sequence of events in reactor vessel and containment failure, the uncertainty band attached to level 2 results is considerably larger than that associated with a level 1 analysis.

A completed PRA provides the characteristics of a given plant at a specific point in time. Changes in operating procedure or components can change PRA results. When a "living PRA" is maintained, all such changes are evaluated and, when applicable, incorporated in the PRA.¹⁴³ Successful implementation of this approach requires a computer code capable of (1) computing the necessary event and fault tree probabilities for appropriately supplied data; (2) easily storing and retrieving the PRA models, data, and results; and (3) allowing easy modification of previous models. Several such programs are available.¹⁴³

6.9 VAPOR CONTAINER ANALYSIS

6.9.1 Containment Systems

The primary system of a Western PWR is surrounded by a containment structure designed to contain any radioactive material released in a LOCA.* The containment structure is most commonly a large steel pressure vessel. This may be surrounded by a concrete structure that provides some shielding and secondary containment. The secondary containment reduces leakage to the atmosphere. Alternatively, the vapor containment may be provided entirely by a prestressed concrete vessel. In most cases, the vapor containers have been cylindrical vessels. However, as noted in Chapter 1, some designs have used spherical vessels.

The most common vapor container design used for PWR units has been the large dry type [see Fig. 6.13(a)]. This design simply provides a sufficiently large free volume so that the final pressure reached is not excessive (e.g., 4 to 5 bar). The normal operating pressure inside the container may be at or below atmospheric.

Alternatively, the vapor suppression design may be used. This design uses water in the solid or liquid form to remove sensible and latent heat from the containment gases. In BWR plants, the vapor suppression is obtained by forcing the steam released to flow through a large pool of water. Vapor suppression pools have not been used in any current (1995) PWR plant. When vapor suppression has been used on PWRs of Western design, the vapor suppression was accomplished by means of ice condensers. In this scheme, a number of large vertical chests containing ice cubes are placed in an annular region around the periphery of the vapor container. Conventional refrigeration equipment keeps the ice frozen under normal

*Although the more recent Russian VVER plants have vapor containers, the early VVER plants do not.

circumstances. The vapor container compartments are arranged so that steam released from a reactor system rupture must flow through the ice chests before escaping into the free volume of the vapor container [see Fig. 6.13(b)]. Final pressures of slightly less than 2 bar are usually obtained.

The Russian VVER plants use a vapor container arrangement similar to that for an ice condenser containment. However, the ice chests are replaced by bubble-cap towers containing trays of cool water.¹⁴⁴ Condensation of the steam bubbling through the water trays substantially lowers the final pressure attained. Plants using one of the vapor suppression schemes require considerably smaller containment volumes than those needed for dry containment. This makes the accumulation of an explosive hydrogen concentration following a LOCA much more likely than in those plants with dry containment. The use of igniters and catalytic recombiners in conjunction with gas circulation to avoid explosive mixtures has been discussed in Sec. 6.8.5.8. Another approach to the problem is the inerting of the original containment atmosphere by replacing the air with nitrogen. While this latter scheme has been used for BWRs, it has not been (as of 1995) applied to any PWR.

6.9.2 Licensing Analysis

The licensing analysis used to determine vapor container size and design pressure normally considers only the design basis accidents. Thus, the vapor container is constructed according to the applicable pressure vessel code with the analyses of a large loss-of-coolant or steam-line break accident setting the operating conditions. In the past, severe accident conditions were not considered in establishing the vapor container design.

6.9.2.1 Large Dry Containment (Approximate Analysis)

A conservative maximum pressure estimate is often obtained by assuming all of the primary coolant or all of the secondary coolant mixes instantaneously with the vapor container air. The pressure reached is then computed by assuming thermodynamic equilibrium and no heat transfer to the environment. Since such a process is adiabatic and at constant internal energy, we may write

$$ME_1 + me_1 = ME_2 + me_2 \quad (6.78)$$

where m = mass of air in container; M = mass of coolant in system; E_1, E_2 = initial and final internal energy of coolant (energy/mass); and e_1, e_2 = initial and final internal energy of containment air (energy/mass).

When a primary system break is analyzed, consideration must be given to the additional energy imparted by the Zircaloy-steam reaction. If we make the simplifying assumption that the total coolant mass involved in this reaction is small, we may then write

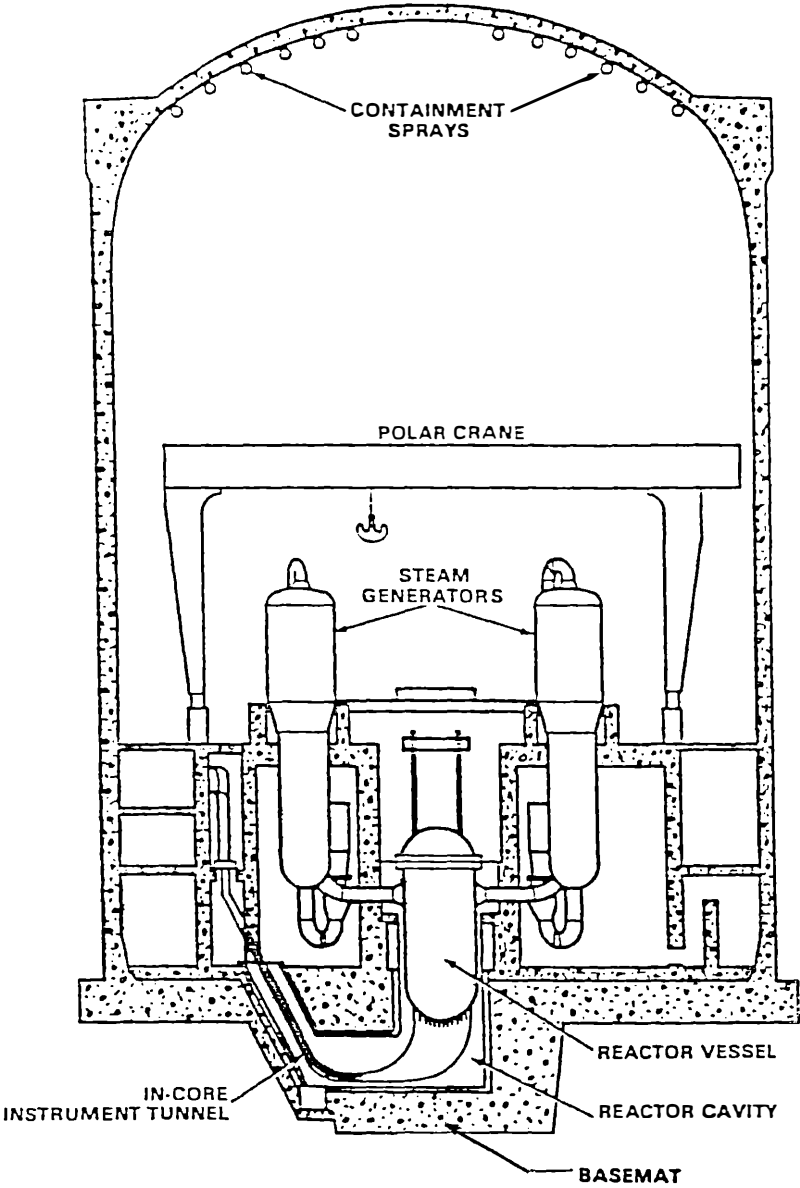


Fig. 6.13(a) Large dry containment [from Ref. 108].

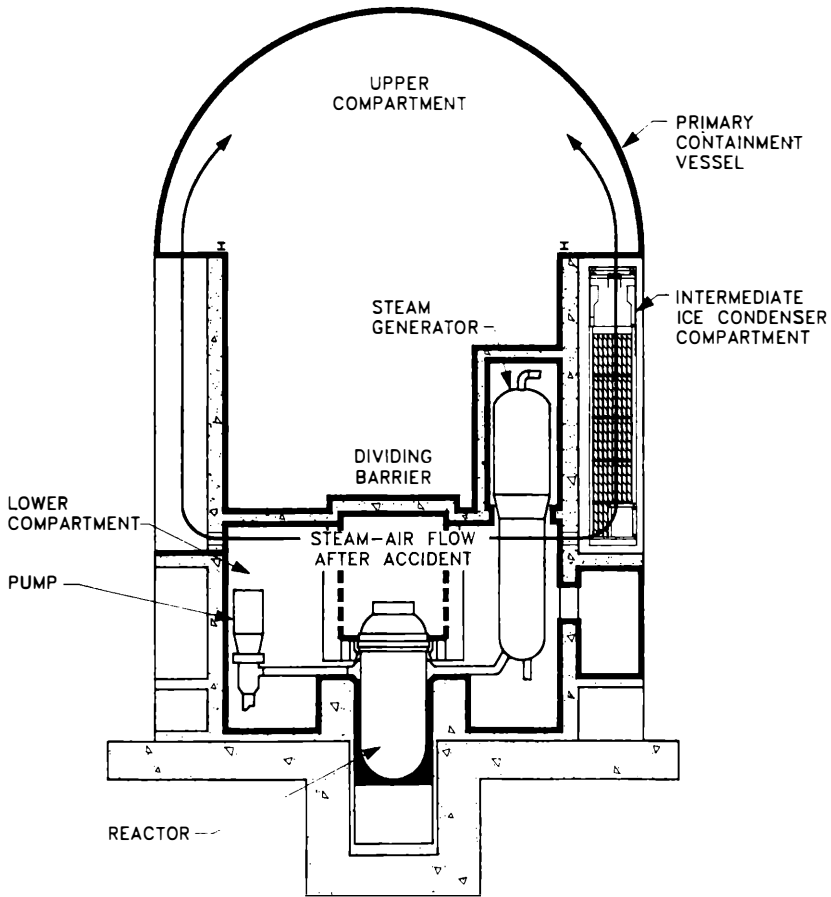


Fig. 6.13(b) Ice condenser containment [from C. W. Forsberg and W. J. Reich, ORNL/TM-11907 (1991)].

$$ME_1 + me_1 + Q_R = ME_2 + me_2 \quad (6.79)$$

where Q_R = total heat released in the Zircaloy-steam reaction. The value of Q_R may be conservatively estimated by assuming that the maximum allowable Zircaloy-steam reaction (17% of cladding reacted) has taken place.

As in previous expositions of this approach,¹⁴⁵ we express e in terms of the specific heat at constant volume, c_v , and take air as an ideal gas. Equation (6.79) then becomes:

$$\frac{Q_R}{M} + E_1 - E_2 = \frac{m}{M}(e_2 - e_1) = \frac{P_1 V_v c_v (T_2 - T_1)}{MR'T_1} \quad (6.80)$$

where

$$m = P_1 V_v / (R' T_1)$$

P_1 initial pressure of air in containment

T_1, T_2 = initial and final air temperatures

R' = R /(molecular weight air)

R = ideal gas constant

V_v vapor container volume.

We may also write

$$M = V_p \rho_p \quad (6.81)$$

$$E_2 = E_f + x E_{fg} \quad (6.82)$$

where ρ_p = coolant density at initial plant conditions; V_p = coolant volume at initial plant conditions; and E_f, E_{fg} = internal energy of saturated liquid and internal energy of evaporation at final pressure (energy/mass). By ignoring the volume of the unvaporized liquid, the quality, x , is approximated by

$$x = V_v / M v_g$$

where v_g = specific volume of steam at final pressure (volume/mass). Upon substitution of the foregoing relation into Eq. (6.80) we obtain

$$V_v / V_p = \frac{(E_1 + (Q_R / M) - E_f) \rho_p}{P_1 v_g (T_2 - T_1) / (R T_1) + E_{fg} / v_g} \quad (6.83)$$

If a value of P_s for the final partial pressure of steam is assumed, the values of E_{fg} , E_f , e_v , and v_g are known. The value of the vapor container volume required to achieve this pressure is obtained from Eq. (6.83). The total vapor container pressure, P_f , is then obtained from

$$P_f = P_s + P_1 (T_2 / T_1) \quad (6.84)$$

where P_s = saturation pressure of coolant at T_2 . By repeating the computation for several values of P_s , the vapor container volume required to achieve the desired value of P_f may be determined.

6.9.2.2 Time-Dependent Pressure Distribution in a Large Dry Containment

The assumption that the containment pressure rises immediately to that set by thermodynamic equilibrium for all material in the system is only a crude approximation. The pressure will, of course, rise over the finite period of primary system blowdown. Further, as inspection of Fig. 6.13(a) will show, the vapor container is really divided into a number of compartments. During the early period of the accident, the pressures in the various compartments will not be equal. It is more realistic to treat the vapor compartments

as a series of interconnected nodes and calculate the time variation in node pressures.

The approach used is essentially that of the “node and branch” method described in Sec. 4.1.2.4.2. Conditions within each node are determined by a simultaneous solution of the mass and energy conservation equations using the assumption of thermodynamic equilibrium.

The branches connecting the nodes provide the resistance to flow. Flow from the primary system into the node containing the pipe break is determined by the primary system blowdown rate. Flow between the various nodes is determined by the pressure differences between the nodes.

For each node we write a mass balance for the steam, water and air¹⁴⁵

$$\frac{\Delta M_s}{\Delta t} = W_{si} - W_{so} + W_v \quad (6.85)$$

$$\frac{\Delta M_w}{\Delta t} = W_{wi} - W_v - W_{wo} \quad (6.86)$$

$$\frac{\Delta M_a}{\Delta t} = W_{ai} - W_{ao} \quad (6.87)$$

where

M_s, M_w, M_a = masses of steam, water, and air in node, respectively

W_{si}, W_{so} = inlet and outlet steam mass flow rate, respectively

W_{wi}, W_{wo} = inlet and outlet water mass flow rate, respectively (if we assume water flows into first node from break but otherwise remains in node, $W_{wo} = 0$)

W_{ai}, W_{ao} = inlet and outlet air mass flow rates, respectively

W_v = flashing or condensing mass flowing between phases.

For simplicity we write the energy balance assuming no heat transfer to the chamber surfaces. This yields

$$\begin{aligned} W_{wi}H_{wi} + W_{si}H_{si} + W_{ai}H_{ai} - W_{so}H_{so} - W_{wo}H_{wo} - W_{ao}H_{ao} = \\ H_{so}\frac{\Delta M_s}{\Delta t} + H_{wo}\frac{\Delta M_w}{\Delta t} + H_{ao}\frac{\Delta M_a}{\Delta t} - \frac{V}{J}\frac{\Delta P}{\Delta t} \end{aligned} \quad (6.88)$$

where

H_{wi}, H_{wo} = initial and outlet water enthalpies, respectively

H_{ai}, H_{ao} = initial and outlet air enthalpies, respectively

H_{si}, H_{so} = initial and outlet steam enthalpies, respectively

J = mechanical equivalent of thermal energy (1.0 in SI units)

V = volume of vapor container

and other symbols have their previous meaning. We may also write a volume balance since the volume of the node does not change. Then

$$\frac{\Delta M_a}{\Delta t} v_a + \frac{\Delta M_w}{\Delta t} v_w + \frac{\Delta m_w}{\Delta t} v_w = 0 \quad (6.89)$$

where v_s, v_w, v_a = specific volume of steam, water, and air, respectively. The specific volumes, v_s, v_w are functions of P and T only

Furthermore, the air partial pressure, P and steam partial pressure, P can be expressed in terms of T . Thus,

$$P = P + P_a = f(T) + \frac{M_a R' T}{V} \quad (6.90)$$

where V_a = volume occupied by air = total volume – water volume. The specific volumes v_s, v_w , and v_a are functions of P and T but by Eq. (6.90) they can be written in terms of T . We may therefore rewrite Eq. (6.85) as

$$\frac{\Delta M_w}{\Delta t} f_1(T) = -\frac{\Delta M_s}{\Delta t} f_2(T) - \frac{\Delta M_a}{\Delta t} f_3(T) \quad (6.91)$$

Since fluid enthalpy is also given in terms of P and T , we may also make use of Eq. (6.90) to write these quantities as functions of T alone. If we then assume that $\Delta M_w / \Delta t$, $\Delta M_s / \Delta t$, and $\Delta M_a / \Delta t$ have the values that held at the previous timestep, substitution of Eq. (6.91) into Eq. (6.88) yields an equation with T as the only unknown. We therefore obtain our first estimate of the new node temperature. An estimate of the new node pressure may then be obtained from Eq. (6.90). The new pressures are then used to obtain revised estimates of the branch flows.

Flows between the nodes are generally obtained assuming the flowing fluid is homogeneous and using conservation of branch momentum given by Eq. (4.79). If desired, the estimates of the flow between nodes during time period Δt may be revised by using an average of the flows at the beginning and end of the time. The mass and energy balances can then be recomputed to give improved estimates of T and P . Iteration may continue until the desired convergence is obtained.

The adiabatic assumption can perhaps be considered reasonable only for the initial blowdown period. If a long-term analysis is required, the various heat sources (e.g., Zircaloy-steam reaction, decay heat) and sinks (e.g., condensation on components and on vapor-container spray) must be included in the energy balance. This considerably complicates the computations since transient conduction within solids must be modeled if the effect of condensation on structures and components is to be evaluated.

A number of computer programs using the node and branch approach (e.g., ZOCO-V,¹⁴⁶ BEACON,¹⁴⁷ CONTEMP¹⁴⁸) are available for vapor containment analysis. Some of the later computer programs (e.g., CON-

TAIN¹⁴⁹) are also designed to follow hydrogen concentration and combustion. These contain a separate mass balance for the hydrogen and a combustion model which is initiated at a defined hydrogen concentration value.

Analyses of conventional PWRs typically show that the steam-line break defines the limiting short-term response while the large LOCA taxes the long-term heat removal capabilities of the containment system.¹⁵⁰

Newer containment analyses programs designed for analyses of advanced PWR designs (e.g., COMPAC¹⁵⁰) tend to use more complex formulations to follow natural circulation within the vapor container. The vapor container compartments are divided into a number of smaller cells and flow is modeled using the porous-media approach.

6.9.2.3 Vapor Suppression Systems

An approximate estimate of the end pressure expected after blowdown in an ice condenser system can be obtained by an approach similar to that used for the dry containment. We may use Eq. (6.79) if we redefine the values of initial water mass and energy. We may write

$$M = M_w + M_i \quad (6.92)$$

$$E_i = (E_w M_w + E_i M_i) / (M_w + M_i) \quad (6.93)$$

$$E_i = c_{pi}(T_o - T_i) - H_{fus} \quad (6.94)$$

where

c_{pi} specific heat of ice

M_i, M_w = total mass of ice chests and water in primary system, respectively

E_i, E_w = internal energy of ice and primary system water at initial conditions, respectively (energy/mass)

T_o, T_i = freezing point of water and initial temperature of ice, respectively

H_{fus} = heat of fusion (energy/mass).

The final pressure may then be estimated using Eqs. (6.83) and (6.84) with these revised definitions.

A realistic estimate of the compartmental pressures during blowdown requires a node and branch approach similar to that used with the large dry containment. The mass and energy balances written for the ice chests require that a separate ice mass be included. In the limiting case of a large LOCA, the ice chests may be expected to melt more or less uniformly from the bottom up.¹⁵¹ Division of each ice chest into a number of vertically

connected nodes (each node equivalent to an ice tray) coupled with the assumption of thermodynamic equilibrium within each node, can then provide a reasonable estimate of system behavior for this case. In this calculation the pressure loss coefficient in the branch momentum equation must be revised as the ice melts. This calculation assumes the steam velocities are high enough to entrain any water from melting ice.

Although melting behavior seems more complicated for steam flows associated with a small LOCA, essentially all the steam is condensed in the ice bed and the containment pressure is not affected by the complexities.¹⁵¹

A more realistic analysis of ice condenser behavior would be obtained by assuming that only the steam, air and water were in thermodynamic equilibrium. The energy balance of Eq. (6.84) would then be modified by adding to the left-hand side the term

$$-(h_c a) V_n (T - T_{ice})$$

where h_c = condensation heat transfer coefficient [energy/(area time)]; a = surface area per unit volume; and V_n = volume of node. In addition, W_i must include the mass of water added by ice melting. Unfortunately, values of $(h_c a)$, as a function of the fraction of the ice remaining, are generally unavailable.

6.9.3 Vapor Containment During Severe Accidents

As noted previously, the designs of the vapor containers for plants constructed prior to 1994 were based solely on steam-line break and LOCA analyses in which ECCS systems were assumed operable. In the case of a severe accident involving a molten or partially molten core, the containment may be challenged by the generation of pressures well above the design value or by direct melt attack on the structure or pressure boundary.

The generation of pressures well in excess of the design value does not necessarily lead to containment failure. ASME code requirements conservatively limit design stresses to a fraction of those which lead to vessel failure. Based on tests of a 1/8 scale dry containment model,¹⁵² actual vapor container failure is not expected until the absolute internal pressure is 3.3 to 5.6 times the design pressure. Low containment leakage rates would be expected to be maintained until the absolute containment pressures reach about 2.4 times the design pressure. Many severe accident scenarios lead to containment pressures below the failure threshold.

To deal with some severe accidents that could produce containment pressures beyond the failure threshold, a number of European plants¹⁵³ have installed filtered vented containment systems (FVCSs). In such a system, a rupture disk vents the vapor container atmosphere through a particle filter before the containment reaches the vapor container failure threshold. Since most of the biologically important fission products are suspended as aerosol

particles, an effective filtering of containment gas prior to release very substantially reduces the off-site radiological consequences.¹⁵³ Note, however, that use of a FVCS does not protect against (1) direct attack of the structure by molten corium or (2) against early containment failure due to missile generation or direct containment heating. Since primary system depressurization can eliminate early containment failures, some designers believe that the additional protection provided by FVCS is worthwhile. Other designers believe that since late overpressure failures contribute only marginally to off-site risk, the high cost of FVCS outweighs the potential benefit.¹⁵⁴

The containment failure mitigation device of greatest significance to the thermal designer is the *core catcher*. A core catcher may be described as a device that would contain any molten corium released from the reactor vessel and prevent contact between the corium and the concrete structure. Core catcher designs have generally been of three types. In one early type,¹⁵⁵ a very large (e.g., 70 ft in diameter, 40 to 80 ft deep) sacrificial bed of low melting, carbonate-free material (SiO_2 , Al_2O_3) is provided below the reactor vessel compartment. The corium would be expected to dissolve into these materials and form eutectics with relatively low melting points. The heat of fusion provided by melting of the sacrificial bed would absorb the decay heat immediately following the accident. Eventually the size of the molten mass would be expected to become sufficiently large so that the decay heat generated could be transferred by conduction to the soil. The compartment containing the sacrificial bed would be lined by a refractory to prevent melt-through of the eutectic.

A second approach¹⁵⁶ uses a refractory bed containing a relatively small sacrificial mass of refractory material and means for transferring heat to the environment. Penetration of the refractory bed would occur in the initial period. However, the corium would be spread over a large enough area so that after a reasonable period, the outermost layer of corium would freeze. Zivi¹⁵⁶ has suggested that the refractory barrier be composed of unenriched UO_2 . He argues that the penetration of the barrier will be reduced since the high density of the molten UO_2 will keep the corium floating above the UO_2 and thus retard mixing.

A more recent approach, which is somewhat similar to that just described, uses a refractory barrier that allows the spreading of the corium over a surface area so large that a very thin (few centimeters) layer of corium is formed.¹⁵⁷ This would permit the rapid resolidification of the corium by conduction to the refractory barrier. After solidification occurs, a final cooldown is obtained by allowing water to flood the region and submerge the solidified corium. Heat is then removed by boiling of the water which transfers heat to the containment heat sink. In the simplest design proposed, a large graphite slab of a single layer of interlocking blocks provides the barrier. An alternative design provides a stack of staggered graphite beams. The requirement for active intervention to flood the

core catcher compartment is a shortcoming of this approach. In the slab design, a passive heat removal system below the graphite slab could provide an alternative to compartment flooding.

The amended version of the German atomic energy act now (1994) requires that, in the event of a severe reactor accident, drastic measures not be required to protect the population outside the confines of the plant site. This implies that the vapor container be able to contain molten corium and to withstand the maximum loads that could be generated by steam explosions, hydrogen detonation/deflagration events, reactor head failure at high system pressure, and containment overpressurization due to corium-basemat interaction or reduction in decay heat removal capacity. This is a difficult task in view of the uncertainties associated with severe accident phenomena.

6.10 SAFETY FEATURES OF ADVANCED DESIGNS

It was noted in Sec. 1.1.3 that advanced reactor designs directed toward improved safety and reliability may be divided into two categories. In the first category are evolutionary technology reactors, which are based on moderate extensions of current technology. The second category consists of the so-called "PRIME" reactors, which feature radical changes in system design and require major technological developments. This section will be confined to consideration of the better developed evolutionary technology reactors using modifications of the conventional loop design.

The design objectives for evolutionary technology reactors were delineated in Sec. 5.7.1. Examination of these objectives clearly shows that two important goals of these designs are to (1) substantially reduce the probability of a severe accident and (2) significantly moderate the consequences of a severe accident should one occur. These goals are in accord with the U.S. NRC requirements for licensing of advanced plants.¹⁵⁸ These requirements stipulate that the designs attempt to prevent severe accidents, contain the radioactivity in case of an accident, and protect the public in the unlikely event of a release.

6.10.1 Redesigned Conventional PWRs

One approach to improved reliability and safety has been through the careful redesign of the active safety systems used in earlier designs.¹⁵⁹ Greater redundancy and diversity are incorporated in the safety systems design. Further, lower core power density, increased pressurizer volume and increased steam generator secondary volume provide increased margin (see Sec. 5.6).

In the most prominent design following this approach, the ECCS system consists of four essentially independent trains (in contrast to the two trains provided in most earlier designs) which take water from an in-containment

refueling water storage tank (IRWST). Containment spray comes from the same source. This eliminates the need to switch from an external source and provides a semiclosed system with continuous recirculation. The emergency coolant flows directly into the reactor vessel rather than into the primary piping.

A four-train emergency feedwater system is provided. Constricting venturis, near the outlets of the injection lines, minimize excess emergency feedwater to a steam generator with a ruptured inlet or outlet line. This eliminates the need for automatic isolation of the failed steam generator.

A safety grade depressurization system is a key feature of this design. The system not only provides a diverse means for heat removal but it rapidly brings the pressure below the shut-off head of the ECCS pumps. This allows the high-capacity ECCS to become effective at an earlier time during a LOCA. The need for a separate high-pressure injection system is eliminated and the core remains covered over a wider spectrum of break sizes. The depressurization system, coupled with ECCS injection, also ensures that the core will remain covered during a total loss of feedwater flow. In addition, the depressurization system should eliminate the potential for containment failure caused by direct containment heating due to high-pressure core melt ejection during a severe accident.

Other severe accident mitigation features include hydrogen igniters that do not require site power for operation and a reactor cavity designed for retention of molten core material. A passive cavity flooding system assures that any molten corium in the reactor cavity can be cooled.

Probabilistic risk assessments of this design approach¹⁵⁹ indicate that the likelihood of a severe accident has been reduced by approximately a factor of 50 compared to earlier conventional designs. This compares favorably with other advanced reactor designs.

6.10.2 Hybrid Safety System Approach

A second approach to advanced LWR design is the “hybrid” safety system concept. This approach incorporates both active and passive safety features.¹⁶⁰ Passive systems handle large and medium LOCAs. Active safety systems deal with very small LOCAs, secondary piping failure, steam generator tube ruptures and other non-LOCA failures. The system configuration is shown in Fig. 6.14.

The operation of the active safety systems is similar to that seen in earlier conventional designs. However, since the systems have a more limited set of functions than in the earlier plants, they are simpler. Containment spray system, low-pressure safety injection pumps and recirculation lines are not needed.

The proponents of this design approach recognize that passive safety systems are highly reliable and largely eliminate human errors. However,

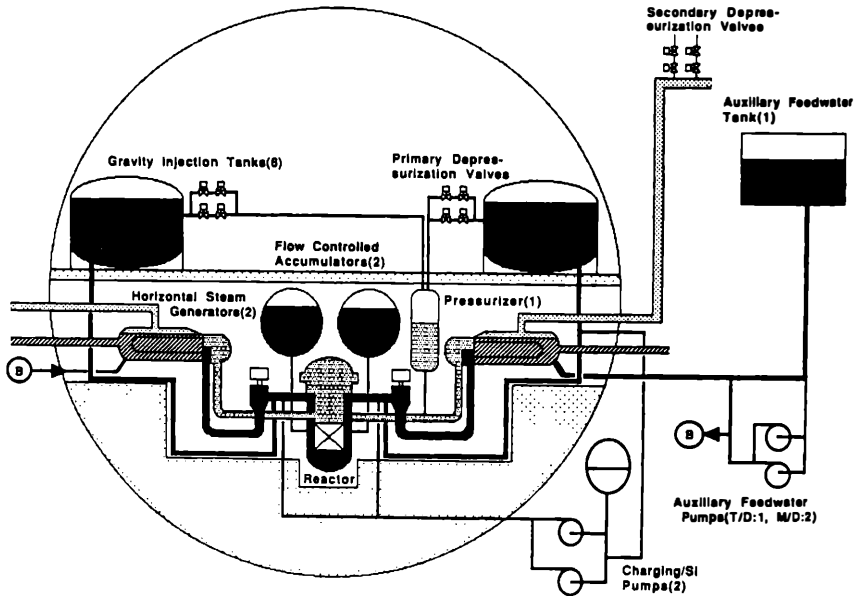


Fig. 6.14 Concept of hybrid safety system [from Ref. 161].

it is argued that accident termination is slow when passive systems are used. Because of this, some small accidents can result in vapor container flooding when a passive system is used but no such flooding occurs with an active safety system.

During a large LOCA the system pressure drops rapidly below 5 MPa, allowing water from high pressure accumulators to enter the primary system. In a small LOCA, a low-pressure signal activates an automatic depressurization system, which rapidly reduces the primary system pressure. The accumulators used for this design contain the vortex control device described in Sec. 5.7.3. The control device initially permits a large water flow but subsequently reduces the water flow as the decay heat decreases. Once the primary pressure is near that of the vapor container, cooling is maintained by water flowing by gravity from the elevated gravity injection tanks.

The secondary side pressure is also decreased at the same time. Water is then fed by gravity to the secondary side of the horizontal steam generators. Natural circulation then allows core cooling for three days via heat removal by the steam generators. The horizontal orientation of the steam generators is designed to avoid a possible siphon break caused by accu-

mulating noncondensable gas in the steam generators. Noncondensable gas is removed by vent lines in the steam generator heads.

As the accident progresses, water continues to flow out the break flooding the lower regions of the vapor container. Approximately 1.5 hr after a guillotine break of the main piping, all of the primary piping is submerged.¹⁶¹ Any pipe break is then under water and no additional steam is released to the containment.

The vapor container consists of a spherical inner steel vessel, a hollow annulus, and an outer secondary container of concrete sandwiched between two steel shells. Radioactive gas, which may leak out of the inner steel vessel, flows through a charcoal filter prior to release to the atmosphere. The heat release from the primary system is initially absorbed by the structural system. After the break is submerged, the heat released from the break is absorbed by the liquid in the containment vessel. At that point, all decay heat is removed by the natural circulation through the steam generators. After three days, it is assumed that further heat removal will be by active components. System analyses show that the peak vapor container pressure remains below 0.5 MPa at all times.

6.10.3 Approaches Placing Primary Reliance on Passive Safety Systems

6.10.3.1 System Configuration

The use of safety systems that are primarily passive is the third approach to advanced PWR safety. The overall configuration of the AP600 plant which uses this approach was described in Sec. 1.1.3 (see Fig. 1.15). The use of canned motor pumps in the primary system allows close coupling of the heat exchangers to the reactor vessel. The vertical piping loops that give rise to loop seal problems during a LOCA are thus eliminated. Additional volume is added in the pressurizer and secondary side of the steam generators to provide additional operating margin. Additional margin is also provided by a lower core power density. Vessel embrittlement is reduced by a core reflector and additional space between the core and reactor vessel.¹⁶²

The passive safety systems for mitigation of a LOCA differs somewhat from those used by the hybrid system previously described. As indicated in Sec. 1.1.3, ECCS flow is initially provided by gravity flow from two core makeup tanks (CMT), which are maintained at system pressure. When the system has depressurized to about 5 MPa, additional flow is provided by two large accumulators. One CMT and core accumulator use a common line, which injects water directly into the vessel downcomer. This assures that at least one CMT and accumulator are available in case of an injection line failure.

Once the system has fully depressurized, long-term core cooling is provided by gravity flow from the in-containment refueling water storage tank (IRWST). The IRWST water can absorb decay heat for about 2 hr before boiling begins. The subsequent drainage of condensate from walls of the air-cooled vapor container into the IRWST maintains the IRWST water level.

During a small LOCA, an automatic depressurization system reduces the system pressure so that accumulator and IRWST water injection is possible. Since the automatic injection system uses remotely operated valves, the safety system cannot be said to be fully passive. The valves used are "fail safe" in that loss of power causes them to open to their safety alignment. Although remotely operating valves are required, all pumps and diesel generators in the safety systems have been eliminated.

Two passive residual heat removal heat exchangers (PRHR HX) are used as protection against transients that upset the steam generator feedwater or the steam systems. Each PRHR HX is connected to the primary system in a natural circulation loop. The loops are normally isolated from the primary system by air-operated valves. The valves fail open if power is lost. Natural circulation through the PRHR heat exchangers provides the required decay heat removal. The heat exchangers are located in the IRWST and cooled by the water contained in the tank. Just as in a LOCA, tank boil-off is condensed on the vapor container and drained back into the IRWST. Each heat exchanger is capable of dissipating all the decay heat. These heat exchangers obviate the need for pumped, safety grade auxiliary feedwater in case of secondary system break.

As may be seen in Fig. 1.15, the steel vapor container is surrounded by a concentric air baffle. Cool air flows downward between the outer concrete shield building and the baffle. Heated air rises in the annular space between the vapor container and baffle. This passive containment cooling by natural circulation of air is supplemented by evaporative cooling during the first few days after an accident. Water is supplied by gravity from tanks located at the top of the concrete shield building.

Advanced Russian VVER designs have also included passive safety systems.¹⁶³ Designs for the 500- to 600-MWe range incorporate accumulator injection, automatic depressurization, and gravity-fed injection for LOCA mitigation. Passive systems are provided for both primary system residual heat removal and vapor container cooling. This design continues the use of horizontal steam generators.

A VVER design for a 1000-MWe plant is also available.¹⁶³ In this unit, vertical steam generators are used. The secondary side of each of the four steam generators is connected to an air-cooled heat exchanger located outside the containment. Natural circulation through this system provides for passive removal of decay particularly during loss of system power. LOCA mitigation is largely passive with initial coolant injection from a tank maintained at system pressure and subsequent injection from a gas-pressurized

accumulator. The concept also provides for (1) vapor container venting through a filter in case of overpressurization and (2) a large bed of concrete for retention of a molten core in the event of a severe accident.

6.10.3.2 *Safety Analyses of Passive System Behavior*

Safety analyses of the AP600^{162,164} indicate an improved performance relative to previous designs with active safety systems. In loss-of-feedwater or feedwater line ruptures, the PRHR HX removes sufficient heat and prevents operation of the pressurizer safety valves. In a steam-line break accident, there is no DNB or adverse effects on the primary system. The core remains covered for most small LOCA accidents (break size up to 8 in. in diameter). The peak cladding temperature in the limiting large LOCA is about 400°C below the allowable maximum of 1200°C. This improvement in large LOCA behavior is primarily due to the use of a lower power density in the core. In the AP600 design, use of the passive residual heat removal system means that no additional water needs to be added to the heat exchangers after the break occurs. This results in a peak pressure after a steam-line break which is below the peak following a LOCA.¹⁶² In many earlier PWR designs, the peak containment pressure following a main steam line was higher than the peak pressure following a LOCA.

Probabilistic risk assessments of the AP600 design indicate a substantial reduction in the predicted core damage frequency relative to PWRs with active safety systems. Core damage frequencies for each initiating event were reduced by at least a factor of 10. The overall core damage frequency was reduced by about two orders of magnitude.

6.11 SAFETY ANALYSIS TECHNIQUES FOR ADVANCED REACTORS

Reactor safety analysis techniques have been developed (1995) to the point where they provide reasonably accurate predictions when used in licensing-type calculations for conventional reactor designs. While these same system analysis techniques have been applied to passive safety system designs, we may expect that modifications will be required as additional experimental results for such systems become available. In particular, a more accurate coupling of the systems analysis to containment pressure is desirable. Gravity head additions from the low-pressure reservoir only become effective when the system pressure approaches the containment pressure.¹⁶⁵

Determination of system behavior over a variety of severe accident scenarios will continue to be of concern. Development and verification of severe accident models (1995) are likely to remain a challenge for some time.

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